

Multilateral Comovement in a New Keynesian World: A Little Trade Goes a Long Way

Online Appendix

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October 26, 2026

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Table A.1: Data Sources and Sample Periods (GDP Per Capita, Quarterly)

	GDP			Population		
	Source	Code	Sample	Source	Code	Sample
AUS	IMF	AUS.B1GQ.Q	1980Q1:2024Q4	OECD	AUS.POP	1981Q3:2024Q2
AUT	IMF	AUT.B1GQ.Q	1995Q1:2024Q4	OECD	AUT.POP	1995Q1:2024Q4
BEL	IMF	BEL.B1GQ.Q	1995Q1:2024Q4	OECD	BEL.POP	1995Q1:2024Q4
BRA	IMF	BRA.B1GQ.Q	1996Q1:2024Q4	Haver	a223pop@EMERGE	1980Q1:2024Q4
CAN	IMF	CAN.B1GQ.Q	1980Q1:2024Q4	OECD	CAN.POP	1981Q1:2024Q4
CHL	IMF	CHL.B1GQ.Q	1996Q1:2024Q4	OECD	CHL.POP	1996Q1:2024Q4
CHN	Haver	h924ngpc@EMERGE	1992Q1:2024Q4	Haver	a924pop@EMERGE	1980Q1:2024Q4
COL	IMF	COL.B1GQ.Q	2005Q1:2024Q4	OECD	COL.POP	2001Q1:2024Q3
DEU	IMF	DEU.B1GQ.Q	1991Q1:2024Q4	OECD	DEU.POP	1991Q1:2024Q4
ESP	IMF	ESP.B1GQ.Q	1995Q1:2024Q4	OECD	ESP.POP	1995Q1:2024Q4
FIN	IMF	FIN.B1GQ.Q	1990Q1:2024Q4	OECD	FIN.POP	1990Q1:2024Q4
FRA	IMF	FRA.B1GQ.Q	1980Q1:2024Q4	OECD	FRA.POP	1980Q1:2024Q4
GBR	IMF	GBR.B1GQ.Q	1980Q1:2024Q4	OECD	GBR.POP	1995Q1:2024Q3
HUN	IMF	HUN.B1GQ.Q	1995Q1:2024Q4	OECD	HUN.POP	1995Q1:2024Q4
IND	IMF	IND.B1GQ.Q	1996Q2:2024Q4	Haver	a534pop@EMERGE	1980Q1:2024Q4
IRL	IMF	IRL.B1GQ.Q	1995Q1:2024Q4	OECD	IRL.POP	1995Q1:2024Q4
ISR	IMF	ISR.B1GQ.Q	1995Q1:2024Q4	OECD	ISR.POP	1995Q1:2024Q4
ITA	IMF	ITA.B1GQ.Q	1996Q1:2024Q4	OECD	ITA.POP	1995Q1:2024Q4
JPN	IMF	JPN.B1GQ.Q	1994Q1:2024Q4	OECD	JPN.POP	1980Q1:2024Q4
KOR	IMF	KOR.B1GQ.Q	1980Q1:2024Q4	Haver	a542pop@EMERGE	1980Q1:2024Q4
LUX	IMF	LUX.B1GQ.Q	1995Q1:2024Q4	OECD	LUX.POP	1995Q1:2024Q3
MEX	IMF	MEX.B1GQ.Q	1993Q1:2024Q4	OECD	MEX.POP	1980Q1:2024Q4
NLD	IMF	NLD.B1GQ.Q	1996Q1:2024Q4	OECD	NLD.POP	1995Q1:2024Q4
NOR	IMF	NOR.B1GQ.Q	1980Q1:2024Q4	OECD	NOR.POP	1995Q1:2024Q4
PHL	IMF	PHL.B1GQ.Q	2000Q1:2024Q3	Haver	a566pop@EMERGE	1980Q1:2024Q1
POL	IMF	POL.B1GQ.Q	1995Q1:2024Q4	OECD	POL.POP	1988Q1:2024Q4
PRT	IMF	PRT.B1GQ.Q	1995Q1:2024Q4	OECD	PRT.POP	1995Q1:2024Q4
RUS	Haver	s922ngpc@EMERGE	2011Q1:2024Q4	Haver	a922pop@EMERGE	1991Q1:2024Q4
SGP	IMF	SGP.B1GQ.Q	1980Q1:2024Q4	Haver	a576pop@EMERGE	1980Q1:2024Q4
SWE	IMF	SWE.B1GQ.Q	1993Q1:2024Q4	OECD	SWE.POP	1993Q1:2024Q4
THA	IMF	THA.B1GQ.Q	2003Q1:2024Q4	Haver	a578pop@EMERGE	1980Q1:2024Q1
USA	IMF	USA.B1GQ.Q	1980Q1:2024Q4	OECD	USA.POP	1980Q1:2024Q4
ZAF	IMF	ZAF.B1GQ.Q	1993Q1:2024Q4	OECD	ZAF.POP	2008Q1:2024Q3

Notes: For conciseness, we omit the IMF.STA.QNEA. prefix and .SA.XDC.Q suffix in the codes for all IMF sources and omit the .PS._T._T suffix in all OECD sources.

A Empirical Motivation

A.1 Data Sources

Tables A.1 and A.2 document the data sources for Figures 1 and 2 in the main text. All series are available from IMF, OECD, or Haver. Since the panel is unbalanced, we also report the sample periods. Tables A.3, A.4, and A.5 document the data sources for the VARs (Figure 3 in the main text), as well as the sample periods. For transparency, we include the full sample of countries. However, as discussed in the main text, we only estimate the VAR for countries that have at least 25 years of data (or 20 years, in a robustness check).

Table A.2: Data Sources and Sample Periods (Prices, Quarterly)

	Prices			Core Prices		
	Source	Code	Sample	Source	Code	Sample
AUS	IMF	AUS.CPI	1980Q1:2024Q4	Haver	h193pcx@G10	1989Q3:2024Q4
AUT	IMF	AUT.CPI	1980Q1:2024Q4	Haver	h122phxc@G10	1990Q1:2024Q4
BEL	IMF	BEL.CPI	1980Q1:2024Q4	Haver	h124phxc@G10	1991Q1:2024Q4
BRA	IMF	BRA.CPI	1980Q1:2024Q4	Haver	h223pcxg@EMERGE	1995Q1:2024Q4
CAN	IMF	CAN.CPI	1980Q1:2024Q4	Haver	s156pcxg@G10	1992Q1:2024Q4
CHL	IMF	CHL.CPI	1980Q1:2024Q4	Haver	h228pcxg@EMERGE	2009Q1:2024Q4
CHN	IMF	CHN.CPI	1993Q1:2024Q4	Haver	h924pcxg@EMERGE	2005Q1:2024Q4
COL	IMF	COL.CPI	1980Q1:2024Q4	Haver	h233pcx@EMERGE	1999Q1:2024Q4
DEU	IMF	DEU.CPI	1980Q1:2024Q4	Haver	h134phxc@G10	1995Q1:2024Q4
ESP	IMF	ESP.CPI	1980Q1:2024Q4	Haver	h184phxc@G10	1992Q1:2024Q4
FIN	IMF	FIN.CPI	1980Q1:2024Q4	Haver	h172phxc@G10	1990Q1:2024Q4
FRA	IMF	FRA.CPI	1980Q1:2024Q4	Haver	h132phxc@G10	1990Q1:2024Q4
GBR	IMF	GBR.CPI	1980Q1:2024Q4	Haver	h112phxb@G10	1988Q1:2024Q4
HUN	IMF	HUN.CPI	1980Q1:2024Q4	Haver	s944pcx@EMERGE	1995Q1:2024Q4
IND	IMF	IND.CPI	1980Q1:2024Q4	Haver	h534pcxg@EMERGE	2011Q1:2024Q4
IRL	IMF	IRL.CPI	1980Q1:2024Q4	Haver	h178phxc@G10	1995Q1:2024Q4
ISR	IMF	ISR.CPI	1980Q1:2024Q4	Haver	h436pcx@EMERGE	1983Q1:2024Q4
ITA	IMF	ITA.CPI	1980Q1:2024Q4	Haver	h136phxc@G10	1990Q1:2024Q4
JPN	IMF	JPN.CPI	1980Q1:2024Q4	Haver	s158pcfx@G10	2010Q1:2024Q4
KOR	IMF	KOR.CPI	1980Q1:2024Q4	Haver	h542pcxg@EMERGE	1990Q1:2024Q4
LUX	IMF	LUX.CPI	1980Q1:2024Q4	Haver	h137phxc@G10	1995Q1:2024Q4
MEX	IMF	MEX.CPI	1980Q1:2024Q4	Haver	h273pcxg@EMERGE	1982Q1:2024Q4
NLD	IMF	NLD.CPI	1980Q1:2024Q4	Haver	h138phxc@G10	1990Q1:2024Q4
NOR	IMF	NOR.CPI	1980Q1:2024Q4	Haver	h142pcx@G10	1996Q1:2024Q4
PHL	IMF	PHL.CPI	2010Q1:2024Q4	Haver	h566pcx@EMERGE	2011Q3:2024Q4
POL	IMF	POL.CPI	1989Q1:2024Q4	Haver	h964phxc@EMERGE	1996Q1:2024Q4
PRT	IMF	PRT.CPI	1980Q1:2024Q4	Haver	h182phxc@G10	1990Q1:2024Q4
RUS	IMF	RUS.CPI	1992Q1:2022Q1	Haver	h922pcxg@EMERGE	2003Q1:2024Q4
SGP	IMF	SGP.CPI	2005Q1:2024Q4	Haver	h576pcx@EMERGE	1990Q1:2024Q4
SWE	IMF	SWE.CPI	1980Q1:2024Q4	Haver	h144pcx@G10	1996Q1:2024Q4
THA	IMF	THA.CPI	2010Q1:2024Q4	Haver	h578pcx@EMERGE	1985Q1:2024Q4
USA	IMF	USA.CPI	1980Q1:2024Q4	Haver	s111ncxj@G10	1980Q1:2024Q4
ZAF	IMF	ZAF.CPI	1980Q1:2024Q4	Haver	h199puxg@EMERGE	2002Q1:2024Q4

Notes: For conciseness, we omit the IMF.STA.CPI. prefix and ._.IX.Q suffix in the codes for all IMF sources.

Table A.3: Data Sources and Sample Periods (U.S. Variables, Monthly)

Variable	Source	Code	Sample
Industrial Production	Haver	ip@USECON	1960Jan:2024Dec
CPI	Haver	pcu@USECON	1960Jan:2024Dec
1-Year Treasury Bond Yield	Haver	FCM1@USECON	1960Jan:2024Dec
Excess Bond premium	Haver	fgzebp@USECON	1973Jan:2024Dec
Imports (Goods)	IMF	USA.MG_CIF_USD.M	1963Jan:2025Mar
Exports (Goods)	IMF	USA.XG_FOB_USD.M	1963Jan:2025Mar
Commodity Price Index	Haver	pzall@USECON	1960Jan:2024Dec
Shadow Rate	Haver	ffedshdw@USECON	1990Jan:2023Jun

Notes: For conciseness, we omit the IMF.STA.IMTS.G001. prefix in the codes for all IMF sources.

Table A.4: Data Sources and Sample Periods (Industrial Production and Prices, Monthly)

	<u>Industrial Production</u>			<u>Prices</u>		
	Source	Code	Sample	Source	Code	Sample
AUS	IMF	PIAUSIND	2000.Jan:2021.Jan	IMF	CPI.AUS.CPI	1960.Mar:2024.Dec
AUT	IMF	PIAUTIND	1996.Jan:2024.Dec	IMF	CPI.AUT.CPI	1960.Jan:2024.Dec
BEL	IMF	PIBELIND	1960.Jan:2024.Dec	IMF	CPI.BEL.CPI	1960.Jan:2024.Dec
BRA	IMF	PIBRAIND	1975.Jan:2023.Nov	IMF	CPI.BRA.CPI	1979.Dec:2024.Dec
CAN	IMF	PICANIND	1960.Jan:2023.Oct	IMF	CPI.CAN.CPI	1960.Jan:2024.Dec
CHL	IMF	PICHLIND	1997.Jan:2019.Sep	IMF	CPI.CHL.CPI	1970.Jan:2024.Dec
CHN	Haver	h924d@EMERGE	1997.Jan:2024.Dec	IMF	CPI.CHN.CPI	1993.Jan:2024.Dec
COL	IMF	PICOLIND	2000.Jan:2019.Sep	IMF	CPI.COL.CPI	1970.Jan:2024.Dec
DEU	IMF	PIDEUIND	1960.Jan:2024.Dec	IMF	CPI.DEU.CPI	1960.Jan:2024.Dec
ESP	IMF	PIESPIND	1961.Jan:2024.Dec	IMF	CPI.ESP.CPI	1960.Jan:2024.Dec
FIN	IMF	PIFININD	1960.Jan:2024.Dec	IMF	CPI.FIN.CPI	1960.Jan:2024.Apr
FRA	IMF	PIFRAIND	1960.Jan:2024.Dec	IMF	CPI.FRA.CPI	1960.Jan:2024.Dec
GBR	IMF	PIGBRIND	1960.Jan:2023.Nov	IMF	CPI.GBR.CPI	1960.Jan:2024.Dec
HUN	IMF	PIHUNIND	1985.Jan:2024.Dec	IMF	CPI.HUN.CPI	1980.Jan:2024.Dec
IND	IMF	PIINDIND	2000.Jan:2017.Nov	IMF	CPI.IND.CPI	1960.Jan:2024.Dec
IRL	IMF	PIIRLIND	1975.Jul:2023.Nov	IMF	CPI.IRL.CPI	1960.Feb:2024.Dec
ISR	Haver	s436dm@EMERGE	1990.Jan:2024.Dec	IMF	CPI.ISR.CPI	1970.Jan:2024.Dec
ITA	IMF	PIITAININD	1960.Jan:2024.Dec	IMF	CPI.ITA.CPI	1960.Jan:2024.Dec
JPN	IMF	PIJPNIND	1960.Jan:2023.Nov	IMF	CPI.JPN.CPI	1960.Jan:2024.Dec
KOR	IMF	PIKORIND	1960.Jan:2023.Nov	IMF	CPI.KOR.CPI	1960.Jan:2024.Dec
LUX	IMF	PILUXIND	2000.Jan:2024.Dec	IMF	CPI.LUX.CPI	1960.Jan:2024.Dec
MEX	IMF	PIMEXIND	1980.Jan:2021.Oct	IMF	CPI.MEX.CPI	1969.Jan:2024.Dec
NLD	IMF	PINLDIND	1960.Jan:2024.Dec	IMF	CPI.NLD.CPI	1960.Apr:2024.Dec
NOR	IMF	PINORIND	1960.Jan:2024.Dec	IMF	CPI.NOR.CPI	1960.Jan:2024.Dec
PHL	Haver	h566dm@EMERGE	1998.Jan:2023.Aug	IMF	CPI.PHL.CPI	2010.Jan:2024.Dec
POL	IMF	PIPOLIND	1997.Jan:2024.Dec	IMF	CPI.POL.CPI	1989.Jan:2024.Nov
PRT	IMF	PIPRTIND	1995.Jan:2024.Dec	IMF	CPI.PRT.CPI	1960.Jan:2024.Dec
RUS	IMF	PIRUSIND	2000.Jan:2022.Jan	IMF	CPI.RUS.CPI	1992.Jan:2024.Dec
SGP	Haver	s576dm@EMERGE	1983.Jan:2024.Dec	IMF	CPI.SGP.CPI	2005.Jan:2024.Dec
SWE	IMF	PISWEIND	1997.Jan:2024.Dec	IMF	CPI.SWE.CPI	1960.Jan:2024.Dec
THA	Haver	h578dm@EMERGE	2000.Jan:2024.Dec	IMF	CPI.THA.CPI	2010.Jan:2024.Dec
ZAF	Haver	s199dm@EMERGE	1963.Jan:2024.Dec	IMF	CPI.ZAF.CPI	1960.Jan:2024.Dec

Notes: For conciseness, we omit the IMF.STA. prefix in the codes for all IMF sources. We also omit the .SA_IX.M suffix from industrial production series from the IMF and the ._.IX.M suffix from all prices series. Details for all 33 countries are reported, but only those with sufficiently long sample periods are used.

Table A.5: Data Sources and Sample Periods (Interest Rates, Monthly)

	<u>Interest Rates</u>		
	Source	Code	Sample
AUS	OECD via FRED	IR3TIB01AUM156N	1968Jan:2024Dec
AUT	OECD via FRED	IR3TIB01ATM156N	1989Jun:2024Dec
BEL	OECD via FRED	IR3TIB01BEM156N	1960Jan:2024Dec
BRA	OECD via FRED	IR3TIB01BRM156N	1995Jan:2024Dec
CAN	OECD via FRED	IR3TIB01CAM156N	1960Jan:2024Dec
CHL	OECD via FRED	IR3TIB01CLM156N	1997Jul:2024Dec
CHN	OECD via FRED	IR3TIB01CNM156N	1997Jun:2024Dec
COL	OECD via FRED	COLIR3TIB01STM	1986Jan:2024Dec
DEU	OECD via FRED	IR3TIB01DEM156N	1960Jan:2024Dec
ESP	OECD via FRED	IR3TIB01ESM156N	1977Jan:2024Dec
FIN	OECD via FRED	IR3TIB01FIM156N	1987Jan:2024Dec
FRA	OECD via FRED	IR3TIB01FRM156N	1970Jan:2024Dec
GBR	OECD via FRED	IR3TIB01GBM156N	1986Jan:2024Dec
HUN	OECD via FRED	IR3TIB01HUM156N	1991Jan:2024Dec
IND	Haver	n534rg3m@EMERGE	1993Jan:2024Dec
IRL	OECD via FRED	IR3TIB01IEM156N	1984Jan:2024Dec
ISR	OECD via FRED	IR3TIB01ILM156N	1992Jan:2024Dec
ITA	OECD via FRED	IR3TIB01ITM156N	1978Oct:2024Dec
JPN	Haver	n158g3m@G10	1988Nov:2024Dec
KOR	OECD via FRED	IR3TIB01KRM156N	1991Jan:2024Dec
LUX	OECD via FRED	IR3TIB01LUM156N	1999Jan:2024Dec
MEX	OECD via FRED	IR3TIB01MXM156N	1997Jan:2024Dec
NLD	OECD via FRED	IR3TIB01NLM156N	1982Jan:2024Dec
NOR	OECD via FRED	IR3TIB01NOM156N	1979Jan:2024Dec
PHL	Haver	n566rg3m@EMERGE	1975Jan:2024Dec
POL	OECD via FRED	IR3TIB01PLM156N	1991Jun:2024Dec
PRT	OECD via FRED	IR3TIB01PTM156N	1985Aug:2024Dec
RUS	OECD via FRED	IR3TIB01RUM156N	1997Jan:2022Feb
SGP	OECD via FRED	IR3TIB01SGM156N	1987Jun:2024Dec
SWE	OECD via FRED	IR3TIB01SEM156N	1982Jan:2024Dec
THA	Haver	n578ri3@EMERGE	2005Jan:2024Dec
ZAF	OECD via FRED	IR3TIB01ZAM156N	1980Dec:2024Dec

Notes: Details for all 33 countries are reported, but only those with sufficiently long sample periods are used.

Table A.6: Correlations Across Specifications

Panel A. Quarter-Over-Quarter					
Specification	GDP Growth	Inflation	Core Infl.	GDP Growth-Inflation	
				Within-Cty	Across-Cty
A1. Full Sample	0.58 (0.05)	0.32 (0.03)	0.30 (0.04)	0.04 (0.02)	0.07 (0.01)
A2. Excl. 2020Q1–2020Q3	0.23 (0.02)	0.31 (0.03)	0.30 (0.04)	0.02 (0.02)	0.05 (0.01)
A3. Excl. 2007Q4–2009Q2 and 2020Q1–2020Q3	0.14 (0.02)	0.31 (0.03)	0.30 (0.05)	−0.02 (0.02)	0.01 (0.01)
A4. Agg. Original EMU and Excl. 2020Q1–2020Q3	0.22 (0.02)	0.28 (0.03)	0.23 (0.05)	0.01 (0.02)	0.04 (0.01)
A5. 2004Q3–2019Q4	0.31 (0.03)	0.29 (0.04)	0.08 (0.02)	0.02 (0.03)	0.08 (0.01)

Panel B. Year-Over-Year					
Specification	GDP Growth	Inflation	Core Infl.	GDP Growth-Inflation	
				Within-Cty	Across-Cty
B1. Full Sample	0.58 (0.04)	0.46 (0.04)	0.38 (0.05)	0.05 (0.03)	0.12 (0.02)
B2. Excl. 2020Q1–2020Q3	0.49 (0.03)	0.45 (0.04)	0.38 (0.05)	−0.01 (0.03)	0.07 (0.02)
B3. Excl. 2007Q4–2009Q2 and 2020Q1–2020Q3	0.45 (0.04)	0.45 (0.04)	0.38 (0.05)	−0.02 (0.03)	0.06 (0.02)
B4. Agg. Original EMU and Excl. 2020Q1–2020Q3	0.46 (0.04)	0.41 (0.05)	0.29 (0.06)	−0.03 (0.04)	0.06 (0.02)
B5. 2004Q3–2019Q4	0.47 (0.04)	0.36 (0.05)	0.11 (0.04)	−0.04 (0.05)	0.08 (0.03)

Notes: Standard errors in parentheses, bootstrapped by country.

A.2 Robustness Checks

Correlations. We re-compute the cross-country correlations under different assumptions: i) excluding the Great Recession (2007Q4 through 2009Q2), ii) including the COVID-19 observations (2020Q1 through 2020Q3), iii) aggregating the E.U. into a single region (using the 11 initial members), iv) restricting the estimation to the subsample covering 2004Q3 through 2019Q4. We then repeat each exercise using year-over-year instead of quarter-over-quarter growth rates. Standard errors are computed using bootstrapping for each country.

Table A.6 reports the full set of results, with line A2 corresponding to the specification in the main text. Panel A shows that the cross-country comovement holds for the range of data configurations with quarter-over-quarter data. In each case, cross-country correlations in GDP growth and inflation are positive and statistically significant. Moreover, cross-country correlations between GDP growth and inflation are also almost all positive and statistically significant, while analogous within-country correlations are not.

Panel B shows that the results become even stronger once we use year-over-year data. The GDP growth correlations become more stable across specifications, headline inflation correlations increase by about 40 percent on average, and core inflation correlations increase by over 25 percent. The across-country GDP growth-inflation correlations also increase substantially. These results suggest that the correlations we report in Section 2 and 6 of the main text are somewhat conservative, as they include some contamination by the high-frequency fluctuations or measurement errors that year-over-year data helps to correct for.

Impulse Responses. With respect to the VAR described in the main text, robustness checks include re-estimating the impulse responses when i) replacing one-year Treasury yields with the shadow rate from [Wu and Xia \(2016\)](#), ii) using an alternative high-frequency monetary policy shock instrument from [Miranda-Agrippino and Ricco \(2021\)](#), iii) including commodity prices or foreign interest rates, iv) lowering the minimum sample length to 20 years, v) extending the sample back to January 1973 where possible. The corresponding impulse responses and interquartile ranges are shown in Panel A of Figure [A.1](#).

In addition to reporting the interquartile range of posterior medians (across countries), we report results in each case for the ‘median’ country, as in [Degasperi et al. \(2026\)](#). To do so, we take one draw from the posterior of the VAR for each country, then find the median of these individual draws across countries. Repeating this for all draws, we obtain a posterior distribution for the ‘median’ country. The responses and 68 percent error bands are shown in Panel B of Figure [A.1](#).

Irrespective of the particular specification we adopt, a contractionary U.S. monetary policy shock decreases production and prices abroad and increases exports relative to imports. While the exact magnitudes can differ, the findings are consistent with those presented in the main text.

B A New Keynesian Global Economy

This section supplements Section 3 of the main text, providing details of the model solution and derivations underlying the log-linearized system and steady-state conditions. We reproduce Figure 4 in the main text to help frame the environment. For completeness, we reproduce the equations from the main text as well.

B.1 Tradable Intermediate and Final Goods

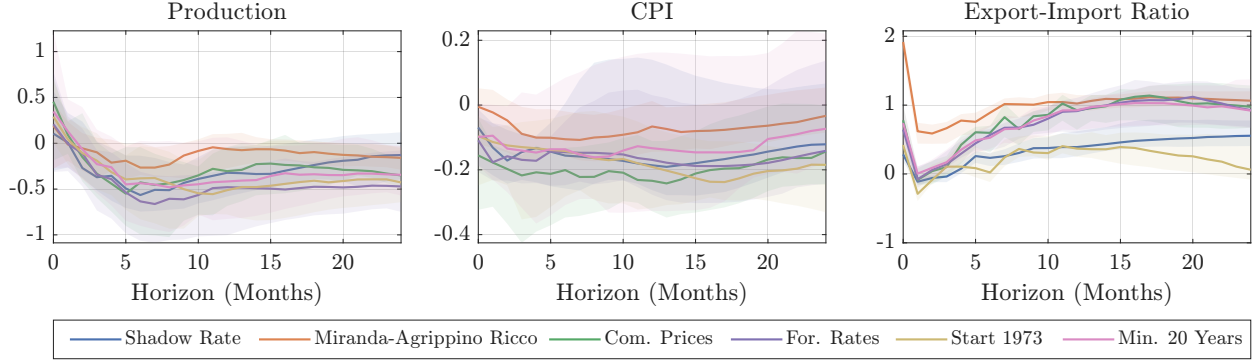
There are N countries indexed by $n \in \{1, \dots, N\}$. Each country produces intermediate and final tradable goods, denoted m and c , respectively.

Goods in sector $k \in \{m, c\}$ are produced using the technology,

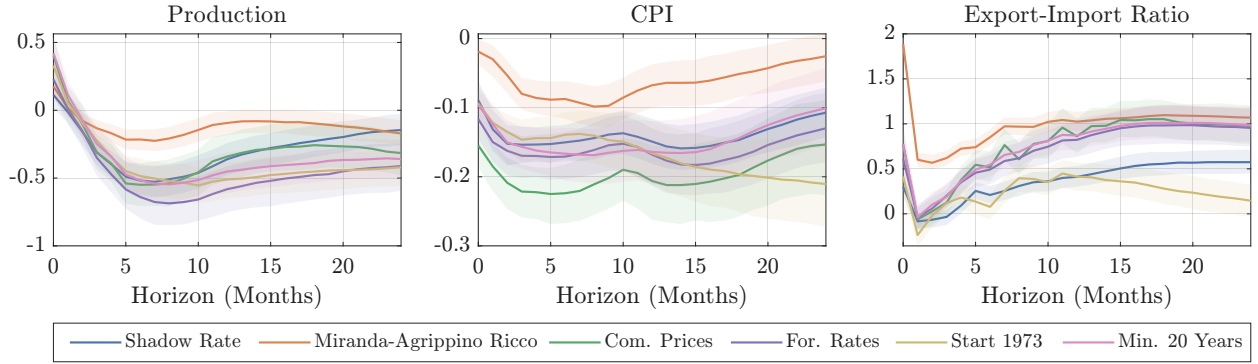
$$q_{n,t}^k(z) = z_n \left(\ell_{n,t}^k(z) \right)^\gamma \left(M_{n,t}^k(z) \right)^{1-\gamma}, \quad \gamma \in (0, 1) \text{ and } k \in \{c, m\}, \quad (\text{A1})$$

Figure A.1: Responses to U.S. Monetary Policy Shock: Alternative Specifications

Panel A: Alternative Specifications



Panel B: Median Country



Notes: Panel A shows distribution of posterior medians for impulse response to 25 basis point U.S. monetary policy shock under different specifications; solid line denotes median across countries, shaded regions denote interquartile range. Panel B shows impulse response of median country to 25 basis point U.S. monetary policy shock under different specifications; solid line denotes posterior median, shaded regions denote 68 percent posterior error bands. In both panels, each color corresponds to one specification.

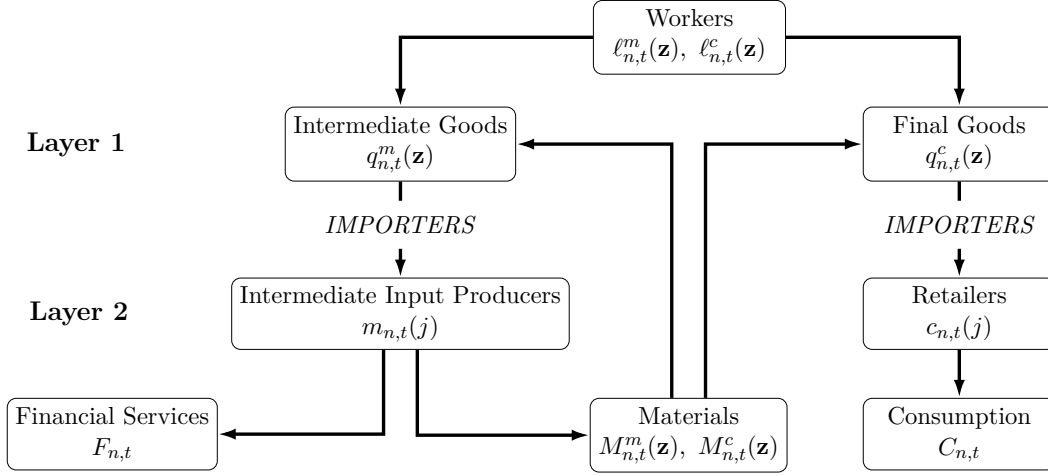
where $\ell_{n,t}^k(\mathbf{z})$ and $M_{n,t}^k(\mathbf{z})$ denote, respectively, labor, and material inputs, and the elements of $\mathbf{z} = \{z_1, \dots, z_N\}$ are jointly drawn from a multivariate Fréchet distribution with shape parameter φ^k , $\Phi^k(\mathbf{z}) = \exp \left\{ - \sum_n (z_n)^{-\varphi^k} \right\}$.

