How Do Sectoral Shocks Shape Future GDP?*

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Abstract

A production sector's size, as measured by its Domar weight, captures the contemporaneous aggregate effect of its productivity shocks. Any future aggregate effects, however, depend on that sector's participation in the investment network. We derive a dynamic generalization of Hulten's theorem in an environment with intermediate-input and investment networks. This generalization, implied by production efficiency alone, decomposes each sector's Domar weight into an *impact* and a *propagation* component. The relative size of these components then determines how persistent the aggregate effects of sectoral shocks are, but cannot be known absent information on the economy's production structure. We show in a tractable structural model that future GDP responses to sectoral shocks can also be described as weighted sums of *all* sectors' Domar weights, with weights primarily dictated by these sectors' network positions and capital shares. Quantifying the model with U.S. production data, we find that i) Domar weights become progressively less informative about the aggregate effects of sectoral shocks as the horizon lengthens and ii) over a three-year horizon, goods-producing sectors have larger cumulative aggregate effects than service-producing sectors, despite goods only accounting for less than one-third of GDP. Model-free local projections of U.S. GDP growth on sectoral TFP growth confirm these findings.

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1 Introduction

A long-standing question in macroeconomics concerns the implications of sectoral shocks for aggregate activity. Intermediate-input linkages propagate these shocks across industries (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, 2012), and investment linkages amplify them over time (vom Lehn and Winberry, 2022; Casal and Caunedo, 2025). Still, the bulk of the literature focuses on either the contemporaneous impact of sectoral disturbances or their longer run consequences. This paper addresses a less explored dimension: the medium run. Specifically, do current shocks to Total Factor Productivity (TFP) meaningfully inform the behavior of GDP growth not just when they arise but over the subsequent two to four years, and if so how?

Two classic ideas provide a natural starting point. First is Hulten's theorem: In an efficient economy, the first-order contemporaneous effect of a sector's productivity on real GDP is governed by its corresponding Domar weight - the ratio of its gross output to GDP (Domar, 1961; Hulten, 1978). This result and the broader production-network literature it supports (Gabaix, 2011; Baqaee and Farhi, 2019) imply that sectoral size and the input-output linkages it reflects determine which industry shocks matter most on impact. Second, the aggregate contemporaneous effects of productivity shocks understate their full importance when capital accumulation propagates these effects over time (Hulten, 1979). Taken together, these ideas suggest that the influence of industry disturbances on future aggregate activity hinges on the interactions between the characteristics of an economy's production networks and investment decisions.

We begin by developing a general theoretical framework that relies solely on *production efficiency*. We impose no restrictions on household preferences and do not require that demand be optimal. Functional forms that describe production need only be constant returns to scale (CRS). This approach contrasts with modern derivations of Hulten's theorem that invoke the first welfare theorem and require Pareto optimality. In a production environment with sectoral endogenous capital accumulation, we derive a dynamic generalization of Hulten's theorem that decomposes each sector's Domar weight into an impact component, reflecting contemporaneous industry effects on aggregate consumption, and a propagation component, capturing the present discounted value of all future consumption responses resulting from changes in the capital stock. Domar weights thus capture the effects of sectoral shocks on current GDP, but the decomposition of Domar weights between impact and propagation components determines how persistent these effects are. In particular, our main theorem establishes that in the period following an industry shock, all continuing aggregate effects operate through the accumulated aggregate capital stock, making the investment network a key determinant of medium-run propagation. However, despite the need to only track aggregate capital, one can no longer assess these propagation effects absent information on the economy's production structure.

To make our theoretical findings operational in the context of U.S. industry data, in addition to knowing the economy's production structure, we address two limitations of the theory in a dynamic context. One is that our general findings, while making no assumptions on demand and requiring only CRS technologies, are derived under perfect foresight. The other is that these findings are expressed in levels which empirically have no well-defined moments, as opposed to growth rates that are stationary. Therefore, to make progress, we study an analytically tractable variant of the canonical multisector growth model anchoring, for example, Foerster, Sarte, and Watson (2011), Atalay (2017), vom Lehn and Winberry (2022), and Casal and Caunedo (2025). Consistent with the sectoral configuration of U.S. production, the model features two distinct production networks: an intermediate-input network and an investment network. In this model variant, the value function is globally linear in the logs of capital and TFP in every industry (i.e., the endogenous and exogenous states), so that the optimal policy functions and impulse responses emerge as exact closed-form expressions that do not depend on the distribution of future shocks. The model's predictions, therefore, hold identically under uncertainty and in (log) first differences (i.e., growth rates).

The closed-form expressions we obtain give rise to an analytical law of motion for real GDP growth that depends on the entire history of past industry innovations, propagated over time through the economy's production structure. In this law of motion, the effects of sectoral shocks on current GDP growth amount to a Hulten (1978)-like term, governed by Domar weights.[1] We then show that the effects of sectoral shocks on future GDP growth are given by weighted sums of *all* Domar weights in the economy. Given a productivity disturbance in a particular sector, the weights in the Domar sums reflect the production attributes of its downstream sectors including their position in the production networks and, importantly, their capital share. Special cases of network architectures provide analytical insights into these findings.

