Q12 Have a secure knowledge and understanding of mathematics to enable them to teach effectively across the age and ability range for which they are trained to teach.

•Understand how the different components of mathematics inter-relate and be able to use this personal understanding to help learners build their own progressive lattices of mathematical understanding.

•Know about the origins of mathematical knowledge, practices and conventions and be able to use this to enrich teaching and to help learners understand the human and multi-cultural roots of mathematics.

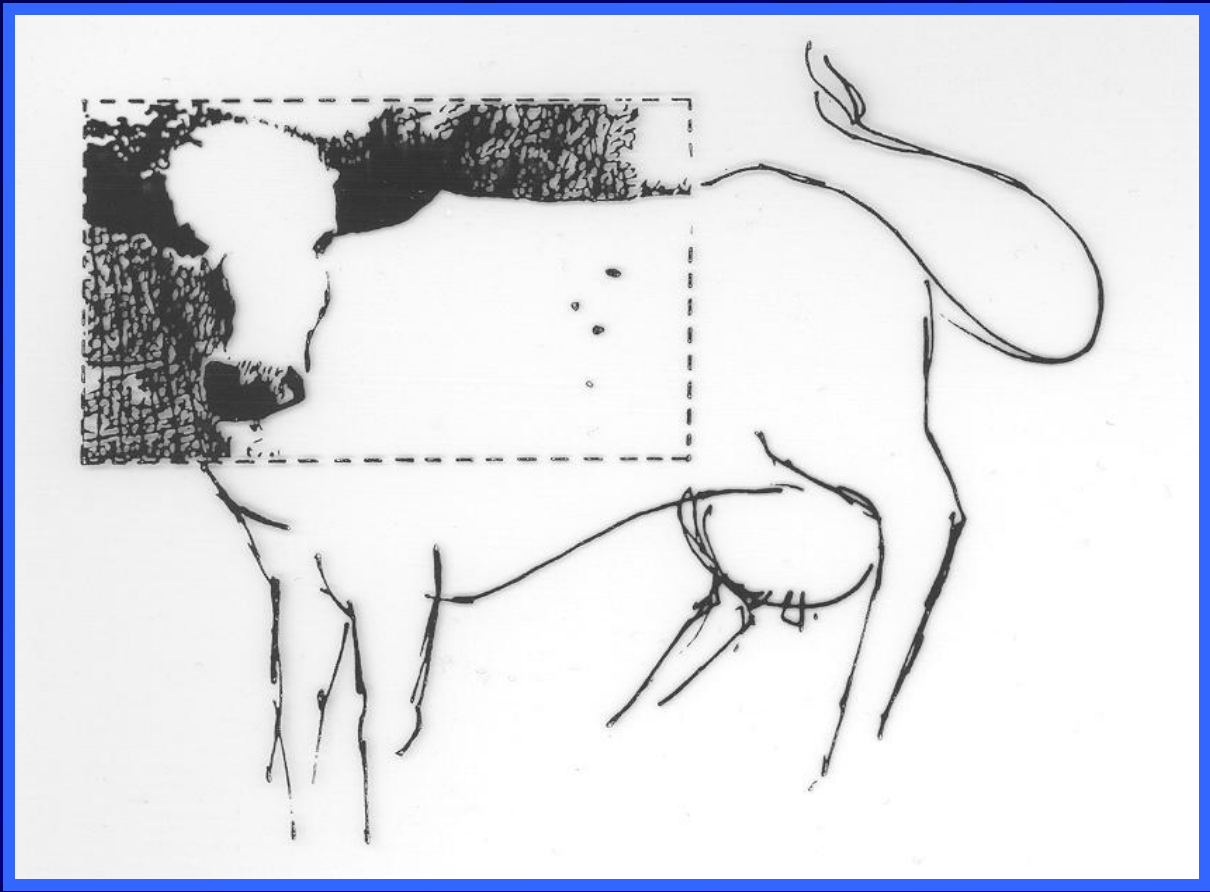
•Use mathematical language, symbols and vocabulary precisely.

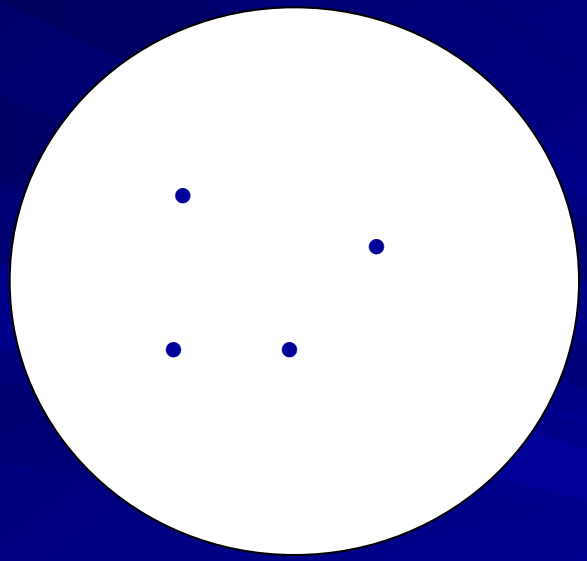
•Be sufficiently mathematically aware and confident to recognise the nature of possible learner misconceptions or insights and be able to plan the next steps in their learning.

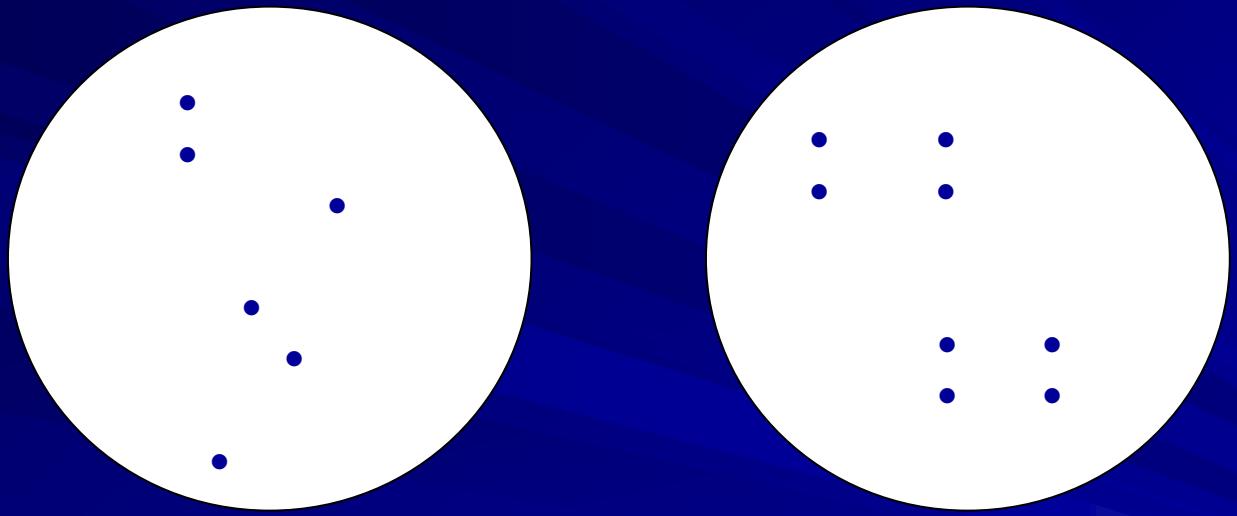
•Be aware of the applications of mathematics to other subjects and real-life situations and be able to use these to motivate learners' confidence and understanding.

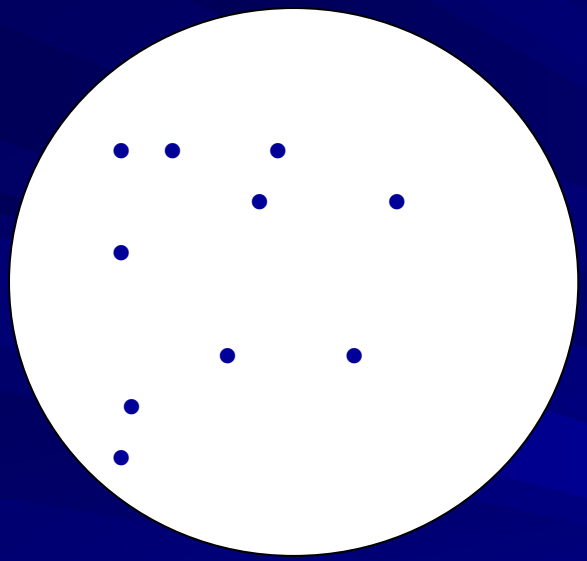
Characteristics of my best teacher

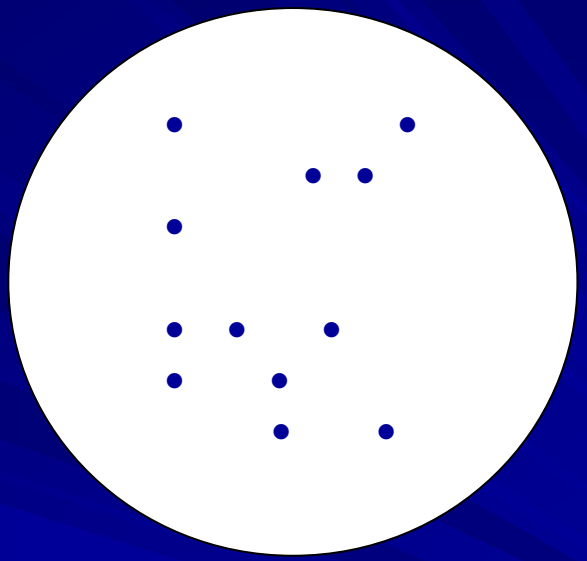
- genuine interest in the subject to the point of passion
- genuine interest in how the subject is learnt
- genuine interest in young people and opening doors of enlightenment and opportunity
- probably slightly unorthodox, but passionate about igniting intellectual and creative curiosity and developing a love of learning
- they see the poet, the scientist, the artist or the mathematician that exists in us and they develop it
- they allow the learner to ask the questions and to follow lines of enquiry
- they make learning fun

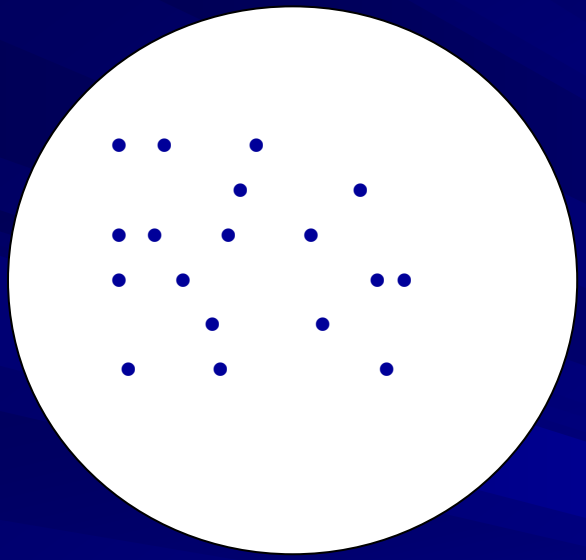


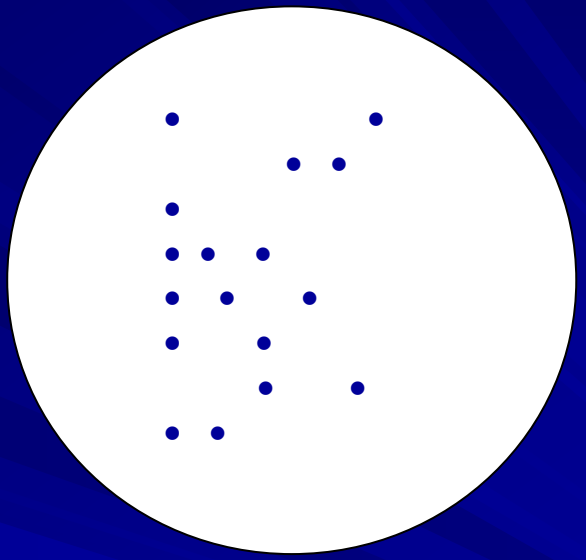




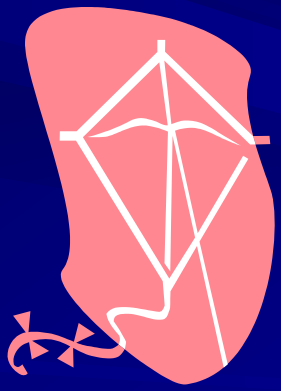


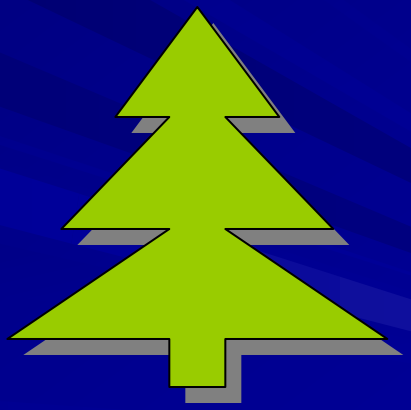






7 2





Counting

Some video clips

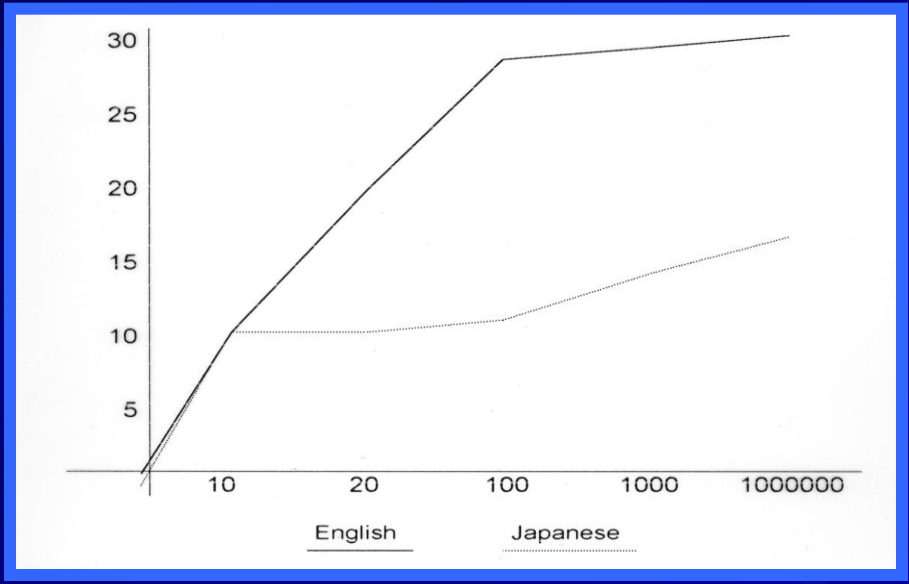
Linguistic comparison of number names

English	one	two	three	four	five	six	seven	eight	nine
Japanese	ichi	ni	san	yon	go	roku	nana	hachi	kyu
	1	2	3	4	5	6	7	8	9
Ten (ty) jyu	10	20	30	40	50	60	70	80	90
Hundred hyaku	100	200	300	400	500	600	700	800	900
Thousand sen	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
ten thou man	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
hun thou jyu man	100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000