As discussed in the main text, the price of traded variety $\mathbf{z}$ charged by country n , denoted $p_{n,t}^k(\mathbf{z})$, is equal to its marginal cost,

$$p_{n,t}^k(\mathbf{z}) \propto \frac{W_{n,t}^\gamma (P_{n,t}^m)^{1-\gamma}}{z_n}, \quad k \in \{c, m\},$$

where $W_{n,t}$ and $P_{n,t}^m$ are, respectively, the nominal wage and the price of materials in n . In addition, cost minimization implies that $W_{n,t} \ell_{n,t}^k(\mathbf{z}) = \frac{\gamma}{1-\gamma} P_{n,t}^m M_{n,t}^k(\mathbf{z})$. Additionally, in units of the local

Figure B.1: Economic Environment in Each Country



currency, the price paid for tradable goods of variety $\mathbf{z}$ in sector k by country n is

$$P_{n,t}^k(\mathbf{z}) = \min_{n'} \left\{ \kappa_{nn'}^k \mathcal{E}_{nn',t} p_{n',t}^k(\mathbf{z}) \right\} = \min_{n'} \left\{ \frac{\kappa_{nn'}^k \mathcal{E}_{nn',t} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma}}{z_{n'}} \right\}, \quad k \in \{c, m\}, \quad (\text{A2})$$

where $\mathcal{E}_{nn',t}$ converts prices quoted in the currency of country n' into that of country n and $\kappa_{nn'}^k$ denotes the iceberg shipping costs.

Denoting total expenditures by country n on tradable goods in sector k from country n' by $X_{nn',t}^k$, we have

$$X_{nn',t}^k = \int \kappa_{nn'}^k \mathcal{E}_{nn',t} p_{n',t}^k(\mathbf{z}) Q_{nn',t}^k(\mathbf{z}) d\Phi^k(\mathbf{z}), \quad k \in \{c, m\}.$$

$X_{n,t}^k \equiv \sum_{n'} X_{nn',t}^k$ then represents country n 's total expenditures on tradable sector k goods from abroad. Under the maintained assumptions, the share of country n 's expenditures on tradable intermediate and final goods from country n' at date t , $\varpi_{nn',t}^k$, is then given by

$$\varpi_{nn',t}^k = \frac{X_{nn',t}^k}{X_{n,t}^k} = \frac{\left(\kappa_{nn'}^k \mathcal{E}_{nn',t} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma} \right)^{-\varphi^k}}{\sum_{n''} \left(\kappa_{nn''}^k \mathcal{E}_{nn'',t} W_{n'',t}^\gamma (P_{n'',t}^m)^{1-\gamma} \right)^{-\varphi^k}}, \quad k \in \{c, m\}. \quad (\text{A3})$$

Traded varieties are assembled in each destination country by importers according to

$$Q_{n,t}^k = \left[\int \left(\sum_{n'} Q_{nn',t}^k(\mathbf{z}) \right)^{\frac{\chi-1}{\chi}} d\Phi^k(\mathbf{z}) \right]^{\frac{\chi}{\chi-1}}, \quad \chi > 1, \quad k \in \{c, m\},$$

where $Q_{nn',t}^k(\mathbf{z})$ denotes the quantity of tradable intermediate or final goods of variety $\mathbf{z}$ imported by n from n' , so that $q_{n',t}^k(\mathbf{z}) = \sum_n \kappa_{nn'}^k Q_{nn',t}^k(\mathbf{z})$. Firms that assemble tradable varieties for

retailers in country n solve

$$\max_{Q_{nn',t}^k(z) \geq 0} P_{n,t}^{Q^k} Q_{n,t}^k - \sum_{n'} \int \mathcal{E}_{nn',t} p_{n',t}^k(z) \kappa_{nn'}^k Q_{nn',t}^k(z) d\Phi^k(z), \quad k \in \{c, m\}.$$

Following the derivations in [Eaton and Kortum \(2002\)](#), the price of the traded final good bundle, $Q_{n,t}^c$, is given by

$$P_{n,t}^{Q^k} = \Gamma \left(\xi^k \right)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^k \mathcal{E}_{nn',t} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma} \right)^{-\varphi^k} \right)^{-\frac{1}{\varphi^k}}, \quad k \in \{c, m\}, \quad (\text{A4})$$

where $\Gamma(\cdot)$ is the Gamma function evaluated at $\xi^k = 1 + \frac{1-\chi}{\varphi^k}$.

B.2 Domestic Intermediate Good Producers and Retailers

Household preferences over final goods sold by different retailers are defined over a consumption bundle,

$$C_{n,t} = \left[\int_0^1 c_{n,t}(j)^{\frac{\varepsilon_{n,t}-1}{\varepsilon_{n,t}}} dj \right]^{\frac{\varepsilon_{n,t}}{\varepsilon_{n,t}-1}}, \quad \varepsilon_{n,t} > 1,$$

where $C_{n,t}$ also denotes aggregate consumption in country n at date t and $\varepsilon_{n,t}$ is a time-varying elasticity of substitution. Similarly, intermediate inputs supplied by producer j in country n at time t , denoted by $m_{n,t}(j)$, are not perfectly substitutable between producers so that

$$\mathcal{M}_{n,t} = \left[\int_0^1 m_{n,t}(j)^{\frac{\varrho_{n,t}-1}{\varrho_{n,t}}} dj \right]^{\frac{\varrho_{n,t}}{\varrho_{n,t}-1}}, \quad \varrho_{n,t} > 1,$$

where $\mathcal{M}_{n,t}$ denotes an aggregate of materials and financial services such that $\mathcal{M}_{n,t} = M_{n,t} + F_{n,t}$, where $\varrho_{n,t}$ is a time-varying elasticity of substitution.

Given a price $P_{n,t}(j)$ of final goods sold by retailer j , the household expenditure minimization problem implies a demand function and price deflator, respectively,

$$c_{n,t}(j) = \left(\frac{P_{n,t}(j)}{P_{n,t}} \right)^{-\varepsilon_{n,t}} C_{n,t},$$

$$P_{n,t} = \left[\int_0^1 P_{n,t}(j)^{1-\varepsilon_{n,t}} dj \right]^{\frac{1}{1-\varepsilon_{n,t}}}.$$

Similarly, given a price $P_{n,t}^m(j)$ for sticky-price intermediate good j , we can derive the demand

function and price deflator, respectively,

$$m_{n,t}(j) = \left(\frac{P_{n,t}^m(j)}{P_{n,t}^m} \right)^{-\varrho_{n,t}} \mathcal{M}_{n,t},$$

$$P_{n,t}^m = \left[\int_0^1 P_{n,t}^m(j)^{1-\varrho_{n,t}} dj \right]^{\frac{1}{1-\varrho_{n,t}}},$$

for the material and financial services aggregate in country n .

Sticky-price retailers and intermediate goods producers purchase bundles of traded varieties from importers, $Q_{n,t}^c$ and $Q_{n,t}^m$, respectively, and process them with the technology,

$$c_{n,t}(j) = A_{n,t} Q_{n,t}^c(j) \quad \text{and} \quad m_{n,t}(j) = A_{n,t} Q_{n,t}^m(j), \quad (\text{A5})$$

where $A_{n,t}$ denotes technical progress in country n . Real marginal costs are

$$mc_{n,t}^k = \frac{P_{n,t}^{Q^k}}{P_{n,t} A_{n,t}}, \quad k \in \{c, m\}, \quad (\text{A6})$$

where we have deflated the marginal cost for both intermediate inputs and final goods by the same deflator $P_{n,t}$.

Following [Calvo \(1983\)](#), in each period, sticky-price intermediate input producers and retailers in country n are unable to adjust their price with probability $\theta_n^k, k \in \{c, m\}$, in which case they index their price increases to steady-state inflation in their respective sectors. For example, retailers set their price using the indexation rule, $P_{n,t}(j) = (1 + \pi_n) P_{n,t-1}(j)$.

Retailers who are able to reset their price optimally choose a price, $P_{n,t}^*$, that satisfies

$$P_{n,t}^* = \arg \max E_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{n,t+k}}{\lambda_{n,t}} (\theta^c)^k \left[(1 + \tau_n^c) \left(\frac{P_{n,t}^* (1 + \pi_n)^k}{P_{n,t+k}} \right)^{1-\varepsilon_{n,t+k}} \right. \\ \left. - mc_{n,t+k}^c \left(\frac{P_{n,t}^* (1 + \pi_n)^k}{P_{n,t+k}} \right)^{-\varepsilon_{n,t+k}} \right] C_{n,t+k},$$

where $\lambda_{n,t}$ is households' marginal utility of consumption in country n , $\beta^k \frac{\lambda_{n,t+k}}{\lambda_{n,t}}$ their stochastic discount factor from t to $t+k$, and τ_n^c is a subsidy given to sticky-price final good retailers.¹ Alternatively,

$$P_{n,t}^* = \frac{1}{1 + \tau_n^c} \frac{E_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{n,t+k}}{\lambda_{n,t}} (\theta^c)^k \varepsilon_{n,t+k} \left(\frac{P_{n,t+k}}{(1 + \pi_n)^k} \right)^{\varepsilon_{n,t+k}} mc_{n,t+k}^c C_{n,t+k}}{E_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{n,t+k}}{\lambda_{n,t}} (\theta^c)^k (\varepsilon_{n,t+k} - 1) \left(\frac{P_{n,t+k}}{(1 + \pi_n)^k} \right)^{\varepsilon_{n,t+k}-1} C_{n,t+k}}.$$

¹We omit the subsidy when describing the model in the main text to simplify notation. However, in our analysis, we assume a subsidy that undoes the effect of the markup in order to free our discussion from monopolistic pricing distortions in steady state.

Substitute for $\lambda_{n,t}$ to obtain

$$p_{n,t}^* = \frac{1}{1 + \tau_n^c} \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k \varepsilon_{n,t+k} \left(\frac{P_{n,t+k}}{P_{n,t}(1+\pi_n)^k} \right)^{\varepsilon_{n,t+k}} m c_{n,t+k}^c C_{n,t+k}^{1-\sigma_n} A_{n,t+k}^{\sigma_n-1}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k (\varepsilon_{n,t+k} - 1) \left(\frac{P_{n,t+k}}{P_{n,t}(1+\pi_n)^k} \right)^{\varepsilon_{n,t+k}-1} C_{n,t+k}^{1-\sigma_n} A_{n,t+k}^{\sigma_n-1}},$$

where $p_{n,t}^* = \frac{P_{n,t}^*}{P_{n,t}}$ and $\mu_{n,t} = \frac{\varepsilon_{n,t}}{\varepsilon_{n,t-1}}$ is a time-varying markup. The aggregate consumption deflator in country n , therefore, is given by

$$\begin{aligned} P_{n,t}^{1-\varepsilon_{n,t}} &= \int_0^1 P_{n,t}(j)^{1-\varepsilon_{n,t}} dj \\ &= (1 - \theta^c) (P_{n,t}^*)^{1-\varepsilon_{n,t}} + \theta^c (1 + \pi_n)^{(1-\varepsilon_{n,t})} P_{n,t-1}^{1-\varepsilon_{n,t}}, \end{aligned}$$

or,

$$1 = (1 - \theta^c) \left(\frac{P_{n,t}^*}{P_{n,t}} \right)^{1-\varepsilon_{n,t}} + \theta^c ((1 + \pi_n)(1 + \pi_{n,t})^{-1})^{1-\varepsilon_{n,t}}.$$

Similarly, domestic producers of intermediate goods maximize their profits and, when able to reset their price optimally, choose a price, $P_{n,t}^{m*}$, that satisfies

$$\begin{aligned} P_{n,t}^{m*} = \arg \max E_t \sum_{k=0}^{\infty} \beta^k \frac{\lambda_{n,t+k}}{\lambda_{n,t}} (\theta^m)^k \left[(1 + \tau_n^m) \left(\frac{P_{n,t}^{m*}(1 + \pi_n)^k}{P_{n,t+k}^m} \right)^{1-\varrho_{n,t+k}} \frac{P_{n,t+k}^m}{P_{n,t+k}} \right. \\ \left. - m c_{n,t+k}^m \left(\frac{P_{n,t}^{m*}(1 + \pi_n)^k}{P_{n,t+k}^m} \right)^{-\varrho_{n,t+k}} \right] \mathcal{M}_{n,t+k}, \end{aligned}$$

where τ_n^m is a subsidy given to sticky-price intermediate input producers. In addition, we have that

$$\frac{p_{n,t}^{m*}}{p_{n,t}^m} = \frac{1}{1 + \tau_n^m} \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k \varrho_{n,t+k} \left(\frac{P_{n,t+k}^m}{P_{n,t}^m(1+\pi_n)^k} \right)^{\varrho_{n,t+k}} m c_{n,t+k}^m \mathcal{M}_{n,t+k} C_{n,t+k}^{-\sigma_n} A_{n,t+k}^{\sigma_n-1}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k (\varrho_{n,t+k} - 1) \left(\frac{P_{n,t+k}^m}{P_{n,t}^m(1+\pi_n)^k} \right)^{\varrho_{n,t+k}-1} \mathcal{M}_{n,t+k} C_{n,t+k}^{-\sigma_n} A_{n,t+k}^{\sigma_n-1} p_{n,t+k}^m},$$

where $p_{n,t}^{m*} = P_{n,t}^{m*}/P_{n,t}$, and $p_{n,t}^m = P_{n,t}^m/P_{n,t}$ is a time-varying relative price. The price of materials or financial services in country n at date t , $P_{n,t}^m$, is then given by

$$\begin{aligned} (P_{n,t}^m)^{1-\varrho_{n,t}} &= \int_0^1 P_{n,t}^m(j)^{1-\varrho_{n,t}} dj \\ &= (1 - \theta^m) (P_{n,t}^{m*})^{1-\varrho_{n,t}} + \theta^m (1 + \pi_n)^{(1-\varrho_{n,t})} (P_{n,t-1}^m)^{1-\varrho_{n,t}}. \end{aligned}$$

B.3 Households

The representative household in country n has preferences defined by

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{(C_{n,t}/A_{n,t})^{1-\sigma_n} - 1}{1-\sigma_n} - \omega_n \frac{L_{n,t}^{1+\eta_n}}{1+\eta_n} \right\},$$

and faces the budget constraint,

$$P_{n,t}C_{n,t} + \frac{B_{n,t}}{R_{n,t}} + E_t \left[\frac{V_{t+1}}{\mathcal{E}_{rn,t}} S_{n,t+1} \right] + P_{n,t}^m F_{n,t} = W_{n,t}L_{n,t} + \Pi_{n,t} + B_{n,t-1} + \frac{1}{\mathcal{E}_{rn,t}} S_{n,t} - T_{n,t}, \quad (\text{A7})$$

where $L_{n,t}$ is labor input in country n , $W_{n,t}$ denotes nominal wages, and $\Pi_{n,t}$ represents nominal firm profits,

$$\Pi_{n,t} = \underbrace{\left[(1 + \tau_n^c) P_{n,t} - \Delta_{n,t}^c MC_{n,t}^c \right] C_{n,t}}_{\text{Retailers' Profits}} + \underbrace{\left[(1 + \tau_n^m) P_{n,t}^m - \Delta_{n,t}^m MC_{n,t}^m \right] \mathcal{M}_{n,t}}_{\text{Intermediate Goods Producers' Profits}},$$

where $\Delta_{n,t}^c = \int_0^1 \left(\frac{P_{n,t}(j)}{P_{n,t}} \right)^{-\varepsilon_{n,t}} dj$ and $\Delta_{n,t}^m = \int_0^1 \left(\frac{P_{n,t}^m(j)}{P_{n,t}^m} \right)^{-\varrho_{n,t}} dj$ are measures of relative price distortions in retail and intermediate goods markets respectively. The term $B_{n,t}$ denotes the quantity of a non-state contingent bond traded domestically by households at price $1/R_{n,t}$ at date t that pays 1 unit of the local currency in the following period.

Countries trade state-contingent securities internationally, $S_{n,t+1}$. These securities are purchased at date t by country n at price V_{t+1} , expressed in the currency of a reference country r . It pays off 1 unit of the reference country currency, translated into local currency units through the exchange rate $\mathcal{E}_{rn,t}$ in the budget constraint (A7). Trading securities internationally is subject to portfolio adjustment costs. Therefore, to carry out these trades, households purchase financial services, $F_{n,t}$, at price, $P_{n,t}^m$.

Financial services cover portfolio adjustment costs that are borne separately for each state contingent security:

$$P_{n,t}^m F_{n,t} = E_t \left[P_{n,t}^m A_{n,t} \Theta_n \left(\frac{V_{t+1}}{\mathcal{E}_{rn,t} P_{n,t}} \frac{S_{n,t+1}}{A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right].$$

The function $\Theta_n(\cdot)$ reflects a cost incurred by country n when adjusting its financial position. We assume that $\Theta_n(0) = \Theta'_n(0) = 0$ and $\Theta''_n(\cdot) = \tau > 0$.

Household optimization implies the following equations:

- The optimal choice of $S_{n,t+1}$ implies

$$\left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} V_{t+1} \left[1 + \frac{P_{n,t}^m}{P_{n,t}} \Theta'_n \left(\frac{V_{t+1} S_{n,t+1}}{\mathcal{E}_{rn,t} P_{n,t} A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] = \beta \frac{\mathcal{E}_{rn,t} P_{n,t} A_{n,t}}{\mathcal{E}_{rn,t+1} P_{n,t+1} A_{n,t+1}} \left(\frac{C_{n,t+1}}{A_{n,t+1}} \right)^{-\sigma_n}.$$

- The optimal choice of $B_{n,t}$ implies

$$\left(\frac{C_{n,t}}{A_{n,t}}\right)^{-\sigma_n} = \beta R_{n,t} E_t \left(\frac{C_{n,t+1}}{A_{n,t+1}}\right)^{-\sigma_n} \frac{P_{n,t} A_{n,t}}{P_{n,t+1} A_{n,t+1}}.$$

- The optimal choice of $C_{n,t}$ implies

$$C_{n,t}^{-\sigma_n} A_{n,t}^{\sigma_n-1} = \lambda_{n,t}. \quad (\text{A8})$$

- Optimal labor supply implies

$$\omega L_{n,t}^{\eta_n} \left(\frac{C_{n,t}}{A_{n,t}}\right)^{\sigma_n} = \frac{W_{n,t}}{A_{n,t} P_{n,t}}.$$

Recall that cost minimization by producers of tradable varieties implied that $W_{n,t} \ell_{n,t}^k(\mathbf{z}) = \frac{\gamma}{1-\gamma} P_{n,t}^m M_{n,t}^k(\mathbf{z})$ for $k \in \{c, m\}$. Aggregating across $\mathbf{z}$ and k , we thus have $W_{n,t} L_{n,t} = \frac{\gamma}{1-\gamma} P_{n,t}^m M_{n,t}$. Moreover, assuming a balanced government budget, $T_{n,t} = \tau^c P_{n,t} C_{n,t} + \tau^m P_{n,t}^m M_{n,t}$. Thus, making the appropriate substitutions in equation (A7), the household budget constraint becomes

$$\begin{aligned} & \Delta_{n,t}^c M C_{n,t}^c C_{n,t} + \Delta_{n,t}^m M C_{n,t}^m M_{n,t} + \frac{B_{n,t}}{R_{n,t}} + \\ & E_t \left[\frac{V_{t+1}}{\mathcal{E}_{rn,t}} S_{n,t+1} \right] + E_t \left[\Delta_{n,t}^m M C_{n,t}^m A_{n,t} \Theta_n \left(\frac{V_{t+1}}{\mathcal{E}_{rn,t} P_{n,t}} \frac{S_{n,t+1}}{A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ & = \frac{W_{n,t} L_{n,t}}{\gamma} + B_{n,t-1} + \frac{1}{\mathcal{E}_{rn,t}} S_{n,t}, \end{aligned}$$

or, given domestic bond market clearing ($B_{n,t} = 0$),

$$\begin{aligned} & \Delta_{n,t}^c M C_{n,t}^c C_{n,t} + \Delta_{n,t}^m M C_{n,t}^m M_{n,t} + \\ & E_t \left[\frac{V_{t+1}}{\mathcal{E}_{rn,t}} S_{n,t+1} \right] + E_t \left[\Delta_{n,t}^m M C_{n,t}^m A_{n,t} \Theta_n \left(\frac{V_{t+1}}{\mathcal{E}_{rn,t} P_{n,t}} \frac{S_{n,t+1}}{A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ & = \frac{W_{n,t} L_{n,t}}{\gamma} + \frac{1}{\mathcal{E}_{rn,t}} S_{n,t}. \end{aligned} \quad (\text{A9})$$

B.4 Market Clearing

The value of traded varieties of final goods produced by country n (in units of the local currency) is the sum of its exports to other countries (and to itself), $\sum_{n'} \mathcal{E}_{nn',t} X_{n'n,t}^c$. Moreover,

$$\begin{aligned} \sum_{n'} \mathcal{E}_{nn',t} X_{n'n,t}^c &= \sum_{n'} \int \mathcal{E}_{nn',t} \mathcal{E}_{n'n,t} p_{n,t}^c(z) \kappa_{n'n}^c Q_{n'n,t}^c(z) d\Phi^c(z) \\ &= \int p_{n,t}^c(z) q_{n,t}^c(z) d\Phi^c(z) \\ &= W_{n,t} \int \ell_{n,t}^c(z) d\Phi^c(z) + P_{n,t}^m \int M_{n,t}^c(z) d\Phi^c(z) = W_{n,t} L_{n,t}^c + P_{n,t}^m M_{n,t}^c, \end{aligned}$$

where $L_{n,t}^c$ and $M_{n,t}^c$ represent, respectively, total labor input and materials employed in the final goods sector. Thus, the sum of the demand from various countries for country n 's varieties of final goods, in units of country n 's currency, $\sum_{n'} \mathcal{E}_{nn',t} X_{n'n,t}^c$, must equal the revenue of its producers, $\int p_{n,t}^c(z) q_{n,t}^c(z) d\Phi^c(z)$, which in turn must equal these producers' input costs (given constant returns and competitive markets), $W_{n,t} L_{n,t}^c + P_{n,t}^m M_{n,t}^c$. Hence,

$$W_{n,t} L_{n,t}^c + P_{n,t}^m M_{n,t}^c = \sum_{n'} \mathcal{E}_{nn',t} \varpi_{n'n,t}^c X_{n'n,t}^c,$$

where $W_{n,t} L_{n,t}^c + P_{n,t}^m M_{n,t}^c$ is then also the total value of traded varieties of final goods produced exported by country n . Moreover, recall above that $W_{n,t} \ell_{n,t}^c(z) = \frac{\gamma}{1-\gamma} P_{n,t}^m M_{n,t}^c(z)$. Therefore, $W_{n,t} L_{n,t}^c + P_{n,t}^m M_{n,t}^c = \frac{1}{\gamma} W_{n,t} L_{n,t}^c$ and

$$W_{n,t} L_{n,t}^c = \gamma \sum_{n'} \mathcal{E}_{nn',t} \varpi_{n'n,t}^c X_{n'n,t}^c.$$

Finally, retailers sell all of their output as final goods for local use. Recall that,

$$\Pi_{n,t} = (1 + \tau_n^c) P_{n,t} C_{n,t} - P_{n,t}^{Q^c} Q_{n,t}^c.$$

Here, $P_{n,t}^{Q^c} Q_{n,t}^c$ represents total expenditures by domestic retailers on bundles of traded varieties, and thus also the revenue of firms producing those bundles. Under competitive markets, this revenue is exhausted by expenditures on varieties imported from abroad to produce $Q_{n,t}^c$, $P_{n,t}^{Q^c} Q_{n,t}^c = X_{n,t}^c$. It then follows that

$$X_{n,t}^c = M C_{n,t}^c \Delta_{n,t}^c C_{n,t},$$

Market clearing for country n goods may then be expressed as:²

$$\underbrace{W_{n,t}L_{n,t}^c}_{\text{Total Value of Final Goods Varieties Produced in } n} = \underbrace{\gamma \sum_{n'} \mathcal{E}_{nn',t} \varpi_{n'n,t}^c MC_{n',t}^c \Delta_{n',t}^c C_{n',t}}_{\text{Total Foreign Demand for Final Goods Varieties from } n}.$$

In the remainder of this appendix, we let $\mathcal{E}_{n,t} \equiv \mathcal{E}_{rn,t}$ for notational convenience and, without loss of generality, take country 1 to be the reference country, ($r = 1$). Thus, under no arbitrage in currency markets, $\mathcal{E}_{rr,t} = \mathcal{E}_{r,t} = \mathcal{E}_{1,t} = 1$. We can then re-write the market clearing equation for final goods in each country as

$$\mathcal{E}_{n,t} W_{n,t} L_{n,t}^c = \gamma \sum_{n'} \mathcal{E}_{n',t} \varpi_{n'n,t}^c MC_{n',t}^c \Delta_{n',t}^c C_{n',t}. \quad (\text{A10})$$

Using similar arguments, market clearing in intermediate goods implies that

$$\mathcal{E}_{n,t} W_{n,t} L_{n,t}^m = \gamma \sum_{n'} \mathcal{E}_{n',t} \varpi_{n'n,t}^m MC_{n',t}^m \Delta_{n',t}^m \mathcal{M}_{n',t}, \quad (\text{A11})$$

where $L_{n,t}^m$ denotes total labor input employed in the intermediate goods production. Finally, labor market clearing requires that

$$L_{n,t} = L_{n,t}^c + L_{n,t}^m. \quad (\text{A12})$$

C Model Summary

C.1 Equilibrium Conditions

Household Budget Constraint:

$$\begin{aligned} & \Delta_{n,t}^c MC_{n,t}^c C_{n,t} + \Delta_{n,t}^m MC_{n,t}^m \frac{1 - \gamma}{\gamma} \frac{W_{n,t}}{P_{n,t}^m} L_{n,t} + \\ & E_t \left[\frac{V_{t+1}}{\mathcal{E}_{n,t}} S_{n,t+1} \right] + E_t \left[\Delta_{n,t}^m MC_{n,t}^m A_{n,t} \Theta_n \left(\frac{V_{t+1}}{\mathcal{E}_{n,t} P_{n,t}} \frac{S_{n,t+1}}{A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ & = \frac{W_{n,t} L_{n,t}}{\gamma} + \frac{1}{\mathcal{E}_{n,t}} S_{n,t}. \end{aligned}$$

Household Optimality Conditions:

²Multiplying both sides by $\mathcal{E}_{rn,t}$ and summing over n , we recover the world resource constraint for final goods,

$$\sum_n \mathcal{E}_{rn,t} W_{n,t} L_{n,t}^c = \gamma \sum_n \mathcal{E}_{rn,t} (W_{n,t} L_{n,t}^c + P_{n,t}^m M_{n,t}^c) = \gamma \sum_{n'} \mathcal{E}_{rn',t} MC_{n',t}^c \Delta_{n',t}^c C_{n',t},$$

using the fact that $\sum_n \varpi_{n'n,t} = 1$.

- International Financial Securities, $S_{n,t+1}$:

$$\begin{aligned} & \left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} V_{t+1} \left[1 + \frac{P_{n,t}^m}{P_{n,t}} \Theta'_n \left(\frac{V_{t+1} S_{n,t+1}}{\mathcal{E}_{n,t} P_{n,t} A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ &= \beta \frac{\mathcal{E}_{n,t} P_{n,t} A_{n,t}}{\mathcal{E}_{n,t+1} P_{n,t+1} A_{n,t+1}} \left(\frac{C_{n,t+1}}{A_{n,t+1}} \right)^{-\sigma_n}. \end{aligned}$$

- Domestic Bonds, $B_{n,t}$: $\left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} = \beta R_{n,t} E_t \left(\frac{C_{n,t+1}}{A_{n,t+1}} \right)^{-\sigma_n} \frac{P_{n,t} A_{n,t}}{P_{n,t+1} A_{n,t+1}}$.
- Consumption, $C_{n,t}$: $\left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} = \lambda_{n,t} A_{n,t}$.
- Labor Supply, $L_{n,t}$: $\omega_n L_{n,t}^{\eta_n} \left(\frac{C_{n,t}}{A_{n,t}} \right)^{\sigma_n} = \frac{W_{n,t}}{A_{n,t} P_{n,t}}$.
- Final Goods: $1 = (1 - \theta^c) \left(\frac{P_{n,t}^*}{P_{n,t}} \right)^{1-\varepsilon_{n,t}} + \theta^c \left(\frac{1+\pi_n}{1+\pi_{n,t}} \right)^{1-\varepsilon_{n,t}}$.
- Intermediate Goods: $1 = (1 - \theta^m) \left(\frac{P_{n,t}^{m*}}{P_{n,t}^m} \right)^{1-\varrho_{n,t}} + \theta^m \left(\frac{1+\pi_n}{1+\pi_{n,t}^m} \right)^{1-\varrho_{n,t}}$.