To assess the quantitative implications of our analysis, we quantify our model economy using U.S. input-output and capital flow tables and other attributes of U.S. production. The implied networks reveal a striking asymmetry: some service-based sectors play a significant role in the intermediate-input network. For example, Wholesale & Retail Trade supplies up to 18 percent of intermediate expenditures in all goods-based sectors. Service sectors feature less prominently in the investment network, where Construction dominates, supplying 80 percent of investment goods to Real Estate and accounting for large investment expenditure shares within Services and Nondurable Goods. The model's impulse responses reflect this asymmetry. When productivity growth changes in an industry, that industry's effect on current GDP growth corresponds to its Domar weight. Thus, sectors with the largest Domar weights (e.g., Durable Goods, Wholesale & Retail) generate the largest contemporaneous aggregate response, with some persistence stemming from their role as intermediate input providers. As the horizon increases, however, the ranking of industry influence on GDP growth generally tilts away from large sectors, with the effects of Construction proving more persistent because of its role in the investment network. In sectors that do not feature prominently in either network, such as Food & Accommodation, the aggregate effects of productivity changes are not only small on impact but also have negligible

[1] In a structural model with unit-elastic technologies and investment linkages, vom Lehn and Winberry (2022) give a detailed account of the contemporaneous aggregate effects of sectoral shocks, and how these effects can deviate from Domar weights when aggregate labor supply is elastic. In contrast, our focus lies instead on the time profile of how sectoral shocks affect aggregate outcomes, and we keep Hulten (1978) as the natural benchmark.

persistence.

The Domar weight decomposition we underscore in our main theorem makes the mechanism transparent: nearly all of Construction's Domar weight (93 percent) reflects its propagation component while the Domar weight of Food & Accommodation predominantly consists of its impact component (e.g., 90 percent). Ultimately, the characteristics of U.S. production imply that i) Domar weights become gradually less informative regarding the aggregate effects of sectoral shocks, rather than immediately after the shocks materialize as implied by a static environment, and ii) over a period of three years, goods-producing sectors have larger cumulative aggregate effects than service-producing sectors despite goods representing less than one-third of the U.S. economy.

We conclude our analysis by exploring our model's predictions using U.S. industry data. Because sectoral TFP growth is exogenous and directly observable as Solow residuals, local projections of GDP growth on sectoral TFP growth serve as the empirical counterpart to impulse response functions in a given structural model. The only supporting empirical assumption needed is that TFP shocks be uncorrelated with other fundamentals, in which case local projection estimates are unbiased. At first glance, sectoral productivity growth seems an unlikely driver of medium-run aggregate dynamics. In particular, we document that in postwar U.S. data, TFP growth within most industries exhibits little to no serial correlation while comovement in TFP growth across industries is negligible. Yet, model-free local projections of U.S. GDP growth on sectoral TFP growth largely replicate our key findings, including the persistent aggregate effects of productivity changes in Construction, the large contemporaneous but less persistent aggregate effects of disturbances in Wholesale & Retail Trade, and the purely contemporaneous aggregate effects of productivity shocks in Food & Accommodation. More generally, the local projections confirm that the aggregate effects of goods-producing sectors, while smaller than those of service-producing industries on impact, eventually overtake services over a three-year horizon. These local projections further confirm that Domar weights become gradually less informative with respect to the aggregate effects of sectoral shocks as the horizon increases.

This paper is organized as follows. Section 2 sets up the general theoretical framework. It describes the aggregate effects of sectoral shocks in a dynamic environment through a generalization of Hulten's theorem, three corollaries, and one proposition. To make these ideas operational, Section 3 presents a structural model informed by U.S. industry data where the aggregate effects of sectoral shocks over time have analytical expressions. We then use this model to study special cases of production network architectures that help untangle the aggregate dynamics stemming from sectoral shocks. Section 4 compares the structural model's dynamic implications against estimates from model-free local projections of GDP growth at different horizons on each sector's current TFP growth. Section 5 provides some concluding remarks. A detailed **online Technical Appendix** contains a comprehensive discussion of the data, detailed derivations of all results discussed in the main text, discussions of departures from our benchmark assumptions, and additional figures and tables complementing our main analysis.

Related Literature

The body of work mapping industry productivity changes into aggregate outcomes by way of production networks has, for the most part, evolved historically along two main lines.

In static environments, Hulten's theorem provides a tight first-order mapping from productivity shocks to real GDP through sales-based (Domar) exposure. Importantly, the insight in Hulten (1978) obviates the need for detailed information about the nature of production linkages among industries, and the pronounced asymmetry in Domar weights underpins the modern literature on the granular origins of aggregate fluctuations (Gabaix, 2011; Yeh, 2025). More recent work refines this static mapping by clarifying how the nature of input-output linkages determines the distribution of Domar weights (Acemoglu et al., 2012) as well as the role of nonlinearities, reallocation, and distortions in rich production networks (Baqaee and Farhi, 2019, 2020; Osotimehin and Popov, 2023).

The second strand of work stresses that productivity changes have *intertemporal* implications. Hulten (1979) emphasizes that with capital accumulation, the consequences of productivity changes cannot be inferred from contemporaneous effects alone. These changes affect investment incentives and, therefore, the entire path of future production and consumption. This notion is explored in multisector environments by Horvath (1998) and Dupor (1999), but in ways that abstract from investment linkages in the economy. In more recent contributions, the investment supply chain mechanisms we highlight are treated as integral to the environment (Foerster, Sarte, and Watson, 2011; Atalay, 2017). In particular, vom Lehn and Winberry (2022) underscore investment linkages as a key mechanism amplifying the effects of idiosyncratic shocks and the outsize role of investment hub industries for macroeconomic fluctuations. In addition, the role of sectoral investment is shown to play a key role in shaping long-run outcomes (Foerster, Hornstein, Sarte, and Watson, 2022; Baqaee and Malmberg, 2025) and in driving economic development and structural change (Casal and Caunedo, 2025).