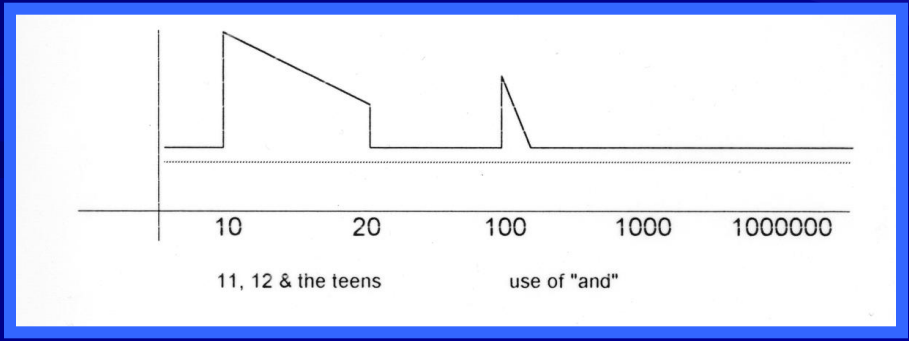
Number Names

Number of words in Counting to	English	Japanese
10	10	10
20	20	10
100	28	11
1000	29	14
1000000	30	16

Linguistic Load



Syntactic Load



mathematics $\neq \Sigma$ (parts)

mathematics $=$ (parts) $\mathcal{R}$ (parts)

mathematical thinking is about establishing the relationships

We stimulate thinking by:

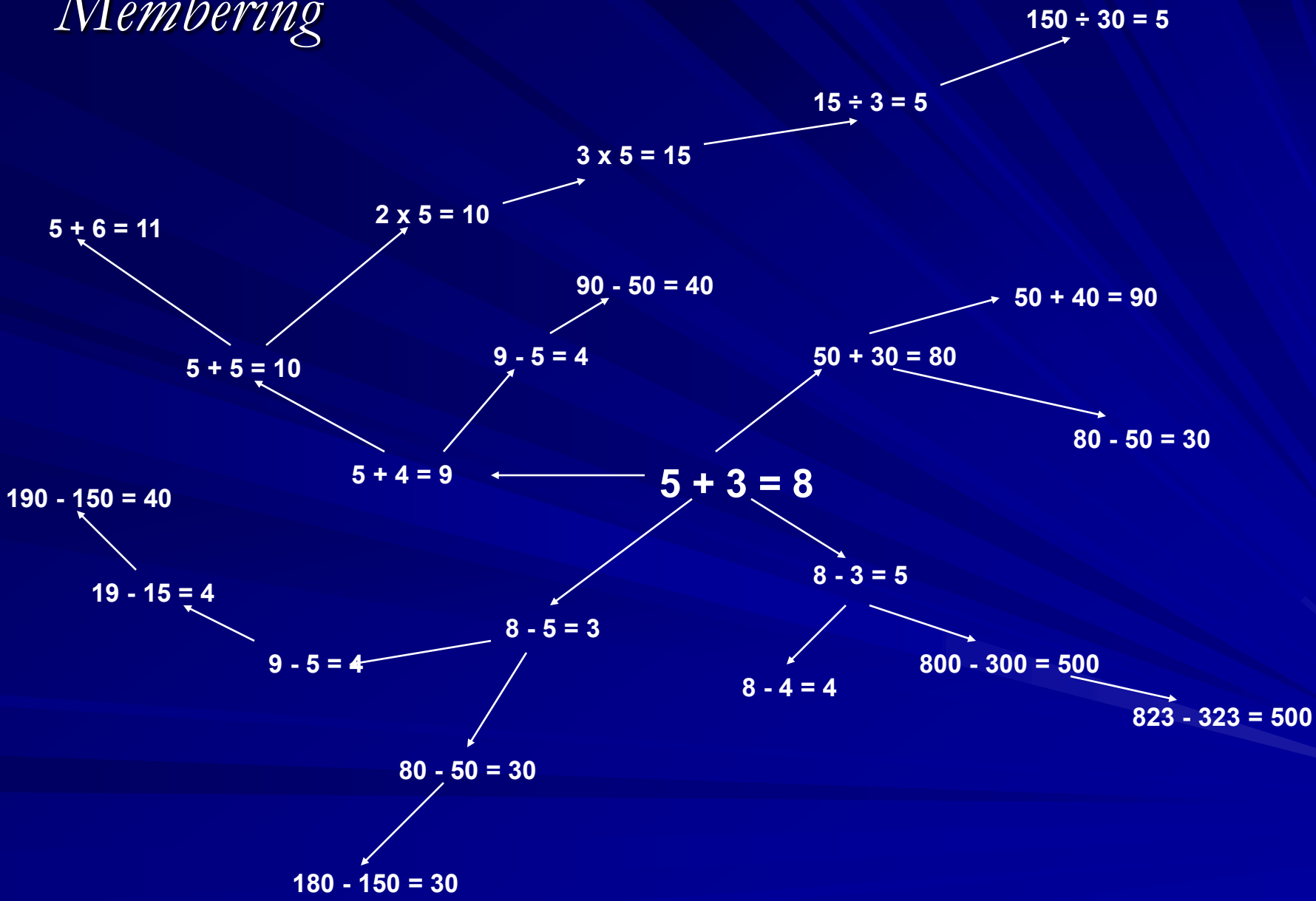
- Providing accessible & extendible images to support connection-making;
- Jolting the brain with contradictions & counter-intuition;
- Not taking the thinking out of situations;
- Going off-piste;
- Providing languages to articulate thinking;
- Providing challenging puzzles & problems;
- Providing open-ended questions.

Big ideas in mathematics

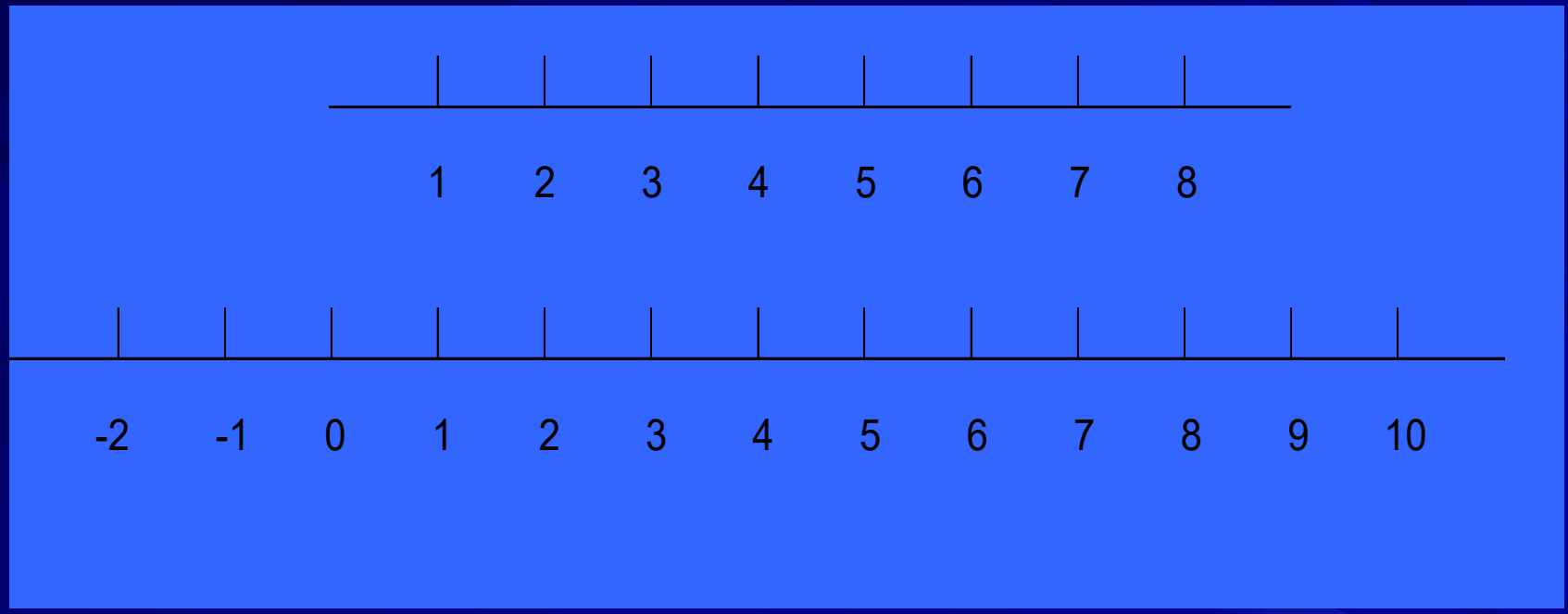
Pupils should appreciate

- that the number system is perfectly regular
- that mathematics is shot through with infinity
- that a lot can be gained from a little
- equivalence
- inverse

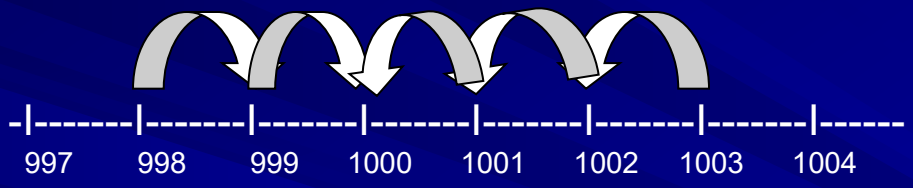
Membering


150 ÷ 30 = 5
15 ÷ 3 = 5
3 x 5 = 15
2 x 5 = 10
5 + 6 = 11
90 - 50 = 40
50 + 40 = 90
9 - 5 = 4
50 + 30 = 80
5 + 5 = 10
80 - 50 = 30
5 + 4 = 9
5 + 3 = 8
190 - 150 = 40
8 - 3 = 5
19 - 15 = 4
8 - 5 = 3
9 - 5 = 4
800 - 300 = 500
8 - 4 = 4
823 - 323 = 500
80 - 50 = 30
180 - 150 = 30

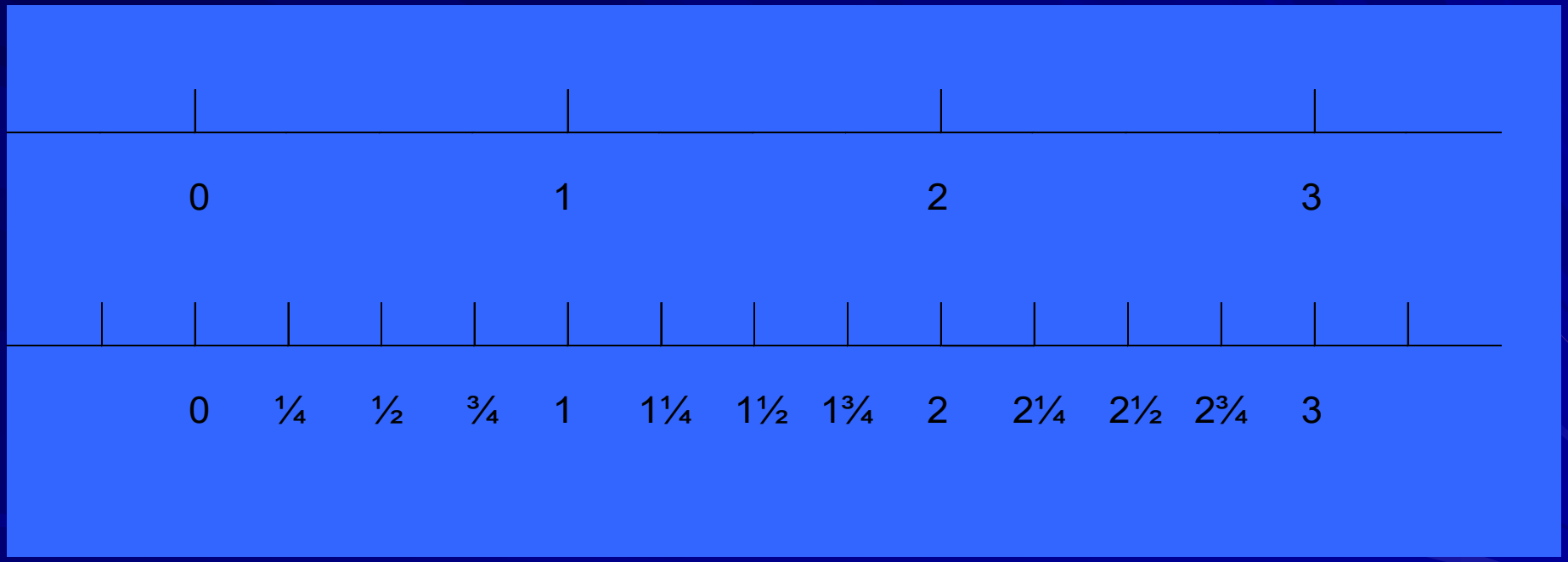
Providing accessible & extendible images to support connection-making

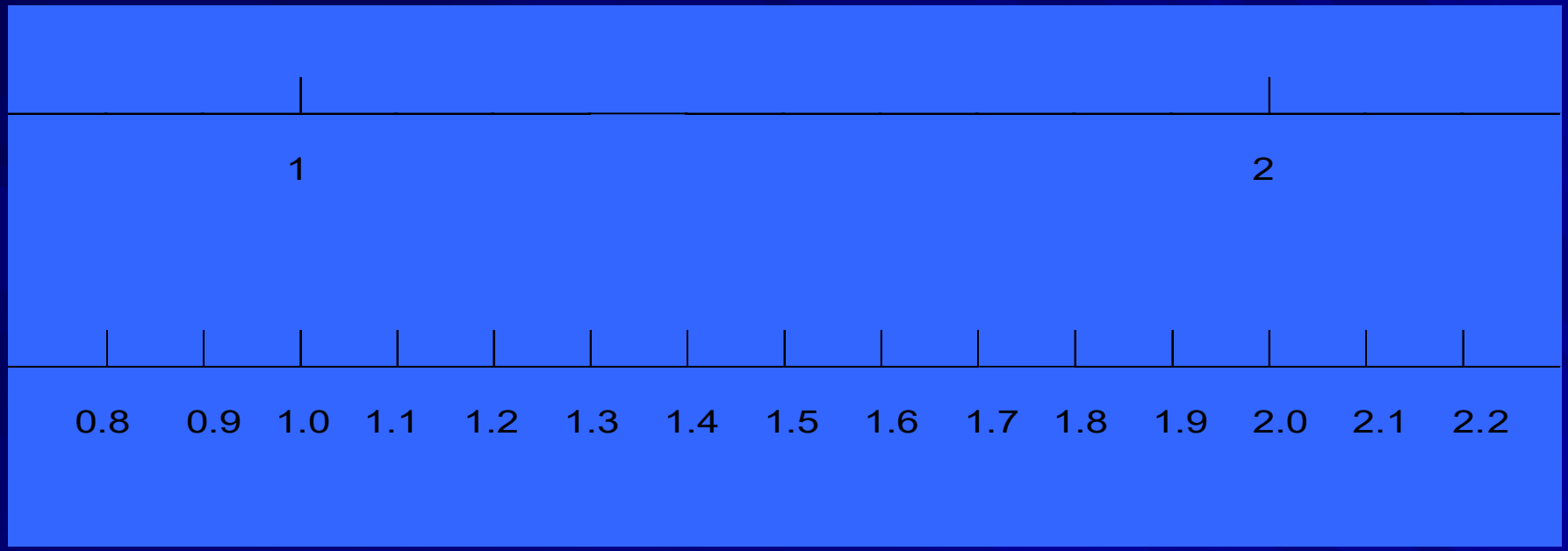

1 2 3 4 5 6 7 8
-2 -1 0 1 2 3 4 5 6 7 8 9 10

For example, the calculation 1003 - 998 is straightforward if the image below can be brought to mind.