Optimal Sticky Prices:

- Final Goods:

$$(1 + \tau_n^c) \frac{P_{n,t}^*}{P_{n,t}} = \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k \varepsilon_{n,t+k} \left(\frac{P_{n,t+k}}{P_{n,t}} \frac{1}{(1+\pi_n)^k} \right)^{\varepsilon_{n,t+k}} m c_{n,t+k}^c C_{n,t+k}^{1-\sigma_n} A_{n,t+k}^{\sigma_n-1}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k (\varepsilon_{n,t+k} - 1) \left(\frac{P_{n,t+k}}{P_{n,t}} \frac{1}{(1+\pi_n)^k} \right)^{\varepsilon_{n,t+k}-1} C_{n,t+k}^{1-\sigma_n} A_{n,t+k}^{\sigma_n-1}}.$$

- Intermediate Goods:

$$\begin{aligned} & (1 + \tau_n^m) \frac{P_{n,t}^{m*}}{P_{n,t}^m} \\ &= \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k \varrho_{n,t+k} \left(\frac{P_{n,t+k}^m}{P_{n,t}^m} \frac{1}{(1+\pi_n)^k} \right)^{\varrho_{n,t+k}} m c_{n,t+k}^m \mathcal{M}_{n,t+k} C_{n,t+k}^{-\sigma_n} A_{n,t+k}^{\sigma_n-1}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k (\varrho_{n,t+k} - 1) \left(\frac{P_{n,t+k}^m}{P_{n,t}^m} \frac{1}{(1+\pi_n)^k} \right)^{\varrho_{n,t+k}-1} \mathcal{M}_{n,t+k} C_{n,t+k}^{-\sigma_n} A_{n,t+k}^{\sigma_n-1} \frac{P_{n,t+k}^m}{P_{n,t+k}}}. \end{aligned}$$

Trade Shares:

- Final Goods: $\varpi_{nn',t}^c = \frac{\left(\kappa_{nn'}^c \mathcal{E}_{n',t} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma} \right)^{-\varphi^c}}{\sum_{n''} \left(\kappa_{nn''}^c \mathcal{E}_{n'',t} W_{n'',t}^\gamma (P_{n'',t}^m)^{1-\gamma} \right)^{-\varphi^c}}$.
- Intermediate Goods: $\varpi_{nn',t}^m = \frac{\left(\kappa_{nn'}^m \mathcal{E}_{n',t} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma} \right)^{-\varphi^m}}{\sum_{n''} \left(\kappa_{nn''}^m \mathcal{E}_{n'',t} W_{n'',t}^\gamma (P_{n'',t}^m)^{1-\gamma} \right)^{-\varphi^m}}$.

Trade Flows:

- Final Goods: $\mathcal{E}_{n,t} W_{n,t} L_{n,t}^c = \gamma \sum_{n'} \mathcal{E}_{n',t} \varpi_{n'n,t}^c MC_{n',t}^c \Delta_{n',t}^c C_{n',t}$.
- Intermediate Goods: $\mathcal{E}_{n,t} W_{n,t} L_{n,t}^m = \gamma \sum_{n'} \mathcal{E}_{n',t} \varpi_{n'n,t}^m MC_{n',t}^m \Delta_{n',t}^m \mathcal{M}_{n',t}$.

Intermediate Goods Sector: $\mathcal{M}_{n,t} = \frac{1-\gamma}{\gamma} \frac{W_{n,t} L_{n,t}}{P_{n,t}^m} + A_{n,t} E_t \left[\Theta_n \left(\frac{V_{t+1} S_{n,t+1}}{\mathcal{E}_{n,t} P_{n,t} A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right]$.

Marginal Cost:

- Retail:

$$MC_{n,t}^c = \frac{P_{n,t}^{Q^c}}{A_{n,t}} = \frac{1}{A_{n,t}} \Gamma(\xi^c)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^c \frac{\mathcal{E}_{n',t}}{\mathcal{E}_{n,t}} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma} \right)^{-\varphi^c} \right)^{-\frac{1}{\varphi^c}}.$$

- Intermediate Inputs:

$$MC_{n,t}^m = \frac{P_{n,t}^{Q^m}}{A_{n,t}} = \frac{1}{A_{n,t}} \Gamma(\xi^m)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^m \frac{\mathcal{E}_{n',t}}{\mathcal{E}_{n,t}} W_{n',t}^\gamma (P_{n',t}^m)^{1-\gamma} \right)^{-\varphi^m} \right)^{-\frac{1}{\varphi^m}}.$$

No Arbitrage in Currency Markets: $\mathcal{E}_{1,t} = 1$.

Definition of $\Delta_{n,t}^k$:

- Final Goods: $\Delta_{n,t}^c = (1 - \theta^c) \left(\frac{P_{n,t}^*}{P_{n,t}} \right)^{-\varepsilon_{n,t}} + \theta^c \left(\frac{1 + \pi_n}{1 + \pi_{n,t}} \right)^{-\varepsilon_{n,t}} \Delta_{n,t-1}^c$.
- Intermediate Goods: $\Delta_{n,t}^m = (1 - \theta^m) \left(\frac{P_{n,t}^{m*}}{P_{n,t}^m} \right)^{-\varrho_{n,t}} + \theta^m \left(\frac{1 + \pi_n}{1 + \pi_{n,t}} \right)^{-\varrho_{n,t}} \Delta_{n,t-1}^m$.

Labor Market Clearing: $L_{n,t} = L_{n,t}^c + L_{n,t}^m$.

Monetary Policy Rule:

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^* = R_n \left(\frac{1 + \pi_{n,t}}{1 + \pi_n} \right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\bar{Y}_{n,t}} \right)^{\phi_{n,Y}}.$$

C.2 Deflated System

To deflate the system of equation described in the previous section, we define the following variables: real wages, $w_{n,t} = W_{n,t}/P_{n,t}$, real marginal cost, $mc_{n,t}^k = MC_{n,t}^k/P_{n,t}$, $k \in \{c, m\}$, the quantity of financial securities held by country n in real terms (i.e., units of country 1 final goods), $s_{n,t} = \frac{S_{n,t}}{P_{1,t}}$, real exchange rates that convert real final goods in country n into units of country 1's final goods, $e_{n,t} = \mathcal{E}_{n,t} \frac{P_{n,t}}{P_{1,t}}$. In addition, $p_{n,t}^m = P_{n,t}^m/P_{n,t}$ and $1 + \pi_{n,t} = P_{n,t}/P_{n,t-1}$ as defined above.

The resulting system of equations in real terms is:

Household Budget Constraint:

$$\begin{aligned} & \Delta_{n,t}^c m c_{n,t}^c C_{n,t} + \Delta_{n,t}^m \frac{m c_{n,t}^m}{p_{n,t}^m} \frac{1-\gamma}{\gamma} w_{n,t} L_{n,t} + \\ & E_t \left[\frac{V_{t+1}}{e_{n,t}} (1 + \pi_{1,t+1}) s_{n,t+1} \right] + E_t \left[\Delta_{n,t}^m m c_{n,t}^m A_{n,t} \Theta_n \left(\frac{V_{t+1}}{e_{n,t}} \frac{(1 + \pi_{1,t+1}) s_{n,t+1}}{A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ & = \frac{w_{n,t} L_{n,t}}{\gamma} + \frac{1}{e_{n,t}} s_{n,t}. \end{aligned}$$

Household Optimality Conditions:

- International Financial Securities, $S_{n,t+1}$:

$$\begin{aligned} & \left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} V_{t+1} \left[1 + p_{n,t}^m \Theta_n' \left(\frac{V_{t+1} (1 + \pi_{1,t+1}) s_{n,t+1}}{e_{n,t} A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ & = \frac{1}{1 + \pi_{1,t+1}} \beta \frac{e_{n,t} A_{n,t}}{e_{n,t+1} A_{n,t+1}} \left(\frac{C_{n,t+1}}{A_{n,t+1}} \right)^{-\sigma_n}. \end{aligned}$$

- Domestic Bonds, $B_{n,t}$: $\left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} = \beta R_{n,t} E_t \left(\frac{C_{n,t+1}}{A_{n,t+1}} \right)^{-\sigma_n} \frac{A_{n,t}}{(1 + \pi_{n,t+1}) A_{n,t+1}}.$
- Consumption, $C_{n,t}$: $\left(\frac{C_{n,t}}{A_{n,t}} \right)^{-\sigma_n} = \lambda_{n,t} A_{n,t}.$
- Labor Supply, $L_{n,t}$: $\omega_n L_{n,t}^{\eta_n} \left(\frac{C_{n,t}}{A_{n,t}} \right)^{\sigma_n} = \frac{w_{n,t}}{A_{n,t}}.$

Aggregate Price Index:

- Retailers: $1 = (1 - \theta^c) (p_{n,t}^*)^{1-\varepsilon_{n,t}} + \theta^c \left(\frac{1+\pi_n}{1+\pi_{n,t}} \right)^{1-\varepsilon_{n,t}}.$
- Intermediate Goods: $1 = (1 - \theta^m) \left(\frac{p_{n,t}^{m*}}{p_{n,t}^m} \right)^{1-\varrho_{n,t}} + \theta^m \left(\frac{1+\pi_n}{1+\pi_{n,t}^m} \right)^{1-\varrho_{n,t}}.$

Optimal Sticky Prices:

- Final Goods:

$$(1 + \tau_n^c) p_{n,t}^* = \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k \varepsilon_{n,t+k} \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s})}{(1 + \pi_n)^k} \right)^{\varepsilon_{n,t+k}} m c_{n,t+k}^c C_{n,t+k}^{1-\sigma_n} A_{n,t+k}^{\sigma_n-1}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k (\varepsilon_{n,t+k} - 1) \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s})}{(1 + \pi_n)^k} \right)^{\varepsilon_{n,t+k}-1} C_{n,t+k}^{1-\sigma_n} A_{n,t+k}^{\sigma_n-1}}.$$

- Intermediate Goods:

$$(1 + \tau_n^m) \frac{p_{n,t}^{m*}}{p_{n,t}^m} = \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k \varrho_{n,t+k} \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s}^m)}{(1 + \pi_n)^k} \right)^{\varrho_{n,t+k}} m c_{n,t+k}^m \mathcal{M}_{n,t+k} C_{n,t+k}^{-\sigma_n} A_{n,t+k}^{\sigma_n - 1}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k (\varrho_{n,t+k} - 1) \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s}^m)}{(1 + \pi_n)^k} \right)^{\varrho_{n,t+k} - 1} \mathcal{M}_{n,t+k} C_{n,t+k}^{-\sigma_n} A_{n,t+k}^{\sigma_n - 1} p_{n,t+k}^m}.$$

Trade Shares:

- Final Goods: $\varpi_{nn',t}^c = \frac{(\kappa_{nn'}^c e_{n',t} w_{n',t}^\gamma (p_{n',t}^m)^{1-\gamma})^{-\varphi^c}}{\sum_{n''} (\kappa_{nn''}^c e_{n'',t} w_{n'',t}^\gamma (p_{n'',t}^m)^{1-\gamma})^{-\varphi^c}}.$
- Intermediate Goods: $\varpi_{nn',t}^m = \frac{(\kappa_{nn'}^m e_{n',t} w_{n',t}^\gamma (p_{n',t}^m)^{1-\gamma})^{-\varphi^m}}{\sum_{n''} (\kappa_{nn''}^m e_{n'',t} w_{n'',t}^\gamma (p_{n'',t}^m)^{1-\gamma})^{-\varphi^m}}.$

Trade Flows:

- Final Goods: $e_{n,t} w_{n,t} L_{n,t}^c = \gamma \sum_{n'} e_{n',t} \varpi_{n'n,t}^c m c_{n',t}^c \Delta_{n',t}^c C_{n',t}.$
- Intermediate Goods: $e_{n,t} w_{n,t} L_{n,t}^m = \gamma \sum_{n'} e_{n',t} \varpi_{n'n,t}^m m c_{n',t}^m \Delta_{n',t}^m \mathcal{M}_{n',t}.$

Intermediate Goods Sector: $\mathcal{M}_{n,t} = \frac{1-\gamma}{\gamma} \frac{w_{n,t}}{p_{n,t}^m} L_{n,t} + A_{n,t} E_t \left[\Theta_n \left(\frac{V_{t+1} (1 + \pi_{1,t+1}) s_{n,t+1}}{e_{n,t} A_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right].$

Marginal Cost:

- Retail: $m c_{n,t}^c = \frac{p_{n,t}^Q}{A_{n,t}} = \frac{1}{A_{n,t}} \Gamma(\xi^c)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^c \frac{e_{n',t}}{e_{n,t}} w_{n',t}^\gamma (p_{n',t}^m)^{1-\gamma} \right)^{-\varphi^c} \right)^{-\frac{1}{\varphi^c}}.$
- Intermediate Inputs: $m c_{n,t}^m = \frac{p_{n,t}^{Q^m}}{A_{n,t}} = \frac{1}{A_{n,t}} \Gamma(\xi^m)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^m \frac{e_{n',t}}{e_{n,t}} w_{n',t}^\gamma (p_{n',t}^m)^{1-\gamma} \right)^{-\varphi^m} \right)^{-\frac{1}{\varphi^m}}.$

No Arbitrage in Currency Markets: $e_{1,t} = 1.$

Definition of $\Delta_{n,t}^k$:

- Final Goods: $\Delta_{n,t}^c = (1 - \theta^c) (p_{n,t}^*)^{-\varepsilon_{n,t}} + \theta^c \left(\frac{1 + \pi_n}{1 + \pi_{n,t}} \right)^{-\varepsilon_{n,t}} \Delta_{n,t-1}^c.$
- Intermediate Goods: $\Delta_{n,t}^m = (1 - \theta^m) \left(\frac{p_{n,t}^{m*}}{p_{n,t}^m} \right)^{-\varrho_{n,t}} + \theta^m \left(\frac{1 + \pi_n^m}{1 + \pi_{n,t}^m} \right)^{-\varrho_{n,t}} \Delta_{n,t-1}^m.$

Labor Market Clearing: $L_{n,t} = L_{n,t}^c + L_{n,t}^m.$

Monetary Policy Rule:

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^* = R_n \left(\frac{1 + \pi_{n,t}}{1 + \pi_n} \right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\bar{Y}_{n,t}} \right)^{\phi_{n,Y}}.$$

C.3 Detrended System

In each country n , $g_n = \frac{A_{n,t}}{A_{n,t-1}} \forall t$ defines the rate of technical progress along a non-stochastic steady-state growth path. As our benchmark case, we assume that each country evolves along its own balanced growth path in the long-run. Thus, in each country n , consumption, real wages and the real price of traded intermediate varieties all grow at rate g_n along its steady-state growth path while, under the maintained preferences and technologies, labor supply and real marginal costs are constant. In addition, our focus is one where no country comes to asymptotically dominate or become insignificant as a consumer of global output. Said differently, the world as a whole features balanced growth in the long run.

For any variable $x_{n,t}$ that grows at rate g_n along country n 's balanced growth path, we define its detrended counterpart as $\tilde{x}_{n,t} = \frac{x_{n,t}}{A_{n,t}}$ (e.g., detrended wages in country n are $\tilde{w}_{n,t} = w_{n,t}/A_{n,t}$). These normalized variables are constant for each country in their detrended steady state. Similarly, shadow prices, $\lambda_{n,t}$, grow at rate $\frac{1}{g_n}$ so that we define $\tilde{\lambda}_{n,t} = \lambda_{n,t}A_{n,t}$. As different countries may grow at different rates in the long run, world balanced growth requires that the real exchange rate, $e_{n,t}$, grows at a rate that reflects that long run difference in technical progress between country n and the reference country. Specifically, we define the detrended real exchange rate, $\tilde{e}_{n,t} = \frac{e_{n,t}A_{n,t}}{A_{1,t}}$, which will be constant along each country n 's balanced growth path.

Given that each country n grows at a constant rate along the balanced growth path, trade costs in every country n' must also grow along that path together with wages. In particular, $\kappa_{nn'}^c = \tilde{\kappa}_{nn'}^c A_{n',t}^{1-\gamma}$ and similarly with $\kappa_{nn'}^m$.

In detrended form, the system becomes the following:

Household Budget Constraint:

$$\begin{aligned} & \Delta_{n,t}^c m c_{n,t}^c \tilde{C}_{n,t} + \Delta_{n,t}^m \frac{m c_{n,t}^m}{p_{n,t}^m} \frac{1-\gamma}{\gamma} \tilde{w}_{n,t} L_{n,t} + E_t \left[\frac{V_{t+1}}{\tilde{e}_{n,t}} (1 + \pi_{1,t+1}) g_{1,t+1} \tilde{s}_{n,t+1} \right] \\ & + E_t \left[\Delta_{n,t}^m m c_{n,t}^m \Theta_n \left(\frac{V_{t+1}}{\tilde{e}_{n,t}} (1 + \pi_{1,t+1}) \tilde{s}_{n,t+1} g_{1,t+1} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ & = \frac{\tilde{w}_{n,t} L_{n,t}}{\gamma} + \frac{1}{\tilde{e}_{n,t}} \tilde{s}_{n,t-1}. \end{aligned}$$

Household Optimality Conditions:

- International Financial Securities, $S_{n,t+1}$:

$$\begin{aligned} & \left(\tilde{C}_{n,t} \right)^{-\sigma_n} V_{t+1} \left[1 + p_{n,t}^m \Theta'_n \left(\frac{V_{t+1}(1 + \pi_{1,t+1}) \tilde{s}_{n,t+1} g_{1,t+1}}{\tilde{e}_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right] \\ &= \beta \frac{1}{(1 + \pi_{1,t+1}) g_{1,t+1}} \frac{\tilde{e}_{n,t}}{\tilde{e}_{n,t+1}} \left(\tilde{C}_{n,t+1} \right)^{-\sigma_n}. \end{aligned}$$

- Domestic Bonds, $B_{n,t}$: $\tilde{C}_{n,t}^{-\sigma_n} = \beta R_{n,t} E_t \tilde{C}_{n,t+1}^{-\sigma_n} \frac{1}{(1 + \pi_{n,t+1}) g_{n,t+1}}$.
- Consumption, $C_{n,t}$: $\tilde{C}_{n,t}^{-\sigma_n} = \tilde{\lambda}_{n,t}$.
- Labor Supply, $L_{n,t}$: $\omega_n L_{n,t}^{\eta_n} \tilde{C}_{n,t}^{\sigma_n} = \tilde{w}_{n,t}$.

Aggregate Price Index:

- Retailers: $1 = (1 - \theta^c) (p_{n,t}^*)^{1 - \varepsilon_{n,t}} + \theta^c \left(\frac{1 + \pi_n}{1 + \pi_{n,t}} \right)^{1 - \varepsilon_{n,t}}$.
- Intermediate Goods: $1 = (1 - \theta^m) \left(\frac{p_{n,t}^{m*}}{p_{n,t}^m} \right)^{1 - \varrho_{n,t}} + \theta^m \left(\frac{1 + \pi_n}{1 + \pi_{n,t}} \right)^{1 - \varrho_{n,t}}$.

Optimal Sticky Prices in Retail and Intermediate Goods Markets:

- Final Goods:

$$(1 + \tau_n^c) p_{n,t}^* = \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k \varepsilon_{n,t+k} \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s})}{(1 + \pi_n)^k} \right)^{\varepsilon_{n,t+k}} m c_{n,t+k}^c \tilde{C}_{n,t+k}^{1 - \sigma_n}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^c)^k (\varepsilon_{n,t+k} - 1) \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s})}{(1 + \pi_n)^k} \right)^{\varepsilon_{n,t+k} - 1} \tilde{C}_{n,t+k}^{1 - \sigma_n}}.$$

- Intermediate Goods:

$$(1 + \tau_n^m) \frac{p_{n,t}^{m*}}{p_{n,t}^m} = \frac{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k \varrho_{n,t+k} \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s}^m)}{(1 + \pi_n)^k} \right)^{\varrho_{n,t+k}} m c_{n,t+k}^m \tilde{\mathcal{M}}_{n,t+k} \tilde{C}_{n,t+k}^{-\sigma_n}}{E_t \sum_{k=0}^{\infty} \beta^k (\theta^m)^k (\varrho_{n,t+k} - 1) \left(\frac{\prod_{s=1}^k (1 + \pi_{n,t+s}^m)}{(1 + \pi_n)^k} \right)^{\varrho_{n,t+k} - 1} \tilde{\mathcal{M}}_{n,t+k} \tilde{C}_{n,t+k}^{-\sigma_n} p_{n,t+k}^m}.$$

Trade Shares:

- Final Goods: $\varpi_{nn',t}^c = \frac{\left(\tilde{\kappa}_{nn'}^c \tilde{e}_{n',t} \tilde{w}_{n',t}^{\gamma} (p_{n',t}^m)^{1 - \gamma} \right)^{-\varphi^c}}{\sum_{n''} \left(\tilde{\kappa}_{nn''}^c \tilde{e}_{n'',t} \tilde{w}_{n'',t}^{\gamma} (p_{n'',t}^m)^{1 - \gamma} \right)^{-\varphi^c}}$.
- Intermediate Goods: $\varpi_{nn',t}^m = \frac{\left(\tilde{\kappa}_{nn'}^m \tilde{e}_{n',t} \tilde{w}_{n',t}^{\gamma} (p_{n',t}^m)^{1 - \gamma} \right)^{-\varphi^m}}{\sum_{n''} \left(\tilde{\kappa}_{nn''}^m \tilde{e}_{n'',t} \tilde{w}_{n'',t}^{\gamma} (p_{n'',t}^m)^{1 - \gamma} \right)^{-\varphi^m}}$.

Trade Flows:

- Final Goods:

$$\tilde{e}_{n,t} \tilde{w}_{n,t} L_{n,t}^c = \gamma \sum_{n'} \tilde{e}_{n',t} \tilde{\omega}_{n'n,t}^c m c_{n',t}^c \Delta_{n',t}^c \tilde{C}_{n',t}. \quad (\text{A13})$$

- Intermediate Goods:

$$\tilde{e}_{n,t} \tilde{w}_{n,t} L_{n,t}^m = \gamma \sum_{n'} \tilde{e}_{n',t} \tilde{\omega}_{n'n,t}^m m c_{n',t}^m \Delta_{n',t}^m \tilde{\mathcal{M}}_{n',t}. \quad (\text{A14})$$

Intermediate Goods Sector: $\tilde{\mathcal{M}}_{n,t} = \frac{1-\gamma}{\gamma} \frac{\tilde{w}_{n,t} L_{n,t}}{p_{n,t}^m} + E_t \left[\Theta_n \left(\frac{V_{t+1}(1+\pi_{1,t+1}) \tilde{s}_{n,t+1} g_{1,t+1}}{\tilde{e}_{n,t}} + \frac{1}{\tau} \xi_{n,t} \right) \right].$

Marginal Cost:

- Retail: $m c_{n,t}^c = \tilde{p}_{n,t}^Q = \Gamma(\xi^c)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\tilde{\kappa}_{nn'}^c \frac{\tilde{e}_{n',t}}{\tilde{e}_{n,t}} \tilde{w}_{n',t}^\gamma (p_{n',t}^m)^{1-\gamma} \right)^{-\varphi^c} \right)^{-\frac{1}{\varphi^c}}.$
- Intermediate Inputs: $m c_{n,t}^m = \tilde{p}_{n,t}^{Q^m} = \Gamma(\xi^m)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\tilde{\kappa}_{nn'}^m \frac{\tilde{e}_{n',t}}{\tilde{e}_{n,t}} \tilde{w}_{n',t}^\gamma (p_{n',t}^m)^{1-\gamma} \right)^{-\varphi^m} \right)^{-\frac{1}{\varphi^m}}.$

No Arbitrage in Currency Markets: $\tilde{e}_{1,t} = 1.$

Definition of $\Delta_{n,t}^k$:

- Final Goods: $\Delta_{n,t}^c = (1 - \theta^c) (p_{n,t}^*)^{-\varepsilon_{n,t}} + \theta^c \left(\frac{1+\pi_n}{1+\pi_{n,t}} \right)^{-\varepsilon_{n,t}} \Delta_{n,t-1}^c.$
- Intermediate Goods: $\Delta_{n,t}^m = (1 - \theta^m) \left(\frac{p_{n,t}^{m*}}{p_{n,t}^m} \right)^{-\varrho_{n,t}} + \theta^m \left(\frac{1+\pi_n}{1+\pi_{n,t}} \right)^{-\varrho_{n,t}} \Delta_{n,t-1}^m.$

Labor Market Clearing:

$$L_{n,t} = L_{n,t}^c + L_{n,t}^m. \quad (\text{A15})$$

Monetary Policy Rule:

$$R_{n,t} = R_{n,t-1}^{\rho_n} R_{n,t}^{*1-\rho_n} e^{\nu_{n,t}} \text{ where } R_{n,t}^* = R_n \left(\frac{1+\pi_{n,t}}{1+\pi_n} \right)^{\phi_{n,\pi}} \left(\frac{Y_{n,t}}{\bar{Y}_{n,t}} \right)^{\phi_{n,Y}}.$$

D Steady-State

From here on, we omit the ‘tildes’ over the variables for notational convenience under the understanding that our final system of equations is expressed in deflated and detrended form.

Our modeling of portfolio adjustment costs does not provide a theory of countries’ long-run net asset positions. As such, we proceed as in [Eaton and Kortum \(2002\)](#) and adopt as a benchmark

a steady state with balanced trade (one that countries have not yet reached). In practice, however, countries' net asset positions have switched from positive to negative or vice versa over our sample period, 2004Q3-2019Q4. Therefore, to carry out quantitative assessments of the effects of productivity or other disturbances, we recover counterfactual steady-state allocations corresponding to balanced trade from a world in which imbalances exist even over long periods. That is, we adapt the approach first developed in [Dekle, Eaton, and Kortum \(2007\)](#) to the steady state of this dynamic setting. Specifically, we first write a variant of the economic environment whose steady state can be matched to observed average regional trade imbalances over our sample. This can be done by setting $\xi_n \neq 0$.³ Then, for any implied set of parameters, we calculate counterfactual steady-state allocations and prices consistent with a world without trade imbalances. Put another way, for any set of parameters, we compute counterfactual steady-state allocations consistent with $\xi_n = 0 \forall n$ in which case real net financial assets are zero in every country, $s_n = \frac{S_n}{P_1} = 0 \forall n$.⁴ The resulting counterfactual world steady state without imbalances is then used to produce a linearized model that can be estimated, in this case using Bayesian methods, in order to assess the effects of various disturbances.

D.1 Steady-State Equations with Unbalanced Trade

The unknowns are: $\mathcal{M}_n, s_n, C_n, mc_n^c, mc_n^m, L_n, L_n^c, L_n^m, w_n, e_n, V, r_n, \lambda_n, p_n^*, p_n^{m*}, \varpi_{nn'}^c, \varpi_{nn'}^m, p^m$. Analogous to the treatment of investment adjustment costs in the investment literature, we assume that portfolio adjustment costs are zero in steady state. In this case, s_n is constant in each country and

$$\Theta_n \left(\frac{V}{e_n} (1 + \pi_1) s_n g_1 + \frac{1}{\tau} \xi_n \right) = \Theta'_n \left(\frac{V(1 + \pi_1) s_n g_1}{e_n} + \frac{1}{\tau} \xi_n \right) = 0.$$

The steady-state equations are then as follows:

Household Budget Constraint: $mc_n^c C_n + \frac{mc_n^m}{p_n^m} \frac{1-\gamma}{\gamma} w_n L_n = \frac{w_n L_n}{\gamma} + (1 - V(1 + \pi_1) g_1) \frac{1}{e_n} s_n.$

Household Optimality Conditions:

- International Financial Securities, $S_{n,t+1}$: $V = \frac{1}{(1+\pi_1)g_1} \beta.$
- Domestic Bonds, $B_{n,t}$: $R_n = \beta^{-1}(1 + \pi_n)g_n.$
- Consumption, $C_{n,t}$: $(C_n)^{-\sigma_n} = \lambda_n.$
- Labor Supply, L_n : $\omega_n L_n^{\eta_n} (C_n)^{\sigma_n} = w_n.$

³The fact that $\xi_n \neq 0$ is exogenous captures the sense in which our modeling of portfolio adjustment costs provides no clear guidance regarding long-run net asset positions

⁴Recall that financial assets are denoted in units of the reference country's currency.

