Commodity Taxation

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Public Economics - Lecture 5

Optimal commodity taxation: overview

Combine lessons on incidence and efficiency costs to optimally design commodity taxes

What is the best way to design commodity taxes given equity and efficiency concerns?

Optimal commodity taxation: overview

- From efficiency perspective, would finance gvmt only through lump-sum tax
- With redistributional motives (equity), would levy individual-specific lump-sum tax
  - Tax higher-ability higher-income individuals a larger lump sum
- Problem: cannot observe individuals' types
- Hence, must tax economic outcomes
  - e.g. income or consumption, which leads to distortions
- Let's start with the analysis of taxation of commodity consumption

Ramsey taxation

- Restrict attention to linear taxes, i.e. $\tau \cdot c$ taxes
- No lump-sum tax available
- We call this a "constrained instruments" optimal tax problem

Ramsey taxation - Government objectives

The government has two objectives

1. **Raise total revenue** of E
   - needed to satisfy some exogenous spending requirement
2. **Maximize social welfare** (minimize utility loss)
   - might care differently about different agents

There will not be any of the inefficiencies that break 1st Welfare Theorem

The economy

1. There are N goods $x_1, \ldots, x_N$, with fixed (exogenous) prices p_i
2. All goods are taxed, such that $q_i = p_i + t_i$
3. Leisure is not taxed, while labor (l) is at rate τ

Let's start with a representative consumer

Representative consumer's problem

The individual solves the following problem

$$V\left(\underline{q}, w\right) = \max_{\{x_i\}_{i=1}^{N}, l} u\left(x_1, \ldots, x_N, l\right)$$

s.t.

$$\sum_i x_i q_i \leq \left(w - \tau\right) l + Z$$

where Z is non-wage income, and w is labor income and $\tilde{w} = w - \tau$

The individual takes prices and taxes as given

- does not internalize the effect of her choices on tax revenues

Representative consumer's problem - Solution

The Lagrangian for the consumer's problem is

$$\mathcal{L} = u\left(x_1, \ldots, x_N, l\right) + \lambda \left[Z + \tilde{w} - \sum_i x_i q_i \right]$$

We call $V\left(\underline{q}, \tilde{w}, Z\right)$ the indirect utility: value of utility at the optimal allocation

- it will be crucial to define the effect of taxes on private surplus

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The first order condition for good i reads

$$u_{x_i} = \lambda q_i.$$

It equates

- the marginal benefit of consuming the good x_i: u_{x_i}
- to its marginal cost given by: price q_i $\times$ marginal utility of money λ
  - indeed, $\partial V\left(\underline{q}, w\right)/\partial Z = \lambda$: λ is private value of receiving \$1 more in Z

The problem's solution delivers demand functions $x\left(q, \tilde{w}, Z\right)$

The Government's problem

Government maximizes social welfare subject to the revenue requirement constraint

$$\max_{\{t_i\}_{i=1}^{N},\tau} V\left(\underline{q}, \tilde{w}, Z\right)$$

s.t.

$$\sum_i t_i x_i + \tau w l \geq E$$

- With representative consumer, social welfare coincides with her indirect utility
- Government internalizes that when changing taxes consumer changes choices

Solving the Government's problem

The Government's Lagrangian is

$$\mathcal{L}_G = V\left(\underline{q}, \tilde{w}, Z\right) + \lambda_G \left[\sum_i t_i x_i + \tau w l - E\right]$$

the FOC for a generic tax-inclusive price is (to simplify, assume l inelastic)

$$\frac{\partial \mathcal{L}_G}{\partial q_i} = \underbrace{\frac{\partial V\left(\underline{q}, \tilde{w}, Z\right)}{\partial q_i}}_{\text{Private Welfare Loss}} + \lambda_G \left[\underbrace{x_i}_{\text{Mechanical Effect}} + \underbrace{\sum_j t_j \frac{\partial x_j}{\partial q_i}}_{\text{Behavioral Response}}\right] = 0$$

Solving the Government's problem

Using an envelope argument we can derive that

$$\frac{\partial V\left(\underline{q}, \tilde{w}, Z\right)}{\partial q_i} = -\lambda x_i$$

which is also known as the Roy's identity

Hence, the formula for optimal taxes becomes (for each i)

$$\frac{1}{x_i} \sum_j t_j \frac{\partial x_j}{\partial q_i} = -\frac{(\lambda_G - \lambda)}{\lambda_G}$$

