STUDYPLUS

A BRIGHTER FUTURE

GCSE Prep Year

HIGHER TIER

ALGEBRA WORKBOOK 37

Name: __________________

Year Group: __________________

Start Date: __________________

www.study-plus.co.uk | info@study-plus.co.uk | 0118 958 56 56

361-363 Oxford Road, Reading, RG30 1AY

Maths Knowledge Organiser Higher Tier
Quadratics Equations

What must I be able to do?

- Solve a quadratic equation by factorising
- Solve a quadratic equation by using the quadratic formula
- Solve a quadratic equation by completing the square
- Identify the significant points of a quadratic function
- Solve a pair of simultaneous equations where one is non linear using an algebraic method
- Solve quadratic inequalities

Key vocabulary

Root	The values of x in a quadratic equation which give a value of y = 0. On a graph, this is where it crosses the x-axis.
Turning point	On a quadratic graph, the turning point is the maximum or minimum point on the curve.
Discriminant	The part of the formula under the square root ($b^2 - 4ac$). It determines how many solutions a quadratic equation will have.

Solving by factorising

Step 1: Rearrange the equation so that one side is equal to 0

Step 2: Factorise the equation

Step 3: Solve each factor equal to 0.

e.g. Solve $x^2 - 6x + 10 = 2$

$x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

Either $x - 4 = 0$ or $x - 2 = 0$

$x = 4$ and $x = 2$

e.g. Solve $2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

Either $2x + 1 = 0$ or $x - 3 = 0$

$2x = -1$

$x = -\frac{1}{2}$ and $x = 3$

The quadratic formula

For a general quadratic equation written $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

e.g. Solve $4x^2 - 8x - 7 = 0$

$a = 4 \quad b = -8 \quad c = -7$

Be careful putting negatives into your calculator. Brackets around the negative number will help.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 4 \times (-7)}}{2 \times 4}$$

$x = 2.66$ and $x = -0.66$ (2.d.p.)

The $b^2 - 4ac$ is known as the **discriminant.**

If $b^2 - 4ac > 0$ then there are 2 unique roots. The graph crosses the x-axis in 2 places.

If $b^2 - 4ac = 0$ then there is a repeated root. The graph touches the x-axis in one spot.

If $b^2 - 4ac < 0$ there are no roots. The graph does not touch the x-axis.

Completing the square

Writing a quadratic equation in the form $(x + p)^2 + r = 0$ is known as completing the square.

e.g. Solve $x^2 + 6x - 8 = 0$

(Half the b value, so $6 \div 2 = 3$)

$(x + 3)^2 - 9 - 8 = 0$

(subtract this value squared as $(x + 3)^2$ multiplied out is $x^2 + 6x + 9$, not $x^2 + 6x$)

$(x + 3)^2 - 17 = 0$ (this is the completed the square form)

$(x + 3)^2 = 17$

$x + 3 = \pm\sqrt{17}$

$x = -3 \pm \sqrt{17}$

When written in the form $(x + p)^2 + r = 0$, you can determine key features of the graph.

The equation of the **line of symmetry** of the curve is $x = -p$

The co-ordinate of the **turning point** of the curve (minimum/maximum point) is $(-p, r)$

Solving quadratic inequalities

Solving a quadratic inequality is very similar to solving a quadratic equation.

Step 1: Solve the equation to find the critical values.

Step 2: Sketch the curve

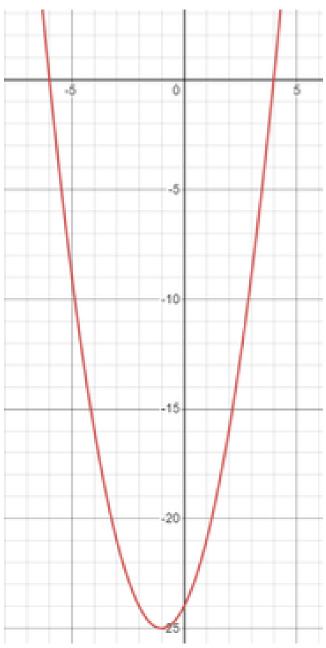
Step 3: Write down the appropriate inequality/inequalities

e.g. Solve $x^2 + 10x - 24 < 0$

Start by solving: $x^2 + 2x - 24 = 0$

$(x + 6)(x - 4) = 0$

$x = -6$ and $x = 4$ (these are the critical values)



The curve is a positive quadratic so is a 'u' shaped parabola.