Aggregate Price Index:

- Retailers: $p_n^* = 1$.
- Intermediate Goods: $p_n^{m*} = p_n^m$.

Optimal Sticky Prices:

- Final Goods: $1 = \frac{1}{1+\tau_n^c} \frac{\varepsilon_n}{\varepsilon_n-1} m c_n^c$.
- Intermediate Goods: $p_n^m = \frac{1}{1+\tau_n^m} \frac{\varrho_n}{\varrho_n-1} m c_n^m$.

Trade Shares:

- Final Goods: $\varpi_{nn'}^c = \frac{\left(\kappa_{nn'}^c e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma}\right)^{-\varphi^c}}{\sum_{n''} \left(\kappa_{nn''}^c e_{n''} w_{n''}^\gamma (p_{n''}^m)^{1-\gamma}\right)^{-\varphi^c}}$.
- Intermediate Goods: $\varpi_{nn'}^m = \frac{\left(\kappa_{nn'}^m e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma}\right)^{-\varphi^m}}{\sum_{n''} \left(\kappa_{nn''}^m e_{n''} w_{n''}^\gamma (p_{n''}^m)^{1-\gamma}\right)^{-\varphi^m}}$.

Trade Flows:

- Final Goods: $e_n w_n L_n^c = \gamma \sum_{n'} e_{n'} \varpi_{n'n}^c m c_{n'}^c C_{n'}$.
- Intermediate Goods: $e_n w_n L_n^m = \gamma \sum_{n'} e_{n'} \varpi_{n'n}^m m c_{n'}^m \mathcal{M}_{n'}$.

Intermediate Goods Sector: $\mathcal{M}_n = \frac{1-\gamma}{\gamma} \frac{1}{p_n^m} w_n L_n$.

Marginal Cost:

- Retail: $m c_n^c = p_n^{Q^c} = \Gamma(\xi^c)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^c \frac{e_{n'}}{e_n} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma} \right)^{-\varphi^c} \right)^{-\frac{1}{\varphi^c}}$.
- Intermediate Inputs: $m c_n^m = p_n^{Q^m} = \Gamma(\xi^m)^{\frac{1}{1-\chi}} \left(\sum_{n'} \left(\kappa_{nn'}^m \frac{e_{n'}}{e_n} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma} \right)^{-\varphi^m} \right)^{-\frac{1}{\varphi^m}}$.

No Arbitrage in Currency Markets: $e_1 = 1$.

Relative Price Distortions:

- Final Goods: $\Delta_{n,t} = 1$.
- Intermediate Goods: $\Delta_{n,t}^m = 1$.

Labor Market Clearing: $L_n = L_n^c + L_n^m$.

D.2 Algorithm for Steady State with Unbalanced Trade, $s_n \neq 0$

We solve for the steady-state (deflated-detrended) values of e_n , w_n , and s_n given parameters and values of nominal GDP, $\mathcal{E}_n W_n L_n$, employment, L_n and trade shares, $\varpi_{nn'}^c$ and $\varpi_{nn'}^m$, calibrated to the 2004-2019 sample periods. The algorithm below also computes implied parameter values for ξ_n , ω_n , $\kappa_{nn'}^c$ and $\kappa_{nn'}^m$. Finally, note that by definition of e_n and w_n , for any country n , $\mathcal{E}_n W_n L_n = P_1 A_1 e_n w_n L_n$.⁵

1. Optimality conditions for domestic bonds imply that $R_n = \beta^{-1} g_n (1 + \pi_n)$. The aggregate price index equation implies that $p_n^* = 1$, and the optimal sticky price equation implies that $mc_n^c = (1 + \tau_n^c) \frac{\varepsilon_n - 1}{\varepsilon_n}$.
2. Set $\xi_n = -\tau V (1 + \pi_1) s_n g_1 / e_n$ so that $\Theta = \Theta' = 0$ in steady state. This implies $V = \frac{\beta}{(1 + \pi_1) g_1}$.
3. Combining the marginal cost for retailers of final goods with the expression for trade shares, and imposing the normalization $\kappa_{nn}^c = 1$, we have

$$e_n mc_n^c = (\varpi_{nn}^c)^{\frac{1}{\varphi^c}} e_n w_n^\gamma (p_n^m)^{1-\gamma} \Rightarrow \frac{\varepsilon_n - 1}{\varepsilon_n} (1 + \tau_n^c) = (\varpi_{nn}^c)^{\frac{1}{\varphi^c}} w_n^\gamma (p_n^m)^{1-\gamma},$$

which we solve for $w_n^\gamma (p_n^m)^{1-\gamma}$.

4. Similarly, combining the marginal cost for intermediate goods producers with the expression for trade-shares, along with $\kappa_{nn}^m = 1$, we have

$$e_n mc_n^m = (\varpi_{nn}^m)^{\frac{1}{\varphi^m}} e_n w_n^\gamma (p_n^m)^{1-\gamma} \Rightarrow \frac{\varrho_n - 1}{\varrho_n} (1 + \tau_n^m) p_n^m = (\varpi_{nn}^m)^{\frac{1}{\varphi^m}} w_n^\gamma (p_n^m)^{1-\gamma},$$

which, given $w_n^\gamma (p_n^m)^{1-\gamma}$ from the previous step, can be solved for p_n^m . Given p_n^m one can then find w_n .

5. Given $\mathcal{E}_1 W_1 L_1$ and L_1 in the data, as well as w_1 from the previous step, calculate $(P_1 A_1) = \frac{\mathcal{E}_1 W_1 L_1}{w_1 L_1}$ (using $e_1 = 1$).
6. Calculate $e_n = \frac{\mathcal{E}_n W_n L_n}{(P_1 A_1) w_n L_n}$.
7. From data on $e_n w_n L_n$, obtain $e_n p_n^m \mathcal{M}_n = e_n p_n^m M_n = \frac{1-\gamma}{\gamma} e_n w_n L_n$, which we can then use to solve for M_n .
8. Write the trade-flow equations for materials in vector form. Using data for $\varpi_{nn'}^m$ and M_n from the previous step, L_n^m :

$$ewL^m = \gamma (\varpi^m)^T e (mc^m) M.$$

⁵For the steady-state algorithms here in the next subsection we omit $\Gamma(\xi^k)$ proportionality term for simplicity since it can be absorbed in κ without loss

9. Solve for $L^c = L - L^m$. Write the trade-flow equations for consumption in vector form and, given $e_n w_n L_n^c$, obtain $e_n m c_n^c C_n$:

$$e(m c^c)C = \frac{1}{\gamma}((\varpi^c)^T)^{-1} e w L^c.$$

10. From the household budget constraint, and given that $V(1 + \pi_1)g_1 = \beta$, solve for s_n ,

$$m c_n^c C_n + \frac{m c_n^m}{p_n^m} \frac{1 - \gamma}{\gamma} w_n L_n = \frac{w_n L_n}{\gamma} + (1 - \beta) \frac{1}{e_n} s_n.$$

11. Solving for ξ_n , from the equations governing the choice of international financial securities transactions, in order for $\Theta' = 0$ in steady state, it must be the case that $\frac{1}{\tau} \xi_n = -\beta s_n / e_n$.
12. Solve for ω_n using the labor supply equations: $\omega_n = L_n^{-\eta_n} C_n^{-\sigma_n} w_n$.
13. Solve for $\kappa_{nn'}^c$ and $\kappa_{nn'}^m$ using the trade share equations and the normalization $\kappa_{nn}^c = \kappa_{nn}^m = 1$:

$$\begin{aligned} \kappa_{nn'}^c &= (\varpi_{nn'}^c)^{-1/\varphi^c} e_n m c_n^c / [e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma}], \\ \kappa_{nn'}^m &= (\varpi_{nn'}^m)^{-1/\varphi^m} e_n m c_n^m / [e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma}]. \end{aligned}$$

D.3 Algorithm for Counterfactual Steady State with Balanced Trade, $s_n = 0$

Conditional on the parameters, $\{\beta, \sigma_n, \omega_n, \eta_n, \varepsilon_n, \varrho_n, g_n, \varphi^c, \varphi^m, \tau^c, \tau^m, \kappa_{nn'}^c, \kappa_{nn'}^m\}$, either obtained or calibrated in Section D.2, we now follow Dekle et al. (2007) and solve for counterfactual allocations consistent with $\xi_n = 0$ which then implies $s_n = 0$. We arrive at a solution by guessing and iterating on a vector of prices.

1. Start with an initial guess for $e_n w_n^\gamma (p_n^m)^{1-\gamma} = (e_n w_n^\gamma (p_n^m)^{1-\gamma})^0$ (in this case, the solution in D.2 consistent with $\xi_n \neq 0$, e_n^* and w_n^*).
2. Using $m c_n^c = (1 + \tau_n^c)(\varepsilon_n - 1)/\varepsilon_n$, we obtain the real (productivity adjusted) exchange rates, e_n , associated with $(e_n w_n^\gamma (p_n^m)^{1-\gamma})^0$ from:

$$e_n m c_n^c = \left(\sum_{n'} (\kappa_{nn'}^c (e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma})^0)^{-\varphi^c} \right)^{-\frac{1}{\varphi^c}}.$$

3. Calculate $m c_n^m = \left(\sum_{n'} (\kappa_{nn'}^m (e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma})^0)^{-\varphi^m} \right)^{-\frac{1}{\varphi^m}} / e_n$.
4. Calculate $p_n^m = \frac{1}{1 + \tau_n^m} \frac{\varrho_n}{\varrho_n - 1} m c_n^m$.
5. Calculate $w_n = [(e_n w_n^\gamma (p_n^m)^{1-\gamma})^0 / (e_n (p_n^m)^{1-\gamma})]^{1/\gamma}$.

6. Given, $s_n = 0$, calculate C_n as a function of L_n from the household budget constraint, $mc_n^c C_n = \left(1 - \frac{mc_n^m}{p_n^m}(1 - \gamma)\right) \frac{w_n L_n}{\gamma}$. Use this expression to substitute for C_n in the labor supply equation, $\omega_n L_n^\eta C_n^{\sigma_n} = w_n$, to obtain L_n given w_n from step 5.

7. Calculate the new trade matrices,

$$\begin{aligned}\varpi_{nn'}^c &= (\kappa_{nn'}^c e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma} / (e_n mc_n^c))^{-\varphi^c}, \\ \varpi_{nn'}^m &= (\kappa_{nn'}^m e_{n'} w_{n'}^\gamma (p_{n'}^m)^{1-\gamma} / (e_n mc_n^m))^{-\varphi^m}.\end{aligned}$$

8. Obtain L_n^c from $w_n e_n L_n^c = \gamma \sum_{n'} \varpi_{n'n}^c e_{n'} C_{n'}$.

9. Obtain a new value for $e_n w_n$, $(e_n w_n)^{\text{new}}$, using the trade-flow equation for intermediate inputs, re-written as

$$\begin{aligned}(e_n w_n)^{\text{new}} (L_n - L_n^c) &= \gamma \sum_{n'} \varpi_{n'n}^m e_{n'} mc_{n'}^m \frac{1 - \gamma}{\gamma} \frac{w_{n'} L_{n'}}{p_{n'}^m} \Rightarrow \\ (e_n w_n)^{\text{new}} &= (1 - \gamma) \sum_{n'} \varpi_{n'n}^m \frac{e_{n'} mc_{n'}^m w_{n'} L_{n'}}{p_{n'}^m} \frac{1}{L_n - L_n^c}.\end{aligned}$$

Obtain $w_n^{\text{new}} = (e_n w_n)^{\text{new}} / e_n$. Rescale all wages, w^{new} , such that $w_1^{\text{new}} = (e_1 w_1)^{\text{new}}$ (i.e, multiply all wages by e_1).

10. Construct $(e_n w_n^\gamma (p_n^m)^{1-\gamma})^{\text{new}}$ and compute

$$\sum_n \left[\frac{(e_n w_n^\gamma (p_n^m)^{1-\gamma})^{\text{new}} - (e_n w_n^\gamma (p_n^m)^{1-\gamma})^0}{(e_n w_n^\gamma (p_n^m)^{1-\gamma})^0} \right]^2.$$

Get an updated guess $(e_n w_n^\gamma (p_n^m)^{1-\gamma})^1$ and iterate to convergence using a numerical solver (e.g., `fsolve` in MATLAB).

D.4 Special Case: Traded Goods as a Single Sector

This case, underlying in part the stylized model version in the main text, is defined by: $\varphi^c = \varphi^m = \varphi$, $\varepsilon_{n,t} = \varrho_{n,t}$, $\theta^m = \theta^c = \theta$, $\kappa_{nn'}^m = \kappa_{nn'}^c = \kappa_{nn'}$ and $\tau^c = \tau^m$.

With $\kappa_{nn'}^c = \kappa_{nn'}^m$ and $\varphi^c = \varphi^m$, we have from the marginal cost equations that $mc_n^m = mc_n^c = mc_n$. Likewise, the trade share equations imply that $\varpi_{nn'}^c = \varpi_{nn'}^m = \varpi_{nn'}$. With $\varepsilon_n = \varrho_n$ and $mc_n^m = mc_n^c$, the optimal sticky price equations imply that $p_n^m = 1$. Furthermore, from the aggregate price index equations, $p_n^{m*} = 1$. With $p_n^m = 1$ and $mc_n^m = mc_n^c$, we have that

$$mc_n^c C_n + \frac{mc_n^m}{p_n^m} \frac{1 - \gamma}{\gamma} w_n L_n + mc_n^m F_n = mc_n C_n,$$

where

$$\mathcal{C}_n \equiv C_n + \frac{1-\gamma}{\gamma} w_n L_n + F_n = C_n + \mathcal{M}_n.$$

Thus the household budget constraint collapses to that in the original model. Finally, given $\varpi_{nn'}^m = \varpi_{nn'}^c = \varpi_{nn'}$ and $mc_n^c = mc_n^m = mc_n$, we can add up the trade flow equations and apply the labor market clearing condition to obtain the aggregate trade flows:

$$\begin{aligned} e_n w_n L_n &= e_n w_n L_n^c + e_n w_n L_n^m \\ &= \gamma \sum_{n'} e_{n'} \varpi_{n'n}^c mc_{n'}^c C_{n'} + \gamma \sum_{n'} e_{n'} \varpi_{n'n}^m mc_{n'}^m \mathcal{M}_{n'} \\ &= \gamma \sum_{n'} e_{n'} \varpi_{n'n} mc_{n'} (C_{n'} + \mathcal{M}_{n'}) = \gamma \sum_{n'} e_{n'} \varpi_{n'n} mc_{n'} \mathcal{C}_{n'}. \end{aligned}$$

E The Linearized System

We now log-linearize the (deflated-detrended) system around its steady state with balanced trade, meaning $s_n = 0$. We use a ‘hat’ over variables to denote log-deviations from steady-state (i.e., $\hat{x}_{n,t} = \log(x_{n,t}) - \log(x_n)$), and dx to denote level deviations from steady state for some variables, x , (i.e., $dx_{n,t} = x_{n,t} - x_n$).

E.1 Derivation of New Keynesian Phillips Curves

From the optimal price setting conditions, we can write for final goods,

$$p_{n,t}^* = \frac{1}{1 + \tau_n^c} \frac{N_{n,t}^c}{D_{n,t}^c},$$

where

$$\begin{aligned} N_{n,t}^c &= \varepsilon_{n,t} mc_{n,t}^c + E_t \Omega_{n,t+1}^c N_{n,t+1}^c, \\ D_{n,t}^c &= (\varepsilon_{n,t} - 1) + E_t \Omega_{n,t+1}^c \frac{1 + \pi_n}{1 + \pi_{n,t+1}} D_{n,t+1}^c, \end{aligned}$$

and $\Omega_{n,t+1}^c = \beta \theta^c \left(\frac{1 + \pi_{n,t+1}}{1 + \pi_n} \right)^{\varepsilon_{n,t+1}} \frac{C_{n,t+1}^{1-\sigma_n}}{C_{n,t}^{1-\sigma_n}}.$

In steady state, $\Omega_n^c = \beta \theta^c$ so that

$$N_n^c = \frac{mc_n^c \varepsilon_n}{1 - \beta \theta^c} \text{ and } D_n^c = \frac{\varepsilon_n - 1}{1 - \beta \theta^c}.$$

Log-linearizing around the steady state then gives

$$\begin{aligned}\widehat{N}_{n,t}^c &= (1 - \beta\theta^c)(\widehat{\varepsilon}_{n,t} + \widehat{mc}_{n,t}^c) + \beta\theta^c E_t \left(\widehat{N}_{n,t+1}^c + \widehat{\Omega}_{n,t+1}^c \right), \\ \widehat{D}_{n,t}^c &= (1 - \beta\theta^c) \frac{\varepsilon_n}{\varepsilon_n - 1} \widehat{\varepsilon}_{n,t} + \beta\theta^c E_t \left(\widehat{D}_{n,t+1}^c - \widehat{\pi}_{n,t+1} + \widehat{\Omega}_{n,t+1}^c \right),\end{aligned}$$

where $\widehat{\pi}_{n,t} = \frac{\pi_{n,t} - \pi}{1 + \pi}$. Bringing it all together gives for final goods,

$$\widehat{p}_{n,t}^* = \widehat{N}_{n,t}^c - \widehat{D}_{n,t}^c = (1 - \beta\theta^c) \left(\widehat{mc}_{n,t}^c - \frac{1}{\varepsilon_n - 1} \widehat{\varepsilon}_{n,t} \right) + \beta\theta^c (\widehat{p}_{n,t+1}^* + \widehat{\pi}_{n,t+1}).$$

Linearizing the price index equation associated with final goods around its steady-state, we obtain

$$\widehat{p}_{n,t}^* = \left(\frac{\theta^c}{1 - \theta^c} \right) \widehat{\pi}_{n,t}^c.$$

Then, defining $\widehat{\mu}_{n,t}^c \equiv -\frac{1}{\varepsilon_n - 1} \widehat{\varepsilon}_{n,t}$, the Phillips Curve for final goods becomes

$$\frac{\theta^c}{1 - \theta^c} \widehat{\pi}_{n,t}^c = (1 - \beta\theta^c)(\widehat{mc}_{n,t}^c + \widehat{\mu}_{n,t}^c) + \beta\theta^c E_t \left[\frac{\theta^c}{1 - \theta^c} \widehat{\pi}_{n,t+1}^c + \widehat{\pi}_{n,t+1} \right],$$

which can also be expressed as

$$\widehat{\pi}_{n,t} = \frac{1 - \theta^c}{\theta^c} (1 - \beta\theta^c)(\widehat{mc}_{n,t}^c + \widehat{\mu}_{n,t}^c) + \beta E_t \widehat{\pi}_{n,t+1}. \quad (\text{A16})$$

For intermediate inputs, we have that

$$\frac{p_{n,t}^{m,*}}{p_{n,t}^m} = \frac{1}{1 + \tau_n^m} \frac{N_{n,t}^m}{D_{n,t}^m},$$

where

$$\begin{aligned}N_{n,t}^m &= \varrho_{n,t} m c_{n,t}^m + E_t \Omega_{n,t+1}^m N_{n,t+1}^m, \\ D_{n,t}^m &= (\varrho_{n,t} - 1) p_{n,t}^m + E_t \Omega_{n,t}^m \frac{1 + \pi_n}{1 + \pi_{n,t+1}^m} D_{n,t+1}^m,\end{aligned}$$

and $\Omega_{n,t}^m = \beta (\theta^m) \left(\frac{1 + \pi_{n,t+1}^m}{1 + \pi_n} \right)^{\varrho_{n,t+1}} \frac{C_{n,t+1}^{-\sigma_n} \mathcal{M}_{n,t+1}}{C_{n,t}^{-\sigma_n} \mathcal{M}_{n,t}}$. In steady state, $\Omega_n^m = \beta \theta^m$ so that

$$N_n^m = \frac{m c_n^m \varrho_n}{1 - \beta \theta^m}, \text{ and } D_n^m = \frac{(\varrho_n - 1) p_n^m}{1 - \beta \theta^m}.$$

Log-linearizing these expressions gives

$$\begin{aligned}\widehat{N}_{n,t}^m &= (1 - \beta\theta^m)(\widehat{\varrho}_{n,t} + \widehat{m}c_{n,t}^m) + \beta\theta^m E_t \left(\widehat{N}_{n,t+1}^m + \widehat{\Omega}_{n,t+1}^m \right), \\ \widehat{D}_{n,t}^m &= (1 - \beta\theta^m)\left(\frac{\varrho_n}{\varrho_n - 1}\widehat{\varrho}_{n,t} + \widehat{p}_{n,t}^m\right) + \beta\theta^m E_t \left(\widehat{D}_{n,t+1}^m - \widehat{\pi}_{n,t+1}^m + \widehat{\Omega}_{n,t+1}^m \right).\end{aligned}$$

The key difference with intermediate goods relative to final goods is that $\widehat{D}_{n,t}^m$ depends on $\widehat{p}_{n,t}^m$. Bringing these expressions together gives

$$\begin{aligned}\widehat{p}_{n,t}^{*,m} - \widehat{p}_{n,t}^m &= \widehat{N}_{n,t}^m - \widehat{D}_{n,t}^m = (1 - \beta\theta^m) \left(\widehat{m}c_{n,t}^m - \widehat{p}_{n,t}^m - \frac{1}{\varrho_n - 1}\widehat{\varrho}_{n,t} \right) \\ &\quad + \beta\theta^m(\widehat{p}_{n,t+1}^{*,m} - \widehat{p}_{n,t+1}^m + \widehat{\pi}_{n,t+1}^m),\end{aligned}$$

where

$$\widehat{p}_{n,t}^{*,m} - \widehat{p}_{n,t}^m = \left(\frac{\theta^m}{1 - \theta^m} \right) \widehat{\pi}_{n,t}^m.$$

As before, we define $\widehat{\mu}_{n,t}^m \equiv -\frac{1}{\varrho_n - 1}\widehat{\varrho}_{n,t}$ so that

$$\frac{\theta^m}{1 - \theta^m}\widehat{\pi}_{n,t}^m = (1 - \beta\theta^m)(\widehat{m}c_{n,t}^m + \widehat{\mu}_{n,t}^m - \widehat{p}_{n,t}^m) + \beta\theta^m E_t \left[\frac{\theta^m}{1 - \theta^m}\widehat{\pi}_{n,t+1}^m + \widehat{\pi}_{n,t+1}^m \right],$$

which can also be expressed as

$$\widehat{\pi}_{n,t}^m = \frac{(1 - \beta\theta^m)(1 - \theta^m)}{\theta^m}(\widehat{m}c_{n,t}^m + \widehat{\mu}_{n,t}^m - \widehat{p}_{n,t}^m) + \beta E_t [\widehat{\pi}_{n,t+1}^m]. \quad (\text{A17})$$

E.2 The Full Linearized System

Household Budget Constraint:

$$\begin{aligned}e_n m c_n^c C \left(\widehat{e}_{n,t} + \widehat{m}c_{n,t}^c + \widehat{C}_{n,t} \right) &+ e_n m c_n^m M \left(\widehat{e}_{n,t} + \widehat{m}c_{n,t}^m - \widehat{p}_{n,t}^m + \widehat{w}_{n,t} + \widehat{L}_{n,t} \right) \\ V(1 + \pi_1)g_1 E_t[ds_{n,t+1}] &= \frac{1}{\gamma}e_n w_n L_n(\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t}) + ds_{n,t}.\end{aligned} \quad (\text{A18})$$

Recall that the above equation is derived under the assumption that $s_n = 0$. This also implies that the terms in $\widehat{e}_{n,t}$ (although we include them) cancel out.

Household Optimality Conditions:

- International Financial Securities, $S_{n,t+1}$:

$$\begin{aligned}
& -\sigma_n \widehat{C}_{n,t-1} + \widehat{V}_t + p_n^m \left[\beta \frac{s_n}{e_n} \tau \left(\widehat{V}_t + \frac{1}{s_n} ds_{n,t} + \widehat{g}_{1,t} + \widehat{\pi}_{1,t} - \widehat{e}_{n,t-1} \right) + d\xi_{n,t-1} \right] \\
& = \widehat{e}_{n,t-1} - \widehat{e}_{n,t} - \widehat{g}_{1,t} - \widehat{\pi}_{1,t} - \sigma_n \widehat{C}_{n,t},
\end{aligned} \tag{A19}$$

- Domestic Bonds, $B_{n,t}$:

$$-\sigma_n \widehat{C}_{n,t} + \sigma_n E_t \widehat{C}_{n,t+1} = \widehat{R}_{n,t} - E_t \widehat{\pi}_{n,t+1} - E_t \widehat{g}_{n,t+1}. \tag{A20}$$

- Consumption, $C_{n,t}$:

$$-\sigma_n \widehat{C}_{n,t} = \widehat{\lambda}_{n,t}. \tag{A21}$$

- Labor Supply, $L_{n,t}$:

$$\eta \widehat{L}_{n,t} + \sigma_n \widehat{C}_{n,t} = \widehat{w}_{n,t}. \tag{A22}$$

The linearized household optimality equations are identical to those in the model where we do not distinguish between traded intermediate inputs and consumption.

Phillips Curve:

- Consumption:

$$\widehat{\pi}_{n,t} = \frac{(1 - \theta^c)(1 - \beta\theta^c)}{\theta^c} (\widehat{m}c_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t [\widehat{\pi}_{n,t+1}], \text{ where } \mu_{n,t} = \frac{\varepsilon_{n,t}}{\varepsilon_{n,t} - 1}. \tag{A23}$$

- Materials:

$$\widehat{\pi}_{n,t}^m = \frac{(1 - \theta^m)(1 - \beta\theta^m)}{\theta^m} (\widehat{m}c_{n,t}^m + \widehat{\mu}_{n,t}^m - \widehat{p}_{n,t}^m) + \beta E_t [\widehat{\pi}_{n,t+1}^m], \tag{A24}$$

where $\mu_{n,t}^m = \frac{\varrho_{n,t}}{\varrho_{n,t} - 1}$ and $p_{n,t}^m = P_{n,t}^m / P_{n,t}$.

- Relative Prices:

$$\widehat{p}_{n,t}^m - \widehat{p}_{n,t-1}^m = \widehat{\pi}_{n,t}^m - \widehat{\pi}_{n,t}. \tag{A25}$$

Trade Shares:

- Consumption:

$$\begin{aligned}
\widehat{\omega}_{nn',t}^c &= -\varphi^c (\widehat{e}_{n',t} + \gamma \widehat{w}_{n',t} + (1 - \gamma) \widehat{p}_{n',t}^m) \\
&+ \varphi^c \sum_{n''} \varpi_{nn''}^c (\widehat{e}_{n'',t} + \gamma \widehat{w}_{n'',t} + (1 - \gamma) \widehat{p}_{n'',t}^m).
\end{aligned} \tag{A26}$$

- Materials:

$$\begin{aligned}\widehat{\varpi}_{nn',t}^m = & -\varphi^m(\widehat{e}_{n',t} + \gamma\widehat{w}_{n',t} + (1-\gamma)\widehat{p}_{n',t}^m) \\ & + \varphi^m \sum_{n''} \varpi_{nn''}^m(\widehat{e}_{n'',t} + \gamma\widehat{w}_{n'',t} + (1-\gamma)\widehat{p}_{n'',t}^m).\end{aligned}\quad (\text{A27})$$

Trade Flows:

- Consumption: $\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t}^c = \sum_{n'} \psi_{n'n}^c \left(\widehat{e}_{n',t} + \widehat{\varpi}_{n'n,t}^c + \widehat{m}c_{n',t} + \widehat{C}_{n',t} \right)$, where $\psi_{n',n}^c = \frac{\varpi_{n'n}^c e_{n'} m c_{n'} C_{n'}}{\sum_{n''} \varpi_{n''n}^c e_{n''} m c_{n''} C_{n''}}$.
- Materials: $\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t}^m = \sum_{n'} \psi_{n'n}^m \left(\widehat{e}_{n',t} + \widehat{\varpi}_{n'n,t}^m + \widehat{m}c_{n',t}^m + \widehat{w}_{n',t} + \widehat{L}_{n',t} - \widehat{p}_{n',t}^m \right)$, where $\psi_{n',n}^m = \frac{\varpi_{n'n}^m e_{n'} m c_{n'}^m M_{n'}}{\sum_{n''} \varpi_{n''n}^m e_{n''} m c_{n''}^m M_{n''}}$.

Marginal Cost:

- Retail:

$$\widehat{e}_{n,t} + \widehat{m}c_{n,t}^c = \sum_{n'} \varpi_{nn'}^c(\widehat{e}_{n',t} + \gamma\widehat{w}_{n',t} + (1-\gamma)\widehat{p}_{n',t}^m). \quad (\text{A28})$$

- Intermediate Inputs:

$$\widehat{e}_{n,t} + \widehat{m}c_{n,t}^m = \sum_{n'} \varpi_{nn'}^m(\widehat{e}_{n',t} + \gamma\widehat{w}_{n',t} + (1-\gamma)\widehat{p}_{n',t}^m). \quad (\text{A29})$$

No Arbitrage: $\widehat{e}_{1,t} = 0$.