This paper in part brings these two lines of research together. We revisit Hulten's theorem but in a dynamic setting with investment, focusing only on production efficiency and imposing no restrictions on technology other than constant returns-to-scale. We then apply the lessons from the dynamic version of Hulten's theorem to a more structural environment and explicitly work out how shocks to individual U.S. production sectors propagate across other sectors and over time. At the one to four-year horizon, these dynamic effects and their aggregate implications turn out to be notably disparate.

Finally, intertemporal considerations in network economies can also arise through lags in the use of materials, as first explored in Long and Plosser (1983). In emerging work, Leng, Liu, Ren, and Tsyvinksi (2025), as well as Taschereau-Dumouchel and Schaal (2025), emphasize and generalize these production lags through time-to-build features in production networks. These features in turn amplify the effects of idiosyncratic shocks and give rise to pronounced endogenous business cycle fluctuations.

2 Capital Accumulation and the Aggregate Effects of Sectoral Shocks

This section describes how changes in sectoral TFP growth, even if largely idiosyncratic and weakly persistent, can nevertheless be informative regarding the behavior of future real GDP. The central finding is a decomposition of each sector's Domar weight into an *impact* component, reflecting the contemporaneous effect of industry productivity on aggregate consumption, and a *propagation* component, that captures changes in the path of future consumption through investment and capital accumulation. We begin with a reformulation of Hulten's theorem in a dynamic setting (Section 2.2), and then extend the associated derivation to characterize the aggregate effects of sectoral productivity at arbitrary horizons (Section 2.3).

Three features of our approach deserve emphasis. First, the results in Sections 2.2–2.3 rely solely on *production efficiency*. We impose no restrictions on household preferences and do not require that consumption be efficient. This approach contrasts with standard modern derivations of Hulten's theorem that invoke the first welfare theorem and, therefore, require that allocations be Pareto optimal (Hulten, 1978; Baqaee and Farhi, 2019).[2] Second, we maintain general constant-returns-to-scale technologies throughout; functional-form restrictions are imposed only in the structural model of Section 3. Third, the dynamic generalization in Section 2.3 examines the level effects of a change in productivity whose future path is known to agents, i.e., under *perfect foresight*. While these assumption are standard in growth-accounting applications of the envelope theorem, they preclude a direct comparison with empirical local projections, which measure responses to unanticipated innovations, since the data are stationary only in growth rates. The structural model of Section 3 resolves these limitations. In particular, the dynamic equilibrium we study gives rise to closed-form impulse response functions that hold identically under uncertainty and in growth rates.

2.1 The Production Environment

Consider an economy with N production sectors indexed by i (or j). In each period t and sector i, firms produce gross output using the constant-returns-to-scale (CRS) technology

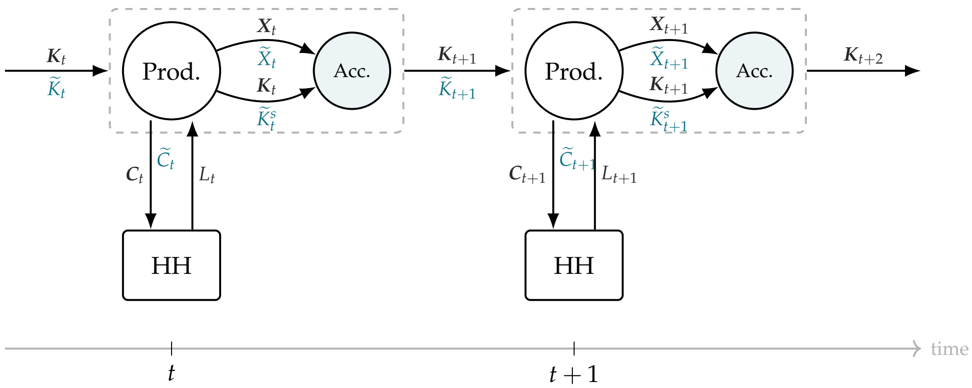
$$Q_{it} = Z_{it}\, \mathcal{F}_i\big(L_{it}, K_{it}, \boldsymbol{M}_{it}\big), \tag{1}$$

where L_{it} and K_{it} denote labor and capital inputs respectively, $\boldsymbol{M}_{it} = (M_{1it}, \ldots, M_{Nit})$ is a vector of intermediate inputs from all sectors, where M_{jit} denotes the quantity of materials from sector j used by sector i, and Z_{it} is sector-i TFP. Aggregate labor supply, L_t, is exogenous.

New capital goods in sector i are produced using existing capital and investment inputs with

[2] Baqaee and Farhi (2019), in particular, derive Hulten's theorem using a macroeconomic envelope condition resulting from the first welfare theorem, which presumes an efficient competitive equilibrium. Our approach, anchored in production efficiency, instead recovers the same contemporaneous result while naturally extending to a dynamic setting without restrictions on preferences.

Figure 1: **Production and capital accumulation**



the CRS technology

$$K_{it+1} = \mathcal{K}_i\big(K_{it}, \boldsymbol{X}_{it}\big), \tag{2}$$

where $\boldsymbol{X}_{it} = (X_{1it}, \ldots, X_{Nit})$ is a vector of investment goods, with X_{jit} denoting the quantity of sector-j output used as investment in sector i.