Deriving the same formula with a variational argument

Suppose government increases t_i by dt_i, we have effects on

- Government revenues

$$dR = x_i + \sum_j t_j \frac{\partial x_j}{\partial q_i}$$

- Private surplus

$$dU = \frac{\partial V\left(\underline{q}, \tilde{w}, Z\right)}{\partial q_i} = -\lambda x_i$$

At the optimum, the two must cancel out

$$\lambda_G dR + dU = 0$$

and we get the same formula as before

Interpretation of the formula

Use the Slutsky equation to separate income from substitution effects

$$\frac{\partial x_j}{\partial q_i} = \underbrace{\frac{\partial h_j}{\partial q_i}}_{\text{Substitution effect}} - \underbrace{x_i \frac{\partial x_j}{\partial Z}}_{\text{Income effect}}$$

where h_j is Hicksian demand. So the formula derived before becomes

$$\frac{1}{x_i} \sum_j t_j \left(\frac{\partial h_j}{\partial q_i} - x_i \frac{\partial x_j}{\partial Z} \right) = - \frac{(\lambda_G - \lambda)}{\lambda_G}$$

and using symmetry of the Slutsky ($\frac{\partial h_j}{\partial q_i} = \frac{\partial h_i}{\partial q_j}$) we find

$$\frac{1}{x_i} \sum_j t_j \frac{\partial h_i}{\partial q_j} = - \frac{\left(\lambda_G - \lambda - \lambda_G \sum_j t_j \frac{\partial x_j}{\partial Z}\right)}{\lambda_G}$$

Interpretation of the formula

$$\frac{1}{x_i}\sum_j t_j \frac{\partial h_i}{\partial q_j} = -\frac{\overbrace{\left(\lambda_G - \lambda - \lambda_G \sum_j t_j \frac{\partial x_j}{\partial Z}\right)}^{=\theta}}{\lambda_G}$$

RHS is equal for each good i, its numerator θ is the effect of raising a lump-sum tax:

- reduce private surplus by λ
- increases government revenues by 1 mechanically (λ_G)
- lose some revenues due to income effects, proportional to $\frac{\partial x_j}{\partial Z}$

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LHS is the "discouragement" in the consumption of good i (when taxes are small)

- when introduce a tax on j, consumption of i drops by $\frac{\partial h_i}{\partial q_j}$
- so LHS is total reduction of x_i consumption (in % of x_i) caused by tax system
- at optimum, all goods are discouraged in the same way (same RHS)!
- Proportional reduction in demand in response to a equal proportional tax increase

Inverse elasticity rule

Using the definition of "compensated elasticity"

$$\varepsilon^c_{ij} = \frac{\partial h_i}{\partial q_j}\frac{q_j}{x_i}$$

we can write the formula as

$$\sum_j \frac{t_j}{q_j}\varepsilon^c_{ij} = -\frac{\theta}{\lambda_G}$$

if $\varepsilon^c_{ij} = 0$ for $j \neq i$, then

$$\frac{t_i}{q_i} = -\frac{\theta}{\lambda_G \cdot \varepsilon^c_{ii}}$$

which is an inverse elasticity rule! Tax less more elastic goods!

This is to reduce efficiency cost of taxation, inelastic good substitutes best for lump-sum

This concept will always come back in optimal tax problems

Can derive the same conclusion with MVPF logic

Let's write down the MVPF of a change in tax t_i

$$MVPF_i = \frac{WTP}{\text{Government cost}} = \frac{x_j \cdot \partial V / \partial Z}{x_i + \sum_j t_j \frac{\partial x_j}{\partial q_i}}$$

$$\text{(using Slutsky)} = \frac{x_i \lambda}{x_i + \sum_j t_j \left(\frac{\partial h_j}{\partial q_i} - x_i \frac{\partial x_j}{\partial Z} \right)} = \frac{\lambda}{1 - \sum_j t_j \frac{\partial x_j}{\partial Z} + \frac{1}{x_i} \sum_j t_j \frac{\partial h_i}{\partial q_j}}$$

and the blue terms are equal across all goods

At the optimum, MVPF must be equated across taxes, which implies for all $i \neq k$

$$\frac{1}{x_i} \sum_j t_j \frac{\partial h_i}{\partial q_j} = \frac{1}{x_k} \sum_j t_j \frac{\partial h_k}{\partial q_j} = \tilde{\theta}$$