Monetary Policy Rule: $\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)(\phi_\pi\widehat{\pi}_t + \phi_Y\widehat{Y}_t) + \widehat{\nu}_t$.

Labor Market Clearing: $\widehat{L}_t = \frac{L^m}{L}\widehat{L}_t^m + \frac{L^c}{L}\widehat{L}_t^c$.

E.3 System Reduction

We first reduce the system by substituting out the trade shares and the expression for marginal costs. The only equations that change are the trade flow equations:

- Consumption:

$$\begin{aligned}(1+\varphi^c)\widehat{e}_{n,t} + (1+\gamma\varphi^c)\widehat{w}_{n,t} + (1-\gamma)\varphi^c\widehat{p}_{n,t}^m + \widehat{L}_{n,t}^c \\ = \sum_{n'} \psi_{n'n}^c \left((1+\varphi^c)(\widehat{m}c_{n',t}^c + \widehat{e}_{n',t}) + \widehat{C}_{n',t} \right).\end{aligned}$$

- Materials:

$$(1 + \varphi^m)\widehat{e}_{n,t} + (1 + \gamma\varphi^m)\widehat{w}_{n,t} + (1 - \gamma)\varphi^m\widehat{p}_{n,t}^m + \widehat{L}_{n,t}^m \\ = \sum_{n'} \psi_{n'n}^m \left((1 + \varphi^m)(\widehat{m}\widehat{c}_{n',t}^m + \widehat{e}_{n',t}) + \widehat{w}_{n',t} + \widehat{L}_{n',t} - \widehat{p}_{n',t}^m \right).$$

We now stack country-specific variables into vectors, $\widehat{\mathbf{x}}_t = (\widehat{x}_{1,t}, \dots, \widehat{x}_{N,t})'$, and similarly with parameters. The final system of equations in vector form is then given by (with some abuse of notation, and making use of $\mathbf{s} = 0$):

$$\mathbf{m}\mathbf{c}^c\mathbf{C}\mathbf{e} \left(\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^c + \widehat{\mathbf{C}}_t + \widehat{\mathbf{e}}_t \right) + Vg_1(1 + \pi_1)E_t[d\mathbf{s}_{t+1}] \\ = \left(\frac{1}{\gamma} - \frac{\mathbf{m}\mathbf{c}^m}{\mathbf{p}^m} \frac{1 - \gamma}{\gamma} \right) \mathbf{e}\mathbf{w}\mathbf{L} \left(\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t + \widehat{\mathbf{L}}_t \right) - \frac{\mathbf{m}\mathbf{c}^m}{\mathbf{p}^m} \frac{1 - \gamma}{\gamma} \mathbf{e}\mathbf{w}\mathbf{L} (\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^m - \widehat{\mathbf{p}}_t^m) + d\mathbf{s}_t, \quad (\text{A30})$$

$$-\sigma\widehat{\mathbf{C}}_{t-1} + \widehat{\mathbf{V}}_t + \beta\tau \frac{\mathbf{p}^m}{\mathbf{e}} d\mathbf{s}_t + \mathbf{p}^m \times d\boldsymbol{\xi}_{t-1} = \widehat{\mathbf{e}}_{t-1} - \widehat{\mathbf{e}}_t - \widehat{\mathbf{g}}_{1,t} - \widehat{\boldsymbol{\pi}}_{1,t} - \sigma\widehat{\mathbf{C}}_t, \quad (\text{A31})$$

$$-\sigma\widehat{\mathbf{C}}_t + \sigma E_t\widehat{\mathbf{C}}_{t+1} = \widehat{\mathbf{R}}_t - E_t\widehat{\boldsymbol{\pi}}_{t+1} - E_t\widehat{\mathbf{g}}_{t+1}, \quad (\text{A32})$$

$$\boldsymbol{\eta}\widehat{\mathbf{L}}_t + \sigma\widehat{\mathbf{C}}_t = \widehat{\mathbf{w}}_t, \quad (\text{A33})$$

$$\widehat{\boldsymbol{\pi}}_t = \frac{(1 - \boldsymbol{\theta}^c)(1 - \beta\boldsymbol{\theta}^c)}{\boldsymbol{\theta}^c} (\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^c + \widehat{\boldsymbol{\mu}}_t^c) + \beta E_t[\widehat{\boldsymbol{\pi}}_{t+1}], \quad (\text{A34})$$

$$\widehat{\boldsymbol{\pi}}_t^m = \frac{(1 - \boldsymbol{\theta}^m)(1 - \beta\boldsymbol{\theta}^m)}{\boldsymbol{\theta}^m} (\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^m - \widehat{\mathbf{p}}_t^m + \widehat{\boldsymbol{\mu}}_t^m) + \beta E_t[\widehat{\boldsymbol{\pi}}_{t+1}^m], \quad (\text{A35})$$

$$(1 + \varphi^c) \left(\widehat{\mathbf{e}}_t + \frac{1 + \gamma\varphi^c}{1 + \varphi^c} \widehat{\mathbf{w}}_t + \frac{(1 - \gamma)\varphi^c}{1 + \varphi^c} \widehat{\mathbf{p}}_t^m \right) + \widehat{\mathbf{L}}_t^c \\ = (\boldsymbol{\psi}^c)^T \left((1 + \varphi^c)(\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^c + \widehat{\mathbf{e}}_t) + \widehat{\mathbf{C}}_t \right), \quad (\text{A36})$$

$$(1 + \varphi^m) \left(\widehat{\mathbf{e}}_t + \frac{1 + \gamma\varphi^m}{1 + \varphi^m} \widehat{\mathbf{w}}_t + \frac{(1 - \gamma)\varphi^m}{1 + \varphi^m} \widehat{\mathbf{p}}_t^m \right) + \widehat{\mathbf{L}}_t^m \\ = (\boldsymbol{\psi}^m)^T \left((1 + \varphi^m)(\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^m + \widehat{\mathbf{e}}_t) + \widehat{\mathbf{w}}_t + \widehat{\mathbf{L}}_t - \widehat{\mathbf{p}}_t^m \right), \quad (\text{A37})$$

$$\widehat{\mathbf{e}}_t + \widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^c = \boldsymbol{\varpi}^c(\widehat{\mathbf{e}}_t + \gamma\widehat{\mathbf{w}}_t + (1 - \gamma)\widehat{\mathbf{p}}_t^m), \quad (\text{A38})$$

$$\widehat{\mathbf{e}}_t + \widehat{\mathbf{m}}\widehat{\mathbf{c}}_t^m = \boldsymbol{\varpi}^m(\widehat{\mathbf{e}}_t + \gamma\widehat{\mathbf{w}}_t + (1 - \gamma)\widehat{\mathbf{p}}_t^m), \quad (\text{A39})$$

$$\widehat{\mathbf{L}}_t = \frac{\mathbf{L}^m}{\mathbf{L}} \widehat{\mathbf{L}}_t^m + \frac{\mathbf{L}^c}{\mathbf{L}} \widehat{\mathbf{L}}_t^c. \quad (\text{A40})$$

$$\widehat{\mathbf{p}}_t^m - \widehat{\mathbf{p}}_{t-1}^m = \boldsymbol{\pi}_t^m - \boldsymbol{\pi}_t. \quad (\text{A41})$$

$$\widehat{e}_{1,t} = 0. \quad (\text{A42})$$

E.4 Special Case: Traded Goods as a Single Sector

Relative to a model where we do not distinguish between traded final goods and traded intermediate inputs, we have two sets of Phillips curves, (A34) and (A35), two sets of trade equations (A36) and (A37), and two sets of marginal costs (A38) and (A39). In addition, we now have equations for labor market clearing, in (A40), and for the change in relative prices, in (A41).

To recover the model where traded goods are modeled as a single sector, underlying in part the stylized model version motivating Proposition 1, let $\varepsilon_t = \varrho_t$, so the path of the markups is the same.⁶ In addition, suppose $\varphi^c = \varphi^m = \varphi$, $\theta^c = \theta^m = \theta$ and $\kappa_{nn'}^c = \kappa_{nn'}^m \forall n, n'$. Then from the marginal cost equations, (A29) and (A28), $\widehat{mc}_{n,t} = \widehat{mc}_{n,t}^m$, and from the Phillips curve equations, (A23) and (A24), $\pi_{n,t} = \pi_{n,t}^m$ for all t so long as $\widehat{p}_{n,t}^m = 0$. Furthermore, in equation (A25), $\pi_t^c = \pi_t^m$ implies that $\widehat{p}_{n,t}^m = 0$. The trade share equations (A26) and (A27) also imply that $\varpi_{nn',t}^m = \varpi_{nn',t}^c$. With $\widehat{p}_{n,t}^m = 0$, both the marginal cost and trade share equations also collapse to the marginal cost equation in the baseline. The household optimality conditions (A19) to (A22) remain unchanged (and as in the baseline). It also follows immediately that, given $\widehat{p}_{n,t}^m = 0$ the household budget constraint (A18) collapses to the one in the baseline model. Finally, adding up the trade flow equations for final goods and intermediate goods (A13) and (A14) (not log-linearized) and applying the labor market clearing equation (A15), we have that

$$e_{n,t} w_{n,t} L_{n,t} = \gamma \sum_{n'} e_{n',t} (\varpi_{n'n,t} m c_{n',t} \Delta_{n',t} (C_{n',t} + \mathcal{M}_{n',t})).$$

Deflating, detrending and log-linearizing this equation, while using $\widehat{mc}_{n,t}$ and $\widehat{\varpi}_{nn',t}$ to denote the common values of marginal costs and trade shares, yields the log-linearized trade flow equations in the baseline model (recall that $\widehat{\mathcal{M}}_{n',t} = \widehat{M}_{n',t} = \widehat{w}_{n',t} + \widehat{L}_{n',t}$),

$$\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t} = \sum_{n'} \psi_{n'n} \left[\widehat{e}_{n',t} + \widehat{\varpi}_{n'n,t} + \widehat{mc}_{n',t} + \iota_{n'} \widehat{C}_{n',t} + (1 - \iota_{n'}) (\widehat{w}_{n',t} + \widehat{L}_{n',t}) \right],$$

where $\psi_{n'n}$ are export shares and $\iota_n = \frac{C_n}{C_n + M_n}$ is the share of consumption in local expenditures.

F Analytical Results and Derivations

To develop intuition, we now study a version of our model economy under some simplifying assumptions. In particular, we set $\varepsilon_t = \varrho_t$, $\varphi^c = \varphi^m$, $\theta^c = \theta^m$, $\tau_n^c = \tau_n^m$ and $\kappa_{nn'}^c = \kappa_{nn'}^m \forall n, n'$. As discussed above, these simplifying assumptions effectively collapse the traded intermediate goods and final goods sectors into a single sector. For analytical analysis we also remove heterogeneity in preference and policy parameters ($\sigma_n = \sigma$ etc) and assume there are underlying subsidies so that $mc_n = 1$ in steady state, which considerably simplifies equations. Finally, we focus here (and

⁶We set these to be the same even when estimating the full model to avoid introducing additional parameters and shocks for each country.

also in the quantitative estimation) on a model linearized around zero trade deficit steady-state ($s_n = 0$). We also now introduce new notation, with $\mathcal{C}_{n,t} = C_{n,t} + \mathcal{M}_{n,t}$ incorporating the whole of domestic absorption. We present the resulting set of linearized equations below.

F.1 Simplified Linearization

Household Budget Constraint:

$$\frac{1}{\gamma} \widehat{m} \widehat{c}_{n,t} + \widehat{C}_{n,t} + \widehat{e}_{n,t} + V g_1 (1 + \pi_1) E_t \left[\frac{ds_{n,t+1}}{C_n e_n} \right] = \widehat{w}_{n,t} + \widehat{L}_{n,t} + \frac{ds_{n,t}}{C_n e_n}.$$

Household Optimality Conditions:

- International Financial Securities, $S_{n,t+1}$:

$$-\sigma \widehat{C}_{n,t-1} + \widehat{V}_t + \beta \tau \frac{ds_n}{e_n} + d\xi_{n,t-1} = \widehat{e}_{n,t-1} - \widehat{e}_{n,t} - \widehat{g}_{1,t} - \widehat{\pi}_{1,t} - \sigma \widehat{C}_{n,t}.$$

- Domestic Bonds, $B_{n,t}$: $-\sigma \widehat{C}_{n,t} + \sigma E_t \widehat{C}_{n,t+1} = \widehat{R}_{n,t} - E_t \widehat{\pi}_{n,t+1} - E_t \widehat{g}_{n,t+1}$.
- Consumption, $C_{n,t}$: $-\sigma \widehat{C}_{n,t} = \widehat{\lambda}_{n,t}$.
- Labor Supply, $L_{n,t}$: $\eta \widehat{L}_{n,t} + \sigma \widehat{C}_{n,t} = \widehat{w}_{n,t}$.

Phillips Curve: $\widehat{\pi}_{n,t} = \frac{(1-\theta)(1-\beta\theta)}{\theta} (\widehat{m} \widehat{c}_{n,t} + \widehat{\mu}_{n,t}) + \beta E_t [\widehat{\pi}_{n,t+1}]$, where $\mu_{n,t} = \frac{\varepsilon_{n,t}}{\varepsilon_{n,t}-1}$.

Trade Shares: $\widehat{\omega}_{nn',t} = -\varphi(\widehat{e}_{n',t} + \gamma \widehat{w}_{n',t}) + \varphi \sum_{n''} \varpi_{nn''} (\widehat{e}_{n'',t} + \gamma \widehat{w}_{n'',t})$.

Trade Flows: $\widehat{e}_{n,t} + \widehat{w}_{n,t} + \widehat{L}_{n,t} = \sum_{n'} \psi_{n'n} (\widehat{e}_{n',t} + \widehat{\omega}_{n'n,t} + \widehat{m} \widehat{c}_{n',t} + \widehat{C}_{n',t})$, where

$$\psi_{n',n} = \frac{\varpi_{n'n} e_{n'} m c_{n'} C_{n'}}{\sum_{n''} \varpi_{n''n} e_{n''} m c_{n''} C_{n''}}.$$

Domestic Absorption: $\widehat{C}_{n,t} = \frac{C_n}{\widehat{C}_n} \widehat{C}_{n,t} + \frac{1-\gamma}{\gamma} \frac{w_n L_n}{\widehat{C}_n} (\widehat{w}_{n,t} + \widehat{L}_{n,t})$.

Marginal Cost: $\widehat{e}_{n,t} + \widehat{m} \widehat{c}_{n,t} = \sum_{n'} \varpi_{nn'} (\widehat{e}_{n',t} + \gamma \widehat{w}_{n',t})$.

No Arbitrage: $\widehat{e}_{1,t} = 0$.

Monetary Policy Rule: $\widehat{R}_t = \rho \widehat{R}_{t-1} + (1-\rho)(\phi_\pi \widehat{\pi}_t + \phi_Y \widehat{Y}_t) + \widehat{v}_t$.

F.2 System Reduction and International Block of Linearized Model

We reduce this system of equations by applying the domestic absorption equation to eliminate $\mathcal{C}$ and using the relationship $\mathcal{C}_n = C_n + \frac{1-\gamma}{\gamma}w_n L_n$. Next, we combine the equations describing the evolution of trade shares and the marginal cost of sticky-price goods to get $\widehat{w}_{nn',t} = \varphi(\widehat{mc}_{n,t} + \widehat{e}_{n,t} - \gamma\widehat{w}_{n',t} - \widehat{e}_{n',t})$, which we substitute for trade shares, $\widehat{w}_{nn'}$. Furthermore, let $\iota_n = \frac{C_n}{\mathcal{C}_n}$. Finally, we stack country-specific variables into vectors $\widehat{\mathbf{x}}_t = (\widehat{x}_{1,t}, \dots, \widehat{x}_{N,t})'$ and similarly with parameters. The final system of equations in vector form is then given by (with some abuse of notation):

$$\begin{aligned} & \mathbf{C}\mathbf{e} \left(\widehat{\mathbf{m}}\mathbf{c}_t + \widehat{\mathbf{C}}_t + \widehat{\mathbf{e}}_t \right) + Vg_1(1 + \pi_1)E_t[ds_{t+1}] \\ &= \mathbf{e}\mathbf{w}\mathbf{L} \left(\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t + \widehat{\mathbf{L}}_t \right) - \frac{1-\gamma}{\gamma}\mathbf{e}\mathbf{w}\mathbf{L}\widehat{\mathbf{m}}\mathbf{c}_t + ds_t, \end{aligned} \quad (\text{A43})$$

$$\begin{aligned} & -\sigma\widehat{\mathbf{C}}_{t-1} + \widehat{\mathbf{V}}_t + \beta\frac{\tau}{\mathbf{e}}ds_t + d\boldsymbol{\xi}_{t-1} \\ &= \widehat{\mathbf{e}}_{t-1} - \widehat{\mathbf{e}}_t - \widehat{\mathbf{g}}_{1,t} - \widehat{\boldsymbol{\pi}}_{1,t} - \sigma\widehat{\mathbf{C}}_t, \end{aligned} \quad (\text{A44})$$

$$-\sigma\widehat{\mathbf{C}}_t + \sigma E_t\widehat{\mathbf{C}}_{t+1} = \widehat{\mathbf{R}}_t - E_t\widehat{\boldsymbol{\pi}}_{t+1} - E_t\widehat{\mathbf{g}}_{t+1}, \quad (\text{A45})$$

$$\boldsymbol{\eta}\widehat{\mathbf{L}}_t + \sigma\widehat{\mathbf{C}}_t = \widehat{\mathbf{w}}_t, \quad (\text{A46})$$

$$\widehat{\boldsymbol{\pi}}_t = \frac{(1-\boldsymbol{\theta})(1-\beta\boldsymbol{\theta})}{\boldsymbol{\theta}}(\widehat{\mathbf{m}}\mathbf{c}_t + \widehat{\boldsymbol{\mu}}_t) + \beta E_t[\widehat{\boldsymbol{\pi}}_{t+1}], \quad (\text{A47})$$

$$(1 + \varphi) \left(\widehat{\mathbf{e}}_t + \frac{1 + \gamma\varphi}{1 + \varphi}\widehat{\mathbf{w}}_t \right) + \widehat{\mathbf{L}}_t = \boldsymbol{\psi}^T \left((1 + \varphi)(\widehat{\mathbf{m}}\mathbf{c}_t + \widehat{\mathbf{e}}_t) + \boldsymbol{\iota}\widehat{\mathbf{C}}_t + (1 - \boldsymbol{\iota})(\widehat{\mathbf{w}}_t + \widehat{\mathbf{L}}_t) \right), \quad (\text{A48})$$

$$\widehat{\mathbf{e}}_t + \widehat{\mathbf{m}}\mathbf{c}_t = \boldsymbol{\varpi}(\widehat{\mathbf{e}}_t + \gamma\widehat{\mathbf{w}}_t), \quad (\text{A49})$$

$$\widehat{e}_{1,t} = 0. \quad (\text{A50})$$

This set of equations may be framed in terms of two blocks, one familiar from the open economy literature, and the other familiar from the New Keynesian literature. For transparency, we set the subsidy rate to firms producing Calvo goods, $1 + \tau_n$ (with $\tau_n = \tau_n^c = \tau_n^m$) to equal the markup to eliminate any effects stemming from steady-state distortions associated with monopolistic pricing. The long-run marginal cost of Calvo goods is then 1 in all countries (i.e., $mc_n = 1 \forall n$). Recall also that our benchmark steady state is such that $s_n = 0 \forall n$. Then, the international block of equations can be summarized as follows.

Marginal Cost of Producing Sticky-Price Goods: Equation (A38).

Market Clearing for Traded Goods: Rewrite (A48) to obtain

$$\widehat{\mathbf{Y}}_t - \widehat{\mathbf{C}}_t = (\boldsymbol{\psi}^T\boldsymbol{\iota} - \mathbf{I})\widehat{\mathbf{C}}_t - (1 + \varphi)(\mathbf{I} - \boldsymbol{\psi}^T\boldsymbol{\varpi})(\widehat{\mathbf{e}}_t + \gamma\widehat{\mathbf{w}}_t) - (1 - \gamma)\widehat{\mathbf{w}}_t + \boldsymbol{\psi}^T(1 - \boldsymbol{\iota})(\widehat{\mathbf{w}}_t + \widehat{\mathbf{Y}}_t). \quad (\text{A51})$$

Balance of Payments: Rearranging the household budget constraint, linearized around a steady state with balanced trade and no monopolistic trade distortions, gives

$$C \left(\hat{w}_t + \hat{Y}_t - \frac{1}{\gamma} \widehat{mc}_t - \hat{C}_t \right) = \frac{V g_1 (1 + \pi_1) E_t [ds_{t+1}] - ds_t}{e}.$$

Using the fact that $\widehat{mc}_t - \gamma \hat{w}_t = (\varpi - \mathbf{I})(\hat{e}_t + \gamma \hat{w}_t)$ (by re-arranging the marginal cost of sticky-price goods above) we can obtain the balance of payments equation in the main text.

Financial Asset Allocations: In a steady state with balanced trade, the evolution of countries' financial asset positions follows

$$-\sigma \hat{C}_{t-1} + \hat{V}_t + \beta \frac{\tau}{e} ds_t + d\xi_{t-1} = \hat{e}_{t-1} - \hat{e}_t - \hat{g}_{1,t} - \hat{\pi}_{1,t} - \sigma \hat{C}_t.$$

Subtracting the analogous expression written in terms of the reference country gives the financial asset allocation equation in the main text.

F.3 Deviations from Uncovered Interest Rate Parity

Our goods and asset trading equations imply an uncovered interest parity (UIP) condition. Specifically, starting with

$$\sigma E_t [\hat{C}_{t+1} - \hat{C}_t] + E_t [\hat{V}_{t+1}] + \beta \frac{\tau}{e} E_t [ds_{t+1}] + d\xi_t = E_t [\hat{e}_t - \hat{e}_{t+1}] - E_t [\hat{g}_{1,t+1} + \hat{\pi}_{1,t+1}]$$

and substituting $\sigma E_t [\hat{C}_{t+1} - \hat{C}_t] = \hat{R}_t - E_t [\hat{g}_{t+1} - \hat{\pi}_{t+1}]$, we have

$$\hat{R}_t - E_t [\hat{g}_{t+1} - \hat{\pi}_{t+1}] + E_t [\hat{V}_{t+1}] + \beta \frac{\tau}{e} E_t [ds_{t+1}] + d\xi_t = E_t [\hat{e}_t - \hat{e}_{t+1}] - E_t [\hat{g}_{1,t+1} + \hat{\pi}_{1,t+1}].$$

Subtracting the same equation written for the reference country yields the UIP condition:

$$\begin{aligned} E_t \hat{e}_{t+1} - \hat{e}_t = & -\hat{R}_t + E_t [\hat{\pi}_{t+1} + \hat{g}_{t+1}] + \mathbf{1} \left(\hat{R}_{1,t} - E_t [\hat{\pi}_{1,t+1} + \hat{g}_{1,t+1}] \right) \\ & - \beta \tau E_t \left(\frac{1}{e} ds_{t+1} - \mathbf{1} ds_{1,t+1} \right) - (d\xi_t - \mathbf{1} d\xi_{1,t}), \end{aligned} \quad (\text{A52})$$

where $\beta \tau E_t \left(\frac{1}{e} ds_{t+1} - \mathbf{1} ds_{1,t+1} \right)$ are costs arising from endogenous portfolio adjustments and $(d\xi_t - \mathbf{1} d\xi_{1,t})$ are exogenous disturbances that affect countries' trading in global financial markets.

F.4 An Analytical Example

We now derive the solution for the analytical case studied in the main text. Under the assumptions described in the text, the New Keynesian block reduces to:

$$\begin{aligned} -\sigma\hat{C}_t &= -\sigma E_t\hat{C}_{t+1} + \hat{R}_t - E_t\hat{\pi}_{t+1}, \\ \mathcal{A}_{YY}\hat{Y}_t &= \mathcal{A}_{YC}\hat{C}_t + \mathcal{A}_{Ye}\hat{e}_t, \\ \hat{\pi}_t &= \mathcal{A}_{\pi C}\hat{C}_t + \mathcal{A}_{\pi e}\hat{e}_t + \beta E_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{\mu}_t, \\ \sigma(\hat{C}_{1,t} - \hat{C}_t) &= \hat{e}_t, \end{aligned}$$

where $\mathcal{A}_{YY}$, $\mathcal{A}_{YC}$ and $\mathcal{A}_{Ye}$ are matrices of coefficients obtained from using the labor supply equation, (A46), to substitute out $\hat{w}_t$ in the market clearing condition for traded goods, (A48). Similarly, $\mathcal{A}_{\pi C}$ and $\mathcal{A}_{\pi e}$ are matrices obtained by using the resulting market clearing condition for traded goods to eliminate $\hat{Y}_t$ from the global Phillips curve. Specifically, we have that

$$\begin{aligned} \mathcal{A}_{YY} &= \mathbf{I} + (1+\varphi)(\mathbf{I} - \psi^T\varpi), & \mathcal{A}_{\pi C} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(\eta\varpi\mathcal{A}_{YY}^{-1}\mathcal{A}_{YC} + \sigma\varpi), \\ \mathcal{A}_{YC} &= \psi^T - (1+\varphi)(\mathbf{I} - \psi^T\varpi)\sigma, & \mathcal{A}_{\pi e} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(\eta\varpi\mathcal{A}_{YY}^{-1}\mathcal{A}_{Ye} + \varpi - \mathbf{I}), \\ \mathcal{A}_{Ye} &= -(1+\varphi)(\mathbf{I} - \psi^T\varpi), \end{aligned}$$

where $\mathcal{A}_{ij}$ refers to the effect of variable j on variable i . Individual elements of the $\mathcal{A}$ matrices capture the effects of a country n' on another country n . For example, the $(1,2)$ element of $\mathcal{A}_{\pi e}$ refers to the effect of country 2's exchange rate on inflation in country 1.

We now use the monetary policy rule to substitute for the nominal rates, $\hat{R}_t$, and use the [Backus and Smith \(1993\)](#) equation to substitute for the vector of exchange rates, $\hat{e}_t$, which yields

$$\begin{aligned} -\sigma\hat{C}_t &= -\sigma E_t\hat{C}_{t+1} + \phi E_t\hat{\pi}_{t+1} + \sigma\nu_t - E_t\hat{\pi}_{t+1}, \\ \hat{Y}_t &= \mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\hat{C}_t, \\ \hat{\pi}_t &= (\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\hat{C}_t + \beta E_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{\mu}_t, \end{aligned}$$

where $\mathcal{A}_{\pi e}\mathbf{1} = \mathbf{0}$ is used to eliminate the scalar $\hat{C}_{1,t}$.⁷ Observe also that

$$\begin{aligned} \mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye} &= \psi^T, \\ \mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e} &= \frac{(1-\theta)(1-\beta\theta)}{\theta}(\eta\varpi\mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye}) + \sigma\mathbf{I}). \end{aligned}$$

We solve this system of linear rational expectations equations analytically using undetermined coefficients.

⁷In the framework developed by [Eaton and Kortum \(2002\)](#), the matrices of import and export shares, ϖ and ψ^T , are derived as stochastic matrices. Therefore, $(\varpi - \mathbf{I})\mathbf{1} = (\mathbf{I} - \psi^T)\mathbf{1} = \mathbf{0}$. The fact that $\mathcal{A}_{\pi e}\mathbf{1} = \mathbf{0}$ then follows immediately from the definition of $\mathcal{A}_{\pi e}$ and the fact that the product of stochastic matrices is also a stochastic matrix.

Model Solution and Equilibrium: We conjecture that in equilibrium, endogenous variables are linear functions of the states, in this case the exogenous shocks.⁸ Thus, let the solutions for consumption, employment, and inflation be given by

$$\widehat{C}_t = \alpha_{C\nu}\nu_t + \alpha_{C\mu}\widehat{\mu}_t, \quad \widehat{Y}_t = \alpha_{Y\nu}\nu_t + \alpha_{Y\mu}\widehat{\mu}_t, \quad \widehat{\pi}_t = \alpha_{\pi\nu}\nu_t + \alpha_{\pi\mu}\widehat{\mu}_t,$$

where $\alpha_{C\nu}, \alpha_{C\mu}, \alpha_{Y\nu}, \alpha_{Y\mu}, \alpha_{\pi\nu}$, and $\alpha_{\pi\mu}$ are unknown coefficients. To solve for these coefficients, we substitute these solutions into the model equations. It is then straightforward to show that the coefficients describing the solutions for consumption, $\widehat{C}_t$, output, $\widehat{Y}_t$, and inflation, $\widehat{\pi}_t$, as functions of monetary policy and markup shocks satisfy:

$$\begin{aligned} -\alpha_{C\nu} &= \mathbf{I}, & -\sigma\alpha_{C\mu} &= \mathbf{0}, \\ \alpha_{Y\mu} &= \mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\alpha_{C\mu}, & \alpha_{Y\nu} &= \mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye})\alpha_{C\nu}, \\ \alpha_{\pi\nu} &= (\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\alpha_{C\nu}, & \alpha_{\pi\mu} &= (\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\alpha_{C\mu} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\mathbf{I}. \end{aligned}$$

The model solution can thus be written:

$$\widehat{C}_t = -\nu_t, \quad \widehat{Y}_t = -\mathcal{A}_{YY}^{-1}\psi^T\nu_t, \quad \widehat{\pi}_t = -(\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\nu_t + \frac{(1-\theta)(1-\beta\theta)}{\theta}\widehat{\mu}_t.$$

International Comovement Properties in a Global New Keynesian Economy: For simplicity, let all elements of ν_t and $\widehat{\mu}_t$ have unit variances. This is without loss of generality—the proposition holds even when shock variances are away from 1 and country-specific.