Pupils who can calculate 1003 - 998 by mentally seeing and adding 2 to 3are at an advantage over those who are dependent on only using a 'written method', such as decomposition.


0
1
2
3
0
¼
½
¾
1
1¼
1½
1¾
2
2¼
2½
2¾
3


1
2
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2

-44	-43	-42	-41	-40	-39	-38	-37	-36	-35	-34	-33	-32	-31	-30	-29	-28
-34	-33	-32	-31	-30	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18
-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8
-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122
116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132

-44	-43	-42	-41	-40	-39	-38	-37	-36	-35	-34	-33	-32	-31	-30	-29	-28
-34	-33	-32	-31	-30	-29	-28	-27	-26	-25	-24	-23	-22	-21	-20	-19	-18
-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8
-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92
86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122
116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132

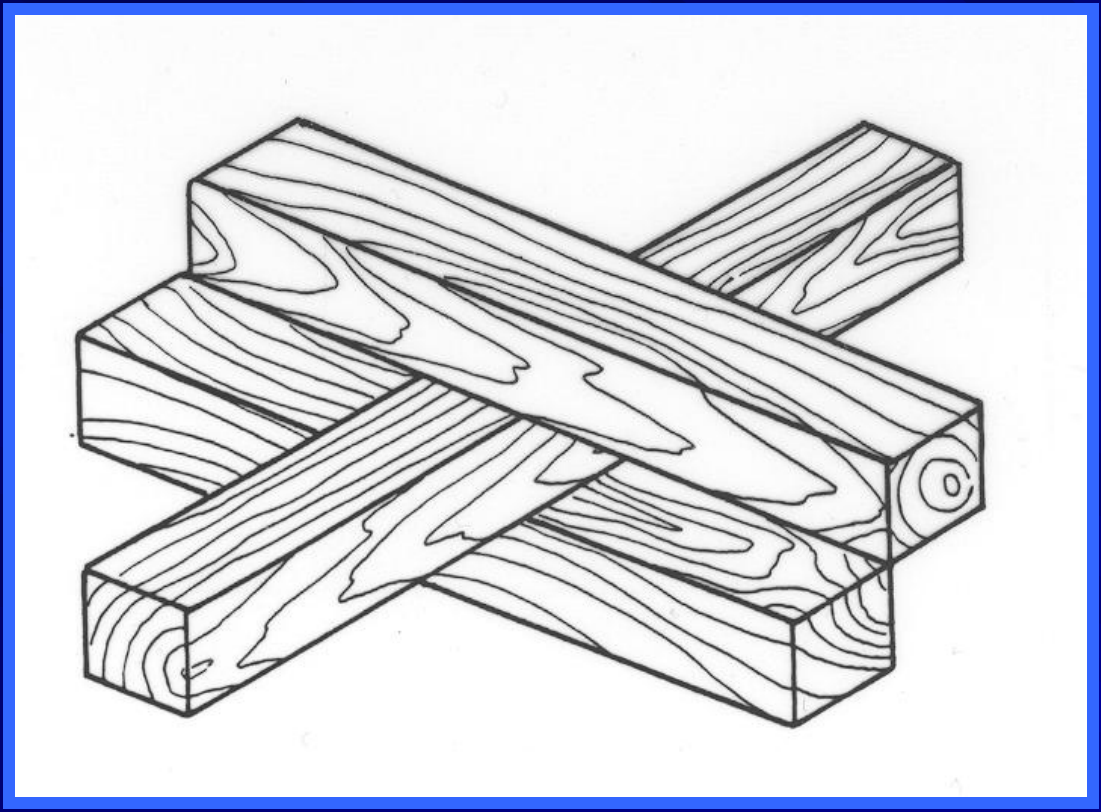
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000
100 000	200 000	300 000	400 000	500 000	600 000	700 000	800 000	900 000

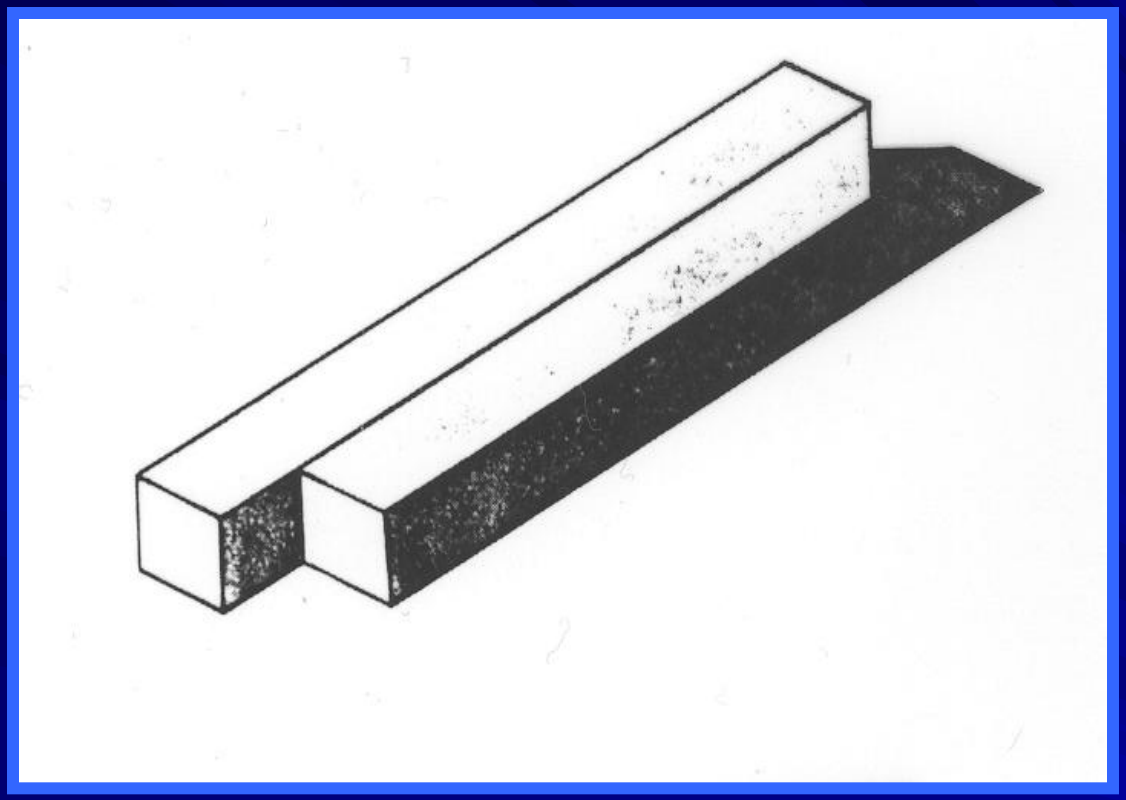
Jolting the brain with contradictions & counter- intuition



Bruno Ernst

‘Impossible Penetrations’

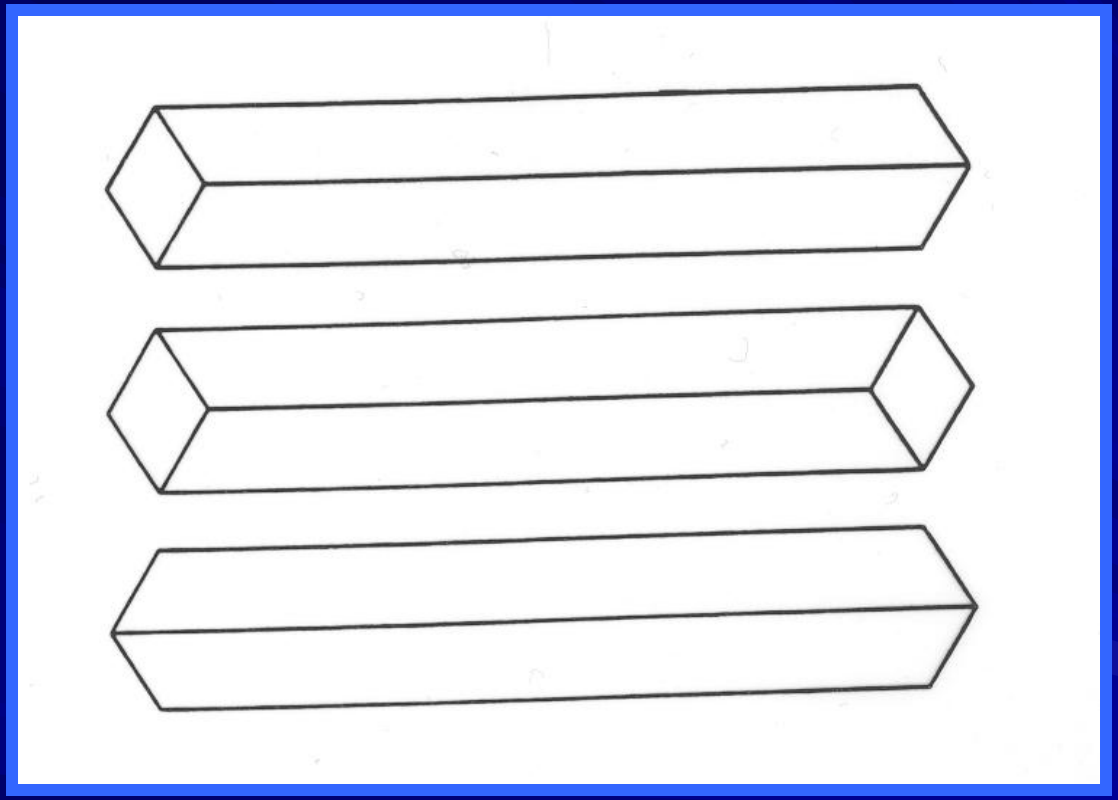
1984

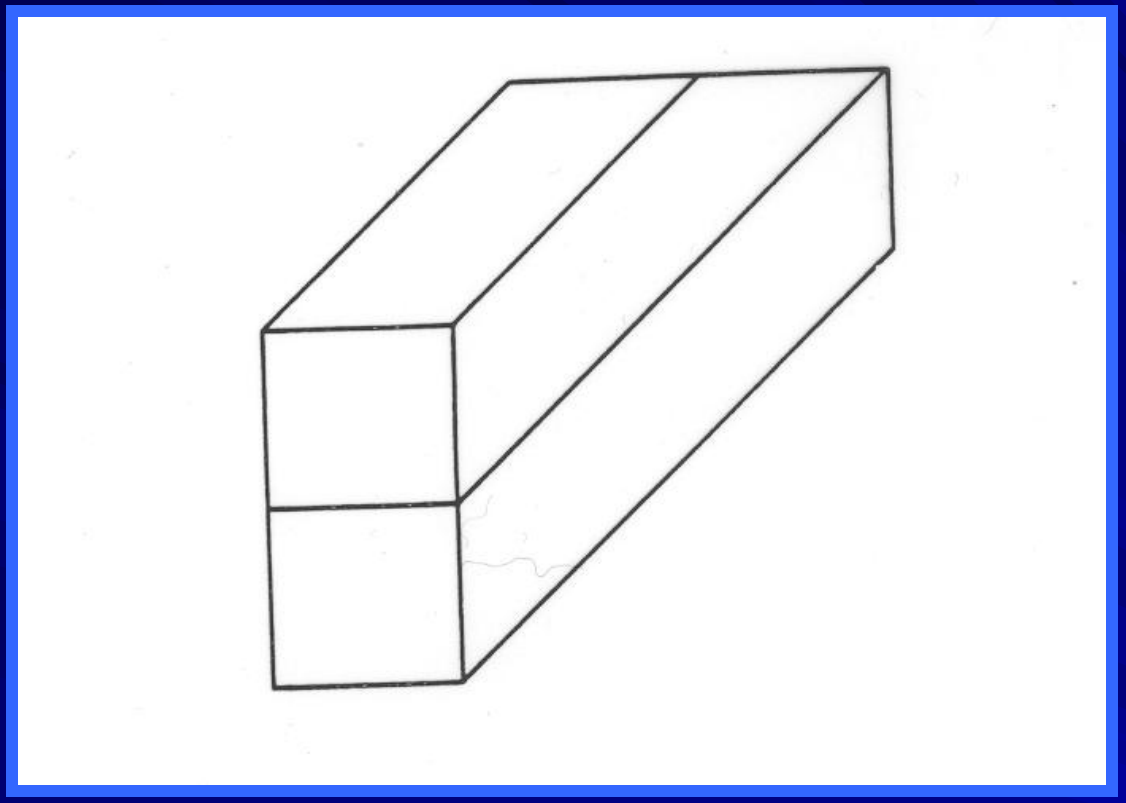


Zenon Kulpa

‘2½ - dimensional beam, or 1=2’