We assume the following market structure, illustrated in Figure 1. At the beginning of period t, firms producing gross output in sector i purchase capital, K_{it}, at price, P^K_{it}, and hire workers at wage rate, W_t. They then produce output, Q_{it}, which they sell at price, P_{it}. Once production is complete, these firms sell their capital at price, $P^K_{it} - R_{it}$, where R_{it} is the rental price of capital in sector i at date t. Thus, $P^K_{it} - R_{it}$ denotes the scrap value of capital used to produce gross output. Firms producing new capital goods then purchase investment inputs, $\boldsymbol{X}_{it}$, along with existing capital to produce the stock of new capital goods, K_{it+1}, which they sell at price P^K_{it+1}. Arbitrage between the asset price at the beginning and end of the period implies that

$$P^K_{it+1} = (1 + r_t)\, P^K_{it+1}, \tag{3}$$

where r_t is the nominal interest rate between t and $t+1$. We define real net output (value added) in sector i as $Y_{it} \equiv Q_{it} - \sum_j M_{jit}$, and nominal GDP as $\widetilde{Y}_t \equiv \sum_i P_{it} Y_{it}$. The first-order conditions associated with efficient production, together with the CRS property of technology, imply that

$$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{X}_t \equiv \sum_i P_{it} C_{it} + \sum_{j,i} P_{jt} X_{jit}, \tag{4}$$

where C_{it} denotes real consumption of sector i goods. A '$\sim$' over a variable represents its nominal value so that $\widetilde{C}_t$ and $\widetilde{X}_t$ denote nominal aggregate consumption and investment, respectively. Appendix B provides the full derivations, including the relationship between nominal investment and the change in the value of the capital stock.

Definition 1 (The change in real GDP)**.** *Consider a change in the economic environment. We define the* *resulting change in real GDP as*

$$d\log Y_t \equiv \sum_i S_{it}\, d\log Y_{it}, \tag{5}$$

where $S_{it} \equiv P_{it}Y_{it}/\widetilde{Y}_t$ *is sector* i*'s value-added share in GDP. Analogously, we define changes in real consumption, real investment, real capital services, and the real capital stock as share-weighted sums of sectoral log changes,*

$$d\log C_t \equiv \sum_i \frac{P_{it}C_{it}}{\widetilde{C}_t}\, d\log C_{it}, \tag{6}$$

$$d\log X_t \equiv \sum_{j,i} \frac{P_{jt}X_{jit}}{\widetilde{X}_t}\, d\log X_{jit}, \tag{7}$$

$$d\log K_t^s \equiv \sum_i \frac{R_{it}K_{it}}{\widetilde{K}_t^s}\, d\log K_{it}, \tag{8}$$

$$d\log K_t \equiv \sum_i \frac{P_{it}^K K_{it}}{\widetilde{K}_t}\, d\log K_{it}, \tag{9}$$

so that

$$d\log Y_t = S_{ct}\, d\log C_t + S_{xt}\, d\log X_t, \tag{10}$$

where $S_{ct} \equiv \widetilde{C}_t/\widetilde{Y}_t$ *and* $S_{xt} \equiv \widetilde{X}_t/\widetilde{Y}_t$.

In addition, Appendix B shows that the share-weighted change in real investment can also be written as

$$S_{xt}\, d\log X_t = S_{kt}\, d\log K_t^s + S_{kt}^+\, d\log K_{t+1} - (1+r_{t-1})\frac{\widetilde{Y}_{t-1}}{\widetilde{Y}_t}\, S_{k,t-1}^+\, d\log K_t, \tag{11}$$

where $\widetilde{K}_t^s \equiv \sum_i R_{it}K_{it}$ is the nominal value of aggregate capital services, $\widetilde{K}_t \equiv \sum_i P_{it}^K K_{it}$ is the nominal value of the aggregate capital stock, $S_{kt} \equiv \widetilde{K}_t^s/\widetilde{Y}_t$ is the income share of capital services, and $S_{kt}^+ \equiv \widetilde{K}_{t+1}/\widetilde{Y}_t$ is the asset-to-income ratio at the end of period t. In other words, a change in real investment decomposes into two components: a change in the current service flow from capital and a change in the continuation value of the capital stock associated with future production.

2.2 Hulten's Theorem in a Dynamic Production Environment

Hulten (1978)'s seminal contribution shows that in a static efficient economy, the first-order effect of a unit productivity shock in sector i on real GDP is that sector's Domar weight. We present a reformulation of this result in the dynamic production environment presented in the previous section, where the theorem applies within each period holding the capital stock fixed.

Consider a change in sector i's productivity at date t, $d\log Z_{it}$, that leaves productivity in all other sectors, aggregate labor supply, and the vector of beginning-of-period capital stocks unchanged. Then,

$$\frac{d\log Y_t}{d\log Z_{it}} = D_{it} \equiv \frac{P_{it}Q_{it}}{\widetilde{Y}_t}. \tag{12}$$

The term, D_{it}, is sector i's *Domar weight*, which is the ratio of its gross output to GDP.

We present two proofs of equation (12) in Appendix B. The first follows the classical approach, constructing a within-period production possibilities frontier and applying the envelope theorem. The second introduces a *single-period dynamic production possibilities frontier* that jointly characterizes the feasible set for current consumption *and* end-of-period capital. This second approach is important because it provides the bridge to our dynamic generalization of Hulten's theorem. In particular, the proof of this dynamic generalization extends this single-period frontier to an arbitrary number of periods.

