# APPENDIX B — CALCULATION REGISTER

*Western Cape Fiscal Autonomy Model v4.0*

*All rand values in constant 2025 prices, using South Africa's GDP deflator, unless otherwise stated. The model's first year, labelled 2025, is 2024 data expressed in constant 2025 prices; it is not a 2025 outturn.*

## Purpose

This appendix documents the calculation methodology used in the Western Cape Fiscal Autonomy Model v4.0. Its purpose is to ensure transparency, replicability and auditability. Every calculation presented in the model is derived from assumptions recorded in Appendix A and data sources recorded in Appendix C. All assumptions are referenced by their Appendix A reference number.

The model comprises eight calculation modules, applied sequentially: the SA Baseline Model, the WC Status Quo Model, the GDP Model, the Revenue Model, the Debt Model, the Spending Model, the Fiscal Multiplier Model, and the Unemployment Model.

## Section 1 — SA Baseline Model

### Purpose

To establish South Africa's fiscal trajectory under the Status Quo, which forms the basis for the WC Status Quo calculations.

### Formula

For each year ***t***:

SA Revenue = *SA GDP × 27% (A2)*

SA Debt Servicing = *SA GDP × 4.0% (A3)* — used only in the derivation of SA Programme Spending; not used in the debt roll-forward below

SA Programme Spending = *SA GDP × 27.3% (A4)*

SA Additional Borrowing = ***SA GDP × 4.3% (A5)***

SA Interest (diagnostic) = ***SA Opening Debt × 2.2% (A12)***

SA Primary Deficit = ***SA Additional Borrowing − SA Interest***

SA Closing Debt = ***SA Opening Debt + SA Additional Borrowing***

SA GDP grows at 0.7% per annum (A6)

SA Interest is calculated as a diagnostic line only and is not separately added to the debt stock. SA Additional Borrowing (the modelled 4.3% deficit) already reflects total government expenditure inclusive of debt-service costs; adding interest a second time would double-count it. This is a stock-flow-consistent identity — see Appendix A, Section 1 Methodology Note.

## Key Outputs (2035)

| Metric | Value |
|---|---|
| SA GDP (2035) | R8,149.1bn |
| SA Gross Loan Debt (2035) | R9,417.3bn |
| SA Debt/GDP (2035) | 115.6% |
| SA Cumulative Interest (2025–2035, diagnostic) | R1,746.7bn |

This path is labelled "historical 4.3% deficit, no consolidation" — it is the arithmetic result of holding the 10-year historical deficit and growth rate constant, not a forecast of South Africa's actual fiscal trajectory or a claim about future Treasury policy. Under this arithmetic, SA's debt/GDP ratio rises from 79.2% (2025) to 115.6% (2035).

# Section 2 — WC Status Quo Model

## Purpose

To establish the Western Cape's fiscal position under existing arrangements, including the quantification of fiscal extraction as a model output.

## Formula

For each year ***t***:

WC GDP grows at 0.8% per annum (A17)

WC Revenue Generated = ***WC GDP × 40.5% (A11)***

WC Total Extraction = ***WC Revenue Generated − WC Spending Received***

WC Debt Servicing (diagnostic) = ***SA Interest × 12% (A7)***

WC Net Extraction = ***WC Total Extraction − WC Debt Servicing***

WC Additional Borrowing = ***SA Additional Borrowing × 12% (A7)***

WC Closing Debt = ***WC Opening Debt + WC Additional Borrowing***

As in Section 1, WC Debt Servicing is a diagnostic figure used only to calculate Net Extraction; it is not separately added to WC's debt stock, for the same stock-flow-consistency reason.

Spending received starts at R297.3bn (A21), the observed 2024/25 base, and grows in line with SA programme spending. The WC fiscal capture rate of 40.5% (A11) is the WC's observed 2024/25 rate — the rate at which its economy generates government revenue — and is the same rate used in the autonomy scenarios. Under the Status Quo, the part of this revenue not returned as spending flows to the national fiscus; that difference is the extraction.