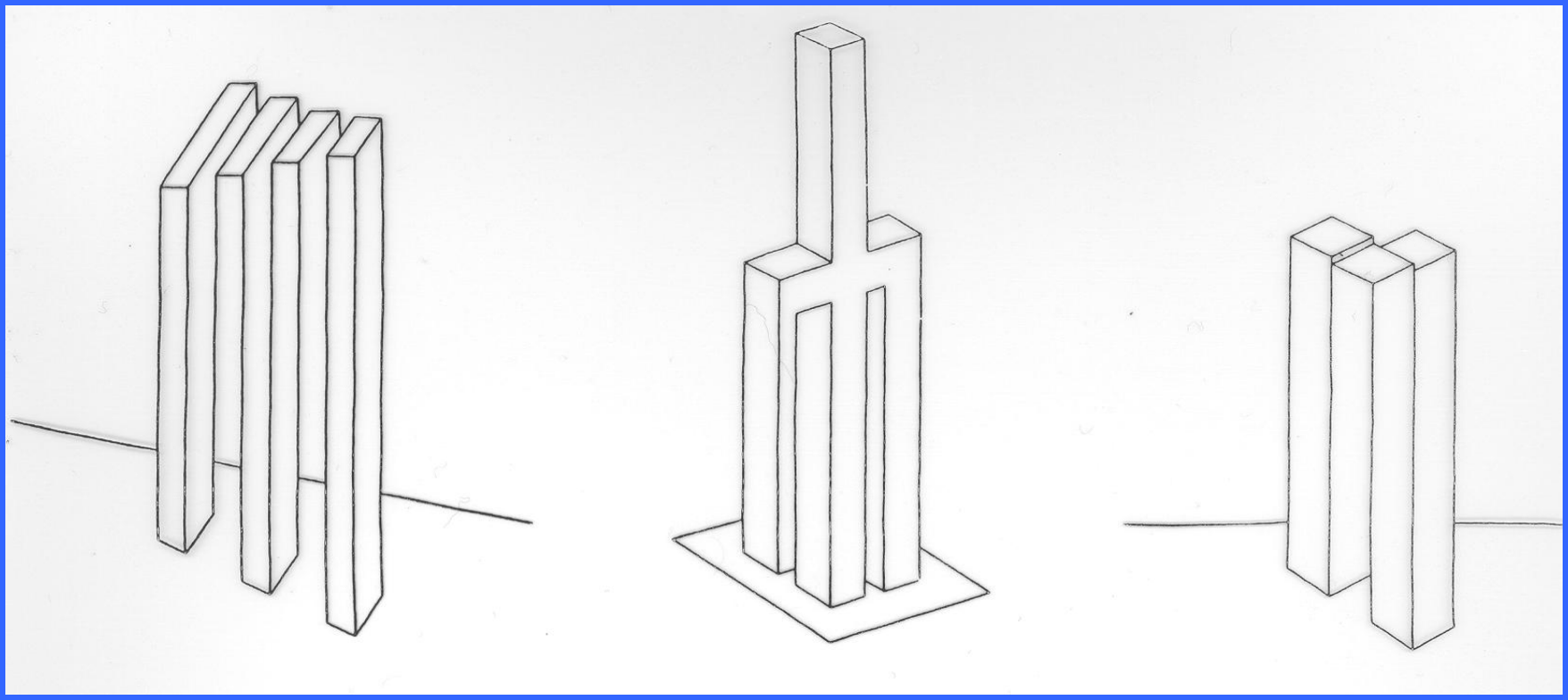
1984

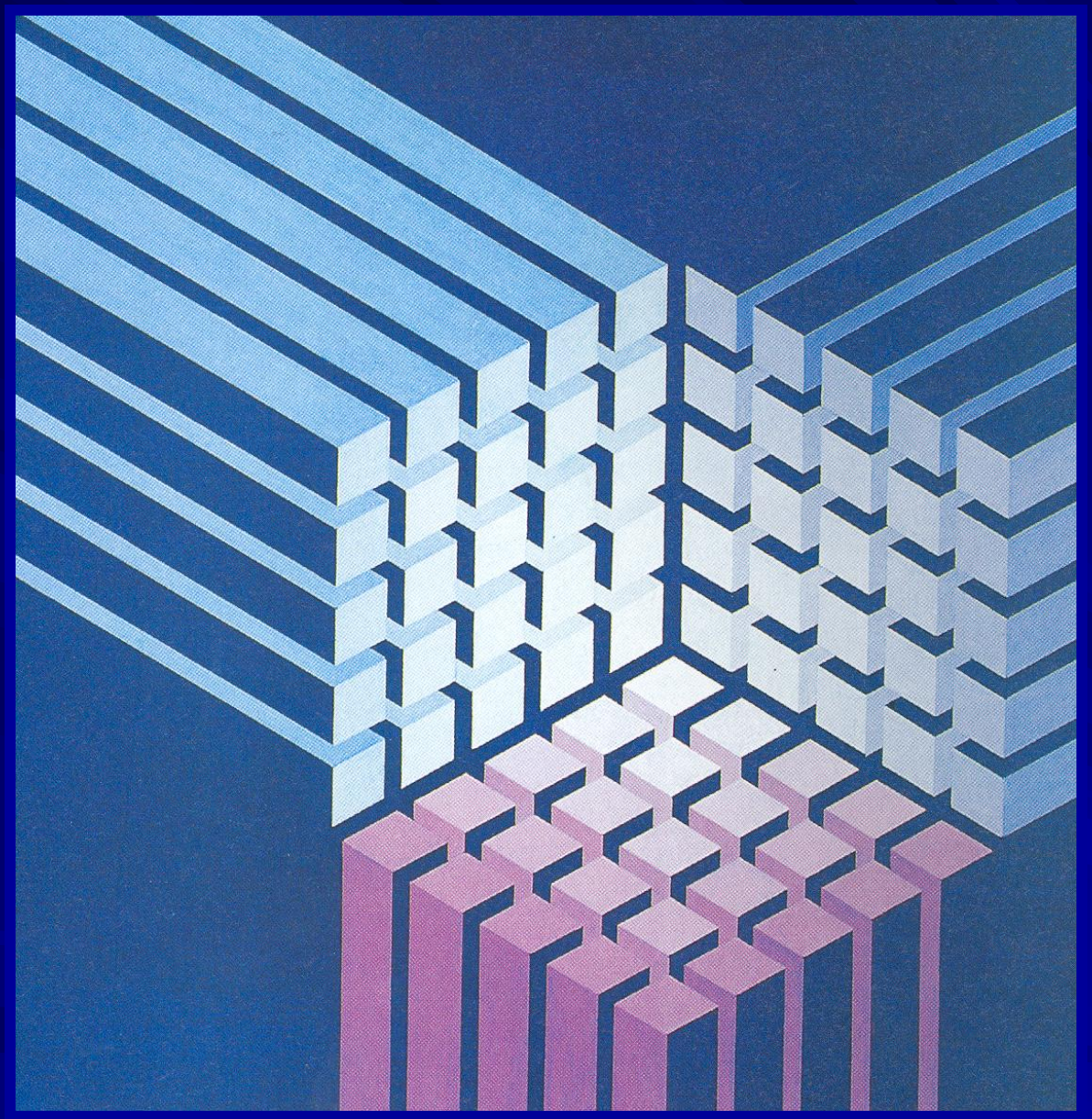




Bruno Ernst

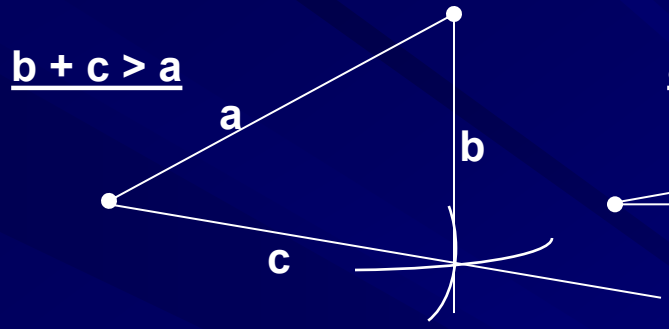
‘Impossible two-bar’

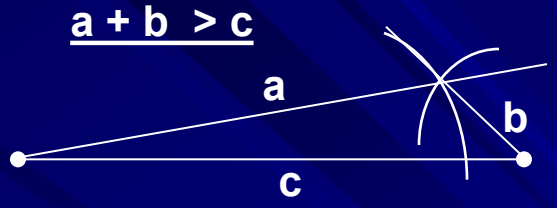


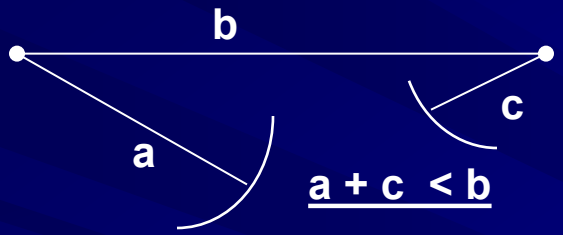


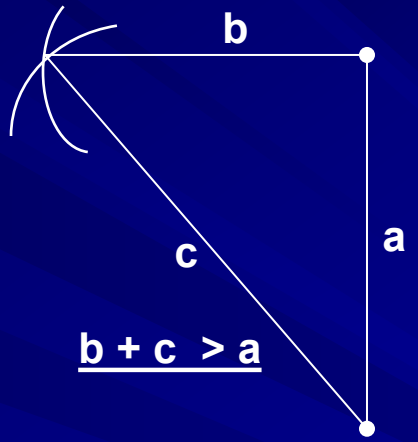
Monika Buch

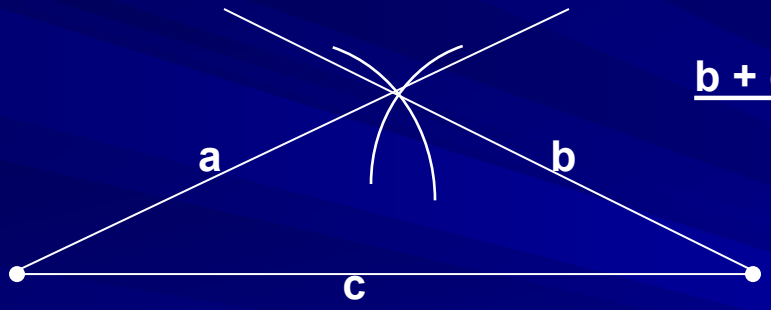
‘Illusory cube’