2.3 The Dynamic Generalization of Hulten's Theorem

We now state our main theoretical result. Consider a shock to the productivity of sector i, known in period t, that materializes at some date $\tau \geq t$. Throughout, τ denotes the date at which the shock occurs, t denotes the date at which it becomes known to agents, and t' indexes the periods over which responses are evaluated. We maintain the assumption that the disturbance is a one-time innovation: sectoral productivity returns to its baseline path after date τ. The following theorem establishes that Domar weights at the date of the shock pin down the entire discounted sequence of responses in aggregate consumption and capital over all subsequent periods.

Theorem 1 (Hulten's Theorem with capital accumulation)**.** *Consider a productivity shock to sector i at date τ, known at date t. Then, for any $T \geq \tau - t$,*

$$D_{i\tau} = \sum_{t\leq t'\leq t+T} \beta_{\tau,t'} \frac{\widetilde{Y}_{t'}}{\widetilde{Y}_\tau} S_{c,t'} \frac{d\log C_{t'}}{d\log Z_{i\tau}} \; + \; \beta_{\tau,t+T} \frac{\widetilde{Y}_{t+T}}{\widetilde{Y}_\tau} S^+_{k,t+T} \frac{d\log K_{t+T+1}}{d\log Z_{i\tau}}, \tag{13}$$

where, for any two dates $s \leq s'$, $\beta_{s,s'} \equiv \prod_{t''=s}^{s'-1}(1+r_{t''})^{-1}$ denotes the discount factor from s to s', with the convention $\beta_{s',s} \equiv 1/\beta_{s,s'}$, so that $\beta_{\tau,t'}$ compounds, rather than discounts, over anticipation periods $t \leq t' < \tau$.

The proof, given in Appendix B.3, relies on constructing a T-period dynamic production possibilities frontier along with applications of the envelope theorem. As such, it is predicated only on *production efficiency*, specifically the condition that the allocations lie on the production possibilities frontier in each period. No assumptions on preferences or the optimality of intertemporal consumption choices are needed. Theorem 1 has three immediate implications.

Corollary 1 (Recovering Hulten's Theorem in its static form)**.** *Setting $\tau = t$ and $T = 0$ recovers Hulten (1978)'s key insight:*

$$D_{it} = S_{ct} \frac{d\log C_t}{d\log Z_{it}} + S^+_{kt} \frac{d\log K_{t+1}}{d\log Z_{it}} = \frac{d\log Y_t}{d\log Z_{it}}. \tag{14}$$

An industry's Domar weight governs the contemporaneous effect of changes in that industry's productivity on GDP. However, in a dynamic economy, this effect now has two components: an immediate effect on aggregate consumption, and an effect on end-of-period capital.

Corollary 2 (Decomposing Domar weights)**.** *Given $T > 0$, and a productivity shock in sector i at date t, each Domar weight can be decomposed into two components,*

$$D_{it} = D_{it}^c + D_{it}^x, \tag{15}$$

where

$$D_{it}^c = S_{ct} \frac{d \log C_t}{d \log Z_{it}}, \tag{16}$$

$$D_{it}^x = \underbrace{\sum_{t'=t+1}^{t+T} \beta_{t,t'} \frac{\widetilde{Y}_{t'}}{\widetilde{Y}_t} S_{c,t'} \frac{d \log C_{t'}}{d \log Z_{it}}}_{\textit{future consumption}} + \underbrace{\beta_{t,t+T} \frac{\widetilde{Y}_{t+T}}{\widetilde{Y}_t} S_{k,t+T}^{+} \frac{d \log K_{t+T+1}}{d \log Z_{it}}}_{\textit{capital continuation value}}. \tag{17}$$

The *impact component*, D_{it}^c, reflects the contemporaneous effect of a change in industry productivity on aggregate consumption. The *propagation component*, D_{it}^x, captures the present discounted value of all future consumption changes. These changes follow from the change in investment prompted by the productivity shock and the resulting evolution of the capital stock. While the Domar weight, D_{it}, is directly observable, its decomposition into D_{it}^c and D_{it}^x depends on the production structure of the economy, namely investment and intermediate-input linkages, sectoral capital shares, and other primitives of industry production. The final corollary characterizes the response of real GDP in periods following the change in industry productivity.

Corollary 3 (Future GDP and capital services)**.** *For $t > \tau$, the response of real GDP at date t to a productivity shock in sector i at date τ is governed entirely by the response in the flow of capital-services,*

$$\frac{d \log Y_t}{d \log Z_{i\tau}} = S_{kt} \frac{d \log K_t^s}{d \log Z_{i\tau}}, \qquad t > \tau. \tag{18}$$

In the periods following the initial shock, productivity in the affected industry has returned to its baseline path. Therefore, any continuing effects on real GDP arise through the accumulated capital stock. Equation (18) makes this precise: the only channel through which a productivity shock that materialized in the past affects current real GDP is given by the resulting change in the aggregate flow of capital services. Corollary 3 thus identifies the mechanism at the heart of this paper: the persistence of the aggregate effects of sectoral shocks operates through capital accumulation.