The roots of the equation are at $x = -6$ and $x = 4$, so this is where it crosses the x axis.

The curve is < 0 (below the x-axis) when it is between $x = -6$ and $x = 4$.

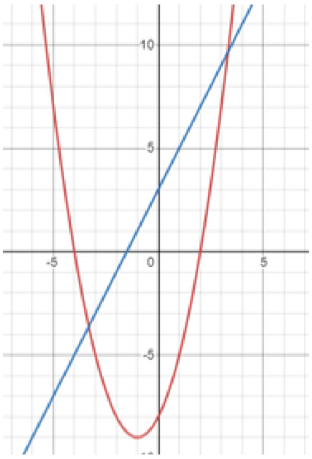
Therefore the solution to $x^2 + 10x - 24 < 0$ is:

$-6 < x < 4$

Note: if the question instead was solve $x^2 + 10x - 24 > 0$ we now need the sections above the x-axis which are not connected and so the solution would have been

$x < -6$ and $x > 4$

Simultaneous equations where one is non-linear



As a non-linear graph will curve, the solution to simultaneous equations with a non-linear equation can have more than 1 answer.

If we are solving a quadratic and a linear graph there are either:

0 solutions – the graphs do not intersect

1 solution – the linear graph is a tangent to the curve and touches only once

2 solutions – the graph crosses twice (as shown on the left)

Solving the equations algebraically allows us to find the exact values of these intersections.

e.g. Solve $y = x^2 + 3x - 8$

$y = 2x + 3$

As both equations are y =, we can equate them

$$x^2 + 3x - 8 = 2x + 3$$

Rearrange so that one side = 0

$$x^2 + x - 11 = 0$$

This does not factorise so using the formula

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$x = 2.8541..$ and $x = -3.854...$

Substitute these back into $y = 2x + 3$

$y = 8.7082...$ and $y = -4.708...$

So the solutions are:

$x = 2.86$ and $y = 8.71$

$x = -3.85$ and $y = -4.71$

e.g. Solve $x^2 + y^2 = 10$ ← This is the equation of a circle

$y = 2x - 5$

This time we need to substitute $y = 2x - 5$ into the top equation.

$$x^2 + (2x - 5)^2 = 10$$

Multiply out the bracket

$$x^2 + 4x^2 - 20x + 25 = 10$$

Simplify and set one side = 0

$$5x^2 - 20x + 15 = 0$$

Factorise and solve

$$5(x^2 - 4x + 3) = 0$$

$$5(x - 3)(x - 1) = 0$$

$x = 3$ and $x = 1$

Substitute back into $y = 2x - 5$

When $x = 3$, $y = 1$

when $x = 1$, $y = -3$

Solutions need to be given in pairs with the correct x and y values matched up.