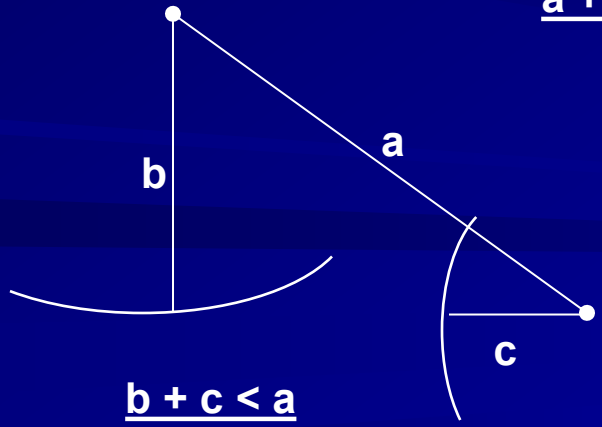


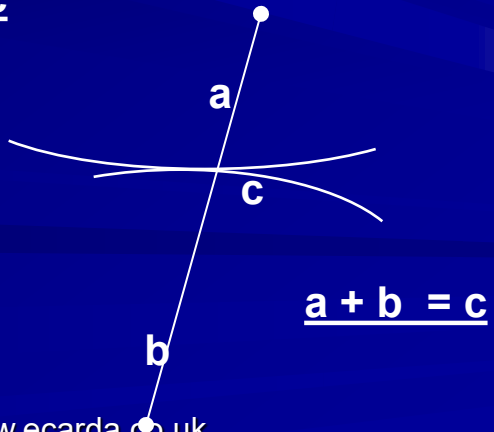


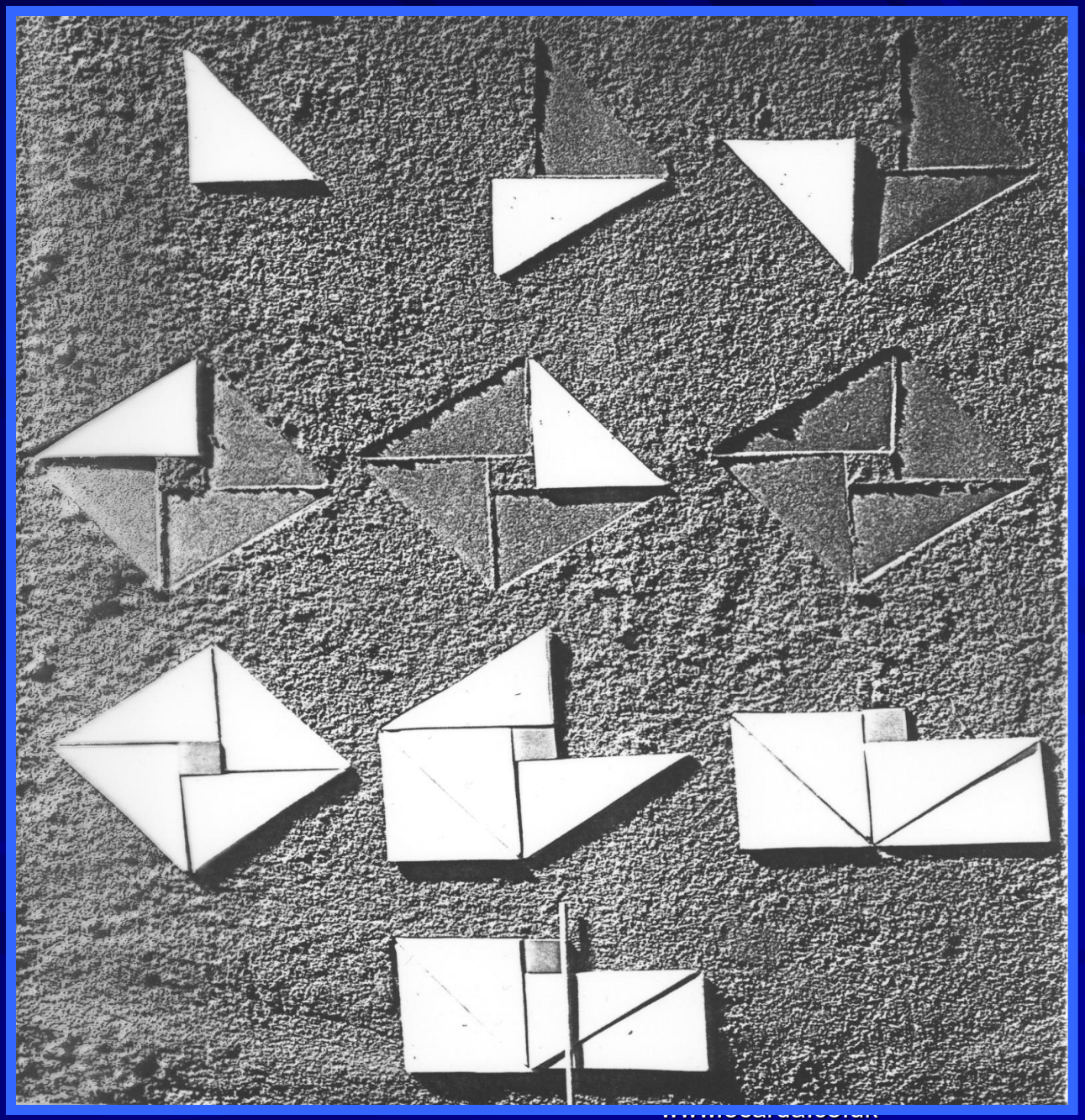






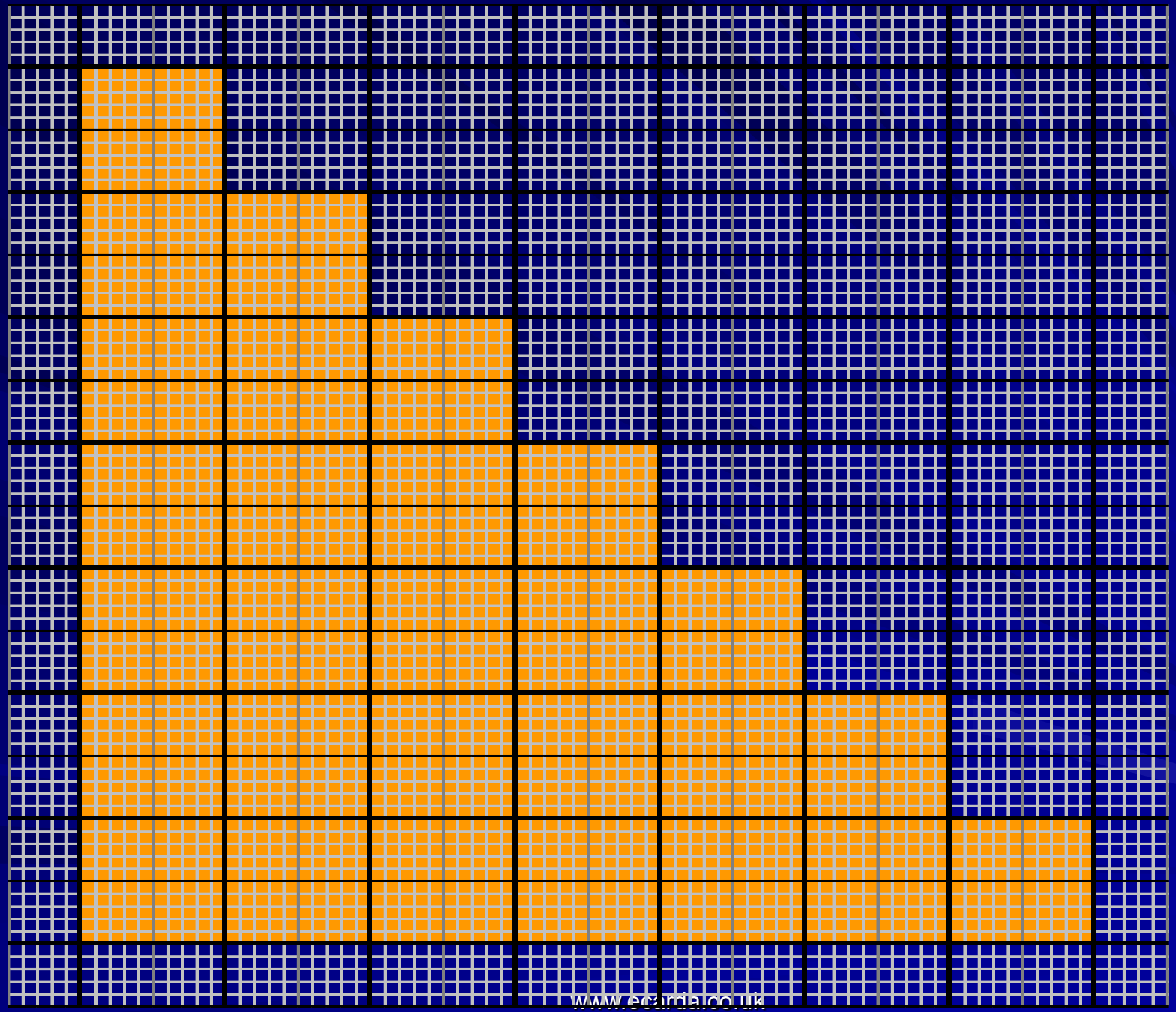


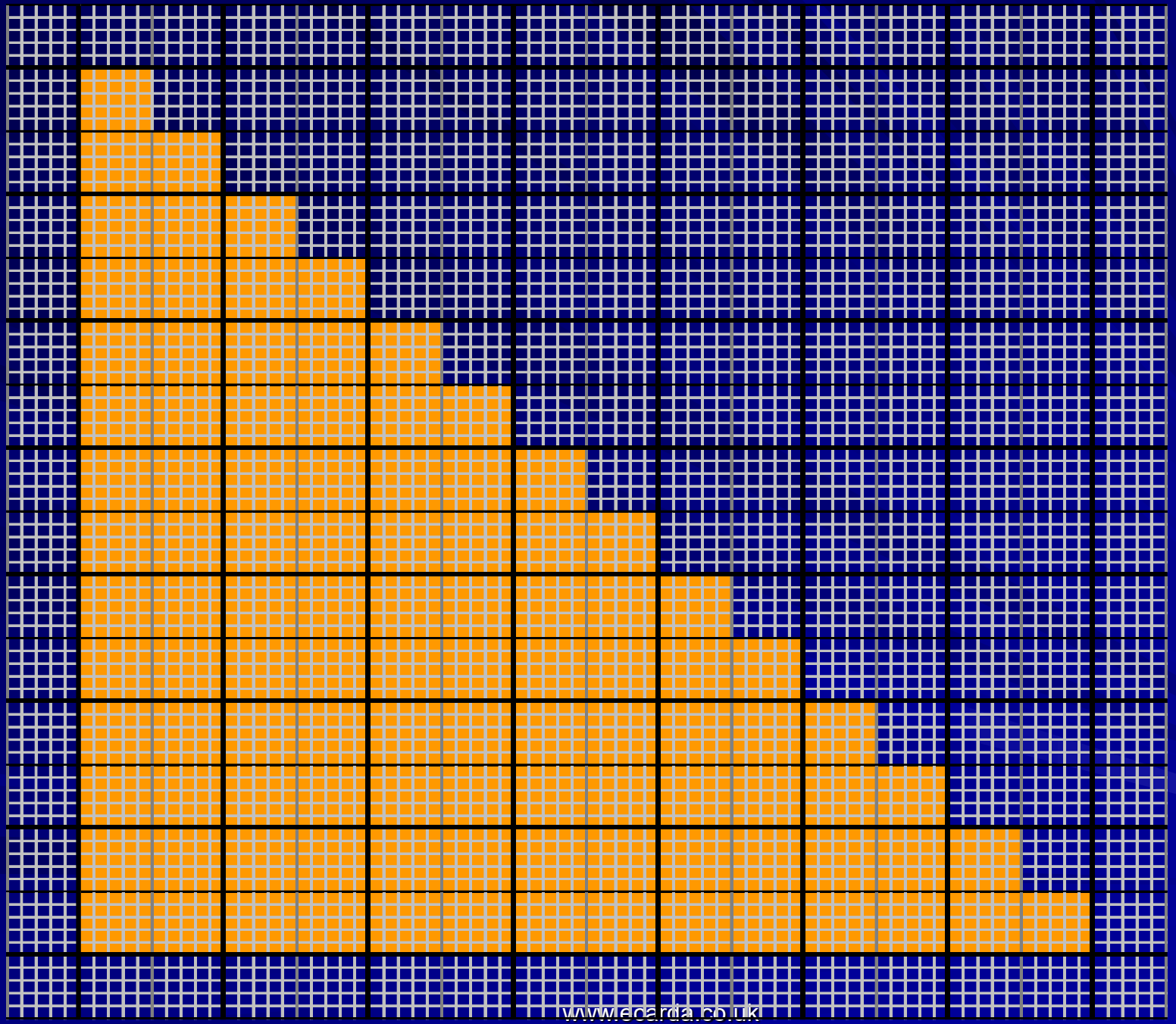


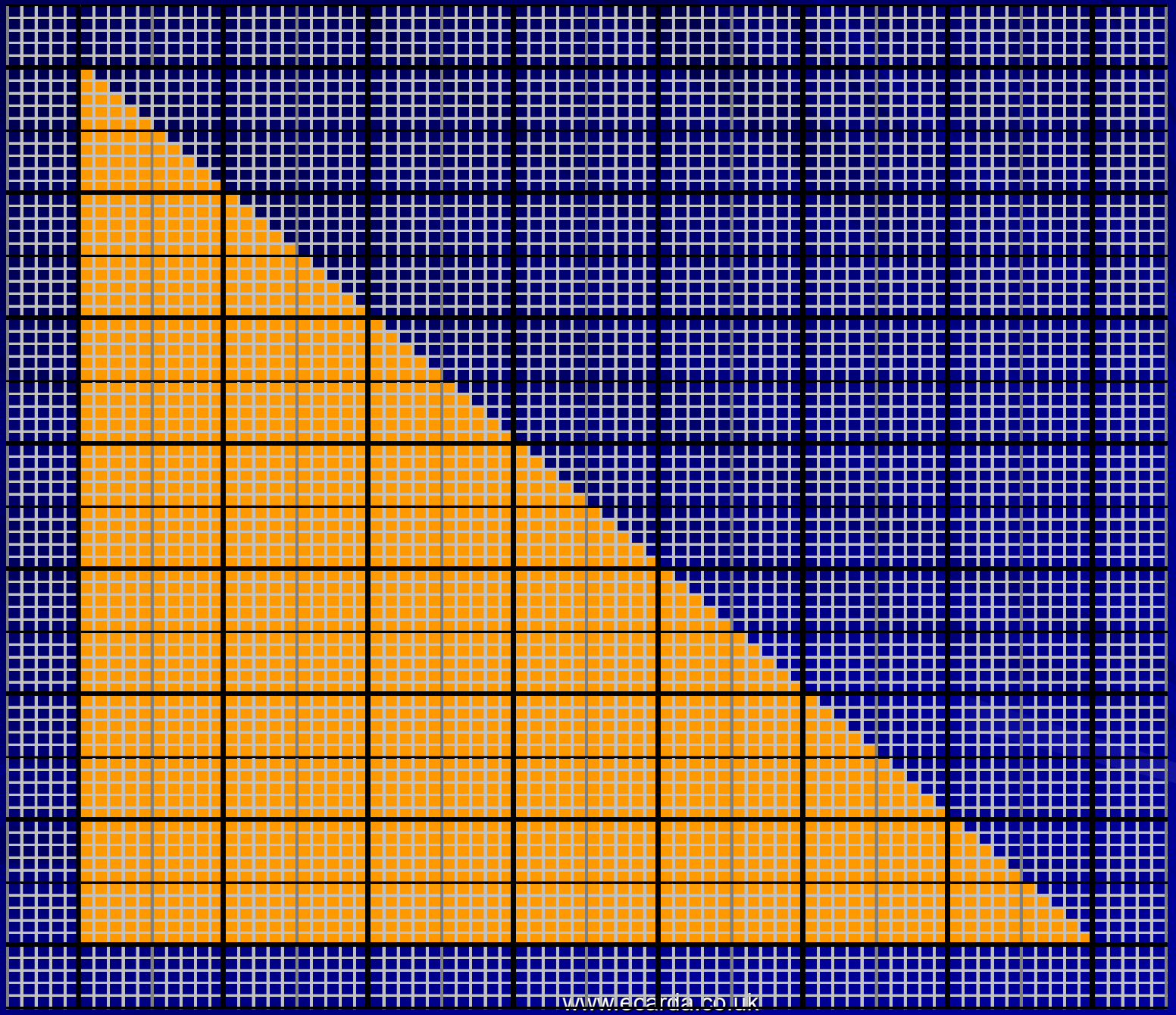


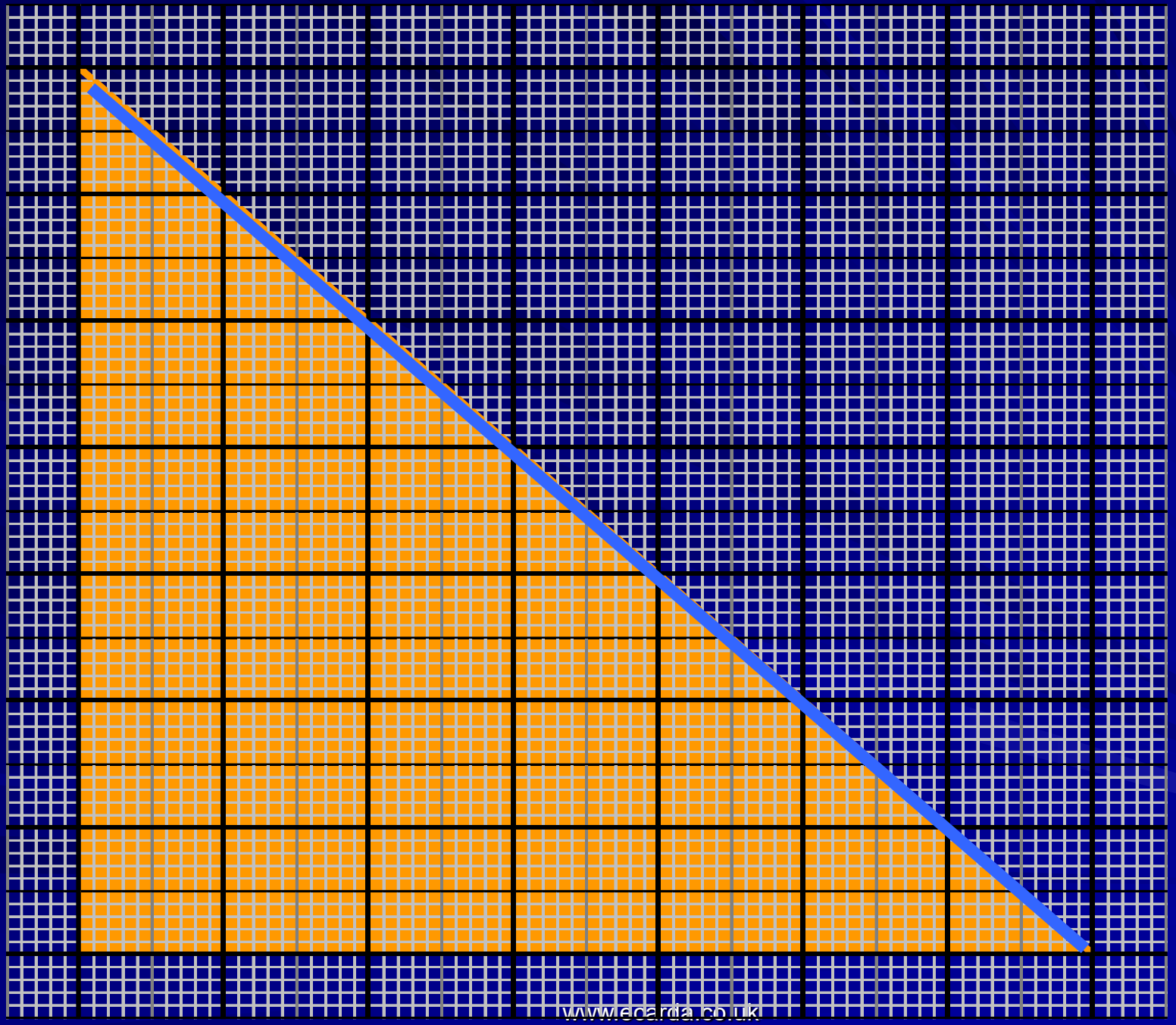
J Bronowski

‘The Ascent of Man’


www.ecarda.co.uk


www.ecarda.co.uk


www.ecards.co.uk


www.ecarda.co.uk

In the game of Ludo each player must throw a six on a die before they can start on the board.