It is worth emphasizing what the corollary rules out. In the periods following the shock, the entire within-period equilibrium adjusts to the altered capital stocks: labor reallocates across sectors even though its aggregate supply is fixed, intermediate inputs are re-routed through the production network, and the composition of final output shifts between consumption and investment. Each of these margins moves sectoral value added, and real GDP aggregates precisely these movements. Corollary 3 states that all such reallocations contribute nothing to real GDP at first order: under production efficiency, they move the economy along the production possibilities frontier, which market prices value at zero. With distortions, by contrast, these reallocations would carry first-order aggregate effects. What remains is a growth-accounting statement with specific and measurable weights: the response of real GDP equals the income share of capital services multiplied by the rental-share-weighted response of sectoral capital stocks, that is, the response of the aggregate capital-services flow.

Because it identifies a single and complete channel, Corollary 3 is also testable. Mechanisms outside our environment—variable capacity utilization, an elastic aggregate labor supply, time-to-build lags in production, or allocative distortions—would each introduce additional channels through which past sectoral shocks affect current GDP. In Section 4.7, we take this restriction directly to the data. Local projections of aggregate capital-services growth on sectoral TFP growth thereby serve as a test of whether the mechanism at the core of the paper is the relevant one.

Key Insights. Theorem 1 and its corollaries deliver several considerations that create comovement between sectoral productivity and aggregate GDP . First, although the Domar weight is directly observable, it is informative only about the *total present discounted value* of current and future consumption responses, not about how these responses are distributed over time. Put another way, two sectors may share the same Domar weight and yet differ markedly in their composition between D^c_{it} and D^x_{it}. Second, sectors that contribute disproportionately to investment in other sectors, rather than to intermediate inputs or final consumption, will tend to have a larger share of their Domar weight reflected in D^x_{it}. Productivity disturbances in such sectors would then have more persistent aggregate effects, precisely the pattern observed in the data for investment hubs industries highlighted by vom Lehn and Winberry (2022). Third, making progress in characterizing D^c_{it} and D^x_{it} separately, and thereby understanding the horizon profile of aggregate responses, requires information regarding the production structure of the economy, most significantly, the intermediate-input network, denoted below by $\mathbf{\Phi}$, the investment network, denoted by $\mathbf{\Omega}$, and sectoral capital shares, denoted by $\boldsymbol{\alpha}$. These considerations motivate the structural model developed in Section 3.

2.4 Consumption Efficiency and Welfare

What about consumption efficiency? While the results we have just presented pertain solely to production efficiency, Domar weights serve only as partial indicators of consumption efficiency. Specifically, consider the production environment of Section 2.1 and suppose that a representative household maximizes the present-discounted utility,

$$\mathcal{V}_t \equiv \sum_{\tau=t}^{\infty} \beta^{\tau-t} L_\tau \mathcal{U}\left(\frac{C_\tau}{L_\tau}\right), \tag{19}$$

given its labor income, where $\mathcal{U}(\cdot)$ is a per-capita utility function and $\beta \in (0,1)$ is households' subjective discount factor.

Proposition 1 (Welfare interpretation of Domar weights)**.** *Under the preferences in equation (19), sector i's Domar weight gives the following weighted response in continuation utility,*

$$D_{it} = \varepsilon_t \frac{S_{ct}}{L_t(\mathcal{U}_t/\mathcal{V}_t)} \frac{\partial \log \mathcal{V}_t}{\partial \log Z_{it}}, \tag{20}$$

where $\varepsilon_t \equiv \partial \log \mathcal{E}(\mathcal{U}_t; \boldsymbol{P}_t)/\partial \log \mathcal{U}_t$ is the utility elasticity of the expenditure function $\mathcal{E}(\cdot;\cdot)$.

The proof is given in Appendix B.5. When preferences take the form $\mathcal{U}(\boldsymbol{C}) = \log \mathcal{C}(\boldsymbol{C})$, where $\mathcal{C}(\cdot)$ is homogeneous of degree one, we have $\varepsilon_t = \mathcal{U}_t$ and equation (20) simplifies to

$$D_{it} = S_{ct} \frac{\partial(\mathcal{V}_t/L_t)}{\partial \log Z_{it}}. \tag{21}$$

In other words, with log utility and a homogeneous consumption aggregator, industry i's Domar weight gives the marginal productivity effect in that industry on per-capita lifetime utility, weighted by the aggregate consumption share of GDP (since aggregate consumption and GDP are now distinct). Proposition 1 then describes the partial link between observable Domar weights and the welfare effects of sectoral productivity disturbances.

2.5 Takeaways and Transition to a Structural Model Economy

In a dynamic economy with capital accumulation, Domar weights fully capture the contemporaneous effect of a sectoral productivity changes on real GDP. However, these weights alone do not tell us how the response of an economy's aggregates unfolds over time. Instead, the composition of Domar weights highlighted in Corollary 2 provides key information in this respect. It separates the within-period consumption effect from a forward-looking propagation component through investment. Thus, productivity disturbances in sectors with large D^x_{it} components (relative to D^c_{it}) generate aggregate responses that are more persistent. That said, knowing the particular split between D^x_{it} relative to D^c_{it} for any sector requires information on the economy's entire sectoral production structure.

To make these results operational, the next section explores a structural model that ties sectoral propagation directly to the primitives of the environment, most notably a Leontief inverse, $\boldsymbol{\mathcal{L}}$, that captures the role of intermediate-input linkages, a matrix of sectoral investment shares, $\boldsymbol{\Omega}$ (i.e., the investment network), and a vector of sectoral capital shares, $\boldsymbol{\alpha}$, that reflects the intensity with which different sectors use capital in production.

