Chris Belthoff

Tessellations

Hardscape Creations

Inspired by the works of

M.C. Escher

Contents

- Origin
  - What inspired the efforts to produce my hardscape designs
- Construction
  - How I created the first examples of pavers based on the designs

The Inspiration

- Ever since I was a child I've been fascinated by mathematics

- In particular, I was always interested in *symmetry* and *patterns*….

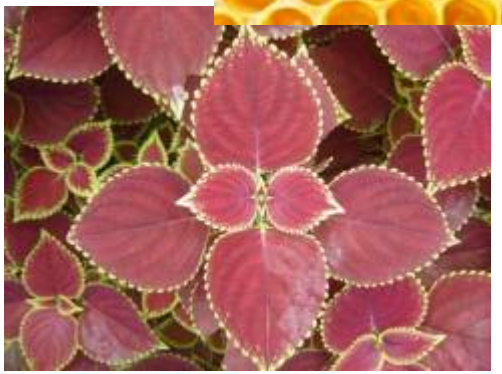
Symmetry and Patterns Have Existed Since Before Man

- Nature exhibits them all the time:

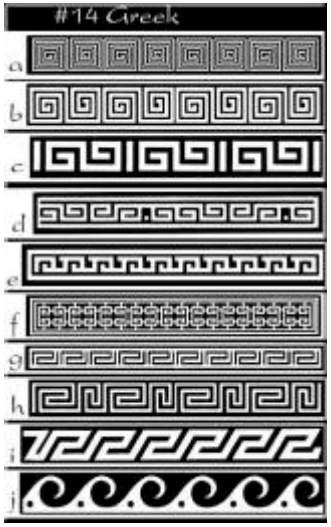








….and Humans Have Creatively Employed Symmetry and Patterns for Centuries



So what is 'Tessellation'

- A **tessellation** is the tiling of a plane using one or more geometric shapes, with ***no overlaps*** and ***no gaps***

- *Tessellation* or *tiling* is the branch of mathematics that studies how shapes, known as *tiles*, can be arranged to fill a plane without any gaps.

- More formally:

  - a tessellation or tiling is a partition of the Euclidean plane into a countable number of closed sets called *tiles*, such that the tiles intersect only on their boundaries. These tiles may be polygons or any other shapes.

  - Many tessellations are formed from a finite number of prototiles; all tiles in the tessellation are congruent to one of the given prototiles.

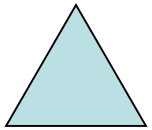
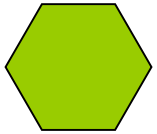
  - If a geometric shape can be used as a prototile to create a tessellation, **the shape is said to be able to *tessellate* or to "*tile the plane*".**

  - Mathematicians have found no general rule for determining if a given shape can tile the plane or not, which means there are many unsolved problems concerning tessellations.

In English, Please….

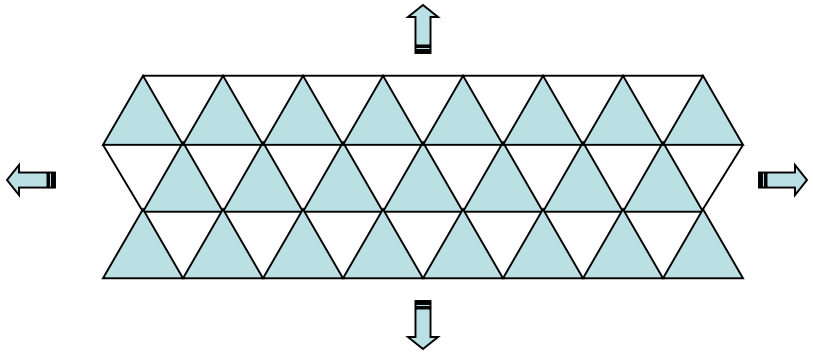
OK… some illustrations to help, starting with simple:

- There are only THREE "regular" tessellations possible
- Using exactly **one** kind of regular polygon arranged **edge-to-edge (and sharing corners)** a tessellation can be made from each of:

1. 3-sided equilateral TRIANGLE 
2. 4-sided SQUARE 
3. 6-sided HEXAGON 

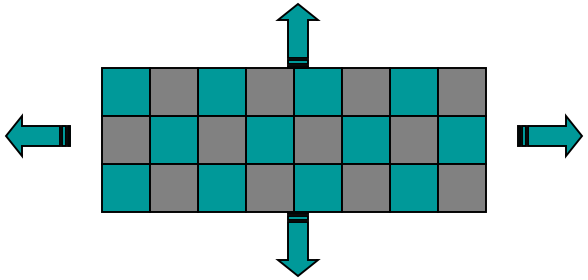
Tiling the Plane…

- Using 3-sided TRIANGLES looks like this (in 2 colors):



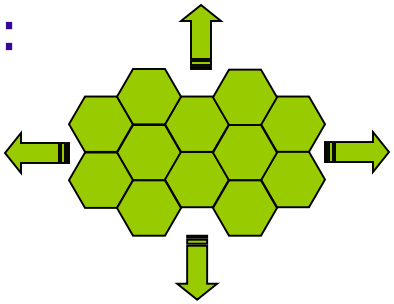
Can be repeated forever as a REGULAR PATTERN

- Using 4-sided SQUARES – like this (2 colors):



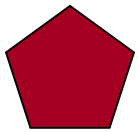
These patterns are all neat and orderly… but

- Using 6-sided HEXAGONS – this:

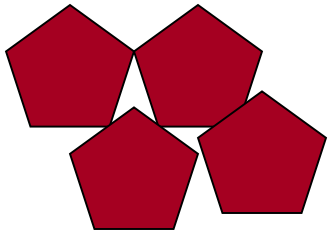


Wait a Second….

- 3-sided shape tessellates, 4-sided shape tessellates, 6-sided shape tessellates…..
- What about **5-SIDED - regular pentagon**??



- **NOPE** – this shape does NOT tessellate….