To characterize comovement in the global New Keynesian economy, observe first that we can write the vector of inflation across countries, $\widehat{\pi}_t$, as a function of countries' output gap, $\widehat{Y}_t$ and exogenous disturbances. In particular, since $\widehat{\pi}_t = -(\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e})\nu_t + \kappa\widehat{\mu}_t$ and $\mathcal{A}_{\pi C} - \sigma\mathcal{A}_{\pi e} = \kappa(\eta\varpi\mathcal{A}_{YY}^{-1}(\mathcal{A}_{YC} - \sigma\mathcal{A}_{Ye}) + \sigma\mathbf{I})$ above, where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, it follows that

$$\widehat{\pi}_t = \kappa\eta\varpi\widehat{Y}_t - \kappa\sigma\nu_t + \kappa\widehat{\mu}_t.$$

We thus have all the ingredients to prove Proposition 1.

Proposition 1. *For any matrix $\mathbf{M}$, let $\mathbf{M} \geq 0$ if and only if $m_{ij} \geq 0 \forall ij$. Then, under the maintained assumptions, $E[\widehat{Y}_t\widehat{Y}_t^T] \geq 0$, $E[\widehat{\pi}_t\widehat{\pi}_t^T] \geq 0$ and $E[\widehat{\pi}_t\widehat{Y}_t^T] \geq 0$.*

Proof. The solution for output in every country is, in vector form,

$$\widehat{Y}_t = -\mathcal{A}_{YY}^{-1}\psi^T\nu_t, \text{ where } \mathcal{A}_{YY}^{-1} = (1-b)(\mathbf{I} - b\psi^T\varpi)^{-1},$$

⁸In this example, we do not allow for equilibria that depend on sunspots. See Galí (2008) for a discussion of the conditions on ϕ , beyond the Taylor principle, that guarantee a unique equilibrium.

with implied variance-covariance matrix given by

$$E \left[\widehat{\mathbf{Y}}_t \widehat{\mathbf{Y}}_t^T \right] = \mathcal{A}_{YY}^{-1} \boldsymbol{\psi}^T \boldsymbol{\psi} (\mathcal{A}_{YY}^{-1})^T.$$

Since $\boldsymbol{\psi}^T \geq 0$, it immediately follows that $E \left[\widehat{\mathbf{Y}}_t \widehat{\mathbf{Y}}_t^T \right] \geq 0$ whenever $\mathcal{A}_{YY}^{-1} \geq 0$.

To show that $\mathcal{A}_{YY}^{-1} \geq 0$, recall that $\mathcal{A}_{YY} = a(\mathbf{I} - b\boldsymbol{\psi}^T \boldsymbol{\varpi})$ where $b = \frac{(1+\varphi)\eta}{1+(1+\varphi)\eta}$ and $a = \frac{1}{1-b}$. Because the matrices of import and export shares, $\boldsymbol{\varpi}$ and $\boldsymbol{\psi}^T$, respectively, are stochastic matrices, so is their product, $\boldsymbol{\psi}^T \boldsymbol{\varpi}$. It follows that $\rho(\boldsymbol{\psi}^T \boldsymbol{\varpi}) = 1$ where $\rho(\boldsymbol{\psi}^T \boldsymbol{\varpi}) = \max_{\Lambda(\boldsymbol{\psi}^T \boldsymbol{\varpi})} \{|\lambda|\}$ such that $\Lambda(\boldsymbol{\psi}^T \boldsymbol{\varpi}) = \{\lambda | \lambda \text{ is an eigenvalue of } \boldsymbol{\psi}^T \boldsymbol{\varpi}\}$, i.e., the spectral radius of $\boldsymbol{\psi}^T \boldsymbol{\varpi}$ is 1. In addition, since $b < 1$, the spectral radius of $b\boldsymbol{\psi}^T \boldsymbol{\varpi}$ is strictly less than 1, $\rho(b\boldsymbol{\psi}^T \boldsymbol{\varpi}) < 1$. We can then write

$$\mathcal{A}_{YY}^{-1} = [a(\mathbf{I} - b\boldsymbol{\psi}^T \boldsymbol{\varpi})]^{-1} = \frac{1}{a} \sum_{m=0}^{\infty} b^m (\boldsymbol{\psi}^T \boldsymbol{\varpi})^m.$$

Given that $\boldsymbol{\psi}^T \boldsymbol{\varpi} \geq 0$, it follows that $\mathcal{A}_{YY}^{-1} \geq 0$ and therefore $E \left[\widehat{\mathbf{Y}}_t \widehat{\mathbf{Y}}_t^T \right] \geq 0$.

The solution for inflation above is given by

$$\begin{aligned} \widehat{\pi}_t &= \kappa \eta \boldsymbol{\varpi} \widehat{\mathbf{Y}}_t - \kappa \sigma \boldsymbol{\nu}_t + \kappa \boldsymbol{\mu}_t \\ &= -\kappa (\eta \boldsymbol{\varpi} \mathcal{A}_{YY}^{-1} \boldsymbol{\psi}^T + \sigma) \boldsymbol{\nu}_t + \kappa \boldsymbol{\mu}_t. \end{aligned}$$

Therefore, we can express the variance-covariance matrix for inflation, $\widehat{\pi}$, as

$$E \left[\widehat{\pi}_t \widehat{\pi}_t^T \right] = \kappa^2 \left[\eta^2 \boldsymbol{\varpi} \mathcal{A}_{YY}^{-1} \boldsymbol{\psi}^T \boldsymbol{\psi} (\mathcal{A}_{YY}^{-1})^T \boldsymbol{\varpi}^T + \eta \sigma (\boldsymbol{\varpi} \mathcal{A}_{YY}^{-1} \boldsymbol{\psi}^T + \boldsymbol{\psi} (\mathcal{A}_{YY}^{-1})^T \boldsymbol{\varpi}^T) + (\sigma^2 + 1) \mathbf{I} \right].$$

Since $\boldsymbol{\varpi} \mathcal{A}_{YY}^{-1} \boldsymbol{\psi}^T = \frac{1}{a} \sum_{m=0}^{\infty} b^m (\boldsymbol{\varpi} \boldsymbol{\psi}^T)^{m+1}$ and $\boldsymbol{\varpi} \boldsymbol{\psi}^T \geq 0$, we have that $E \left[\widehat{\pi}_t \widehat{\pi}_t^T \right] \geq 0$.

Finally, we have

$$E \left[\widehat{\pi}_t \widehat{\mathbf{Y}}_t^T \right] = \kappa \left[\eta \boldsymbol{\varpi} E \left[\widehat{\mathbf{Y}}_t \widehat{\mathbf{Y}}_t^T \right] + \sigma \boldsymbol{\psi} (\mathcal{A}_{YY}^{-1})^T \right] \geq 0$$

since $\boldsymbol{\varpi} E \left[\widehat{\mathbf{Y}}_t \widehat{\mathbf{Y}}_t^T \right] \geq 0$ and $\boldsymbol{\psi} (\mathcal{A}_{YY}^{-1})^T = \frac{1}{a} \sum_{m=0}^{\infty} b^m [(\boldsymbol{\psi}^T \boldsymbol{\varpi})^m \boldsymbol{\psi}^T]^T \geq 0$. $\square$

F.5 Financial Autarky

We now turn to the polar case with financial autarky, $\tau \rightarrow \infty$, while keeping our other simplifying assumptions in place, namely no roundabout production ($\gamma = 1$), symmetric monetary policy, *i.i.d.* shocks, and subsidies that offset the distortions associated with monopolistic competition ($\mathbf{mc} = 1$). For that case, equation (A44), which governs financial asset allocations, imposes that $d\mathbf{s}_t = 0$ for all t so that the trade balance equation (A43) becomes (given that in steady state, $\mathbf{s} = 0$ and $\mathbf{mc} = 1$)

$$\widehat{\mathbf{mc}}_t + \widehat{\mathbf{C}}_t = \widehat{\mathbf{w}}_t + \widehat{\mathbf{Y}}_t.$$

Substituting out $\widehat{\mathbf{m}c}_t$ using equation (A38) yields

$$\widehat{\mathbf{Y}}_t - \widehat{\mathbf{C}}_t = -(I - \boldsymbol{\varpi})(\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t). \quad (\text{A53})$$

Thus, in financial autarky, any real trade-deficit must be met by a terms of trade adjustment.

We can solve for $\widehat{\mathbf{Y}}_t$, $\widehat{\mathbf{e}}_t$ and $\widehat{\mathbf{w}}_t$ as functions of consumption, $\widehat{\mathbf{C}}_t$, using equation (A53), the goods market clearing equation (A48) (with marginal cost substituted out using equation (A38)), and the optimal labor supply condition (A46).

Note that combining the trade balance equation under financial autarky (A53) with the goods market clearing equation, and eliminating marginal cost using (A38), yields

$$-(I - \boldsymbol{\varpi})(\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t) = (\boldsymbol{\psi}^T - I)\widehat{\mathbf{C}}_t - (1 + \varphi)(I - \boldsymbol{\psi}^T \boldsymbol{\varpi})(\widehat{\mathbf{w}}_t + \widehat{\mathbf{e}}_t).$$

With some rearrangement, we obtain

$$((1 + \varphi)(I - \boldsymbol{\psi}^T \boldsymbol{\varpi}) - (I - \boldsymbol{\varpi}))(\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t) = (\boldsymbol{\psi}^T - I)\widehat{\mathbf{C}}_t. \quad (\text{A54})$$

Now, observe that the resulting linear system of equations (A54) is collinear, (i.e., the matrices pre-multiplying the variables on both sides of (A54) have one eigenvalue equal to 0, since $\boldsymbol{\varpi}$ and $\boldsymbol{\psi}$ are stochastic matrices). Therefore, the equations making up this linear system need one additional constraint, in this case supplied by the exchange rate normalization equation, $\widehat{e}_{1,t} = 0$. With this additional constraint, it becomes possible to obtain an analytical solution in the limit where $\varphi \rightarrow 0$ ⁹. In particular, the expression simplifies to

$$(\boldsymbol{\psi}^T - I)\boldsymbol{\varpi}(\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t) = -(\boldsymbol{\psi}^T - I)\widehat{\mathbf{C}}_t.$$

This suggests a solution of the form,

$$\widehat{\mathbf{e}}_t + \widehat{\mathbf{w}}_t = -\boldsymbol{\varpi}^{-1}\widehat{\mathbf{C}}_t.$$

Substituting this expression back into equation (A53) yields

$$\widehat{\mathbf{Y}}_t = \widehat{\mathbf{C}}_t + (I - \boldsymbol{\varpi})\boldsymbol{\varpi}^{-1}\widehat{\mathbf{C}}_t = \boldsymbol{\varpi}^{-1}\widehat{\mathbf{C}}_t. \quad (\text{A55})$$

Since $\widehat{\mathbf{C}}_t = -\boldsymbol{\nu}_t$ in this example economy, it follows that, under financial autarky, $\tau \rightarrow \infty$ (and in the limit where $\varphi \rightarrow 0$),

$$\widehat{\mathbf{Y}}_t = -\boldsymbol{\varpi}^{-1}\boldsymbol{\nu}_t.$$

It is straightforward to check that for $N = 2$ the covariance of output is negative so long as $\boldsymbol{\varpi}$

⁹To keep steady-state trade matrices constant as $\varphi \rightarrow 0$, we also let trade costs increase at compatible rates so that $\kappa_{nn'}^{-\varphi} \forall n, n'$ remains constant.

is invertible. When $N = 2$, ϖ is a 2×2 row-stochastic matrix with positive entries:

$$\varpi = \begin{pmatrix} \varpi_{11} & 1 - \varpi_{11} \\ 1 - \varpi_{22} & \varpi_{22} \end{pmatrix}, \quad \varpi_{11}, \varpi_{22} \in (0, 1).$$

Then $\det(\varpi) = \varpi_{11} + \varpi_{22} - 1$, which is nonzero because ϖ is invertible (as the limit $-\varpi^{-1}\nu_t$ presupposes), and

$$\varpi^{-1} = \frac{1}{\varpi_{11} + \varpi_{22} - 1} \begin{pmatrix} \varpi_{22} & -(1 - \varpi_{11}) \\ -(1 - \varpi_{22}) & \varpi_{11} \end{pmatrix}.$$

Since $E[\nu_t \nu_t^T] = \mathbf{I}$,

$$E[\hat{Y}_t \hat{Y}_t^T] = \varpi^{-1}(\varpi^{-1})^T,$$

whose off-diagonal entry is

$$(E[\hat{Y}_t \hat{Y}_t^T])_{12} = -\frac{\varpi_{22}(1 - \varpi_{22}) + \varpi_{11}(1 - \varpi_{11})}{(\varpi_{11} + \varpi_{22} - 1)^2} < 0,$$

since $\varpi_{11}, \varpi_{22} \in (0, 1)$. The sign of $\det(\varpi)$ does not matter, because it enters squared.

We now show the proof for Proposition 2, showing that if ϖ is diagonal-dominant, then both ϖ^{-1} and the covariance matrix for $\hat{Y}_t$ will feature negative off-diagonal elements:

Proposition 2. *Consider the case of financial autarky, $\tau \rightarrow \infty$. Then, given an import share matrix, ϖ , $\lim_{\varphi \rightarrow 0} \hat{Y}_t = -\varpi^{-1}\nu_t$. If ϖ is diagonally dominant ($\varpi_{ii} > 1/2 \forall i$) and every country imports from at least one other country (for each i , $\varpi_{ij} > 0$ for some $j \neq i$), then, in the limit $\varphi \rightarrow 0$: (i) a monetary expansion in some country lowers output in at least one other country (ϖ^{-1} has negative off-diagonal terms); and (ii) some pair of countries has negatively correlated output (the covariance matrix $E[\hat{Y}_t \hat{Y}_t^T]$ has negative off-diagonal terms).*

Proof. The limit $\lim_{\varphi \rightarrow 0} \hat{Y}_t = -\varpi^{-1}\nu_t$ follows from the equilibrium conditions under financial autarky ($\tau \rightarrow \infty$) as $\varphi \rightarrow 0$ derived in this appendix section above.

Part (i): negative off-diagonal terms of ϖ^{-1} . Since ϖ is strictly diagonally dominant, it is invertible (Horn and Johnson, 2013, Theorem 6.1.10). Since $\varpi \mathbf{1} = \mathbf{1}$, multiplying both sides by ϖ^{-1} implies that $\varpi^{-1} \mathbf{1} = \mathbf{1}$, so

$$\sum_k (\varpi^{-1})_{ik} = 1 \quad \text{for all } i. \tag{A56}$$

We show that every row of ϖ^{-1} contains a negative off-diagonal entry. Fix a row i and suppose, to the contrary, that $(\varpi^{-1})_{ik} \geq 0$ for all $k \neq i$.

By assumption, there is $j \neq i$ with $\varpi_{ij} > 0$. The (i, j) entry of $\varpi^{-1}\varpi = \mathbf{I}$ gives

$$(\varpi^{-1})_{ii} \varpi_{ij} + \sum_{k \neq i} (\varpi^{-1})_{ik} \varpi_{kj} = 0.$$

The sum is nonnegative and $\varpi_{ij} > 0$, so $(\varpi^{-1})_{ii} \leq 0$. By (A56), $\sum_{k \neq i} (\varpi^{-1})_{ik} = 1 - (\varpi^{-1})_{ii}$.

The (i, i) entry of $\varpi^{-1}\varpi = \mathbf{I}$ gives

$$(\varpi^{-1})_{ii} \varpi_{ii} + \sum_{k \neq i} (\varpi^{-1})_{ik} \varpi_{ki} = 1.$$

For $k \neq i$, diagonal dominance gives $\varpi_{ki} \leq 1 - \varpi_{kk} < 1/2$. Since $(\varpi^{-1})_{ik} \geq 0$ for $k \neq i$,

$$1 \leq (\varpi^{-1})_{ii} \varpi_{ii} + \frac{1}{2}(1 - (\varpi^{-1})_{ii}) = \frac{1}{2} + (\varpi^{-1})_{ii}(\varpi_{ii} - \frac{1}{2}) \leq \frac{1}{2},$$

where the last step uses $(\varpi^{-1})_{ii} \leq 0$ and $\varpi_{ii} > 1/2$. This is a contradiction. Hence each row of ϖ^{-1} has at least one negative off-diagonal entry.

Part (ii): negative off-diagonal terms of $E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T]$. We cannot sign the entries of $E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T]$ directly, but its inverse $\varpi^T \varpi$ is nonnegative, which is enough to show that each row must contain a negative off-diagonal entry. Since $E[\nu_t \nu_t^T] = \mathbf{I}$, in the limit

$$E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T] = \varpi^{-1}(\varpi^{-1})^T, \quad (E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T])^{-1} = \varpi^T \varpi.$$

The entries of $\varpi^T \varpi$ are $(\varpi^T \varpi)_{kj} = \sum_n \varpi_{nk} \varpi_{nj} \geq 0$. The diagonal entries of $E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T]$ are $\sum_k [(\varpi^{-1})_{ik}]^2 > 0$, since no row of the invertible matrix ϖ^{-1} is zero.

Fix a row i and, by assumption, take $j \neq i$ with $\varpi_{ij} > 0$. Then $(\varpi^T \varpi)_{ij} \geq \varpi_{ii} \varpi_{ij} > 0$. The (i, j) entry of $E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T] \varpi^T \varpi = \mathbf{I}$ gives

$$(E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T])_{ii} (\varpi^T \varpi)_{ij} + \sum_{k \neq i} (E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T])_{ik} (\varpi^T \varpi)_{kj} = 0.$$

The first term is strictly positive, so some term in the sum is negative. Since $(\varpi^T \varpi)_{kj} \geq 0$, this requires $(E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T])_{ik} < 0$ for some $k \neq i$. Hence each row of $E[\hat{\mathbf{Y}}_t \hat{\mathbf{Y}}_t^T]$ has at least one negative off-diagonal entry. $\square$

Proposition 3. *Let ϖ be a row-stochastic import share matrix with all entries strictly positive. Let $\underline{\varpi} := \min_i \varpi_{ii}$ be the smallest diagonal term in ϖ and $\varpi_* := \min_{n \neq n'} \varpi_{nn'}$ the smallest off-diagonal term. If*

$$\underline{\varpi} > 1 + \frac{\varpi_*}{4} \left(1 - \sqrt{1 + \frac{4}{\varpi_*}} \right), \quad (\text{A57})$$

then ϖ is invertible and all of the off-diagonal terms of ϖ^{-1} are negative.

Proof. Condition (A57) implies diagonal dominance. The right-hand side of (A57) equals $1 - s^*$, where

$$s^* := \frac{\sqrt{\varpi_*^2 + 4\varpi_*} - \varpi_*}{4}. \quad (\text{A58})$$

Since $(\varpi_* + 2)^2 = \varpi_*^2 + 4\varpi_* + 4 > \varpi_*^2 + 4\varpi_*$, we have $s^* < 1/2$, so (A57) implies $\underline{\varpi} > 1/2$, and hence $2(1 - \underline{\varpi}) < 1$.

Row sums of $\mathbf{I} - \boldsymbol{\varpi}$. Row r of $\mathbf{I} - \boldsymbol{\varpi}$ has diagonal entry $1 - \varpi_{rr} > 0$ and off-diagonal entries $-\varpi_{rl}$. Since $\sum_{l \neq r} \varpi_{rl} = 1 - \varpi_{rr}$ by row-stochasticity,

$$\sum_l |(\mathbf{I} - \boldsymbol{\varpi})_{rl}| = (1 - \varpi_{rr}) + \sum_{l \neq r} \varpi_{rl} = 2(1 - \varpi_{rr}) \leq 2(1 - \underline{\varpi}) \quad \text{for every row } r. \quad (\text{A59})$$

Row sums of $(\mathbf{I} - \boldsymbol{\varpi})^k$. We show by induction that, for every $k \geq 1$,

$$\sum_l |[(\mathbf{I} - \boldsymbol{\varpi})^k]_{rl}| \leq [2(1 - \underline{\varpi})]^k \quad \text{for every row } r. \quad (\text{A60})$$

The case $k = 1$ is (A59). For $k \geq 2$, write $(\mathbf{I} - \boldsymbol{\varpi})^k = (\mathbf{I} - \boldsymbol{\varpi})(\mathbf{I} - \boldsymbol{\varpi})^{k-1}$. For each row r ,

$$\begin{aligned} \sum_l |[(\mathbf{I} - \boldsymbol{\varpi})^k]_{rl}| &= \sum_l \left| \sum_p (\mathbf{I} - \boldsymbol{\varpi})_{rp} [(\mathbf{I} - \boldsymbol{\varpi})^{k-1}]_{pl} \right| \leq \sum_p |(\mathbf{I} - \boldsymbol{\varpi})_{rp}| \sum_l |[(\mathbf{I} - \boldsymbol{\varpi})^{k-1}]_{pl}| \\ &\leq [2(1 - \underline{\varpi})]^{k-1} \sum_p |(\mathbf{I} - \boldsymbol{\varpi})_{rp}| \leq [2(1 - \underline{\varpi})]^k, \end{aligned}$$

where the first inequality is the triangle inequality, the second uses (A60) for $k - 1$, and the third uses (A59). Since a single entry is at most the absolute sum of its row, (A60) implies

$$|[(\mathbf{I} - \boldsymbol{\varpi})^k]_{ij}| \leq [2(1 - \underline{\varpi})]^k \quad \text{for all } i, j. \quad (\text{A61})$$

Neumann series. Since $2(1 - \underline{\varpi}) < 1$, (A61) implies that the Neumann series converges, so $\boldsymbol{\varpi}$ is invertible and

$$\boldsymbol{\varpi}^{-1} = \sum_{k=0}^{\infty} (\mathbf{I} - \boldsymbol{\varpi})^k = \mathbf{I} + (\mathbf{I} - \boldsymbol{\varpi}) + \sum_{k=2}^{\infty} (\mathbf{I} - \boldsymbol{\varpi})^k. \quad (\text{A62})$$

Off-diagonal entries. For $i \neq j$, the $k = 0$ term of (A62) contributes zero and the $k = 1$ term contributes $(\mathbf{I} - \boldsymbol{\varpi})_{ij} = -\varpi_{ij}$. By the triangle inequality, (A61), and the geometric series formula, the remaining terms satisfy

$$\left| \sum_{k=2}^{\infty} [(\mathbf{I} - \boldsymbol{\varpi})^k]_{ij} \right| \leq \sum_{k=2}^{\infty} [2(1 - \underline{\varpi})]^k = \frac{[2(1 - \underline{\varpi})]^2}{1 - 2(1 - \underline{\varpi})} = \frac{4(1 - \underline{\varpi})^2}{2\underline{\varpi} - 1}.$$

Combining, and using $\varpi_{ij} \geq \varpi_*$,

$$(\boldsymbol{\varpi}^{-1})_{ij} \leq -\varpi_{ij} + \frac{4(1 - \underline{\varpi})^2}{2\underline{\varpi} - 1} \leq -\varpi_* + \frac{4(1 - \underline{\varpi})^2}{2\underline{\varpi} - 1}. \quad (\text{A63})$$

Sign. Setting $s = 1 - \underline{\varpi} \in (0, 1/2)$, the right-hand side of (A63) is negative if and only if $4s^2 <$

$\varpi_*(1 - 2s)$, i.e. $4s^2 + 2\varpi_*s - \varpi_* < 0$. The left-hand side is increasing in s for $s > 0$, and its positive root is s^* in (A58). So the right-hand side of (A63) is negative if and only if $s < s^*$, i.e. $\underline{\varpi} > 1 - s^*$. Since

$$1 - s^* = 1 - \frac{\varpi_*}{4} \left(\sqrt{1 + \frac{4}{\varpi_*}} - 1 \right) = 1 + \frac{\varpi_*}{4} \left(1 - \sqrt{1 + \frac{4}{\varpi_*}} \right),$$

this is exactly condition (A57). Therefore $(\varpi^{-1})_{ij} < 0$ for all $i \neq j$. $\square$

G Producer Currency Pricing

In conventional New Keynesian Open Economy (NKOE) models, e.g., Galí and Monacelli (2005) and Clarida et al. (2002), sticky price producers use local inputs to produce and sell their goods abroad. These goods are then assembled into final goods in the different countries in which they are imported. In that class of models, exchange rate fluctuations pass through to final goods prices perfectly. In contrast, in our framework, price stickiness applies after trade takes place so that, in line with Devereux and Yetman (2010) for instance, exchange rate fluctuations do not pass through entirely to local final goods prices.

This section discusses in greater detail how the location of price stickiness in the production chain affects our findings. To that end, we extend the baseline model by including an additional layer of monopolistic firms with sticky prices upstream from tradable goods producers: producer currency pricing (PCP). These firms hire workers, purchase materials, and sell varieties of inputs to Eaton and Kortum (2002) firms that produce tradable goods. They can also be interpreted as unions who negotiate sticky wages on worker's behalf (see, for example, Rubbo (2023) for a similar mapping). For simplicity, as in Section 5 of the main text, we abstract from roundabout production ($\gamma = 1$), and include a subsidy to eliminate steady-state markup distortions ($\mathbf{mc} = 1$).

With this extension, there are now three layers:

- **Layer 1:** Firms that hire workers to produce varieties of intermediate goods with sticky prices in the local currency. This is the new layer of sticky price intermediate goods.
- **Layer 2:** Firms that purchase intermediate inputs from Layer 1 and produce tradable goods that can be sold at home or abroad as in Eaton and Kortum (2002). This is Layer 1 of Figure 4 in the main text (or Figure B.1 in Section B of this appendix).
- **Layer 3:** Firms that purchase final goods produced by Layer 2, at home or abroad, and assemble them into final goods to be consumed by households domestically. These are the sticky price retailers in the baseline model, i.e., Layer 2 of Figure 4 in the main text (or Figure B.1 in Section B of this appendix).

Layer 1 in this section is the new element relative to the model in the main text. The resulting model nests the simplified version of our baseline model in the main text if prices in Layer 1 are

perfectly flexible. When prices in Layer 3 are flexible, we obtain a model akin to standard NKOE setups with PCP.

For consistency with the analytical insights discussed in the main text, we consider a forward-looking monetary policy rule as in [Clarida et al. \(2002\)](#), abstract from financial frictions (setting $\tau = 0$), and focus on *i.i.d.* monetary policy shocks. In this model extension, households and final goods producers remain identical to the baseline. There are also no changes in the problem of retailers (Layer 3). Key changes pertain to the introduction of a new layer of intermediate input producers (Layer 1), which then affects the equations describing the behavior of firms producing tradable (EK) goods (now Layer 2). Thus, we focus below on these two layers.

G.1 Intermediate Goods Producers

Layer 1 (New Sticky Price Intermediate Inputs): We denote the output of a variety i produced by firms in Layer 1 by $x_{n,t}(i)$ and the price at which those varieties are sold to Layer 2's tradable goods firms by $T_{n,t}(i)$. The production function for these varieties is

$$x_{n,t}(i) = \ell_{n,t}(i),$$

where, as before, $\ell_{n,t}(i)$ denotes labor used in producing variety i in country n at time t .

The production function for tradable goods produced by Layer 2 firms is now

$$q_{n,t}(\mathbf{z}) = z_n \left[\int x_{n,t}(i)^{\frac{v-1}{v}} di \right]^{\frac{v}{v-1}},$$

where $q_{n,t}(\mathbf{z})$ is the production for variety $\mathbf{z}$ in country n at time t , z_n is the productivity parameter of variety $\mathbf{z}$ in country n (drawn as before from a Fréchet distribution) and v is the elasticity of substitution between varieties.

These technologies imply demand functions for varieties i given by

$$x_{n,t}(i) = \left(\frac{T_{n,t}(i)}{T_{n,t}} \right)^{-v} \sum_{n'} \int \frac{\kappa_{n'n} Q_{n'n,t}(\mathbf{z})}{z_n} d\Phi(\mathbf{z}) = \left(\frac{T_{n,t}(i)}{T_{n,t}} \right)^{-v} X_{n,t},$$

where $Q_{n'n,t}(\mathbf{z})$ is the demand for variety $\mathbf{z}$ in country n for sale in country n' , $\Phi(\mathbf{z})$ is the *c.d.f.* for varieties of type $\mathbf{z}$ (given by the Fréchet distribution, as in the main text) and $T_{n,t} = \left(\int T_{n,t}(i)^{1-v} di \right)^{\frac{1}{1-v}}$ is the price index associated with a cost-minimizing bundle of input varieties i .