## Extraction Definition

**Total Extraction:** *All revenue raised in the WC that is not returned as government spending. Equivalent to the Econometrix fiscal outflow figure. In 2025: R140.0bn.*

**Net Extraction:** *Total Extraction minus the WC's legitimate debt servicing obligation. Represents the purely discretionary outflow above inherited debt obligations. In 2025: R125.2bn.*

### Key Outputs

| Metric | 2025 | 2035 | Cumulative |
|---|---|---|---|
| Total Extraction | R140.0bn | R154.8bn | R1,619.9bn |
| Net Extraction | R125.2bn | R131.3bn | R1,410.3bn |
| WC Debt Servicing (diagnostic) | R14.7bn | R23.5bn | — |
| WC Closing Debt | R723.2bn | R1,130.8bn | — |
| WC Debt/GDP | 66.9% | 96.7% | — |

## Section 3 — GDP Model (Autonomous Scenarios)

### Purpose

To project WC GDP under each autonomous scenario, incorporating both the explicit growth premium and the additional GDP generated by the fiscal multiplier.

### Formula

For each year $t$:

Underlying GDP (GDP before the multiplier): $\text{Underlying GDP}_t = \text{Underlying GDP}_{t-1} \times (1 + \text{Base Rate} + \text{Premium})$

Multiplier Addition: $\text{GDP Addition}_t = \text{Additional Spending}_{t-1} \times 0.16$ (A28)

GDP: $\text{GDP}_t = \text{Underlying GDP}_t + \text{GDP Addition}_t$

The multiplier is applied as a level effect: each year's additional spending lifts the following year's GDP once, and the lift is not carried into the underlying growth path. It therefore lasts only as long as the additional spending continues. Underlying GDP grows at the base rate plus the scenario's premium only.

| Scenario | Base Rate | Premium | Combined |
|---|---|---|---|
| Status Quo | 0.8% | 0% | 0.8% |
| Conservative (no reform premium; fiscal retention plus multiplier) | 0.8% | 0% | 0.8% + multiplier |
| Central | 0.8% | 2.0% | 2.8% + multiplier |
| Benchmarked | 0.8% | 4.0% | 4.8% + multiplier |

## Key Outputs (2035 GDP)

| Scenario | 2035 GDP |
|---|---|
| Status Quo | R1,169.8bn |
| Conservative | R1,194.8bn |
| Central | R1,464.1bn |
| Benchmarked | R1,784.6bn |

For Conservative, the entire GDP gap over Status Quo (R25.0bn, about 2%, in 2035) is attributable to the fiscal multiplier, since the 0% premium (A18) means Conservative's

underlying GDP is identical to Status Quo's; the retention effect of ending extraction, independent of growth, is visible instead in this scenario's spending and debt figures (Sections 5 and 6).

# Section 4 — Revenue Model (Autonomous Scenarios)

## Purpose

To calculate total annual government revenue retained by the WC under fiscal autonomy.

## Formula

WC Revenue = ***WC GDP × 40.5% (A11)***

The WC fiscal capture rate of 40.5% (the observed 2024/25 rate, Appendix A, A11) is applied in the Status Quo and all autonomous scenarios and held constant for ten years. Under autonomous scenarios the WC retains this revenue in full; under the Status Quo it does not.

# Section 5 — Debt Model (Autonomous Scenarios)

## Purpose

To project the WC's debt position under each autonomous scenario, incorporating mandatory interest charges, debt repayment, and the 30% debt sustainability threshold.

## Step 1 — Interest Charge

Interest = ***Opening Debt × 2.2% (A12)***

Interest is charged on the opening balance before any revenue allocation. This is a mandatory prior obligation under autonomous scenarios.

## Step 2 — Revenue Allocation

Revenue remaining after the interest charge is allocated as follows, subject to the threshold rule:

Remainder = ***WC Revenue − Interest***

Debt Repayment Allocation = ***Remainder × 25% (A15)***

Spending Allocation = ***Remainder × 75% (A16)***

## Step 3 — Threshold Rule

The Debt Target for each year is calculated as:

Debt Target = ***WC GDP × 30% (A14)***

The threshold rule operates as follows:

- ***If Opening Debt > Debt Target: apply 25%/75% split. Cap repayment at the amount needed to reach the target if the full 25% would overshoot.***
- ***If Opening Debt ≤ Debt Target: Debt repayment = MAX(0, Opening Debt – Debt Target). All remaining revenue flows to services.***

## Step 4 — Closing Debt

Closing Debt = ***Opening Debt – Debt Repayment***

Interest (Step 1) is paid in cash from revenue each year; it is not added to the closing debt balance. Because interest is paid rather than capitalised, once the required repayment (Step 3) reaches zero the debt stock is held flat in real terms — it does not continue to grow. The 30% threshold (A14) itself continues to rise every year with GDP, so debt/GDP falls further below 30% every year after the threshold is first reached — not because debt grows more slowly than the target, but because debt has stopped growing entirely while the target keeps rising. This is the no-borrowing rule functioning as intended, not a model error — reintroducing borrowing solely to hold the ratio at exactly 30% would contradict the no-deficit rule that is itself a deliberate design feature of the autonomous scenarios.