- WHY it does not, is a boring math reason…..but simply put:
  - For any **regular shape to Tessellate,** it must have an interior angle that can evenly be divided into 360 (360 degrees in a full circle…if you remember high school math)
  - For a regular Triangle: each interior angle is 180 degrees – which works
  - For a regular Square: each interior angle is 90 degrees – also works
  - Regular Hexagon: each interior angle is 120 degrees – also works
  - Regular PENTAGON: each interior angle is 108 degrees – DOES NOT WORK

Two Curious Short Detours….

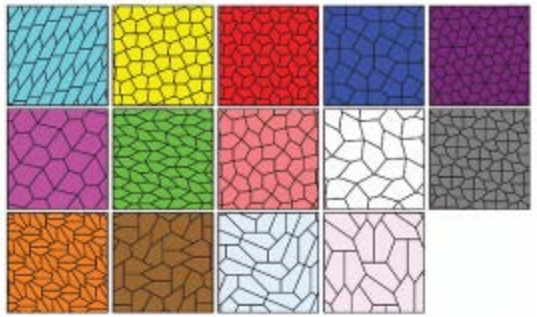
1. Interestingly, while a regular 5-sided Pentagon does NOT Tessellate a FLAT plane….

- There IS a *combination* of 5-Sided Pentagons and 6-Sided Hexagons that DO tessellate a ***curved surface***.

- You've likely seen it dozens of times, and haven't thought much of it… here's a picture:

2. If you like pentagons….You CAN Tessellate a flat plane …. with ***IRREGULAR Pentagons***….

- In fact there are **exactly FOURTEEN** different types of irregular convex pentagons that can 'Tile the Plane':

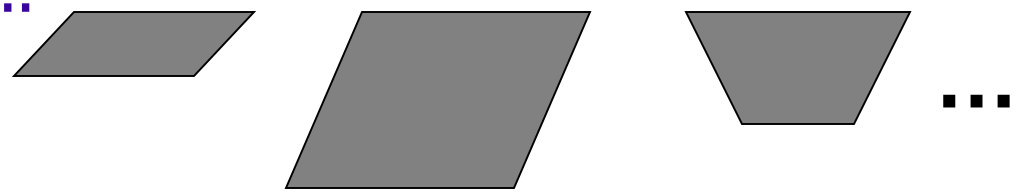


Why 14? – Don't ask ☺.

What About Other Shapes?

- Of course, you can 'tile the plane' with any proper rectangle…

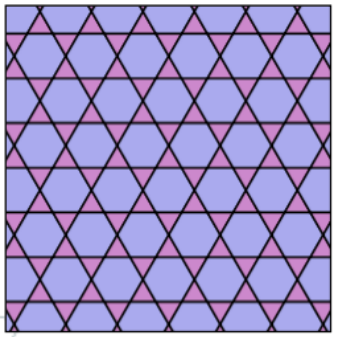
- In fact, *any* quadrilateral (a shape with 4 sides, but not necessarily having interior angles of 90 degrees) can tile the plane…

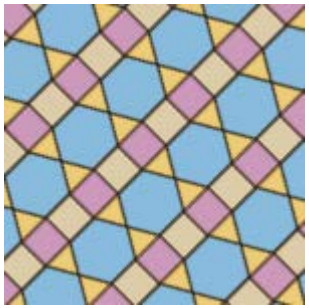
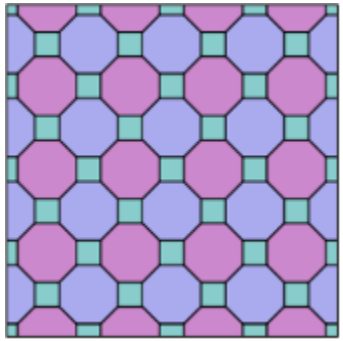


- BUT - *many* other types of tessellations are possible, differing in the constraints that are chosen to apply.

Using Multiple Shapes

- Things can get more elaborate when 'tiling the plane' with 2 or more shapes – here are just a few examples:



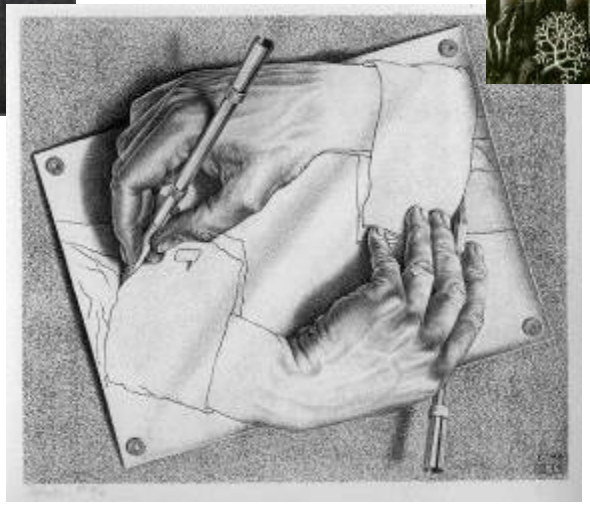



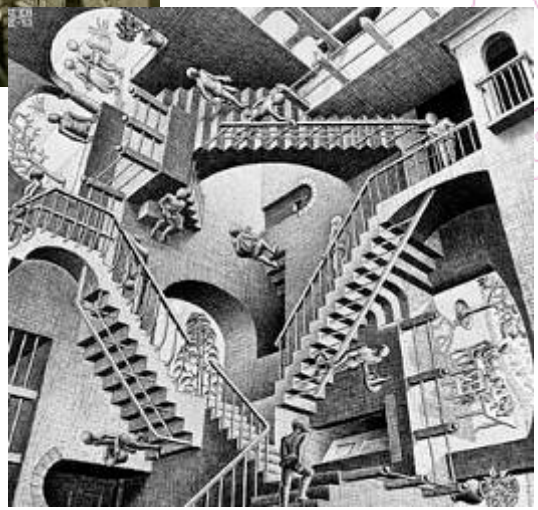
- There's a lot of funky math underlying the possibilities and rules of multi-shape tiling…which I will NOT discuss (yay)

So… Who is this M.C. Escher?