On which roll of the die is a player most likely to throw a six?

First? Second? Third? Fourth? etc

Not taking the thinking out of situations

Maths Questions

A compulsory question from the "short question" section of a mathematics O-Level paper in 1974:

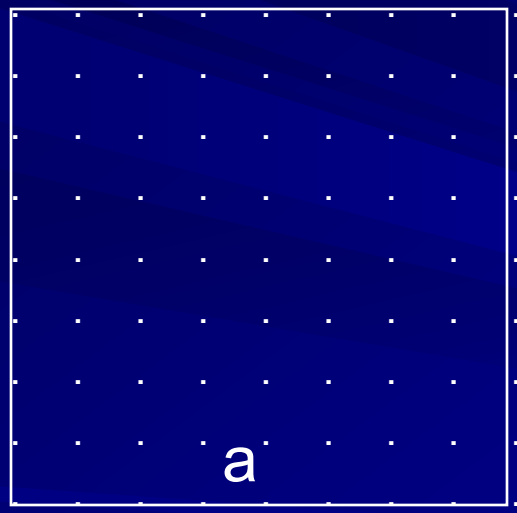
- A road haulage firm estimated that the cost of a particular journey was £10.40 and that 30% of this was the cost of the fuel used. What was the new cost of the journey if the price of a gallon of fuel was raised from 52p to 56p?

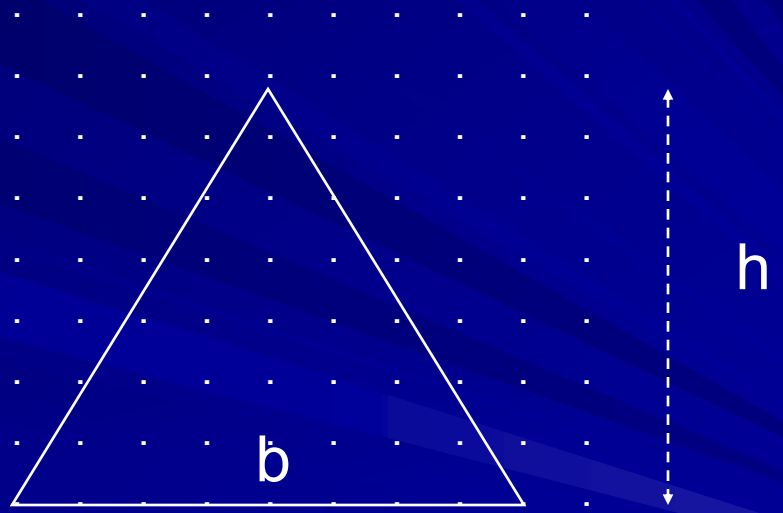
A compulsory question from a 2003 higher-tier mathematics GCSE paper:

- Petra has 6¾ metres of ribbon. She makes 6 blouses and uses 2/5 of a metre of ribbon on each blouse. How much ribbon does she have left?

$A = a^2$

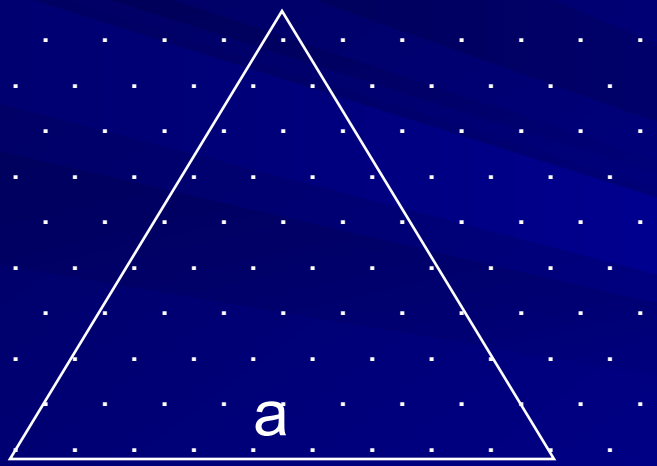
$A = \frac{1}{2} (b \times h)$

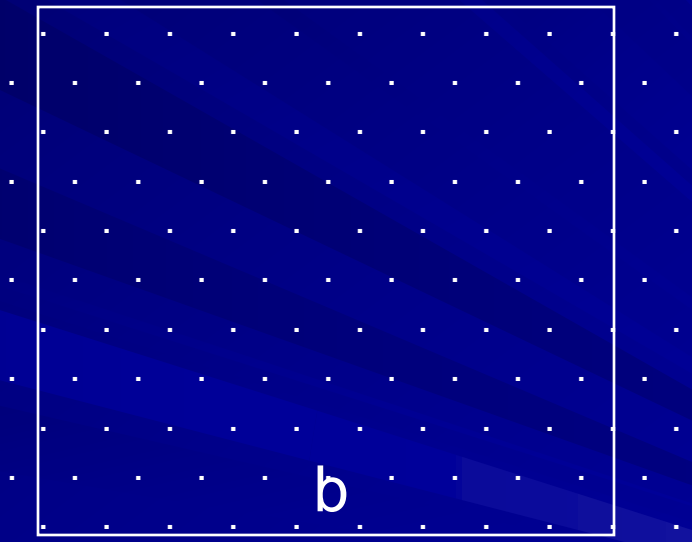




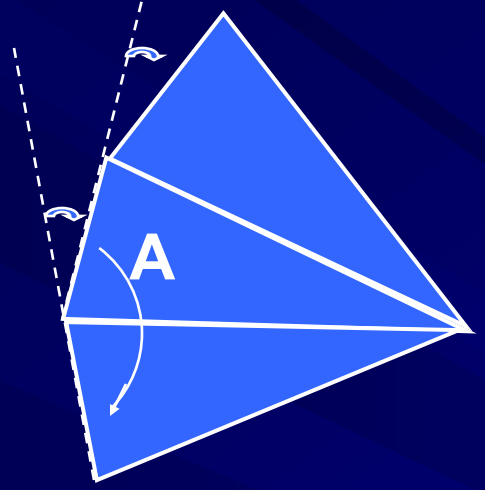
$A = a^2$

$A = ?$





Going off-piste



Let the internal angle of a n-gon =A

Then A = 180 - 360/n

Given n then A is calculable

Given A then n is calculable

For n = 3, 4, 5, 6

A = ?

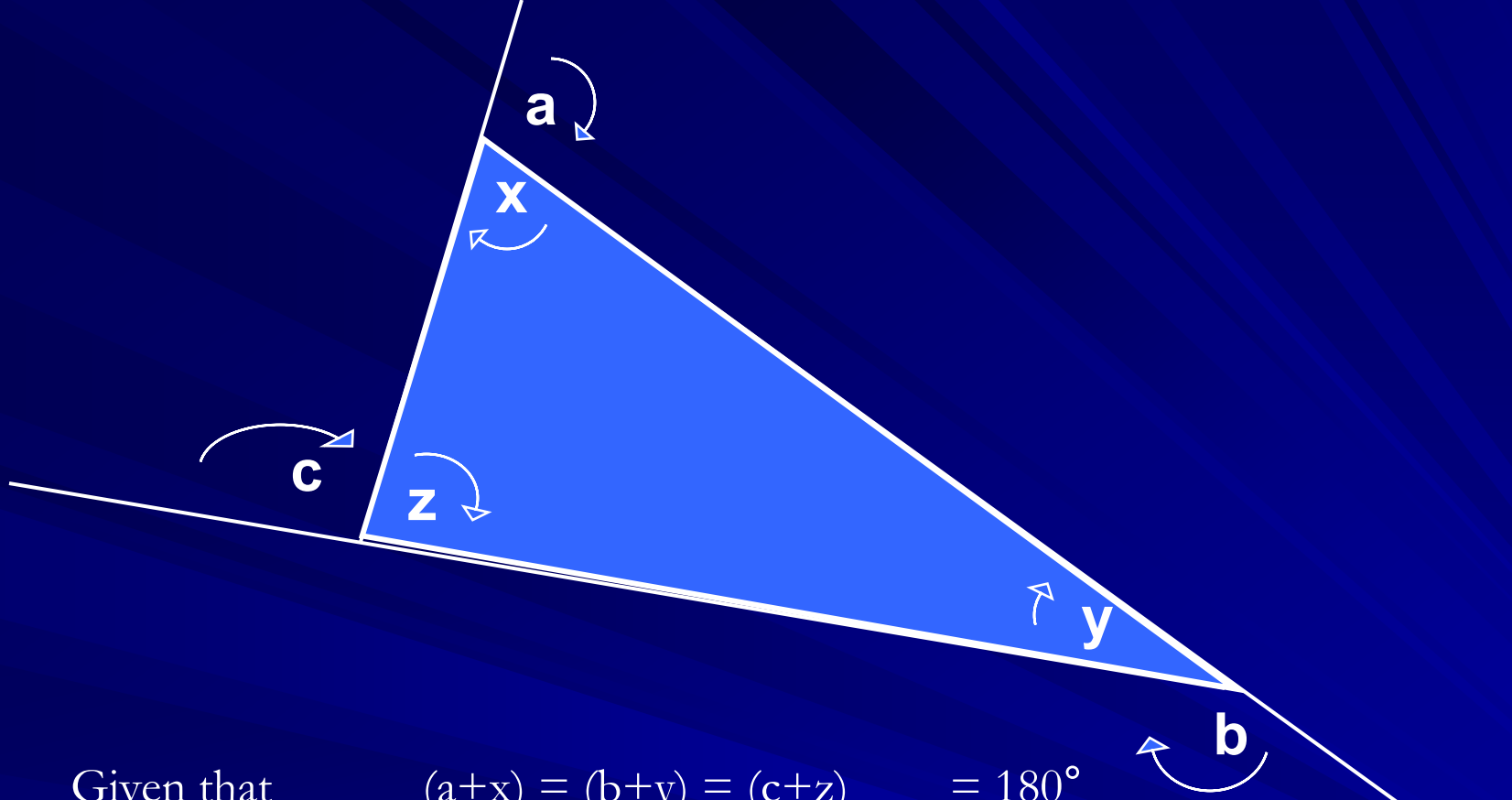
For n = -5, 2.5, 1.5, 3.75

A = ? Discuss

For A = 30°, 80°

n = ? Discuss

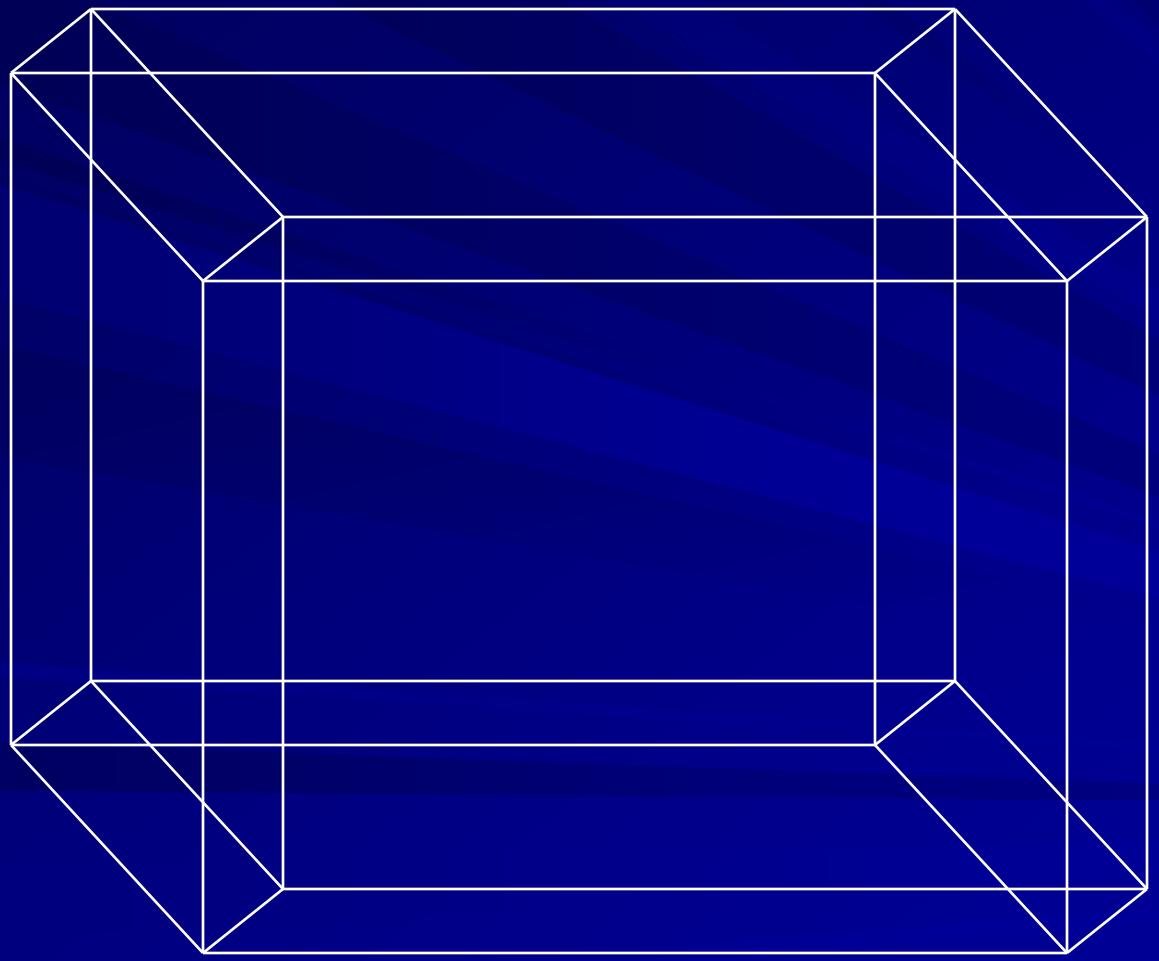
Equations can model situations. Sometimes the model applies only over a certain range.