### Key Outputs (2035)

| Scenario | Closing Debt | Debt/GDP | Threshold Year |
|---|---|---|---|
| Status Quo | R1,130.8bn | 96.7% | Never |
| Conservative | R333.1bn | 27.9% | 2028 |
| Central | R353.9bn | 24.2% | 2028 |
| Benchmarked | R357.6bn | 20.0% | 2027 |

## Section 6 — Spending Model (Autonomous Scenarios)

### Purpose

To calculate annual government spending and cumulative spending under each scenario.

### Formula

Annual Spending = ***WC Revenue − Interest − Debt Repayment***

This enforces the balanced budget constraint — the WC spends only what it raises after servicing debt. Once the threshold is reached and debt repayment ceases, all revenue after interest flows to services.

### Status Quo Spending

Under the Status Quo, spending received is not derived from this formula. It is taken directly from the Econometrix 2024/25 reconciliation of the WC fiscal position (R297.3bn, the observed 2024/25 base, A21) and grows in line with SA programme spending. The WC has no control over this figure.

### Key Outputs

| Scenario | 2025 Spending | 2035 Spending | Cumulative | vs Status Quo (2035) |
|---|---|---|---|---|
| Status Quo | R297.3bn | R318.8bn | R3,387.1bn | — |
| Conservative | R316.9bn | R476.5bn | R4,631.0bn | +49.5% |
| Central | R316.9bn | R585.0bn | R5,215.9bn | +83.5% |
| Benchmarked | R316.9bn | R714.7bn | R5,856.6bn | +124.2% |

All three autonomous scenarios produce spending above the Status Quo from the first year of autonomy. The significant step-change in spending occurs when the debt threshold is reached and all revenue after interest flows to services.

## Section 7 — Fiscal Multiplier Model

### Purpose

To capture the additional GDP generated by increased government spending under fiscal autonomy, and its downstream effects on revenue, debt and employment.

### Formula

Additional Spending vs Status Quo *(year t)* = ***Autonomous Spending (t) – Status Quo Spending (t)***.

Status Quo Spending is WC spending received; under autonomy the WC gives up all spending received, including equitable-share transfers and grants, so this is a like-for-like comparison.

Multiplier GDP Addition *(year t+1)* = ***Additional Spending (t) × 0.16 (A28)***

This is a level effect, not compounded (see Section 3).

The one-year lag reflects the finding in Du Rand, Hollander and van Lill (2023) that the peak multiplier effect occurs four quarters after a spending shock. The multiplier is applied only to the increment above Status Quo spending, not to total spending, and only to government services/consumption spending. Infrastructure and capital spending are not modelled; the same paper reports an average cumulative government investment multiplier of −0.118 for South Africa. Lasting gains in productive capacity are represented only by the reform premiums (A19, A20).

### Assumption Note

The study this figure is drawn from is contextually different from the Western Cape scenario it is applied to here, and is used as the best available data rather than an ideal match: the model assumes that a large, ongoing increase in provincial spending, funded directly by additional revenue rather than debt, will provide some level of economic impetus. 0.16 is a national, deficit-financed estimate applied here to retained revenue. Leakage to the rest of the country would push a provincial multiplier lower; retaining revenue that previously left the province could push it higher. The bias could run either way. With no multiplier, 2035 GDP versus the Status Quo would be about 0% (Conservative), +21.7% (Central) and +47.6% (Benchmarked), against +2.1%, +25.2% and +52.6% with it. Kemp and Hollander (2020, Empirical Estimates of Fiscal Multipliers for South Africa, WIDER Working Paper 2020/91) — a separate study using a different methodology and dataset — identify a short-term range of 0.13 to 0.61, and a long-term present-value range of −0.24 to 1.06, for South African fiscal multipliers. The model uses Du Rand, Hollander and van Lill's (2023) central government consumption estimate of 0.16, which sits near the lower end of Kemp and Hollander's short-term range.

## Section 8 — Unemployment Model

### Purpose

To project expanded unemployment under each scenario using the Wesgro employment elasticity of growth.

