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WORKBOOK 37

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Maths Knowledge Organiser Higher Tier
Quadratics Equations

What must I be able to do?	Key vocabulary	
Solve a quadratic equation by factorising Solve a quadratic equation by using the quadratic formula Solve a quadratic equation by completing the square Identify the significant points of a quadratic function Solve a pair of simultaneous equations where one is non linear using an algebraic method	Root	The values of x in a quadratic equation which give a value of y = 0. On a graph, this is where it crosses the x-axis.
	Turning point	On a quadratic graph, the turning point is the maximum or minimum point on the curve.
	Discriminant	The part of the formula under the square root (b^2 – 4ac). It determines how many solutions a quadratic equation will have.
Solve quadratic inequalities		

Solving by factorising

Step 1: Rearrange the equation so that one side is equal to 0

Step 2: Factorise the equation

Step 3: Solve each factor equal to 0.

e.g. Solve $x^2 - 6x + 10 = 2$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

Either $x - 4 = 0$ or $x - 2 = 0$

$x = 4$ and $x = 2$

e.g. Solve $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

Either $2x + 1 = 0$ or $x - 3 = 0$

$2x = -1$

$x = \frac{-1}{2}$ and $x = 3$

The quadratic formula

For a general quadratic equation written

then $x = \frac{- \quad ^2 \quad -}{2a}$

e.g. Solve $4x^2 - 8x - 7 = 0$

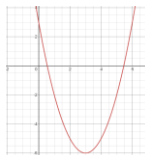
Be careful putting negatives into your calculator. Brackets

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times}}{4 \times (-7)}$$

x = 2.66 and x = -

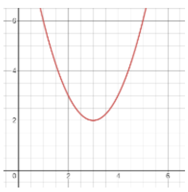
The b^2 – 4ac is known as the discriminant.

If $b^2 - 4ac > 0$ then there are 2 unique roots. The graph crosses the x-axis in 2



If $b^2 - 4ac = 0$ then there is a repeated root. The graph touches

If $b^2 - 4ac < 0$ there are no roots. The graph does not touch the x- axis.



Completing the square

Writing a quadratic equation in the form $(x + p)^2 + r = 0$ is known as completing the square.

e.g. Solve $x^2 + 6x - 8 = 0$

(Half the b value, so 6

$\div 2 = 3$) $(x + 3)^2 - 9 - 8 = 0$

(subtract this value squared as $(x + 3)^2$ multiplied out is

$(x + 3)^2 - 17 = 0$ (this is the completed the

$(x + 3)^2 =$

17 $x + 3$

$= \pm\sqrt{17}$

When written in the form $(x + p)^2 + r = 0$, you can determine key features of the graph.

The equation of the line of symmetry of the curve is $x = - p$

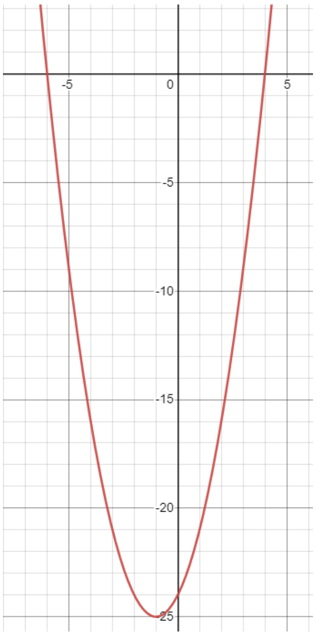
Solving quadratic inequalities

Solving a quadratic inequality is very similar to solving a quadratic equation.

Step 1: Solve the equation to find the critical values. Step 2: Sketch the curve

e.g. Solve $x^2 + 10x - 24 < 0$

$x^2 + 2x - 24 = 0$ $(x + 6)(x - 4) = 0$



The curve is a positive quadratic so is a 'u' shaped parabola.

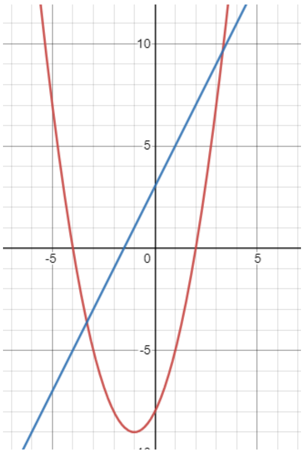
The roots of the equation are at $x = -6$ and $x = 4$, so this is where it crosses the x axis.

The curve is < 0 (below the x-axis) when it is between $x = -6$ and $x = 4$. Therefore the solution to $x^2 + 10x - 24 < 0$ is:

Note: if the question instead was solve $x^2 + 10x - 24 > 0$ we now need the sections above the x-axis which are not connected and so the solution would have been

$x < -6$ and $x > 4$

Simultaneous equations where one is non-linear



As a non-linear graph will curve, the solution to simultaneous equations with a non-linear equation can have more than 1 answer.