Following the convention in classic NKOE models (e.g., [Obstfeld and Rogoff, 1995](#)), firms producing intermediate varieties, $x_{n,t}(i)$, set their prices for one period at a time. They differ in whether they set their period t prices in period t (after all period t shocks are known) or in period $t - 1$. Single-period price-setting implies that we do not need to keep track of the relative prices between varieties in Layers 1 and 3 as a state variable, allowing us to conveniently extend the analytics in Section 5 of the paper.

Prices of intermediate varieties, $x_{n,t}(i)$, are set in the local currency. Thus, firms setting period t prices in period $t - 1$ choose their price, $T_{n,t}(i)$, to solve

$$\max_{T_{n,t}(i)} E_{t-1} [(T_{n,t}(i) - W_{n,t}) x_{n,t}(i)],$$

subject to the demand function above. The optimal choice of $T_{n,t}(i)$ for these firms is

$$T_{n,t}(i) = \frac{v}{v-1} E_{t-1} [W_{n,t}],$$

where $W_{n,t}$ is the nominal wage in country n , time t .

The optimal choice for firms choosing prices in period t is

$$T_{n,t}(i) = \frac{v}{v-1} W_{n,t}.$$

Denote the fraction of early price-setting firms by ϑ . Then

$$T_{n,t}^{1-v} = \left(\frac{v}{v-1} \right)^{1-v} \left[\vartheta (E_{t-1} [W_{n,t}])^{1-v} + (1 - \vartheta) (W_{n,t})^{1-v} \right].$$

Dividing by $P_{n,t-1}$, and using small caps to denote real variables in local currency (for simplicity, in the derivations below, we abstract from the stochastic time-trend), we have

$$[t_{n,t}(1 + \pi_{n,t})]^{1-v} = \left(\frac{v}{v-1} \right)^{1-v} \left[\vartheta (E_{t-1} [w_{n,t}(1 + \pi_{n,t})])^{1-v} + (1 - \vartheta) (w_{n,t}(1 + \pi_{n,t}))^{1-v} \right].$$

Log-linearizing (where a ‘hat’ now denotes log deviation from steady state) yields

$$\hat{t}_{n,t} = \vartheta E_{t-1} (\hat{w}_{n,t} + \hat{\pi}_{n,t}) + (1 - \vartheta) (\hat{w}_{n,t} + \hat{\pi}_{n,t}) - \hat{\pi}_{n,t}.$$

Observe that absent early price setters, $\vartheta = 0$, the price of inputs to tradables mimic production costs, $\hat{t}_{n,t} = \hat{w}_{n,t}$.

Layer 2 (Tradable Intermediate Goods): Layer 2 consists of firms that produce tradable intermediate varieties and have heterogeneous productivity levels, as described in [Eaton and Kortum \(2002\)](#) and comparable to the tradable goods layer of our baseline model. The local price of a variety z in country n of a good sourced in n' is

$$p_{nn',t}(z) = \frac{\kappa_{nn'} \mathcal{E}_{nn',t} T_{n',t}}{z_{n'}} = \frac{\kappa_{nn'} \mathcal{E}_{n',t} T_{n',t}}{\mathcal{E}_{n,t} z_{n'}},$$

where $\mathcal{E}_{nn',t}$ is the nominal exchange rate that converts prices in country n' currency to country n .

Following the usual derivations laid out in [Eaton and Kortum \(2002\)](#), we have that

$$\mathcal{E}_{n,t} P_{n,t}^Q \propto \left[\sum_{n'} (\kappa_{nn'} \mathcal{E}_{n',t} T_{n',t})^{-\varphi} \right]^{-\frac{1}{\varphi}},$$

where, as in the main text, $P_{n,t}^Q$ is the price of the bundle of tradable varieties used in country n and φ is the shape parameter of the Fréchet distribution governing the distribution of productivity parameters, $\mathbf{z}$. In real terms, we have that

$$e_{n,t} p_{n,t}^Q \propto \left[\sum_{n'} (\kappa_{nn'} e_{n',t} t_{n',t})^{-\varphi} \right]^{-\frac{1}{\varphi}}.$$

One can then calculate the share of expenditures in country n sourced from country n' :

$$\varpi_{nn',t} = \frac{(\kappa_{nn'} t_{n',t} e_{n',t})^{-\varphi}}{\sum_{n''} (\kappa_{nn''} t_{n'',t} e_{n'',t})^{-\varphi}}.$$

Log-linearizing (setting $mc_{n,t} = p_{n,t}^Q$ without loss of generality; they could differ because of productivity levels), we obtain

$$\begin{aligned} \hat{e}_{n,t} + \widehat{mc}_{n,t} &= \sum_{n'} \varpi_{nn'} (\hat{e}_{n',t} + \hat{t}_{n',t}), \\ \widehat{\varpi}_{nn',t} &= -\varphi (\hat{e}_{n',t} + \hat{t}_{n',t} - \hat{e}_{n,t} - \widehat{mc}_{n,t}). \end{aligned}$$

Observe that these equations for marginal costs and trade-shares are similar to those of the baseline model in the main text, except for the price of inputs to tradables, $\hat{t}_{n,t}$, substituting for the wage rate, $\hat{w}_{n,t}$. In particular, without PCP, $\vartheta = 0$ and $\hat{t}_{n,t} = \hat{w}_{n,t}$ so that the two sets of expressions become identical and reproduce the mechanisms in the main text.

Market clearing for country n goods requires that

$$\mathcal{E}_{n,t} T_{n,t} L_{n,t} = \sum_{n'} \varpi_{n'n,t} \mathcal{E}_{n',t} \Delta_{n',t} MC_{n',t} C_{n',t},$$

where $\Delta_{n',t}$ is a measure of price distortions in country n' . In real terms, this expression becomes

$$e_{n,t} t_{n,t} L_{n,t} = \sum_{n'} \varpi_{n'n,t} e_{n',t} \Delta_{n',t} mc_{n',t} C_{n',t}.$$

Log-linearizing yields

$$\hat{e}_{n,t} + \hat{t}_{n,t} + \hat{L}_{n,t} = \sum_{n'} \psi_{n'n} (\hat{e}_{n',t} + \widehat{mc}_{n',t} + \hat{C}_{n',t} + \widehat{\varpi}_{n'n,t}),$$

which, substituting out the expression for $\widehat{\omega}_{nn',t}$, gives us

$$(1 + \varphi) (\widehat{e}_{n,t} + \widehat{t}_{n,t}) + \widehat{L}_{n,t} = \sum_{n'} \psi_{n'n} \left((1 + \varphi) (\widehat{e}_{n',t} + \widehat{m}\widehat{c}_{n',t}) + \widehat{C}_{n',t} \right).$$

Note that this last expression collapses to the equivalent equation in the baseline model when $\vartheta = 0$ so that $\widehat{t}_{n,t} = \widehat{w}_{n,t}$.

G.2 Linearized Equilibrium Equations

Stacking country-specific variables into vectors, $\widehat{\mathbf{x}}_t = (\widehat{x}_{1,t}, \dots, \widehat{x}_{N,t})'$, and similarly with parameters, the final system of equations in vector form is given by (with some abuse of notation):

$$\widehat{\mathbf{Y}}_t - \widehat{\mathbf{C}}_t + \widehat{\mathbf{t}}_t - \widehat{\mathbf{m}}\widehat{\mathbf{c}}_t = Vg_1(1 + \pi_1)E_t[d\mathbf{s}_{t+1}/(\mathbf{e} \times \mathbf{C})] - d\mathbf{s}_t/(\mathbf{e} \times \mathbf{C}), \quad (\text{A64})$$

$$\mathbf{1}\widehat{C}_{1,t} - \widehat{\mathbf{C}}_t = \frac{1}{\sigma}\widehat{\mathbf{e}}_t, \quad (\text{A65})$$

$$-\sigma\widehat{\mathbf{C}}_t + \sigma E_t\widehat{\mathbf{C}}_{t+1} = \widehat{\mathbf{R}}_t - E_t\widehat{\boldsymbol{\pi}}_{t+1} - E_t\widehat{\mathbf{g}}_{t+1}, \quad (\text{A66})$$

$$\eta\widehat{\mathbf{Y}}_t + \sigma\widehat{\mathbf{C}}_t = \widehat{\mathbf{w}}_t, \quad (\text{A67})$$

$$\widehat{\boldsymbol{\pi}}_t = \frac{(1 - \theta)(1 - \beta\theta)}{\theta}\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t + \beta E_t[\widehat{\boldsymbol{\pi}}_{t+1}], \quad (\text{A68})$$

$$\widehat{\mathbf{Y}}_t - \widehat{\mathbf{C}}_t = (\boldsymbol{\psi}^T - \mathbf{I})\widehat{\mathbf{C}}_t - (1 + \varphi)(\mathbf{I} - \boldsymbol{\psi}^T\boldsymbol{\omega})(\widehat{\mathbf{e}}_t + \widehat{\mathbf{t}}_t), \quad (\text{A69})$$

$$\widehat{\mathbf{e}}_t + \widehat{\mathbf{m}}\widehat{\mathbf{c}}_t = \boldsymbol{\omega}(\widehat{\mathbf{e}}_t + \widehat{\mathbf{t}}_t), \quad (\text{A70})$$

$$\widehat{e}_{1,t} = 0, \quad (\text{A71})$$

$$\widehat{\mathbf{t}}_t = \vartheta E_{t-1}[\widehat{\mathbf{w}}_t + \widehat{\boldsymbol{\pi}}_t] + (1 - \vartheta)(\widehat{\mathbf{w}}_t + \widehat{\boldsymbol{\pi}}_t) - \widehat{\boldsymbol{\pi}}_t. \quad (\text{A72})$$

With $\gamma = 1$, $\tau = 0$, $\mathbf{s} = 0$, and no financial or markup shocks ($d\boldsymbol{\xi}_t = 0$, $\widehat{\boldsymbol{\mu}}_t = 0$), equations (A66) through (A68), as well as (A71), are identical to their counterparts in the baseline model (with $\widehat{\mathbf{Y}}_t = \widehat{\mathbf{L}}_t$ when $\gamma = 1$), and equation (A65) is the Backus and Smith (1993) condition implied by its counterpart, (A44). Equations (A64), (A69) and (A70) are the same as their baseline counterparts, (A43), (A48) and (A49), but with the price of inputs to tradables, $\widehat{\mathbf{t}}_t$, replacing wages, $\widehat{\mathbf{w}}_t$. In the budget constraint, this reflects that household income from producing tradable goods now includes the profits of Layer 1 firms, so that it is valued at $T_{n,t}$ rather than $W_{n,t}$.¹⁰

¹⁰Substituting out $\widehat{\mathbf{m}}\widehat{\mathbf{c}}_t$ using (A70) gives $\widehat{\mathbf{Y}}_t - \widehat{\mathbf{C}}_t + (\mathbf{I} - \boldsymbol{\omega})(\widehat{\mathbf{e}}_t + \widehat{\mathbf{t}}_t) = Vg_1(1 + \pi_1)E_t[d\mathbf{s}_{t+1}/(\mathbf{e} \times \mathbf{C})] - d\mathbf{s}_t/(\mathbf{e} \times \mathbf{C})$, the balance of payments in the main text with the terms of trade now defined over $\widehat{\mathbf{t}}_t$.

G.3 Spillovers from Idiosyncratic Demand Shocks: An Analytical Example

Following steps analogous to those in Appendix F.4 and under the same assumptions, we have

$$\widehat{C}_t = -\nu_t, \quad (\text{A73})$$

$$\widehat{Y}_t = -(1-b)[I - b(I - \Psi)]^{-1} \left[\psi^T + \frac{(1+\varphi)\vartheta(1+\kappa)\sigma}{1+\vartheta\kappa} \Psi \right] \nu_t, \quad (\text{A74})$$

$$\widehat{\pi}_t = (I + \vartheta\kappa\varpi)^{-1} \kappa \left((I - \vartheta\varpi) \sigma \widehat{C}_t + (1 - \vartheta)\eta\varpi \widehat{Y}_t \right), \quad (\text{A75})$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$ and

$$\begin{aligned} \Psi &= (1 + \vartheta\kappa)(I - \psi^T \varpi)(I + \vartheta\kappa\varpi)^{-1}, \\ b &= \frac{(1 + \varphi)\eta}{1 + (1 + \varphi)\eta + \frac{\vartheta}{1-\vartheta}(\kappa + 1)}. \end{aligned} \quad (\text{A76})$$

In particular, $\widehat{Y}_t$ can be expressed again through a Neumann series,

$$\begin{aligned} \widehat{Y}_t &= -(1-b) \left(I + b(I - \Psi) + b^2(I - \Psi)^2 + b^3(I - \Psi)^3 + \dots \right) \\ &\quad \times \left[\psi^T + \frac{(1 + \varphi)\vartheta(1 + \kappa)\sigma}{1 + \vartheta\kappa} \Psi \right] \nu_t \end{aligned} \quad (\text{A77})$$

With $\vartheta = 0$, this collapses to equation (19) in the main text, with $b = (1 + \varphi)\eta / (1 + (1 + \varphi)\eta)$ and $I - \Psi = \psi^T \varpi$. With $\vartheta > 0$, there are two key changes that dampen the knock-on effects of trade.

Pattern of Knock-On Effects ($I - \Psi$): The first change is that shocks propagate through the trade network according to powers of $I - \Psi$ rather than $\psi^T \varpi$. To understand the difference, recall the expenditure switching term,

$$(1 + \varphi)(I - \psi^T \varpi)(\widehat{e}_t + \widehat{t}_t), \quad (\text{A78})$$

in the trade equation (A69). Substituting $\widehat{t}_t$ using equation (A72) and using the *i.i.d.* assumption on monetary policy shocks, this becomes

$$(1 + \varphi)(I - \psi^T \varpi)(\widehat{e}_t + (1 - \vartheta)\widehat{w}_t - \vartheta\widehat{\pi}_t). \quad (\text{A79})$$

In words, the real costs of tradable goods in country n now depend not only positively on wages, but also negatively on inflation. Specifically, because a fraction ϑ of tradable goods input prices have been set in advance, higher inflation reduces the real cost of tradable goods.

Using the labor supply and Backus-Smith conditions to substitute out $\widehat{w}_t$, the term becomes

$$(1 + \varphi)(1 - \vartheta)(I - \psi^T \varpi)\eta \left(\widehat{Y}_t + \frac{\vartheta}{(1 - \vartheta)\eta}(\widehat{e}_t - \widehat{\pi}_t) \right).$$

Given past price levels, $\widehat{e}_t - \widehat{\pi}_t$ moves with changes in the *nominal* exchange rate, so a nominal depreciation makes a country's tradable goods cheaper and diverts expenditure towards it, a beggar-thy-neighbor force. Inflation in turn depends on conditions abroad, which is captured by the term, $(\mathbf{I} + \vartheta \kappa \boldsymbol{\varpi})^{-1}$ in equation (A75). In particular, the slope of the Phillips curve, κ , reflects that inflation depends on marginal costs (equation (A68)), which themselves depend on the import matrix, $\boldsymbol{\varpi}$.

In short, price stickiness in tradable goods inputs attenuates the knock-on effects because shocks no longer pass through perfectly to tradable good prices via wages. Instead, the real price of these tradable goods may fall with inflation, dampening knock-on effects in each round. The dependence of marginal costs on inflation is what changes how shocks propagate through the network, leading to dependence on $\mathbf{I} - \boldsymbol{\Psi}$ rather than $\boldsymbol{\psi}^T \boldsymbol{\varpi}$.

Compounding of Knock-On Effects (b): The second change is that the scalar, b , in equation (A76) now declines with ϑ , owing to an additional $\frac{\vartheta}{1-\vartheta}(\kappa + 1)$ term in the denominator. In other words, the scale of expenditure switching is decreasing in the stickiness of tradable goods inputs. The greater the fraction of firms unable to set their prices in period t , the less is the increase in the price of tradable goods inputs in response to an expansionary demand shock. Consequently, the terms of trade response is muted and knock-on effects are reduced. In the limit, $b \rightarrow 0$ as $\vartheta \rightarrow 1$, i.e., all knock-on effects vanish.

The additional $\frac{\vartheta}{1-\vartheta}(\kappa + 1)$ is also increasing in κ . As $\kappa \rightarrow \infty$, final goods prices become fully flexible and this model extension converges to an environment similar to that of conventional NKOE models. In this case, for any positive level of PCP, $\vartheta > 0$, $b \rightarrow 0$. To see why the knock-on effects vanish, observe that when $\kappa \rightarrow \infty$, $\widehat{m\mathbf{c}}_t = 0$, and equation (A70) implies that we can write the terms of trade as:

$$(\mathbf{I} - \boldsymbol{\varpi}) (\widehat{e}_t + \widehat{t}_t) = \widehat{t}_t = (\boldsymbol{\varpi}^{-1} - \mathbf{I}) \widehat{e}_t. \quad (\text{A80})$$

Therefore, the terms of trade, now $\widehat{t}_t$, move one for one with the real exchange rate $\widehat{e}_t$, which is pinned down by consumption due to the Backus and Smith (1993) equation (A65). Since consumption depends directly on the exogenous monetary policy shock (equation (A73)), the terms of trade are divorced from changes in factor prices (here wages) that otherwise underlie the successive rounds of expenditure switching driving the knock-on effects of trade. Without these knock-on effects, demand shocks can only generate global comovement through the direct effects of trade, i.e., a little trade no longer goes a long way.

H Estimation

H.1 Calibrated Steady States

We calibrate the steady-state import share matrices, ϖ^c and ϖ^m , using annual data for 2004–2019. For the E.U., which consists of the 11 initial members of the Euro area (Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain), all variables below are sums over member countries.

1. **Trade data:** Bilateral imports by partner, in current U.S. dollars, are taken from the World Integrated Trade Solution (WITS) database, compiled by the World Bank, by stage-of-processing product group. Imports of final goods, $X_{nn',t}^c$, are imports of consumer goods. Imports of intermediate goods, $X_{nn',t}^m$, are the sum of imports of intermediate goods and capital goods. Imports by the E.U. include imports from other E.U. members.
2. **Absorption:** Using World Bank national accounts in current U.S. dollars, let $\tilde{Y}_{n,t} = Y_{n,t} - G_{n,t} - I_{n,t}$, where Y is GDP, G is general government final consumption expenditure, and I is gross fixed capital formation. Absorption of final goods is $D_{n,t}^c = \tilde{Y}_{n,t} - NX_{n,t}$, where NX is net exports of goods and services. Absorption of intermediate goods is $D_{n,t}^m = \tilde{Y}_{n,t}$, where this last equality follows from calibrating $\gamma = 1/2$ so that absorption of intermediate goods is equal to that of final goods under balanced trade.
3. **World import shares:** Let $\varpi_{nn',t}^{k*} = X_{nn',t}^k / D_{n,t}^k$ denote country n 's import share from $n' \neq n$, and let $\varpi_{nn,t}^{k*} = 1 - \sum_{n' \neq n} X_{nn',t}^k / D_{n,t}^k$ denote its domestic share, where the sum runs over all partner countries in the world.
4. **Sample import shares:** We set $\varpi_{nn,t}^k = \varpi_{nn,t}^{k*}$ and distribute the remaining imports proportionally across the other countries in the sample, i.e., for $n' \neq n$,

$$\varpi_{nn',t}^k = \frac{\varpi_{nn',t}^{k*}}{\sum_{n'' \in \mathcal{S}, n'' \neq n} \varpi_{nn'',t}^{k*}} \left(1 - \varpi_{nn,t}^k\right),$$

where $\mathcal{S}$ is the set of countries used in the estimation. Each row of ϖ_t^k thus sums to one.

5. **Averaging:** The steady-state matrices are the averages over the sample, $\varpi^k \equiv \frac{1}{T} \sum_{t=1}^T \varpi_t^k$, $k \in \{c, m\}$.

Table 2 of the main text reports these trade matrices. Panels A and B of Table H.1 additionally report $(\varpi^c)^{-1}$ and $(\varpi^m)^{-1}$. All the off-diagonal elements are negative, indicating that in the analytical version of our model in Section F above and Section 5 of the main text, demand shocks at home generate negative spillovers abroad under either of these trade matrices.

The wage bill, $\mathcal{E}_n W_n L_n$, and labor input, L_n , also need to be calibrated in the steady state. We calibrate the wage bill to match average GDP in current U.S. dollars over the sample period, taken

Table H.1: Inverse Trade Matrices and Calibrated Steady States

Panel A: Inverse of Final Goods Import Trade Matrix $((\varpi^c)^{-1})$

	Canada	China	E.U.	Japan	U.S.
Canada	1.144	-0.024	-0.020	-0.006	-0.093
China	-0.000	1.026	-0.012	-0.009	-0.005
E.U.	-0.002	-0.079	1.133	-0.016	-0.037
Japan	-0.000	-0.023	-0.011	1.039	-0.005
U.S.	-0.012	-0.017	-0.011	-0.005	1.045

Panel B: Inverse of Intermediate Goods Import Trade Matrix $((\varpi^m)^{-1})$

	Canada	China	E.U.	Japan	U.S.
Canada	1.326	-0.041	-0.029	-0.006	-0.249
China	-0.013	1.417	-0.159	-0.178	-0.067
E.U.	-0.005	-0.183	1.356	-0.032	-0.135
Japan	-0.001	-0.068	-0.016	1.109	-0.024
U.S.	-0.029	-0.045	-0.034	-0.009	1.117

Panel C: Steady States

		Canada	China	E.U.	Japan	U.S.
400 ($g_n - 1$)	Productivity Growth	0.78	0.78	0.78	0.78	0.78
400 π_n	Inflation	1.75	2.63	1.54	0.32	2.03
L_n	Employment	0.03	1.70	0.22	0.12	0.26
$\mathcal{E}_n W_n L_n$	Wage Bill	1.56	7.83	12.04	5.14	16.39

Notes: See text for construction of trade matrices in Panels A and B. In Panel C, productivity growth and inflation are annualized and calibrated based on quarterly averages of per capita GDP growth and inflation, respectively, over the 2004Q3 to 2019Q4 sample period. Total employment, L_n , is in trillions of hours, calibrated to the product of employment and average hours, using annual data from the Penn World Table over the 2004 to 2019 sample period. The wage bill, $\mathcal{E}_n W_n L_n$, is calibrated to average GDP in trillions of current U.S. dollars, using annual data from the World Bank and OECD over the 2004 to 2019 sample period.

from the World Bank and OECD National Accounts data files. We define labor as the product of employment and average hours, with data taken from the Penn World Tables. While these are potentially non-stationary variables, only their relative values matter for our estimation. Thus the growth rates are immaterial so long as the ratio of the variables across countries remains fairly constant over the sample period. Panel C of Table H.1 reports the values we find for each country.

H.2 Priors

Table H.2 reports the priors. See Section 6 for further discussion.

H.3 Data and Measurement Equations

We estimate the remaining variables using Bayesian methods. We compute the likelihood using the Kalman filter with data on the following observables:

Table H.2: Priors

Parameter		Distribution	Mean	Std Dev
<u>Global Parameters</u>				
$100(\beta^{-1} - 1)$	Discount Rate	Gamma	0.20	0.10
φ^c, φ^m	Fréchet Parameter (Trade Elasticity Minus One)	Gamma	5.50	2.00
τ	Financial Frictions	Gamma	5.00	3.00
ρ_x^G	Global Shock Persistence	Beta	0.50	0.20
<u>Country-Specific Parameters</u>				
σ_n	Inverse Intertemporal Elasticity of Substitution	Gamma	2.00	0.50
η_n	Inverse Labor Supply Elasticity	Gamma	2.00	0.50
θ_n	Calvo Price Stickiness	Beta	0.70	0.10
ρ_n	Taylor Rule: Persistence	Beta	0.75	0.10
$\phi_{\pi,n}$	Taylor Rule: Inflation Coefficient	Normal	1.50	0.50
$\phi_{y,n}$	Taylor Rule: Output Coefficient	Normal	0.12	0.10
$\phi_{\Delta y,n}$	Taylor Rule: Output Growth Coefficient	Normal	0.12	0.10
$\tilde{\rho}_{x,n}$	Idiosyncratic Shock Persistence	Beta	0.50	0.20
$100\tilde{\sigma}_{x,n}$	Idiosyncratic Shock Standard Deviation	Inverse Gamma	0.70	4.00
$\lambda_{x,n}$	Factor Loading	Normal	0.20	0.50

1. **Per Capita Real Output Growth:** Denote the real gross domestic product and population of country n in period t by $GDP_{n,t}$ and $POP_{n,t}$, respectively. We define

$$\text{Per Capita Real Output Growth} \equiv 100 \left[\log \frac{GDP_{n,t}}{POP_{n,t}} - \log \frac{GDP_{n,t-1}}{POP_{n,t-1}} \right].$$

2. **Annualized Inflation:** Denote CPI price level of country n in period t by $CPI_{n,t}$. We define

$$\text{Annualized Inflation} \equiv 400 \log \frac{CPI_{n,t}}{CPI_{n,t-1}}.$$

3. **Interest Rate and One-Year Government Bond Yields:** The interest rate for China is the interbank rate from the OECD. The one-year government bond yields are directly obtained from Haver. Both rates are annualized and in percentage points.

4. **Nominal Exchange Rate Growth:** Denote the nominal exchange rate between country n and the U.S. by $EX_{n,t}$. We define

$$\text{Nominal Exchange Rate Growth} \equiv 100 \log \frac{EX_{n,t}}{EX_{n,t-1}}.$$

All data are from Haver and CPI for the E.U. is harmonized based on Eurostat. The E.U. constitutes the original 11 countries in the currency union: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain.

For the model validation in Table 4, we additionally use data on:

1. **Consumption:** Data for China from the [Federal Reserve Bank of Atlanta](#); all other consumption series from Haver.
2. **Interest Rates (Outside China):** Call money or interbank rate from Federal Reserve Economic Data (FRED) and the OECD.

The measurement equations in the Kalman filter are as follows:

$$\begin{aligned}
\text{Per Capita Real Output Growth} &= 100(\hat{L}_{n,t} - \hat{L}_{n,t-1} + \hat{g}_{n,t}), \\
\text{Annualized Inflation} &= 400\hat{\pi}_{n,t}, \\
\text{Interest Rate} &= 400\hat{R}_{n,t}, \\
\text{One-Year Government Bond Yield} &= 100 \sum_{h=0}^3 E_t [\hat{R}_{n,t+h}] \\
\text{Nominal Exchange Rate Growth} &= 100(\hat{e}_{n,t} - \hat{e}_{n,t-1} - \hat{\pi}_{n,t} + \hat{\pi}_{1,t} - \hat{g}_{n,t} + \hat{g}_{1,t}).
\end{aligned}$$

We demean each of the variables, so that to only use information on their variations. In particular, we abstract from the steady-state relationships that the model implies across variables as they may not adequately reflect, for instance, the exchange rate and interest rate regime in China.

H.4 Sequential Monte Carlo Algorithm

This section summarizes the adaptive sequential Monte Carlo (SMC) algorithm that we use to estimate the model. The algorithm is taken directly from [Cai et al. \(2021\)](#). As with Markov chain Monte Carlo (MCMC) methods, the end goal is to obtain a set of draws from the posterior.

The SMC algorithm begins with draws from the prior, referred to as *particles*, then iteratively constructs particle approximations to *bridge distributions* that serve as intermediate steps between the prior and posterior. In stage $n - 1$, we have particles $\{\theta_{n-1}^i, W_{n-1}^i\}_{i=1}^N$, where θ_{n-1}^i denotes the parameter draw and W_{n-1}^i denotes the weight. To move to stage n , we follow three steps:

1. **Correction:** Reweighting the stage $n - 1$ particles to reflect the stage n distributions.
2. **Selection:** Resampling the particles to avoid an uneven distribution of weights.
3. **Mutation:** Propagating particles forward using an MCMC algorithm.

The algorithm is initialized with θ_0^i drawn from the prior and $W_0^i = 1$. It concludes when the (weighted) distribution of particles approximates the posterior.

Details of Sequential Monte Carlo Steps: We now describe the SMC steps in more detail.

1. **Correction:** The correction step reflects importance sampling between the stage $n - 1$ and stage n distributions. It starts with particles $\{\theta_{n-1}^i, W_{n-1}^i\}_{i=1}^N$ and updates the weights W_{n-1}^i to

$$\bar{W}_n^i \equiv \frac{w_n^i W_{n-1}^i}{\frac{1}{N} \sum_{i=1}^N w_n^i W_{n-1}^i}.$$

The incremental weights w_n^i are defined by

$$w_n^i = \frac{p_n(\theta_{n-1}^i)}{p_{n-1}(\theta_{n-1}^i)},$$

where $p_n(\theta_{n-1}^i)$ is the pdf of the stage n bridge distribution at θ_{n-1}^i , which we discuss below. This step yields particles $\{\theta_{n-1}^i, \bar{W}_n^i\}_{i=1}^N$.