## Starting Point

WC Expanded Unemployment (2025) = ***24.7% (A24)***

## Formula

$EUR_t$ = ***$EUR_{t-1}$ – (0.52 (A26) × (1 − $EUR_{t-1}$) × $\Delta GDP_t$) + 0.0014 (A25), floored at 5.0% (A27)***

Where 0.52 is the Wesgro employment elasticity (A26), ΔGDP is the actual growth rate above the 0.8% baseline in each year including the fiscal multiplier contribution, (1 − $EUR_{t-1}$) converts the employment-growth elasticity into a year-on-year change in the unemployment rate — a mathematical requirement of that conversion, since the same percentage growth in employment moves a larger number of people, in level terms, off the unemployment rolls as the employed share of the labour force rises — and +0.0014 is the observed historical trend of +0.14 percentage points per annum (A25), derived from Stats SA QLFS data (Q1 2015: 23.3%; Q1 2025: 24.7%) and applied uniformly to all scenarios as a conservative assumption.

A separate risk is that the Wesgro elasticity (A26) is itself documented as implicitly incorporating historical labour force dynamics including participation rate changes, net migration and demographic shifts — the same broad category of forces likely driving the +0.14pp drift. Some overlap between the two terms cannot be ruled out with the data available; if present, it would cause the model to understate rather than overstate the unemployment improvement in the autonomy scenarios, since the drift is applied on top of, not instead of, the elasticity-driven improvement. This works in the opposite direction to a second effect: the (1 − EUR) conversion assumes the additional jobs do not themselves enlarge the labour force, whereas faster growth would likely draw in new work-seekers and migrants (the expanded definition already counts discouraged work-seekers, but not new entrants or in-migrants). That effect would cause the model to overstate the improvement, most noticeably in Benchmarked. Neither effect can be quantified with the available data.

Under the Status Quo, actual growth equals the base rate of 0.8% and the elasticity term is approximately zero — only the drift applies. Under Conservative, the multiplier contribution above 0.8% produces improvement that partially offsets the drift. Under Central and Benchmarked, the explicit growth premium plus the multiplier contribution generates material improvement each year.

### Frictional Floor

A frictional unemployment floor of 5.0% (A27) is applied across all scenarios. This follows the IMF's general concept of a natural rate of unemployment and sits at the midpoint of the commonly cited 4–6% range in mainstream macroeconomic literature; no South African-specific estimate of this floor exists in the available literature. The floor is applied here to the expanded unemployment definition rather than the IMF's narrow-definition convention, which is the stricter of the two conditions — see Appendix A, A27, for the full derivation and caveats. No scenario reaches this floor by 2035.

### Key Outputs (2035)

| Scenario | Expanded Unemployment (2035) |
|---|---|
| Status Quo | 26.1% |
| Conservative | 25.3% |
| Central | 16.7% |
| Benchmarked | 7.4% |

## Section 9 — Model Limitations

The following limitations apply to this model:

- *The model does not incorporate transition costs arising from any move to fiscal autonomy or sovereign independence.*
- *The fiscal multiplier of 0.16 is derived from South African national data and is applied here to a provincial economy funded by retained revenue; the bias could run either way (see Section 7).*

- *The model does not incorporate the employment effect of increased government spending as a separate channel — only through the GDP multiplier feeding into the Wesgro elasticity.*
- *The Wesgro employment elasticity of 0.52 is applied as a constant over the full modelling period. In practice, elasticity may vary with economic conditions and labour market structure.*
- *The +0.14 percentage point annual unemployment drift (A25) is applied uniformly across all scenarios, including Central and Benchmarked. Whether this drift would hold, narrow, or widen at growth rates above the 0.8% baseline it was measured from is not established by the available two-point data; it is carried forward unchanged as a conservative assumption.*
- *The model assumes fiscal autonomy is fully implemented from 2025 and that all revenue is retained as assumed. Partial or delayed implementation would produce proportionally smaller benefits.*
- *Currency arrangements, trade-friction effects, capital-flight effects and global economic shocks are outside the scope of the model.*
- *Infrastructure and capital spending are not modelled (see Section 7).*
- *The model does not allow additional jobs to enlarge the labour force; faster growth would likely draw in new work-seekers and migrants (see Section 8).*
- *The comparator growth premiums are derived from historical post-reform experience and may not be fully replicable under different global conditions.*