If we are solving a quadratic and a linear graph there are either:

0 solutions – the graphs do not intersect

1 solution – the linear graph is a tangent to the curve and touches only once

2 solutions – the graph crosses twice (as shown on the left)

Solving the equations algebraically allows us to find the exact values of these intersections.

e.g. Solve $y = x^2 + 3x - 8$
$y = 2x + 3$

As both equations are y =, we can equate them

$$x^2 + 3x - 8 = 2x + 3$$

Rearrange so that one side = 0

$$x^2 + x - 11 = 0$$

This does not factorise so using the formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$x = 2.8541..$ and $x = -3.854…$

Substitute these back into

$y = 2x + 3$ $y = 8.7082…$ and $y = -4.708…$

So the solutions are:

$x = 2.86$ and $y = 8.71$ $x = -3.85$ and $y = -4.71$

This is the equation of a circle

e.g. Solve $x^2 + y^2 = 10$
$y = 2x - 5$

This time we need to substitute $y = 2x - 5$ into the top equation.

$$x^2 + (2x - 5)^2 = 10$$

Multiply out the bracket

$$x^2 + 4x^2 - 20x + 25 = 10$$

Simplify and set one side = 0

$$5x^2 - 20x + 15 = 0$$

Factorise and solve

$$5(x^2 - 4x + 3) = 0$$

$$5(x - 3)(x - 1) = 0$$

$x = 3$ and $x = 1$ Substitute back into $y = 2x - 5$

When $x = 3$, $y = 1$ when $x = 1$, $y = -3$

Solutions need to be given in pairs with the correct x and y values matched up.

Knowledge Organiser - Solving Quadratics by Factorising

Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2+bx+c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2-3x+7$ Examples of non-quadratic expressions: $2x^3-5x^2$ $9x-1$
2. Factorising Quadratics	When a quadratic expression is in the form x^2+bx+c find the two numbers that **add to give b** and **multiply to give c**.	$x^2+7x+10=(x+5)(x+2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2+2x-8=(x+4)(x-2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2-b^2}$ can be factorised to give $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}-\boldsymbol{b})$	$x^2-25=(x+5)(x-5)$ $16x^2-81=(4x+9)(4x-9)$
4. Solving Quadratics ($ax^2=b$)	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2=98$ $x^2=49$ $x=\pm 7$
5. Solving Quadratics ($ax^2+bx=0$)	**Factorise** and then **solve = 0**.	$x^2-3x=0$ $x(x-3)=0$ $x=0 \text{ or } x=3$
6. Solving Quadratics by Factorising ($a=1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2+3x-10=0$ Factorise: $(x+5)(x-2)=0$ $x=-5 \text{ or } x=2$
7. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form ax^2+bx+c 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2+5x-4$ 1. $6\times -4=-24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2+8x-3x-4$ 4. Factorise in pairs: $2x(3x+4)-1(3x+4)$ 5. Answer = $(3x+4)(2x-1)$
8. Solving Quadratics by Factorising ($a \neq 1$)	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2+7x-4=0$ Factorise: $(2x-1)(x+4)=0$ $x=\frac{1}{2} \text{ or } x=-4$

SOLVING QUADRATIC SIMULTANEOUS EQUATIONS

One of the equations contains a quadratic term

Solve each pair of equations simultaneously:

1) $y = x + 3$
$y = x^2 + 5x - 2$

2) $y = x + 3$
$y = x^2 - 3x - 2$

3) $y = x + 18$
$y = x^2 - 2x - 10$

4) $y = x^2 - 3x - 2$
$y = 2x - 8$

5) $y = x^2 - 3$
$y = x - 3$

6) $y = x + 3$
$y = x^2 - 3$

7) $y = 2x - 1$
$y = x^2 - 2x + 2$

8) $y - 2x = 4$
$y = x^2 + x - 2$

9) $y = 3x + 9$
$y = 2x^2 + 9x + 1$

With more than one quadratic term

Solve each pair of equations simultaneously:

1) $x^2 + y^2 = 9$
   $y = x + 3$

2) $y = x - 3$
   $x^2 + y^2 = 9$

3) $x^2 + y^2 = 25$
   $y = x + 5$

4) $y = x - 4$
   $x^2 + y^2 = 16$

5) $y - x + 7 = 0$
   $x^2 + y^2 - 49 = 0$

6) $x - 4y + 1 = 0$
   $x^2 - 4xy + y^2 = 13$

7) $y = -x^2 + 5x + 2$
   $y = 3x^2 - x - 2$

8) $y + x^2 = 5x$
   $y + x + 2 = 3x^2 - 2$

9) $2x + y = 7$
   $x^2 - y^2 = 8$

PROBLEM SOLVING

1) Line A and Line B intersect at coordinates C and D.
Find C and D if the equations of the lines are as follows:

$Line\ A: x + y + 1 = 4$

$Line\ B: x^2 + 3y - 27 = 0$

2) Solve the simultaneous equations:

$y = 5x - 1$

$y = (x + 1)^2$

3) The line 'L' and the curve 'C' intersect at the points A and B

$L: y - x = 4$

$C: y - x^2 - 4 = 3x$

Find the distance between the points A and B