- Maurits Cornelis Escher (1898-1972) is one of the world's most famous graphic artists, and was born in Leeuwarden (Holland).
- He is known for his mathematically inspired woodcuts, lithographs, and mezzotints.
- His art often features impossible constructions, explorations of infinity, architecture, and tessellations.
- During his lifetime, he made 448 lithographs, woodcuts and wood engravings and over 2000 drawings and sketches.
- Like some of his famous predecessors - Michelangelo, Leonardo da Vinci, Dürer and Holbein - M.C. Escher was left-handed.

Some of Escher's Works





Escher's Work Related to 'Tessellations'

- Escher became fascinated by tessellations (what he also called the ***Division of the Plane)*** when he first visited the **Alhambra**, a famous 14th century Moorish castle in Granada, Spain in 1922.

What Escher Saw

- Islamic art exhibits very sophisticated patterns… here's just a *few* of the things Escher saw at the Alhambra

Regular Division Drawings

- Inspired by the intricate patterns at Alhambra, Escher devised ways to create amazing patterns of 1 or more shapes that resembled real-world items, typically animals
- He captured many of these patterns in what he termed '***Regular Division Drawings***'
- During the Second World War, he vigorously pursued his hobby, drawing 62 of the total of 137 ***Regular Division Drawings*** he would make in his lifetime.

Regular Division Drawings Using 1 Shape

- His early drawings appear somewhat crude-looking…..

Regular Division #1

Regular Division #2

Regular Division #3

Regular Division Drawings Using 1 Shape

- But later ones are much more sophisticated

Regular Division #18

Regular Division #66

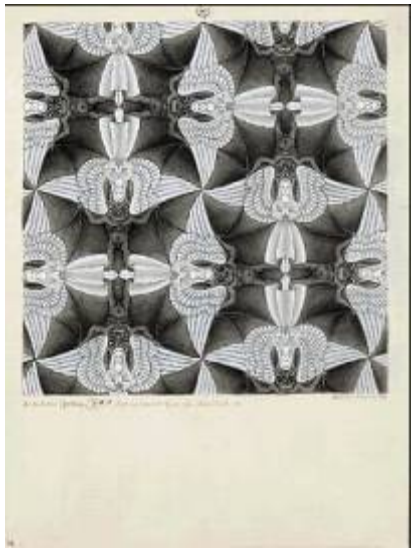
Regular Division #76

Using More than 1 Shape

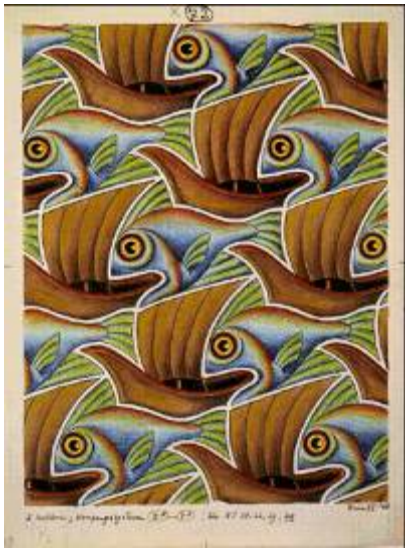
- By employing 2 or more shapes to 'Tile the Plane', Escher created some amazing drawings (remember, this was before any computers or even calculators… these were all drawn by hand), here are just a few:

Regular Division #22

Regular Division #42



Regular Division #45



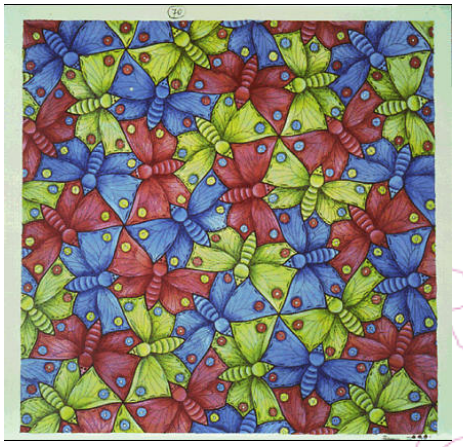
Regular Division #72

My Favorites…

- I always liked these Escher drawings:

Regular Division #11
Seahorses

Regular Division #25
Lizards



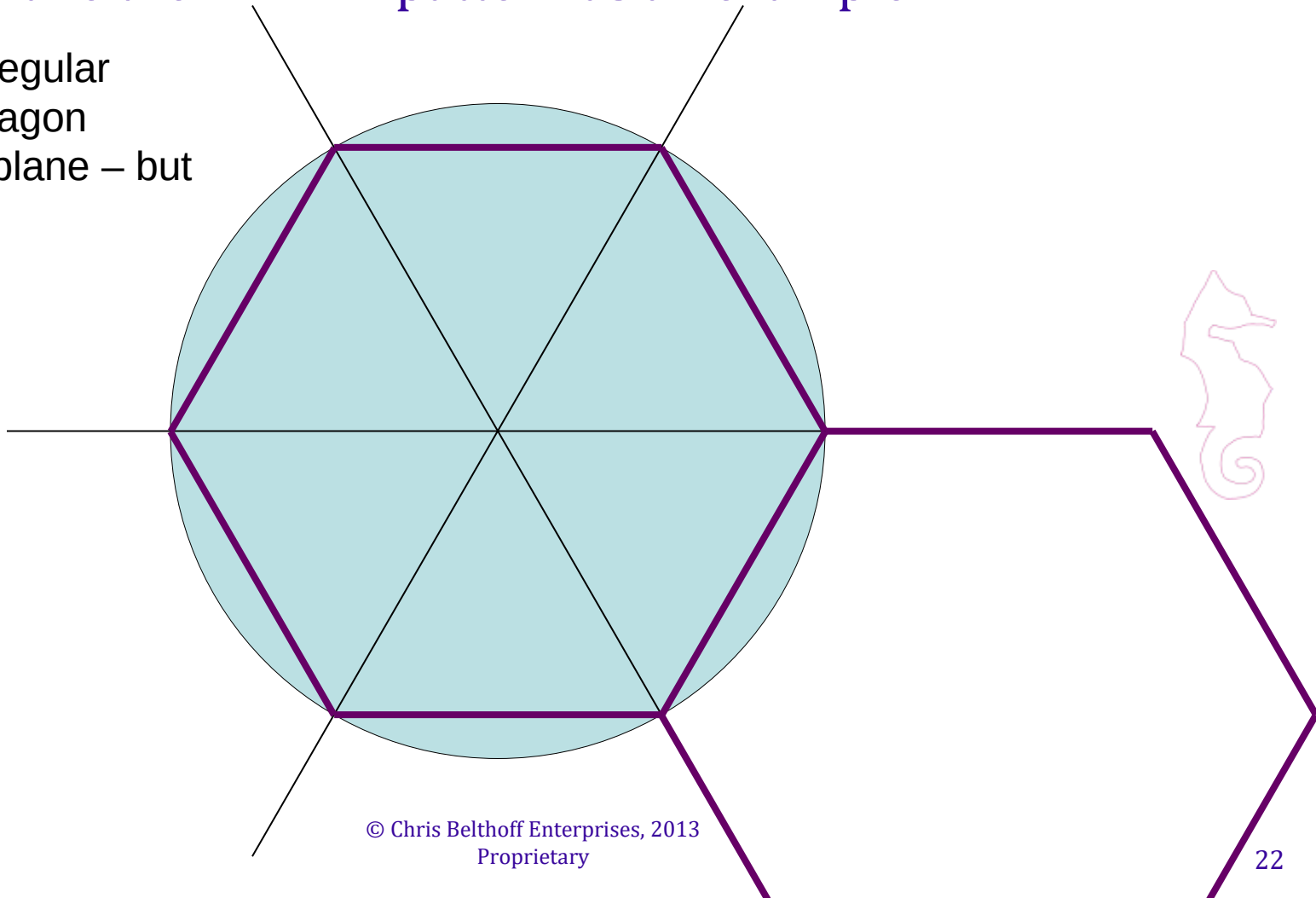
Regular Division #70
Butterflies

How Did Escher Draw These?

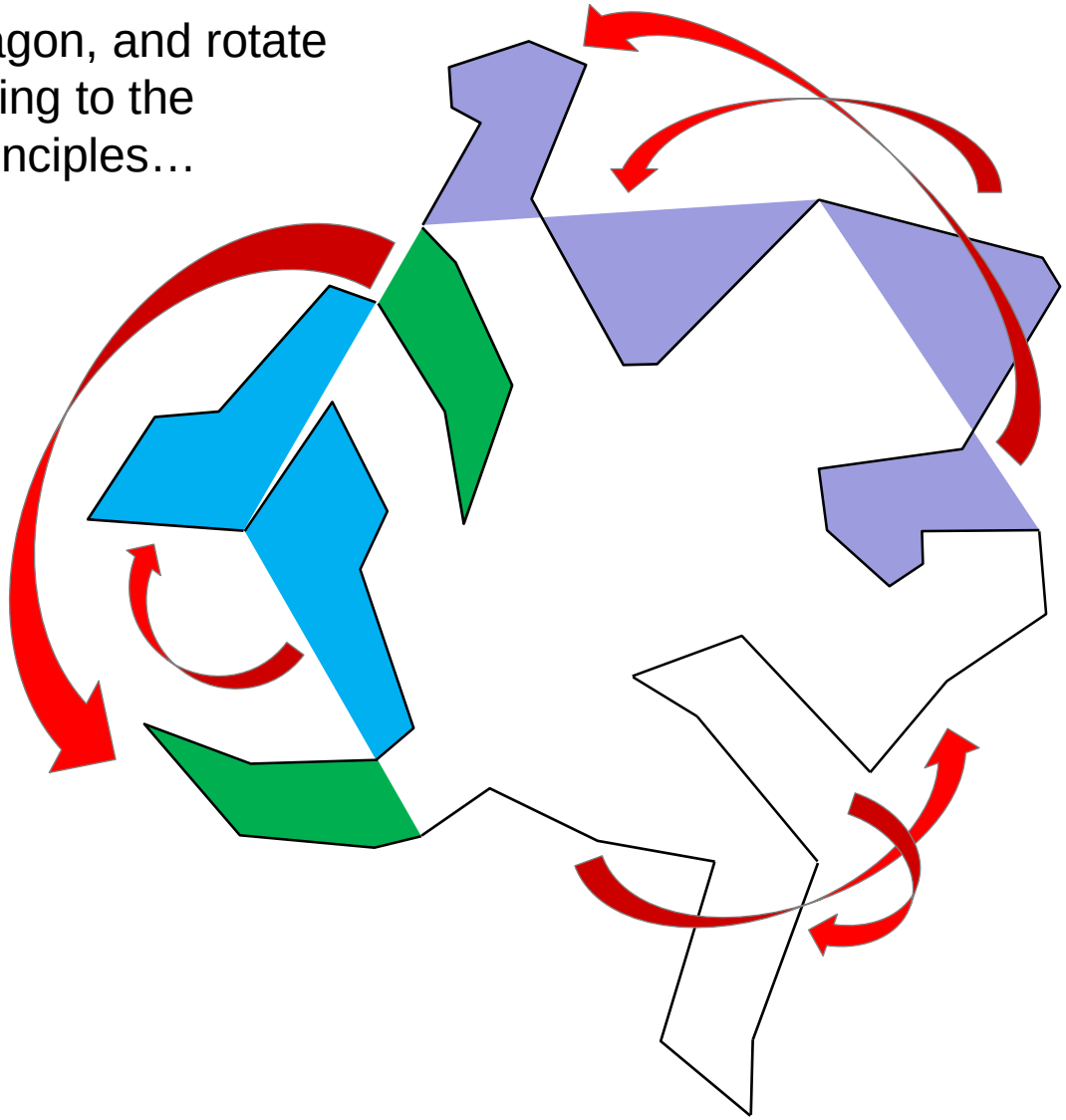
There are underlying geometry 'principles' Escher used to create all of these patterns….