Knowledge Organiser - Solving Quadratics by Factorising

Topic/Skill	Definition/Tips	Example
1. Quadratic	A quadratic expression is of the form $\boldsymbol{ax^2 + bx + c}$ where a, b and c are numbers, $\boldsymbol{a \neq 0}$	Examples of quadratic expressions: x^2 $8x^2 - 3x + 7$ Examples of non-quadratic expressions: $2x^3 - 5x^2$ $9x - 1$
2. Factorising Quadratics	When a quadratic expression is in the form $x^2 + bx + c$ find the two numbers that **add to give b** and **multiply to give c**.	$x^2 + 7x + 10 = (x + 5)(x + 2)$ (because 5 and 2 add to give 7 and multiply to give 10) $x^2 + 2x - 8 = (x + 4)(x - 2)$ (because +4 and -2 add to give +2 and multiply to give -8)
3. Difference of Two Squares	An expression of the form $\boldsymbol{a^2 - b^2}$ can be factorised to give $(\boldsymbol{a + b})(\boldsymbol{a - b})$	$x^2 - 25 = (x + 5)(x - 5)$ $16x^2 - 81 = (4x + 9)(4x - 9)$
4. Solving Quadratics $(ax^2 = b)$	Isolate the x^2 term and square root both sides. Remember there will be a **positive and a negative solution**.	$2x^2 = 98$ $x^2 = 49$ $x = \pm 7$
5. Solving Quadratics $(ax^2 + bx = 0)$	**Factorise** and then **solve = 0**.	$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0 \text{ or } x = 3$
6. Solving Quadratics by Factorising $(a = 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $x^2 + 3x - 10 = 0$ Factorise: $(x + 5)(x - 2) = 0$ $x = -5 \text{ or } x = 2$
7. Factorising Quadratics when $a \neq 1$	When a quadratic is in the form $ax^2 + bx + c$ 1. Multiply a by c = ac 2. Find two numbers that add to give b and multiply to give ac. 3. Re-write the quadratic, replacing bx with the two numbers you found. 4. Factorise in pairs – you should get the same bracket twice 5. Write your two brackets – one will be the repeated bracket, the other will be made of the factors outside each of the two brackets.	Factorise $6x^2 + 5x - 4$ 1. $6 \times -4 = -24$ 2. Two numbers that add to give +5 and multiply to give -24 are +8 and -3 3. $6x^2 + 8x - 3x - 4$ 4. Factorise in pairs: $2x(3x + 4) - 1(3x + 4)$ 5. Answer = $(3x + 4)(2x - 1)$
8. Solving Quadratics by Factorising $(a \neq 1)$	**Factorise** the quadratic in the usual way. **Solve = 0** Make sure the equation = 0 before factorising.	Solve $2x^2 + 7x - 4 = 0$ Factorise: $(2x - 1)(x + 4) = 0$ $x = \frac{1}{2} \text{ or } x = -4$

SOLVING QUADRATIC SIMULTANEOUS EQUATIONS

One of the equations contains a quadratic term

Solve each pair of equations simultaneously:

1) $y = x + 3$
$y = x^2 + 5x - 2$

2) $y = x + 3$
$y = x^2 - 3x - 2$

3) $y = x + 18$
$y = x^2 - 2x - 10$

4) $y = x^2 - 3x - 2$
$y = 2x - 8$

5) $y = x^2 - 3$
$y = x - 3$

6) $y = x + 3$
$y = x^2 - 3$

7) $y = 2x - 1$
$y = x^2 - 2x + 2$

8) $y - 2x = 4$
$y = x^2 + x - 2$

9) $y = 3x + 9$
$y = 2x^2 + 9x + 1$

With more than one quadratic term

Solve each pair of equations simultaneously:

1) $x^2 + y^2 = 9$
   $y = x + 3$

2) $y = x - 3$
   $x^2 + y^2 = 9$

3) $x^2 + y^2 = 25$
   $y = x + 5$

4) $y = x - 4$
   $x^2 + y^2 = 16$

5) $y - x + 7 = 0$
   $x^2 + y^2 - 49 = 0$

6) $x - 4y + 1 = 0$
   $x^2 - 4xy + y^2 = 13$

7) $y = \ - x^2 + 5x + 2$
   $y = 3x^2 - x - 2$

8) $y + x^2 = 5x$
   $y + x \ + 2 = 3x^2 - 2$

9) $2x + y = 7$
   $x^2 - y^2 = 8$

PROBLEM SOLVING

1) Line A and Line B intersect at coordinates C and D.
Find C and D if the equations of the lines are as follows:

$Line\ A: x + y + 1 = 4$

$Line\ B: x^2 + 3y - 27 = 0$

2) Solve the simultaneous equations:

$y = 5x - 1$

$y = (x + 1)^2$

3) The line 'L' and the curve 'C' intersect at the points A and B

$L: y - x = 4$

$C: y - x^2 - 4 = 3x$

Find the distance between the points A and B