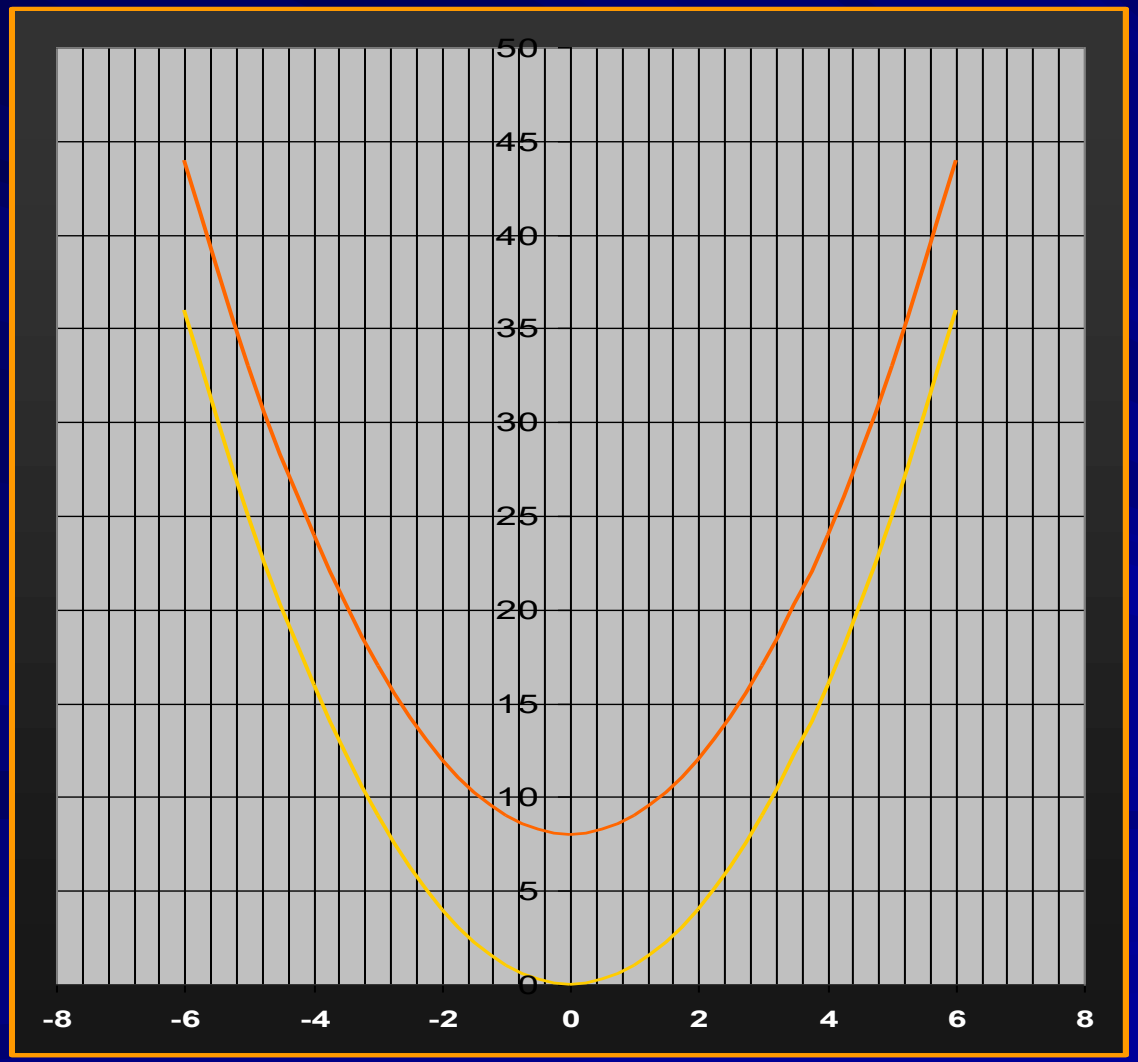
Given that	(a+x) = (b+y) = (c+z)	= 180°
Then	(a+x) + (b+y) + (c+z)	= 540°
Then	(a + b +c) + (x + y + z)	= 540°

And if	a + b + c	= 360°
Then	x + y + z	= 180°

Hyperparallelopiped



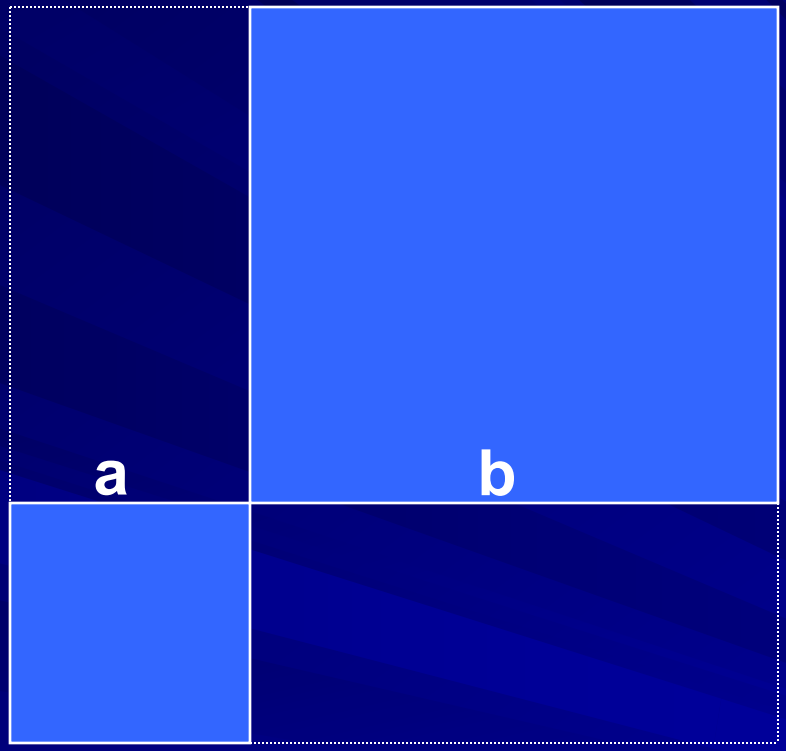
Transforming $y = x^2$



Providing languages to articulate thinking

- If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

  Euclid, Elements, 11.4, 300BC



$$(a + b)^2 = a^2 + b^2 + 2ab$$

Seeing the central dot:

(3x1) + 1 (3x2) + 1 (3x3) + 1 3n +1

Seeing the central column:

2 + (2x1) 3 + (2x2) 4 + (2x3) (n+1) + 2n

3n +1 = (n+1) +2n

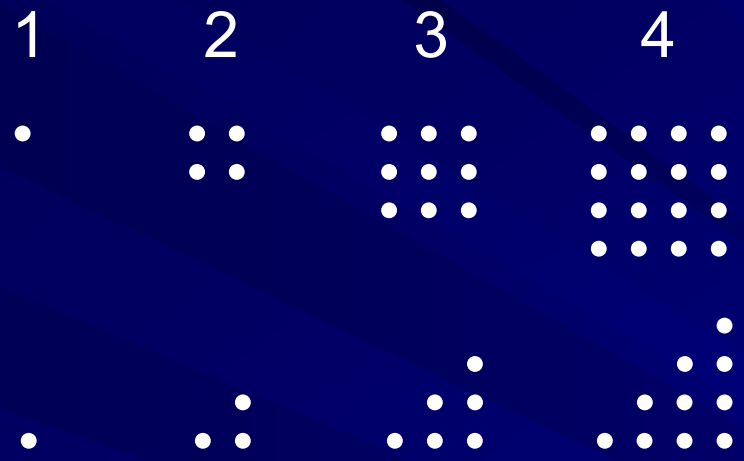
viz: an entry point into simplification

Expressing as a table:

x	1	2	3	4
y	4	7	10	13

x ⇨ 3x + 1 or y = 3x +1

viz: an entry point into graphical representation a possible entry point into inverse functions



$2^2 + 1^2 = 2^2$, $3^2 + 2^2 = 3^2$, $4^2 + 3^2 = 4^2$ …… $r^2 + (r-1)^2 = r^2$

$2^2 - 1^2 = 2$, $3^2 - 2^2 = 3$, $4^2 - 3^2 = 4$ …… $r^2 - (r-1)^2 = r$

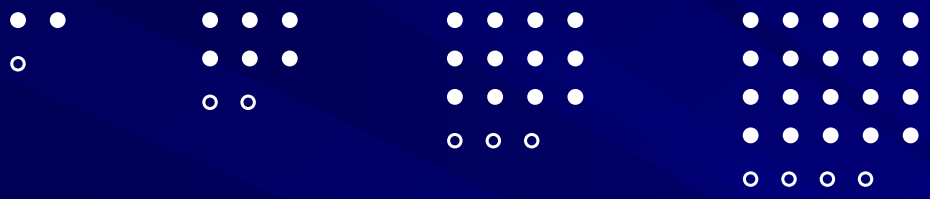
$\Rightarrow 2r^2 = r^2 + r$

$\Rightarrow r^2 = \frac{r^2 + r}{2}$

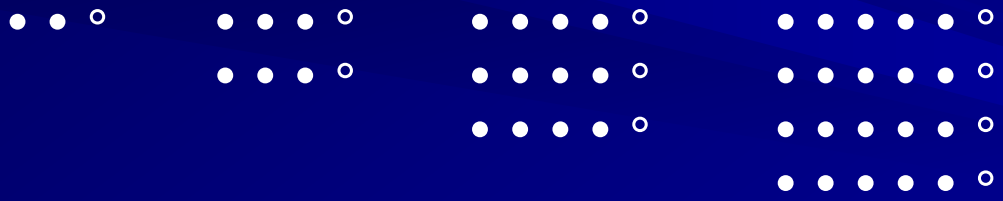
$= \frac{r(r+1)}{2}$

$$\Rightarrow \sum_{1}^{r} n = \frac{r(r+1)}{2}$$

2 3 4 5



$2^2 - 1$ $3^2 - 1$ $4^2 - 1$ $5^2 - 1$ $n^2 - 1$



3 x 1 4 x 2 5 x 3 6 x 4 (n + 1) (n - 1)

$n^2 - 1 = (n + 1)(n - 1)$

negative

$1 \times 3 = 2^2 - 1$

$2 \times 4 = 3^2 - 1$

$3 \times 5 = 4^2 - 1$

$4 \times 6 = 5^2 - 1$

fractional

larger numbers

Methods of seeing and expressing patterns and relationships

	words	diagrams	symbols	pictures	graphs	tables
geometric						
arithmetic						
statistical						
algebraic						

Providing challenging puzzles & problems

Take a 10cm x 10cm sheet of squared paper; cut out a single square from each corner and fold each side up to form an open box. How many unit centimetre cubes fit into your box?

On a new 10cm x 10cm sheet cut a two-square out of each corner and repeat the experiment.

Continue.

What is the maximum possible capacity of the box?

Maths Questions

- If a cow and a goat could eat all the grass in a field in 45 days, and the cow and a goose could eat all the grass in the same field in 60 days, and the goat and the goose could eat all the grass in the field in 90 days, how long would it take the goat, cow and goose to eat all the grass when turned into the field together?

- Which is the worse fit:

  a square peg in a round hole or a round peg in a square hole?

```
  CROSS
+ ROADS
-------
 DANGER
```

Work out the value of each letter in this addition sum

Maths Questions

- An alien visits Earth at midnight. Her clock is divided into ten hours. Each hour is divided into a hundred minutes. She asks you what time it is. You tell her it is midnight. She sets her clock with both hands pointing upwards to ten.

What does the alien's clock say when Earth time is:

- Six o'clock
- Nine o'clock
- 1:30
- 4:00

What is Earth time when the alien's clock says:

- 2:50
- 6:50
- 4:00

Asking open-ended questions

- A data set of whole numbers has a mode of 3, a median of 4 and a mean of 5. Work with a friend to find as many data sets as you can in ten minutes.

- My sum is 17 what is my highest product?

The Basketball Challenge

Your group is the board of directors of a highly successful mixed basketball team. One of your best scorers has moved to another team. You have £100,000 to spend to replace the player.

In your group you have the chair person, the scout, the press secretary and the accountant.

- The chair person makes sure you agree on a decision;
- The scout has some information on players you are interested in;
- The press secretary is going to tell everyone whom they have selected and why;
- The accountant will make sure that the club's money has been used properly.

In your group, sort out who is who.

You have 15 minutes to make your decision. You will then have some time to explain your decision to everyone else. The press secretary does this.

HINTS

- You might want to calculate one or more of the mean, median and mode for each player. One may be more useful than others.
- In making a decision, are you looking for consistency or occasional flair? Is there a best combination of the two?
- You might want to consider the spread of scores made by each player.
- Don't forget the cost.

Good Luck! - I look forwards to hearing your report.

This is the scouts information. The scout has watched the last 8,9,10 games of 4 particular players. The scout has made a spreadsheet.

Player		Pat Bowland	Jo Kamau	Sam Smith	Yan Lee
Game	1	4	7	2	3
	2	4	0	2	0
	3	4	0	2	0
	4	5	10	2	4
	5	0	0	8	2
	6	2	2	8	2
	7	0	3	0	1
	8	3	0	0	1
	9	-	0	1	5
	10	-	6	2	-
Cost		£100,000	£100,000	£100,000	£75,000

I have two discs with numbers written on each face of each disc.

I toss the two discs and add together the numbers shown.

The different totals I record are 13, 11, 10 & 8.

What are the numbers on each face of the two discs?