Take the LIZARD pattern as an example:

Start with a regular "perfect" hexagon (Will tile the plane – but boring)

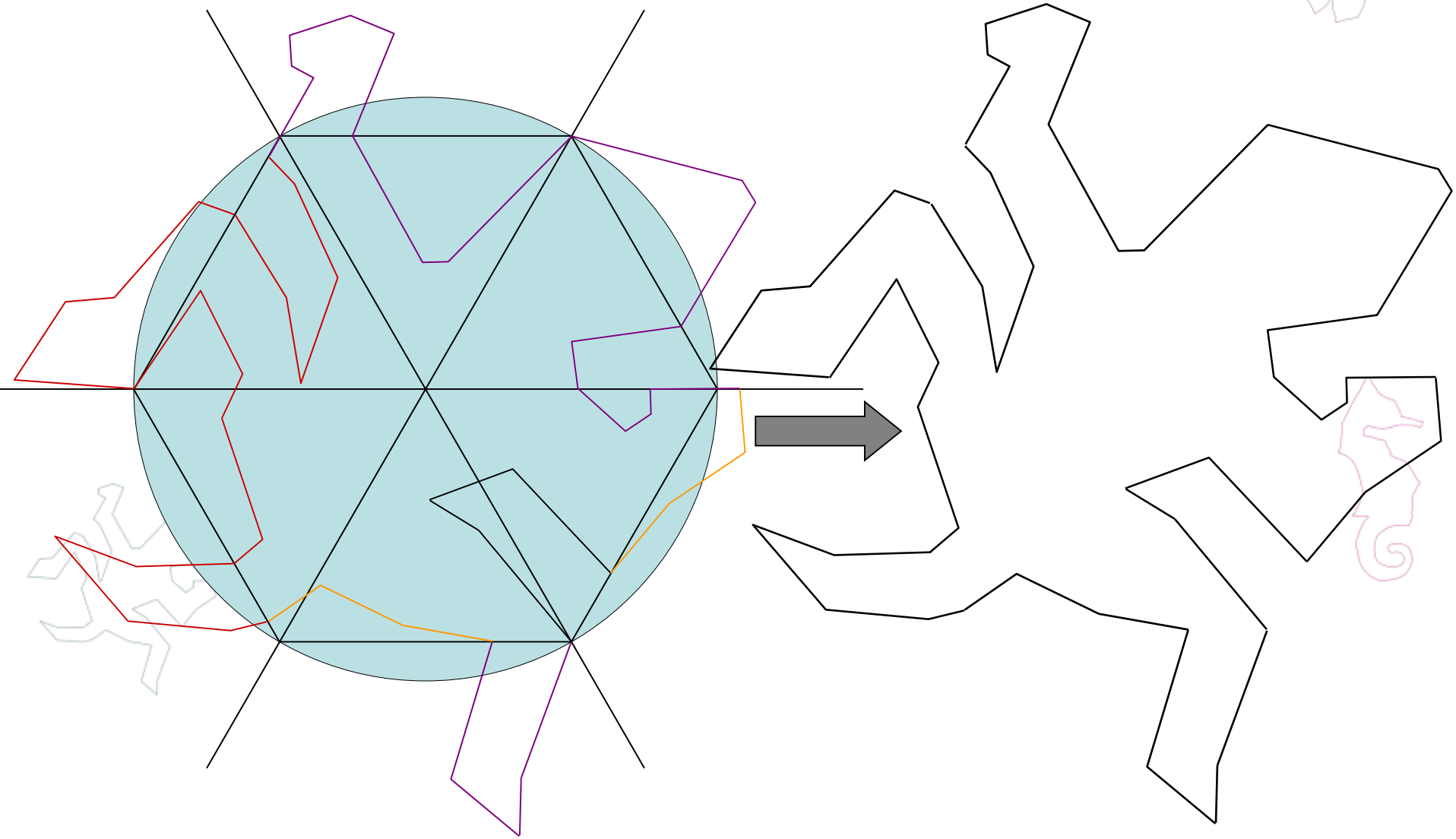


Take some ‘cuts’ out of the original hexagon, and rotate them according to the geometry principles…