2. **Selection:** The selection step resamples particles $\{\theta_{n-1}^i, \bar{W}_n^i\}_{i=1}^N$ based on weights $\bar{W}_n^i$ to obtain $\{\hat{\theta}_{n-1}^i, W_{n-1}^i\}_{i=1}^N$, resetting the weights to $W_{n-1}^i = 1$ for all i . The selection step faces a trade-off—it adds noise through resampling but equalizes particle weights, which increases the accuracy of importance sampling. Hence, following [Herbst and Schorfheide \(2015\)](#) and [Cai et al. \(2021\)](#), we only carry out the selection step if the effective sample size,

$$\widehat{ESS}_n \equiv N / \left[\frac{1}{N} \sum_{i=1}^N \left(\bar{W}_n^i \right)^2 \right], \quad (\text{A81})$$

falls below a threshold $\underline{N}$. The effective sample size measures how uneven the weights are. If the weights are all equal to one, then $\widehat{ESS}_n = N$. If the weight on all but one particle is zero, then $\widehat{ESS}_n = 1$.

3. **Mutation:** The mutation step changes the parameter values $\hat{\theta}_n^i$ so that they do not remain in the initial set of values. In particular, we generate N independent MCMC chains using Metropolis-Hastings. These can be parallelized. [Herbst and Schorfheide \(2015\)](#) discuss how to select proposal densities. The weights remain unchanged. This yields particles $\{\theta_n^i, W_n^i\}_{i=1}^N$.

Adaptively Choosing Bridge Distributions: What remains is the choice of bridge distributions p_n . We use a likelihood tempering approach, in which the stage n bridge distribution is

$$p_n(\theta) \propto \pi(\theta) \mathcal{L}(\theta|Y)^{\phi_n},$$

where $\pi(\cdot)$ is the prior, $\mathcal{L}(\cdot|Y)$ is the likelihood given data Y , and ϕ_n is an exponent that we choose adaptively. The algorithm begins with $\phi_0 = 0$, which corresponds to the prior, and terminates when $\phi_n = 1$, which corresponds to the posterior.

To choose ϕ_n , we rely on the effective sample size, defined analogously to (A81):

$$\widehat{ESS}(\phi) \equiv N / \left[\frac{1}{N} \sum_{i=1}^N \left(\widetilde{W}_n^i \right)^2 \right], \quad (\text{A82})$$

where $\widetilde{W}_n^i \equiv \frac{w^i(\phi)W_{n-1}^i}{\frac{1}{N} \sum_{i=1}^N w^i(\phi)W_{n-1}^i}$ and $w^i(\phi) \equiv \mathcal{L}(\theta|Y)^{\phi-\phi_{n-1}}$. We choose ϕ to obtain a target effective sample size, so that $\widehat{ESS}(\phi) = \alpha \widehat{ESS}_{n-1}$. The tuning parameter $\alpha \in (0, 1)$ determines how fast the effective sample size decays. A larger value of α improves accuracy at the cost of computational time.

Tuning Parameter Choices: We take $N = 25,000$ particles, set $\alpha = 0.97$, and use the selection threshold $\underline{N} = N/2$. These are all in line with values used in Cai et al. (2021).

Speed and Comparison to Metropolis-Hastings: Our SMC estimation takes 57 hours on an Intel 2.00 GHz processor with 16 cores using MATLAB. To evaluate the computational advantages of SMC in our setting and the challenges of estimating our model, it is useful to consider a standard Metropolis-Hasting MCMC benchmark. This is the standard approach for estimating New Keynesian models using MCMC (e.g., An and Schorfheide, 2007; Smets and Wouters, 2007).

To that end, we obtain a set of draws using Metropolis-Hastings MCMC and ask how long it would take to have an effective sample size comparable to the number of particles we used in our SMC. we take the initial draw to be the posterior mean of the posterior obtained from SMC algorithm and take the proposal density to be the covariance of the SMC algorithm's posterior multiplied by a scaling constant (chosen to give an acceptance rate of between 0.25 and 0.30). Therefore, we avoid having to run burn-in draws and have a reasonable proposal density.

We take 10^5 draws, saving every 10 draws. With this sample of 10^4 draws, we can approximate the effective sample size for parameter i :

$$N_{i,eff} \equiv \frac{N_{keep}}{1 + 2 \sum_{h=1}^H \widehat{\rho}_{i,h}},$$

where $N_{keep} = 10^4$ is the number of draws we saved, $\widehat{\rho}_{i,h}$ is the autocorrelation between draw t and draw $t + h$, and we take $H = 10^3$. We then find that the approximate time taken to obtain 2.5×10^4 effective draws (matching the number of SMC particles) would be

$$\frac{2.5 \times 10^4}{\overline{N}_{eff}} \mathcal{T} = 913 \text{ hours}$$

where $\mathcal{T} = 0.74$ hours is the time taken by the MCMC to obtain $\overline{N}_{eff} \equiv \frac{1}{121} \sum_{i=1}^{121} N_{i,eff}$ effective draws. The above calculation suggests that plain vanilla MCMC would take over 5 weeks. The number above is conservative since we utilized the SMC output to tune the MCMC algorithm. We

would otherwise have needed a substantial number of burn-in draws, further increasing the run time. In short, a standard MCMC estimation would be essentially infeasible, taking many times longer than the SMC approach we use.

H.5 Supplementary Results

Posterior Parameter Estimates: Figures H.1 to H.3 show the posteriors and priors of all parameters. The posterior means of the country-specific parameters fall around the ranges found in closed-economy settings—the inverse intertemporal elasticities of substitution, σ_n , lie between 0.94 and 2.75, the inverse labor supply elasticities, η_n , lie between 2.01 and 3.01, and the Calvo price setting parameters, θ_n , lie between 0.67 and 0.79.¹¹ Nevertheless, there is meaningful heterogeneity across regions, suggesting a role for the likelihood in driving these parameter estimates.

Country-Specific Steady-State Productivity Growth: In our baseline estimation, we assumed a common steady-state productivity growth across countries, reflecting that all countries will eventually converge to the same steady-state growth path determined by the rate of global technological progress in the infinite future. However, there are low frequency differences in GDP growth across countries, reflected in substantially different average growth rates over our sample period. To capture these differences, we consider a robustness check where we calibrate steady state productivity growth in all countries to their sample average annualized growth rates of 0.71, 7.94, 0.79, 0.71, and 0.90 percent for Canada, China, the E.U., Japan, and the U.S., respectively.

Table H.3 summarizes the results. Overall, the results remain largely similar to the estimation in the main text—trade has a substantial role in explaining the cross-country correlations, driven in large part by the knock-on effects (as captured by the $\eta = 0$ counterfactual in line A5), financial flows (as captured by the $\tau = 10^6$ counterfactual in line A6, and demand shocks (line A7). Moreover, the role of trade is essentially eliminated when we aggregate Canada, China, the E.U., and Japan into a single region.

Nevertheless, there remain some quantitative differences. For example, in line A8 we find that supply shocks now drive a limited amount of GDP growth comovement instead of inflation comovement. Similarly, in line A10, we find that with flexible prices, financial autarky, and country-specific TFP shocks only, we are now able to generate about 30 percent of the GDP growth comovement. The differences arise in part due to the estimates for the monetary policy rules. The country-specific steady-state growth rates change the path of the output gap that the likelihood seeks to fit. Consequently, the monetary policy rules change relative to our benchmark estimation in the

¹¹Estimating New Keynesian models, [Smets and Wouters \(2007\)](#) and [Herbst and Schorfheide \(2015\)](#) report posterior means for σ_n of 1.38 and 2.83, respectively, for the U.S.; [Iiboshi, Shintani, and Ueda \(2018\)](#) report a posterior mean of 1.55 for Japan. Across a broader range of methodologies, [Havráněk \(2015\)](#) reports a mean estimate of between 0.3 and 0.4 for $1/\sigma_n$. For η_n , [Smets and Wouters \(2007\)](#) and [Justiniano et al. \(2010\)](#) find estimates of 1.83 and 3.79, respectively, for the U.S.; [Iiboshi et al. \(2018\)](#) find a posterior mean of 3.92 for Japan. For θ_n , [Justiniano et al. \(2010\)](#) report a median estimate of 0.84 for the U.S.; [Sugo, Ueda, et al. \(2007\)](#) find a mean estimate of 0.88 for θ_n using data from Japan; [Galí, Gertler, and Lopez-Salido \(2001\)](#) reports estimates of between 0.67 and 0.92 for Europe.

Figure H.1: Priors and Posteriors (Preferences, Production, and Monetary Policy)

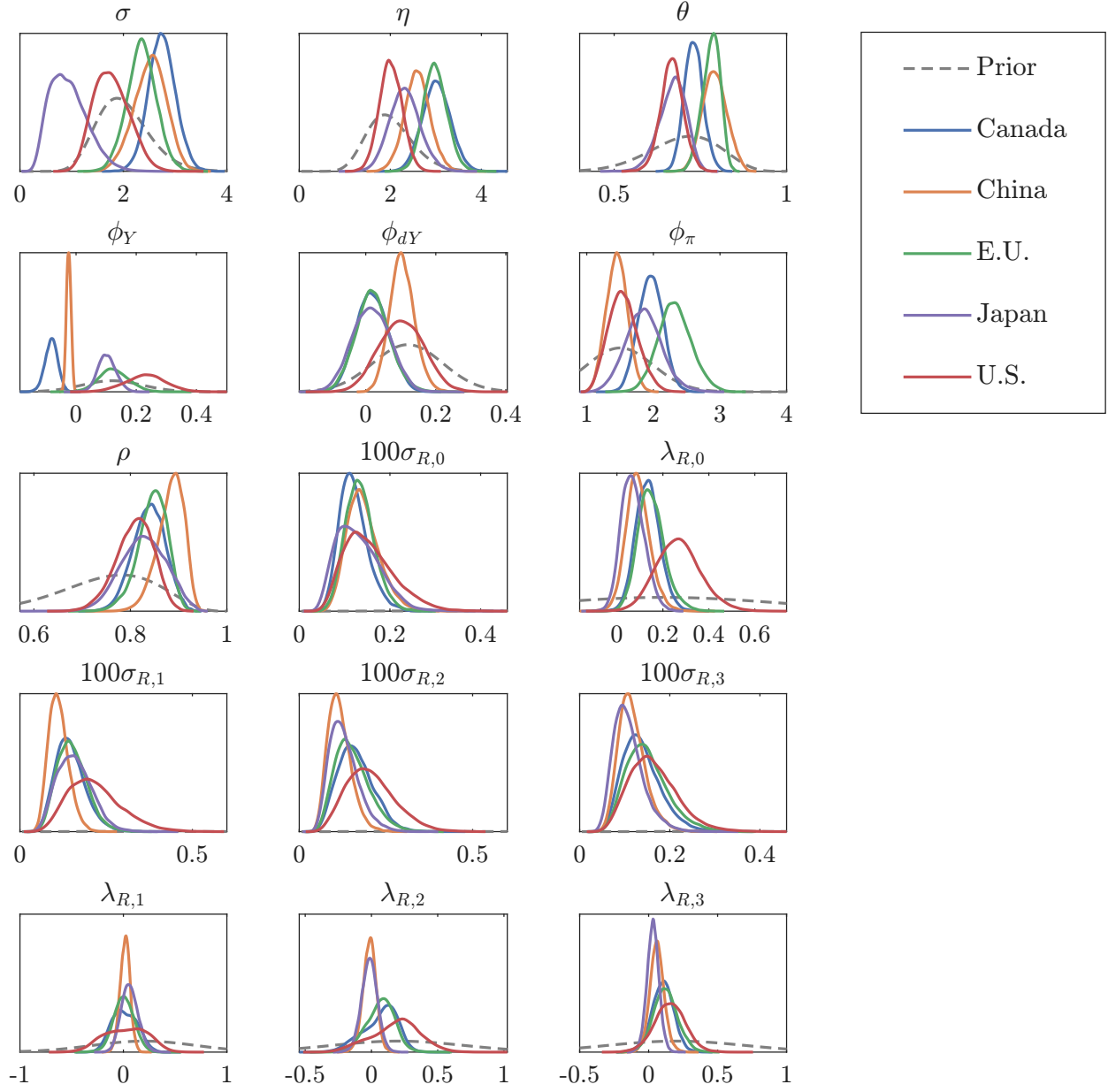


Figure H.2: Priors and Posteriors (Productivity, Markup, and Financial Shocks)

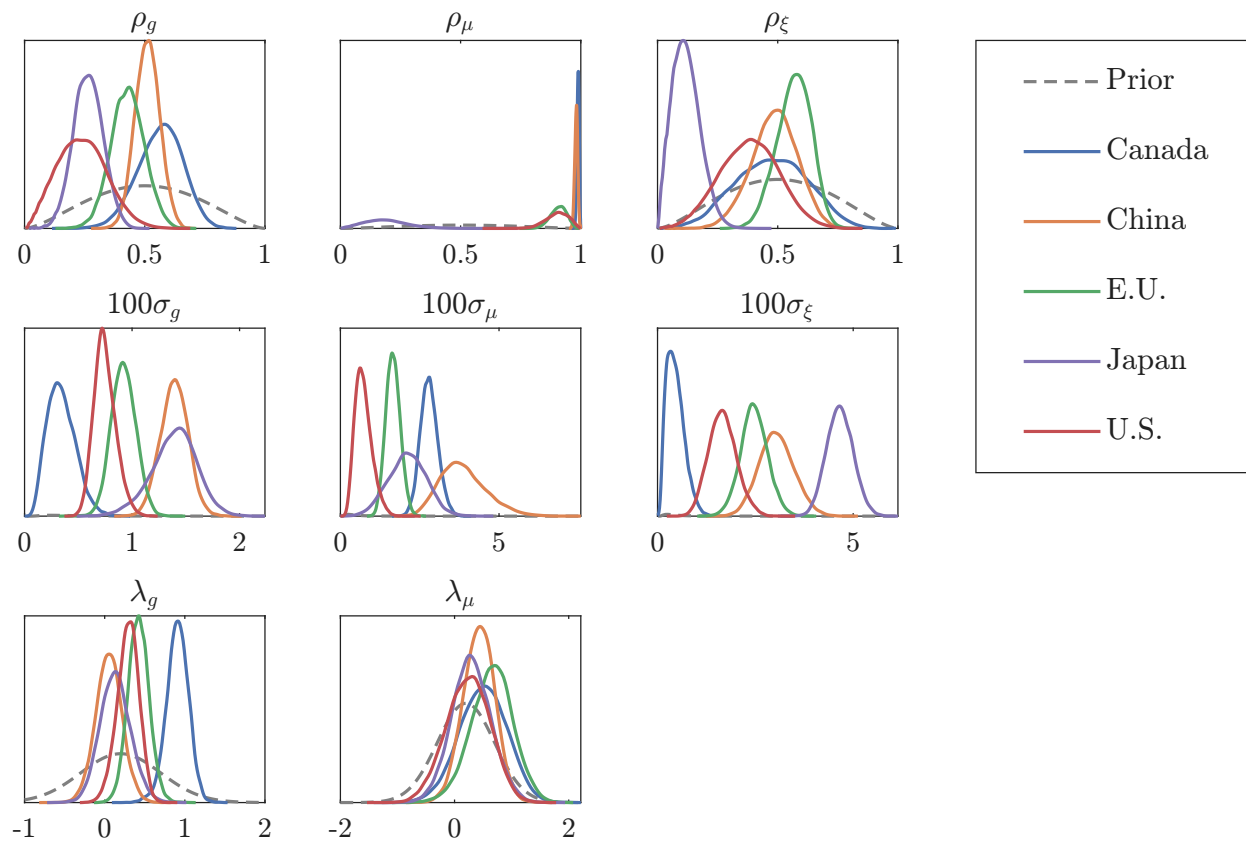


Figure H.3: Priors and Posteriors (Common Parameters)

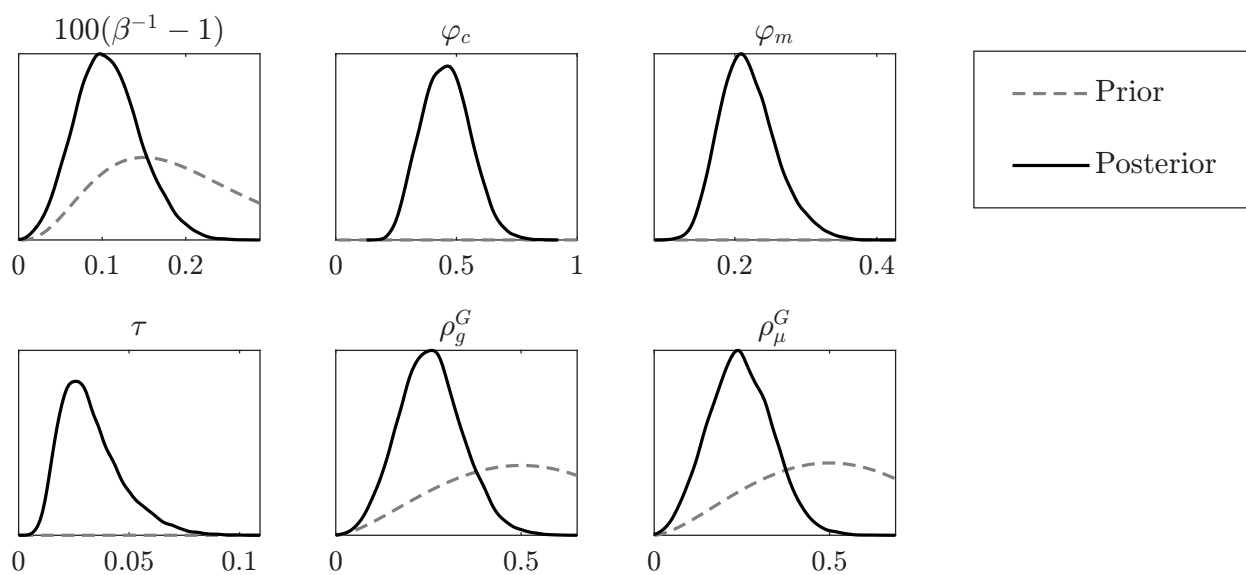


Table H.3: Results With Country-Specific Steady-State Productivity Growth

Panel A: Five Country

Model	Mean Cross-Country Correlation			
	GDP Growth	Inflation	GDP Growth– Inflation	Consumption Growth
<u>Benchmarks</u>				
A1. Data	0.39	0.45	0.25	0.18
A2. Baseline	0.39	0.45	0.25	−0.01
A3. No Trade (Re-estimated)	0.39	0.45	0.25	0.39
<u>Role of Trade</u>				
A4. No Trade (Counterfactual)	0.10	0.31	0.14	0.10
A5. Infinite Labor Elasticity ($\eta = 0$)	−0.04	0.19	0.08	−0.04
A6. Financial Autarky ($\tau = 10^6$)	−0.10	0.10	0.01	0.15
<u>Role of Country-Specific Shocks</u>				
A7. Monetary Shocks Only	0.30	0.06	0.05	−0.10
A8. TFP and Markup Shocks Only	0.12	−0.00	0.10	0.04
A9. Financial Shocks Only	−0.11	−0.20	−0.05	−0.17
<u>Alternative Mechanisms</u>				
A10. Flexible Price, Financial Autarky, Country-Specific TFP Shocks Only	0.12	0.05	0.05	0.12
A11. No Materials ($1 - \gamma = 0$)	0.30	0.42	0.21	0.10
A12. Symmetric Monetary Policy	0.39	0.47	0.27	−0.02

Panel B: Two Country

Model	Mean Cross-Country Correlation			
	GDP Growth	Inflation	GDP Growth– Inflation	Consumption Growth
B1. Data	0.56	0.72	0.35	0.49
B2. Baseline	0.56	0.72	0.35	0.57
B3. No Trade (Counterfactual)	0.59	0.72	0.37	0.59

Panel C: Model Validation Moments

Moment	Data	Baseline	No Trade (Re-estimated)	No Trade (Counterfactual)
<u>Targeted</u>				
1. autocorr(Δe_t)	0.35	0.35	0.35	0.57
2. std(Δe_t)	3.61	3.61	3.61	3.29
<u>Untargeted</u>				
3. std(ΔC_t)	0.66	1.05	0.74	0.99
4. corr($\Delta C_t, \Delta Y_t$)	0.56	0.66	1.00	1.00
5. corr($\Delta C_t - \Delta C_{US,t}, \Delta e_t$)	0.19	0.04	−0.03	0.14
6. Fama Coefficient	−1.10	−1.83	−0.89	−1.43

Notes: All correlations in Panels A and B are averaged across country pairs; all moments in Panel C are averaged across countries.

main text, which in turn dictates the response of monetary policy and associated spillovers when a shock hits a given country.

Panel C of Table H.3 presents the same set of model validation moments as Table 3 in the main text. Overall, the fit is relatively similar except for the Backus and Smith (1993) correlation in line 5, where the estimated model now undershoots the empirical correlation by a greater degree.

Role of the Great Recession: Given the prominence of the Great Recession during our sample period and the role of trade the literature has found for the period (e.g., Bems, Johnson, and Yi, 2013; Eaton, Kortum, Neiman, and Romalis, 2016; Mukoyama, Nakakuni, and Nirei, 2024), it is an important data point for our estimation. Nevertheless, a natural question is whether there was something particular about the distribution of shocks during this period that is driving our results.

We first check if demand shocks were particularly prominent during the Great Recession. To that end, we take the smoothed shocks from our model, normalize them to mean zero and standard deviation one (so that their scales are comparable), and plot a 4-quarter moving average of the absolute values of these standardized shocks. This represents a smoothed path for the size of the shocks, shown in Figure H.4.

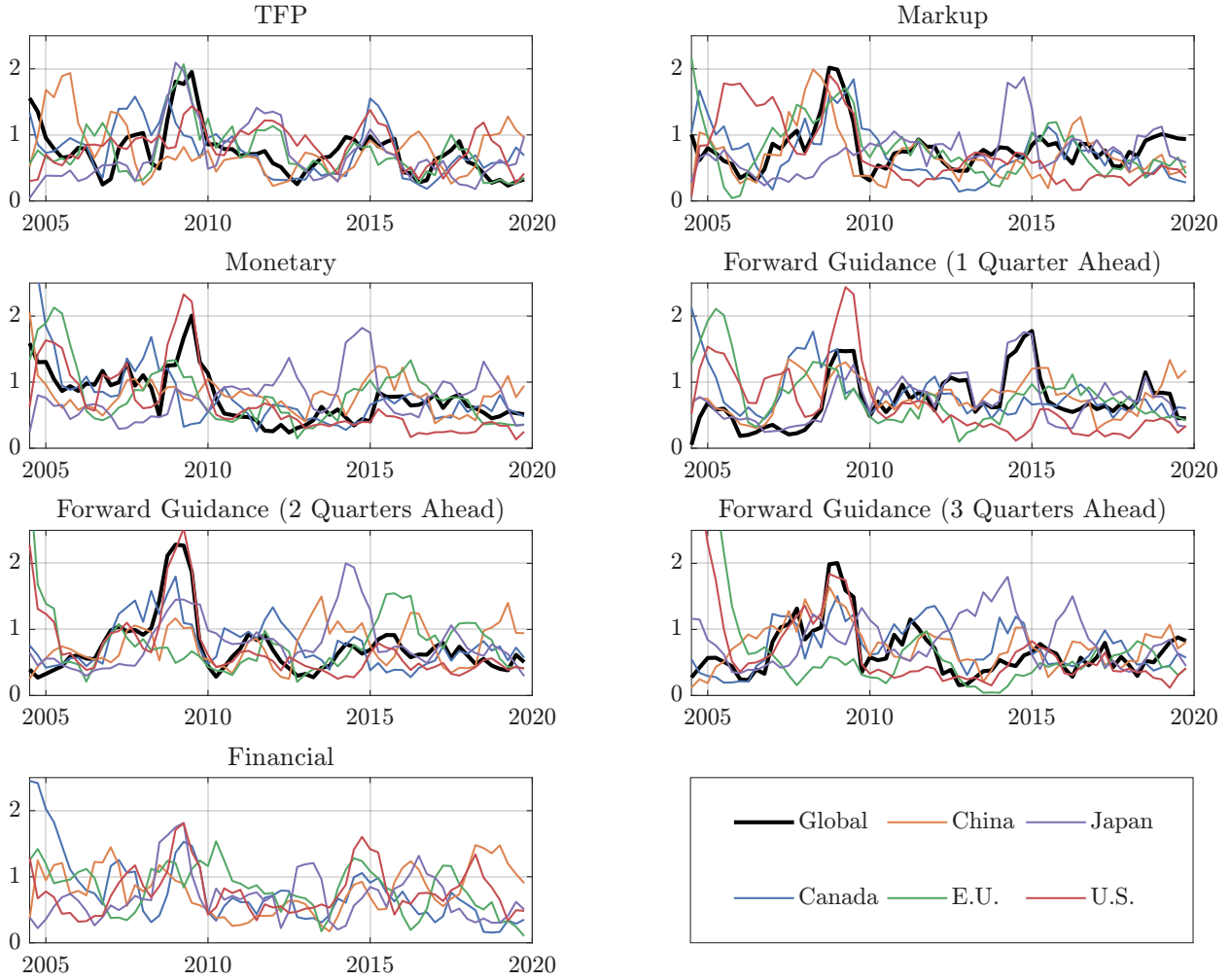
Two key observations emerge. First, while there are large shocks during the Great Recession, these are not limited to country-specific demand (monetary or forward guidance) shocks. Instead, the model finds spikes in the size of global shocks as well as TFP and markup shocks during this period. Second, There are spikes in demand shocks outside the Great Recession, such as for Japan around 2014. Therefore, the Figure H.4 suggests that our results are not driven solely by an outsized role of country-specific demand shocks during the Great Recession.

As a further check, we reproduce Panel A of Table 4 when we exclude the Great Recession observations in our counterfactuals. We keep all parameters unchanged from the baseline estimation in the main text. Thus, any deviations from the full sample can be traced back to differences in the shock distributions for different subsamples.

The results in Table H.4 reinforce the message that our results are not reliant on a specific composition of shocks during the Great Recession period. Specifically, we continue to find a large role for trade, with GDP correlations becoming slightly negative and inflation correlations falling by almost 30 percent when we shut down trade. Moreover, the subsequent counterfactuals support the mechanisms that we have emphasized—the correlations fall substantially without knock-on effects (line 5) and financial integration (line 6). Each set of country-specific shocks on their own (lines 7 to 9) imply relatively similar cross-country correlations to the baseline in Table 4 of the main text. Finally, we find that materials (line 11) and asymmetric monetary policy (line 12) are not the central forces driving the cross-country correlations, and instead that the cross-country correlations turn negative once we shut down price rigidity, financial integration, and non-TFP shocks.

While the degree of comovement in the data (line 1) falls once we exclude the Great Recession, the numbers are similar to line A3 of Table A.6, where we excluded the Great Recession but

Figure H.4: Relative Size of Shocks Over Sample Period



included a longer sample period and larger panel of countries. In that sense, the comovement across the five regions we use for estimation does not substantially differ from the broader data. In addition, some of the decline in comovement could originate from smaller global shocks outside the Great Recession. Indeed, in Figure H.4, we find that all the global shocks spike in size around the Great Recession (to different degrees and with slightly different timing). Moreover, in Table H.4 the average cross-country correlations fall more in percentage terms when we shut down trade than in our baseline that included the Great Recession (line 4 of Table H.4 versus line A4 of Table 4 of the main text). Finally, we note that cross-country comovement in GDP growth and inflation has been of interest to the literature even prior to the Great Recession; see references in footnote 1 in the main text.

Table H.4: Counterfactuals Outside the Great Recession (2007Q4–2009Q2)

Model	<u>Mean Cross-Country Correlation</u>			
	GDP Growth	Inflation	GDP Growth– Inflation	Consumption Growth
<u>Benchmarks</u>				
1. Data	0.15	0.25	0.02	0.11
2. Baseline	0.15	0.25	0.02	–0.09
3. No Trade (Re-estimated)	0.15	0.25	0.02	0.15
<u>Role of Trade</u>				
4. No Trade (Counterfactual)	–0.07	0.18	–0.01	–0.07
5. Infinite Labor Elasticity ($\eta = 0$)	–0.09	0.02	–0.03	–0.12
6. Financial Autarky ($\tau = 10^6$)	–0.15	0.03	–0.05	–0.05
<u>Role of Country-Specific Shocks</u>				
7. Monetary Shocks Only	0.29	–0.08	0.06	–0.11
8. TFP and Markup Shocks Only	0.04	0.23	–0.07	–0.00
9. Financial Shocks Only	–0.13	–0.04	–0.08	–0.15
<u>Alternative Mechanisms</u>				
10. Flexible Price, Financial Autarky, Country-Specific TFP Shocks Only	–0.08	–0.14	–0.08	–0.08
11. No Materials ($1 - \gamma = 0$)	0.10	0.25	–0.01	–0.02
12. Symmetric Monetary Policy	0.14	0.22	0.01	–0.09

Notes: All correlations averaged across country pairs.

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