3 A Structural Model

The general results established in Section 2 highlight a decomposition of Domar weights where all aggregate effects beyond the initial productivity shock operate through endogenous adjustments in investment. To make these results quantitatively operational and gain insight into the aggregate effects of sectoral shocks over time, we proceed with a tractable variant of the canonical multisector growth model. This model is convenient since it can be directly mapped into input-output and fixed asset tables that represent an economy's production networks. We show that the resulting model economy delivers exact closed-form expressions for the law of motion of GDP growth, analytical impulse-response functions, and analytical expressions for the sectoral decomposition of Domar weights.

Several aspects of the model warrant notice. First, unlike the general framework of Section 2 that required perfect foresight, the model we study features *certainty equivalence*. Therefore, the closed-form policy functions we derive hold under uncertainty. Second, the structural environment we study delivers findings that can be framed in terms of growth rates that are well-described empirically. These two equilibrium properties mean that the model's impulse response functions can be directly compared to empirical local projections of the type estimated in Section 4. Third, the model's solution is exact and represents a global characterization of the equilibrium rather than a log-linearization around a steady state. The value function of the representative household is globally linear in the logs of sectoral capital and TFP, extending the analytical tradition of Long and Plosser (1983) to an economy with two distinct production networks and nontrivial capital accumulation. The two distinct networks consist of an intermediate-input network, $\boldsymbol{\Phi}$, and an investment network, $\boldsymbol{\Omega}$, that play different roles in the propagation of shocks over time.

3.1 Economic Environment

A representative household maximizes its lifetime utility,

$$\mathcal{V} = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \log C_{\tau}, \tag{22}$$

where $C_\tau \equiv \prod_i C_{i\tau}^{\theta_i}$ is a Cobb–Douglas consumption aggregate with $\theta_i > 0$ and $\sum_i \theta_i = 1$, which serves as the numéraire.

In each sector i, gross output is assembled by combining value added with intermediate inputs according to

$$Q_{it} = Z_{it} \left(\frac{Y_{it}}{\gamma_i} \right)^{\gamma_i} \prod_j \left(\frac{M_{jit}}{(1-\gamma_i)\phi_{ji}} \right)^{(1-\gamma_i)\phi_{ji}}, \tag{23}$$

where value added is produced from capital and labor using the Cobb–Douglas technology,

$$Y_{it} = \left(\frac{K_{it}}{\alpha_i}\right)^{\alpha_i} \left(\frac{L_{it}}{1-\alpha_i}\right)^{1-\alpha_i}, \tag{24}$$

where Z_{it} is sector-i gross-output TFP, $\gamma_i \in (0,1)$ is the value-added share in gross output, and $\phi_{ji} > 0$ represents sector i's intermediate-input expenditure shares such that $\sum_j \phi_{ji} = 1$. The matrix, $\boldsymbol{\Phi} = [\phi_{ij}]$, then captures the economy's *intermediate-input network*.

Capital in sector i evolves according to

$$K_{it+1} = K_{it}^{1-\zeta_i} X_{it}^{\zeta_i}, \tag{25}$$

$$X_{it} = \chi_i \prod_j \left(\frac{X_{jit}}{\omega_{ji}}\right)^{\omega_{ji}}, \tag{26}$$

where X_{jit} denotes sector-j output used as investment by sector i, with corresponding expenditure share, $\omega_{ji} \geq 0$, such that $\sum_j \omega_{ji} = 1$. In addition, total investment X_{it} in each sector is subject to adjustment costs governed by the parameter, $\zeta_i \in (0,1]$.[3] The matrix, $\boldsymbol{\Omega} = [\omega_{ij}]$, then characterizes the economy's *investment network*.

Throughout the paper, we denote the log of any variable 'X' by its lowercase, 'x,' and use the following notation to define the vectors, $\boldsymbol{c}_t \equiv (c_{it})$, $\boldsymbol{k}_t \equiv (k_{it})$, $\boldsymbol{z}_t \equiv (z_{it})$, etc., and the model parameters, $\boldsymbol{\theta} \equiv (\theta_i)$, $\boldsymbol{\alpha}_d \equiv \text{diag}(\alpha_i)$, $\boldsymbol{\zeta}_d \equiv \text{diag}(\zeta_i)$, $\boldsymbol{\gamma}_d \equiv \text{diag}(\gamma_i)$, $\boldsymbol{\Phi} \equiv [\phi_{ij}]$, $\boldsymbol{\Omega} \equiv [\omega_{ij}]$, etc.

For now, TFP in each sector follows a random walk with sector-specific drift,

$$\boldsymbol{z}_t = \boldsymbol{z}_{t-1} + \boldsymbol{g}^z + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim i.i.d.\ \mathcal{N}\left(0, \text{diag}(\sigma_i^2)\right), \tag{27}$$

where the innovations, ε_{it}, are independent across sectors and over time.[4] Finally, goods markets clear in each sector, $C_{it} + \sum_j M_{ijt} + \sum_j X_{ijt} = Q_{it}\ \forall i$, as does the labor market, $\sum_i L_{it} = L_t$.