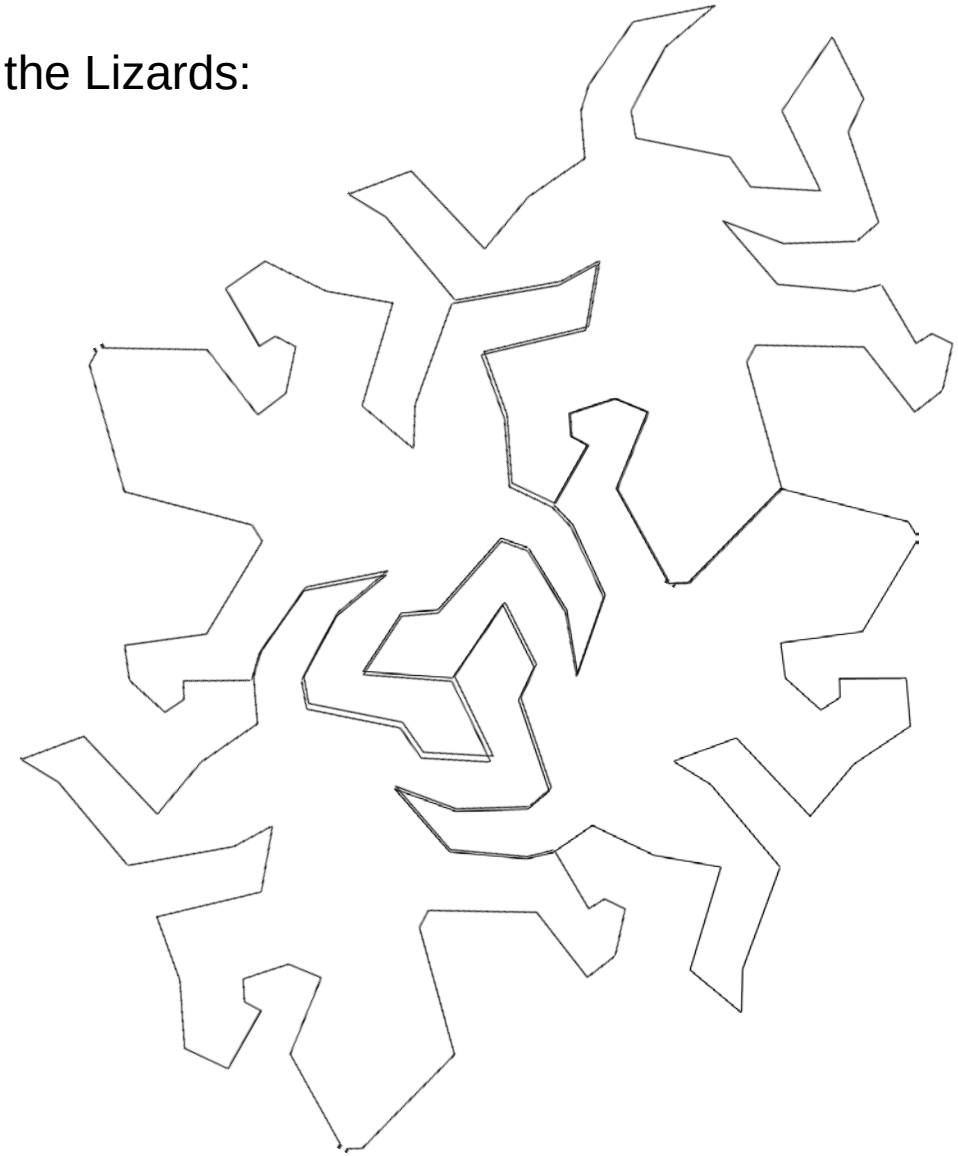


This is the part that took artistic creativity, combined with great math and drawing skills

The resulting shape, if properly drawn, will TESSELLATE



Here's a few of the Lizards:

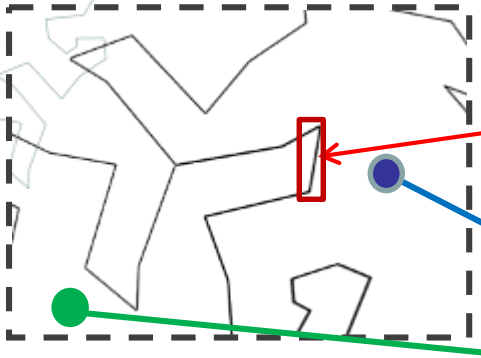


My Vision

- Ever since I first saw Escher drawings I imagined how cool it would be to produce these types of patterns in the real world, for example:
  - As PAVING STONES
  - On walls
  - In tiling patterns throughout a house (bathroom, kitchen, laundry room…)

But there's a problem….

- All of Escher's drawings are… well, just *drawings….*
- The Escher shapes 'tile the plane' – but in the *abstract*, 2 dimensional world
- The Key Issue: In all of Escher's Tessellations, the shapes literally *share their edges*



For example:
THIS line
is part of
this Lizard
AND
THIS Lizard

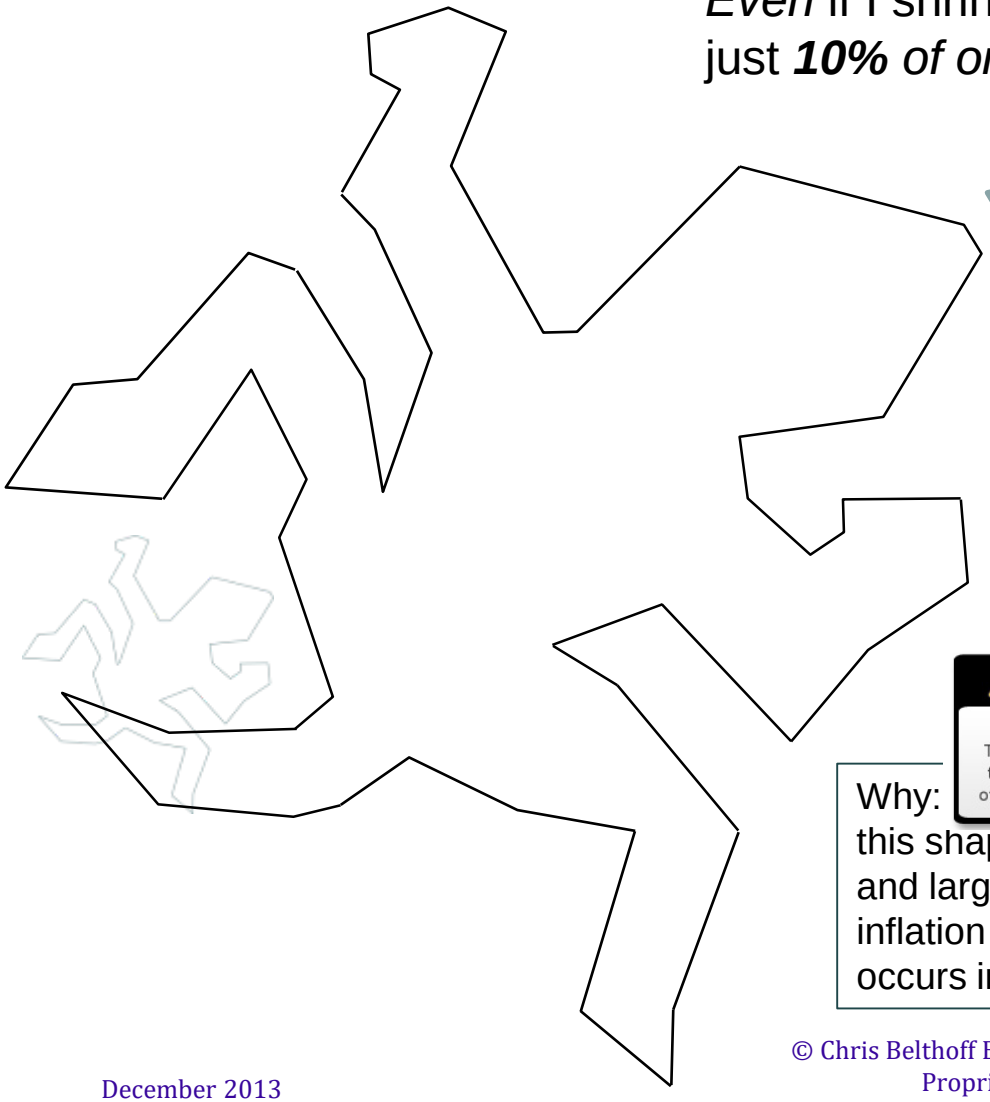
This is not possible in the real physical world; 2 objects cannot occupy the same place

What To Do?