Some descriptions of mathematics

It is the goal of mathematics to identify and describe sources of order, and the relations between the various kinds of order that occur.

Andrew Gleason,
Harvard 1984

Mathematics is the classification and study of all possible patterns. Pattern covers almost any kind of regularity that can be recognised by the mind.

W W Sawyer, 1955

Mathematics makes the invisible visible.

Keith Devlin

Mathematics is what mathematicians do.

Keith Devlin

Our brains are disposed or programmed to make sense of experience; to find pattern and form; to seek connections and relationships. In this sense we are all born mathematical thinkers.

If education is about realising human potential then any educative process should be structured around this assertion.

Mathematics is more than a list of 'things to be learnt'; it is about unlocking the infinite possibilities of mathematical thinking.

An enlightened society should be striving to describe a curriculum for the mathematician rather than for mathematics.

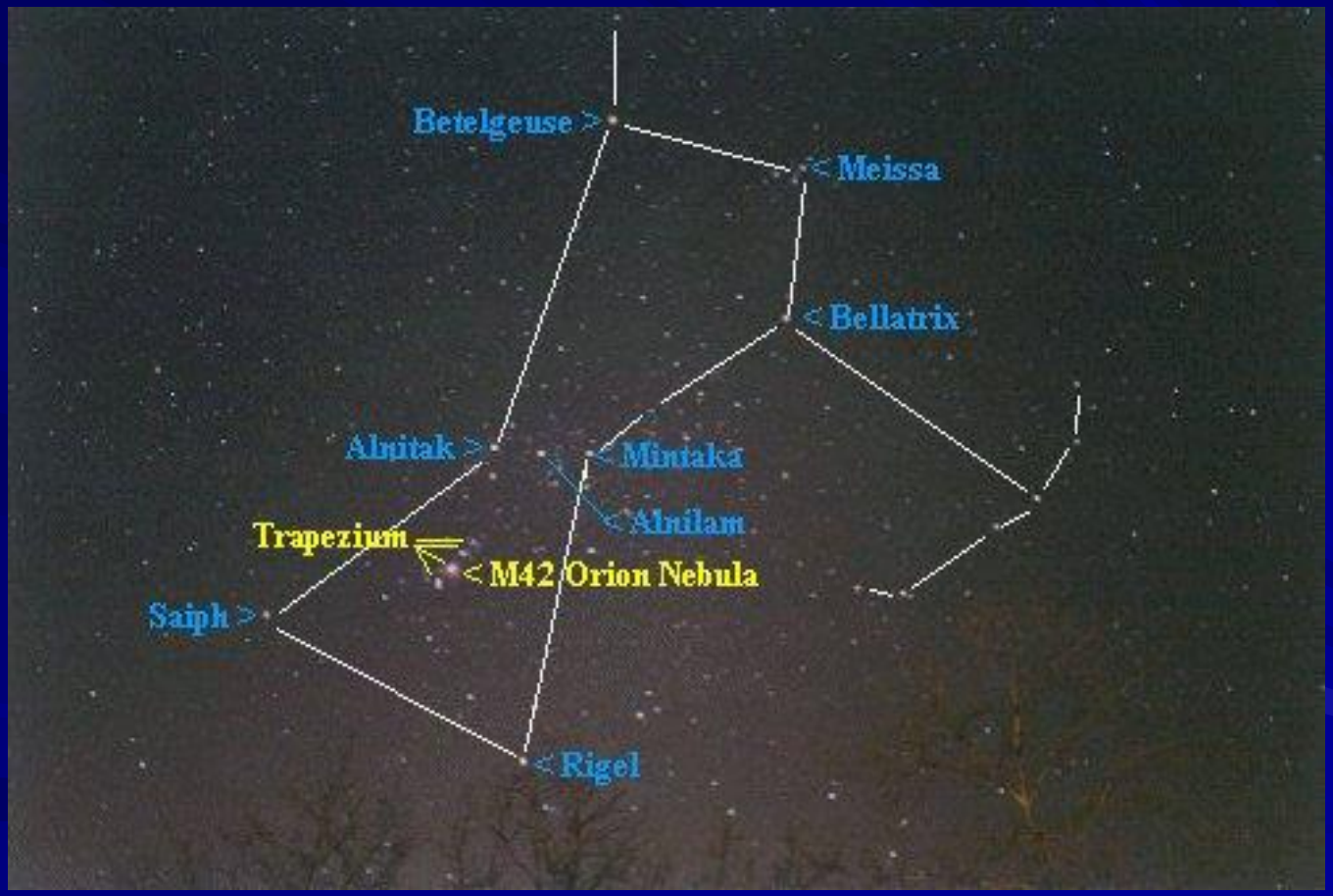
Peter Lacey

Some thoughts on learning mathematics

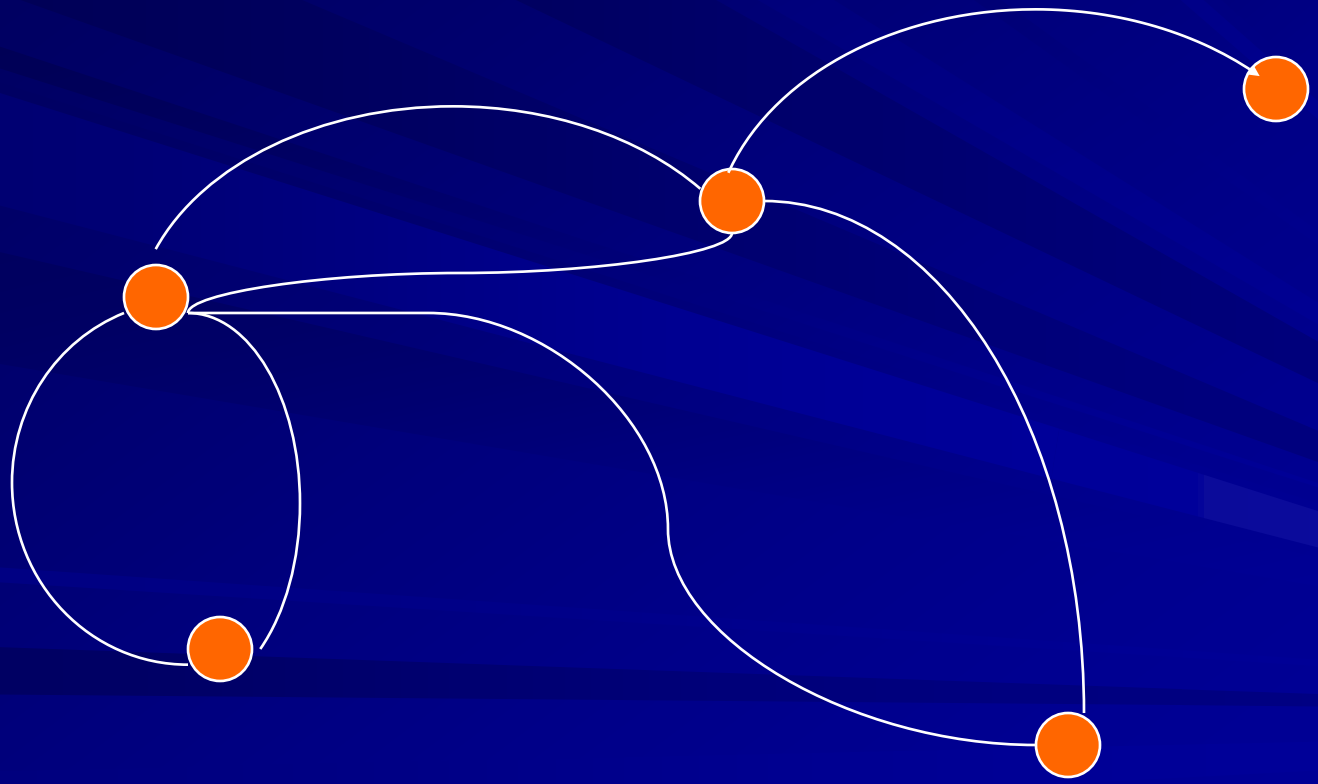
- Seeing in formation
- Membering & re-membering
- Some process models

Stars in the night sky

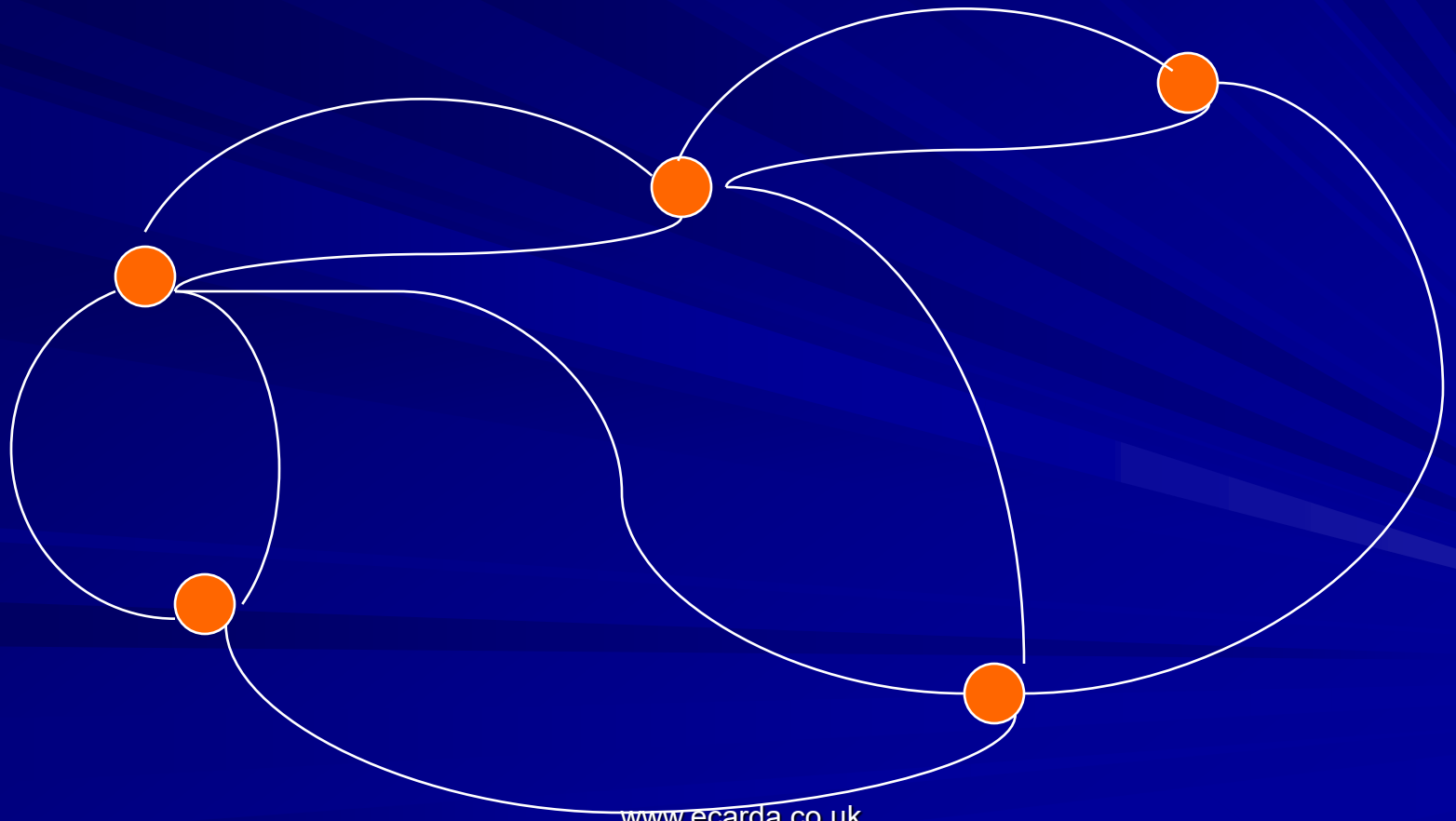
Orion, the hunter



Membering



Re-membering



learner	teacher
imagining	providing images
realising	questioning discussing
practising	repetition application

About Learning

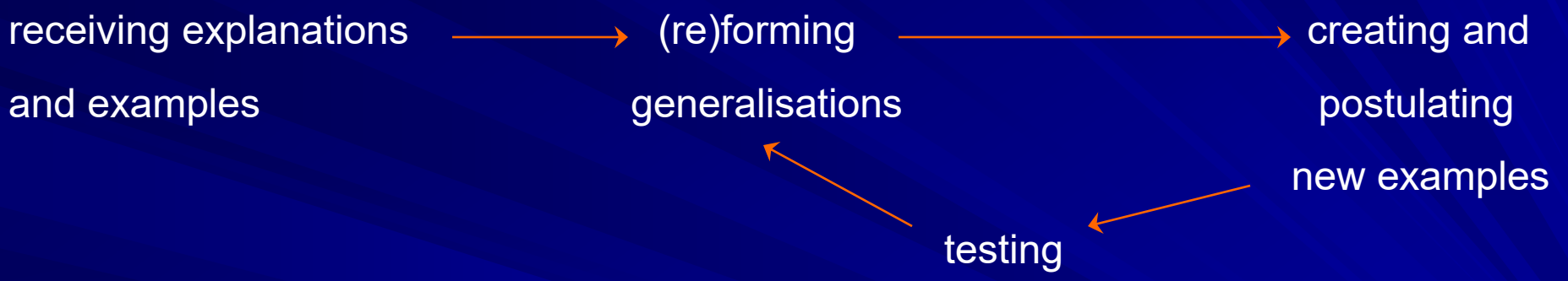
superficial → profound

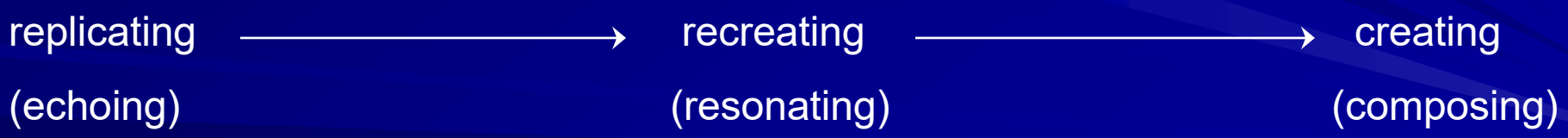
Information and experience → knowing { that, how to, when to } → understanding → wisdom

connecting to current understanding (assimilating) → reforming understanding (accommodating)

receiving linear streams → creating holistic, interconnected multidimensional maps of understanding

About Learning





About Learning

existing knowledge → Mathematical reasoning → new knowledge

maths understanding → technical fluency

maths understanding → technical fluency

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