Heterogeneous individuals and redistributive concerns

Let's consider an economy with multiple individual types $i = 1, \dots I$

Government SWF attaches a Pareto weight ψ_i to each i, there are f_i types i

- $\psi_i > \psi_{i'}$ if government values utils of i more
- redistributionary concerns imply $\psi_{poor} > \psi_{rich}$

The government problem becomes

$$\max_{\{q_j\}_{j=1}^{N}} \sum_{i=1}^{I} \psi_i f_i V_i\left(\underline{q}, Z_i\right)$$

s.t.

$$\sum_i f_i \left[\sum_j t_j x_{ij} + \tau w l_i\right] \geq E$$

Optimal tax with heterogeneous individuals

The FOC for q_k on the government Lagrangian delivers

$$\sum_{i=1}^{I} \psi_i f_i \frac{\partial V_i}{\partial q_k} + \lambda_G \left[\sum_{i=1}^{I} f_i \left(x_{ik} + \sum_j t_j \frac{\partial x_{ij}}{\partial q_k} \right) \right] =$$

$$- \sum_{i=1}^{I} \psi_i f_i x_{ik} \frac{\partial V_i}{\partial Z_i} + \lambda_G \left[\sum_{i=1}^{I} f_i x_{ik} \left(1 - \sum_j t_j \frac{\partial x_{ij}}{\partial Z_i} \right) + \sum_{i=1}^{I} \sum_j f_i t_j \frac{\partial h_{ik}}{\partial q_j} \right] = 0$$

so that

$$\sum_i \sum_j f_i t_j \frac{\partial h_{ik}}{\partial q_j} = \sum_{i=1}^{I} f_i x_{ik} \left(\frac{\psi_i}{\lambda_G} \frac{\partial V_i}{\partial Z_i} - 1 + \sum_j t_j \frac{\partial x_{ij}}{\partial Z_i} \right)$$

Optimal tax with heterogeneous individuals

$$\sum_i \sum_j f_i t_j \frac{\partial h_{ik}}{\partial q_j} = \sum_{i=1}^{I} f_i x_{ik} \overbrace{\left(\frac{\psi_i}{\lambda_G} \frac{\partial V_i}{\partial Z_i} - 1 + \sum_j t_j \frac{\partial x_{ij}}{\partial Z_i} \right)}^{\tilde{\theta}_i}$$

becomes

$$\mathbb{E}_i \left[\sum_j t_j \frac{\partial h_{ik}}{\partial q_j} \right] = \mathbb{E}_i \left[x_{ik} \tilde{\theta}_i \right]$$

LHS change in aggregate demand from proportional increase in all taxes

RHS proportional to covariance **between:**

- "social marginal utility of income" $\tilde{\theta}_i$: social value of giving \$1 to i
- AND consumption of good k

Optimal tax with heterogeneous individuals

$$\sum_i \sum_j f_i t_j \frac{\partial h_{ik}}{\partial q_j} = \sum_{i=1}^{I} f_i x_{ik} \overbrace{\left(\frac{\psi_i}{\lambda_G} \frac{\partial V_i}{\partial Z_i} - 1 + \sum_j t_j \frac{\partial x_{ij}}{\partial Z_i} \right)}^{\tilde{\theta}_i}$$

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LHS change in aggregate demand from proportional increase in all taxes

RHS proportional to covariance **between**:

- "social marginal utility of income" $\tilde{\theta}_i$: social value of giving \$1 to i
- AND consumption of good k

RHS is exactly equal to covariance if there is lump-sum tax:

- Optimality for lump-sum $\sum_i f_i \tilde{\theta}_i \lambda_G = 0$, so $\mathbb{E}_i \left[\tilde{\theta}_i \right] = 0$

Interpret the covariance terms

$$\mathbb{E}_i \left[\sum_j t_j \frac{\partial h_{ik}}{\partial q_j} \right] = \text{Cov}_i \left(\tilde{\theta}_i, x_{ik} \right)$$

Discourage more goods consumed more by low $\tilde{\theta}$ types, and viceversa

- Low $\tilde{\theta}$ types are those with low social marginal utility of income

If $\psi_{\text{poor}} > \psi_{\text{rich}}$ tax goods consumed by rich more

If $\tilde{\theta}_i$ is the same across all individuals, just use the lump-sum if available

- However, if $\tilde{\theta}_i$ heterogeneous might want commodity tax even if we have lump-sum