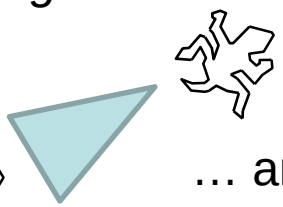
- Since the physical world requires at least a bit of 'space' between real objects, one quick thought is:
  - *Hey, just shrink down this 'Escher lizard shape' thingy by, say, 5% and all will be fine – you will end up with as much space as you need between each lizard*
- Funny thing is – this does NOT WORK
- *In fact, no matter how extreme the reduction is of the original shape….once you make copies of this drawing and organize them – they tile the plane EXACTLY as the originals did*

Example:

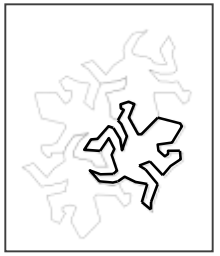
Here's the first drawing of the Escher Lizard as shown earlier….



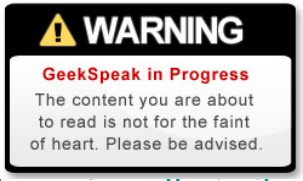
Even if I shrink it down, to only just ***10%*** *of original size*….



… and make copies & arrange….



…and they will mathematically Tessellate again.



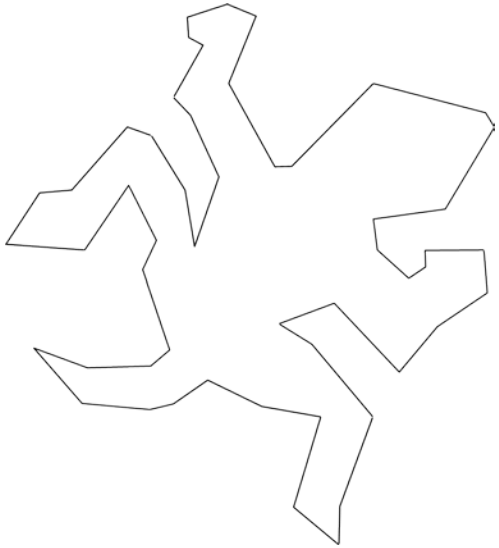
Why:
this shape is self-similar…the same patterns occur at smaller and larger scales. Thus, tiling can be obtained through inflation (or deflation) and any finite patch from the tiling occurs infinitely many times.

So, What to DO?….

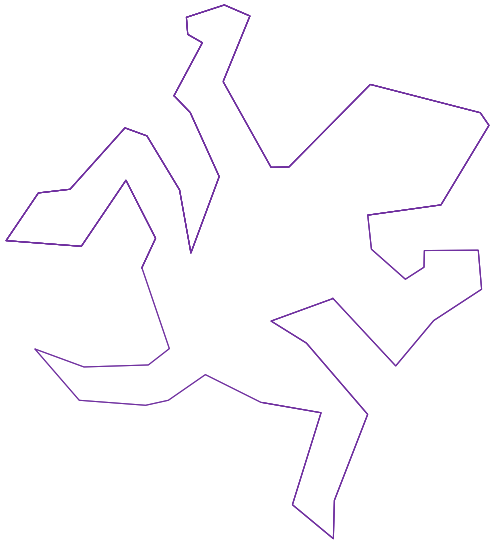
- The mathematical shapes used in Escher's drawings *cannot* be used to successfully create physical objects that form patterns, such as tiles or pavers…

- *HOWEVER*…
  - via a process I have created
  - that takes the actual Escher mathematical drawings as 'inspirations',
  - I have created shapes that CAN be used to produce real-world physical items (such as pavers)
  - which:

Are manufacturable
Will properly cover surfaces
Retain the spirit of the various 'forms' such as lizard, butterfly, etc.



Original Escher shape.
This shape will tessellate,
but only in the abstract
mathematical realm.

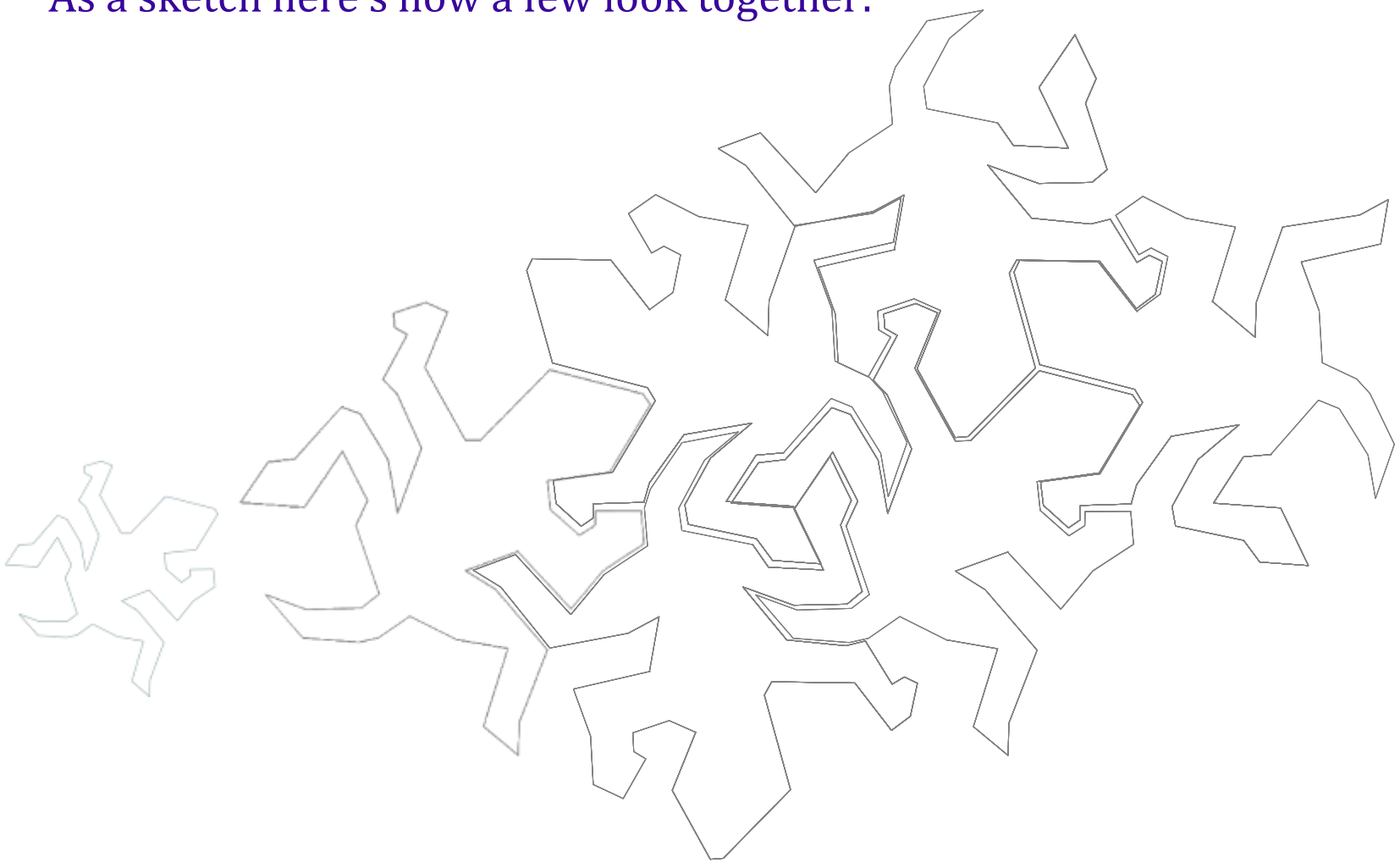


HERE is the Chris Belthoff shape which CAN be used in the real physical world of hardscaping (pavers, tiles, etc.).

Look *Identical*? - NOPE
These are NOT the same shapes

Chris Belthoff Lizards

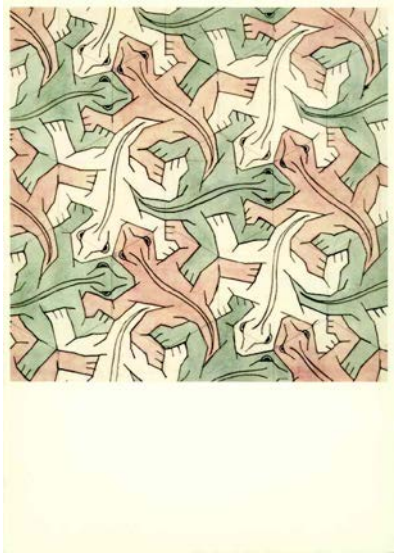
As a sketch here's how a few look together:

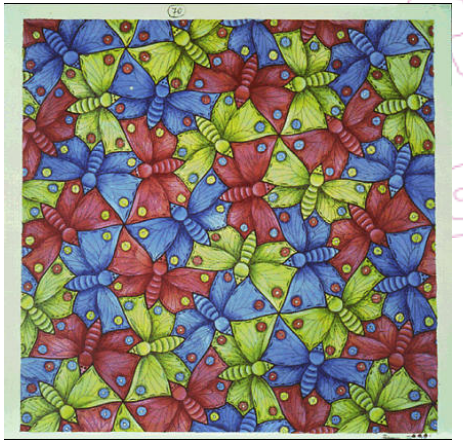


Does It Really Work?

- To validate that my designs work for real-world uses such as pavers, tiling of walls, etc., I decided to manufacture some paving stones in the shapes of:
  - Butterflies
  - Lizards
  - Seahorses

  3 of my favorite Escher 'Division of the Plane' sketches…..





Construction

- Using my designs, I had 2 inch thick granite slabs cut using water-jet technology:

Here is the lizard…

…and here are the seahorse & butterfly

- Each shape is approximately 12 inches on longest dimension (but they can be any size)

Butterfly Details

- For each design, the shape itself is important… but I also wanted each to have *surface details* similar to the Escher drawings
- To do this – I used a rotary tool and hand-carved the top surface of each granite shape

Here's the butterfly granite shape during my addition of the surface details…

Molds

- Using these ‘positive’ granite prototype shapes, with the surface details added, I created urethane molds of each:

Butterfly mold; these are extremely elastic and can be bent and stretched without tearing or breaking

Molds (cont.)

Seahorse and lizard molds

- With the carved granite ‘prototypes’, as many molds can be produced as desired

A Butterfly Paver

- Working with cement and a coloring agent, here's an example of one 'pour' of the mold:

Seahorse

- I experimented with a different 'coloring' technique for one of the seahorse pavers…. Putting color along the edges of the mold:

But – Will they 'WORK'??

- While each individual cement paver 'looks' nice… I wanted to ensure that I could use multiple copies of my design shapes to properly create a paver surface
- For cost and efficiency considerations, I had a quantity of each shape water-jet cut from 2 inch thick extruded polystyrene sheets….

Lizard Pavers

- Here's a bunch of the lizard pavers laid in a paver pattern (next to the granite prototype):

Looking good!

What's Next…

- As I create this document, it is snowing outside and 24 degrees….
- I will be manufacturing larger quantities of the cement pavers of all three shapes through the winter (in various colors)
- In spring of 2014 (once the ground thaws…) I will construct several paths and patios on my property….and will document my results