3.2 Equilibrium and Policy Functions

Since the economy has no frictions, equilibrium allocations are efficient. We can then obtain these allocations by solving the social planner's problem. Specifically, we maximize the expected value of households' lifetime utility, conditional on information available at date t, subject to the technologies, (24) - (25), the process defining the evolution of sectoral productivity, (27), and market clearing. We conjecture that the value function is linear in the (log of the) state variables,

$$\mathcal{V}(\boldsymbol{k}_t, \boldsymbol{z}_t) = \boldsymbol{B}_k' \boldsymbol{k}_t + \boldsymbol{B}_z' \boldsymbol{z}_t + B, \tag{28}$$

[3]When $\zeta_i = 0$, adjustment costs are infinite and capital becomes a static factor of production, in which case sectoral shocks cannot propagate over time. In the steady state, $K_i = \chi_i^{1/\zeta_i} \prod_j (X_{ji}/\omega_{ji})^{\omega_{ji}}$, where $1/\chi_i^{1/\zeta_i}$ becomes the investment rate needed to maintain a constant capital stock.

[4]While this specification treats sectoral TFP growth as cross-sectionally and serially uncorrelated, the quantitative analysis in Section 4 allows for modest serial correlation in sectoral TFP growth where appropriate.

where a 'prime' denotes the transpose of a vector or a matrix, while $\boldsymbol{k}_t \equiv (\log K_{1t}, \cdots, \log K_{Nt})'$, and $\boldsymbol{z}_t \equiv (\log Z_{1t}, \cdots, \log Z_{Nt})'$. The $N \times 1$ vectors, $\boldsymbol{B}_k$ and $\boldsymbol{B}_z$, as well as the scalar, B, ultimately reflect primitives of the economic environment, $\boldsymbol{\theta}$, $\boldsymbol{\alpha}_d$, $\boldsymbol{\gamma}_d$, $\boldsymbol{\Phi}$, $\boldsymbol{\Omega}$, etc. Appendix C provides a full derivation of the model solution and verifies that the value function, (28), has the correct form.

We highlight two properties of the model economy. First, the (log) linearity of the value function implies *certainty equivalence* so that the first-order conditions characterizing efficient allocations are the same as those under perfect foresight. Second, the optimal policy functions are (log) linear, Markovian, and globally exact rather than linear approximations around a steady state.

In equilibrium, the trajectories of sectoral consumption and capital are given by,

$$\boldsymbol{c}_t = \boldsymbol{\mathcal{L}}\left(\boldsymbol{z}_t + \boldsymbol{\alpha}_d \boldsymbol{\gamma}_d \, \boldsymbol{k}_t\right) + \text{const.}, \tag{29}$$

$$\boldsymbol{k}_{t+1} = (\boldsymbol{I} - \boldsymbol{\zeta}_d)\boldsymbol{k}_t + \boldsymbol{\zeta}_d \boldsymbol{x}_t, \tag{30}$$

$$\boldsymbol{x}_t = \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}(\boldsymbol{z}_t + \boldsymbol{\alpha}_d \boldsymbol{\gamma}_d \, \boldsymbol{k}_t) + \text{const.}, \tag{31}$$

where the *Leontief inverse*,

$$\boldsymbol{\mathcal{L}} \equiv \left(\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{\gamma}_d)\boldsymbol{\Phi}'\right)^{-1}, \tag{32}$$

embodies the direct and indirect effects of intermediate-input linkages. Put another way, the element, $\mathcal{L}_{ij}$, gives sector j's total gross-output requirement per unit of value added in sector i, accounting for all rounds of intermediate-input demand created by intermediate input linkages. Hence the Leontief inverse, which plays a key role in shaping the 'influence vector' emphasized by Acemoglu et al. (2012), now also helps shape the dynamics of sectoral variables.

The log linearity of the policy functions and the random walk nature of sectoral TFP growth in equation (27) mean that real quantities inherit a unit root in levels but are stationary and well-behaved in growth rates. In particular, first differencing the optimal policy functions immediately yields the state space representation governing the dynamics of the growth rates of real consumption and capital,

$$\Delta \boldsymbol{c}_t = \boldsymbol{\mathcal{L}} \, \Delta \boldsymbol{z}_t + \boldsymbol{\mathcal{L}} \, \boldsymbol{\alpha}_d \boldsymbol{\gamma}_d \, \Delta \boldsymbol{k}_t, \tag{33}$$

$$\Delta \boldsymbol{k}_{t+1} = \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}} \, \Delta \boldsymbol{z}_t + \boldsymbol{\mathcal{J}} \, \Delta \boldsymbol{k}_t, \tag{34}$$

where

$$\boldsymbol{\mathcal{J}} \equiv \boldsymbol{I} - \boldsymbol{\zeta}_d + \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}} \, \boldsymbol{\alpha}_d \boldsymbol{\gamma}_d. \tag{35}$$

Equation (33) indicates that the dynamics of real consumption growth in each sector respond to changes in both exogenous and endogenous state variables, $\Delta \boldsymbol{z}_t$ and $\Delta \boldsymbol{k}_t$, intermediated by the Leontief inverse, $\boldsymbol{\mathcal{L}}$. Moreover, the response of sectoral consumption growth to changes in capital stocks in different industries is further weighted by their corresponding capital shares, $\boldsymbol{\alpha}_d$. At the time a productivity shock arises in an industry, $\Delta \boldsymbol{z}_t = \boldsymbol{\varepsilon}_t$, the capital stock is fixed so that $\Delta \boldsymbol{k}_t = 0$. In the following period, however, equation (34) shows that capital in each sector adjusts since sectoral investment also responds to the productivity shock, in a way that reflects the investment network, $\boldsymbol{\Omega}$, and the Leontief inverse, $\boldsymbol{\mathcal{L}}$. This adjustment in sectoral capital stocks then persists in subsequent periods according to the transition matrix, $\boldsymbol{\mathcal{J}}$.

3.3 Propagation and The Dynamics of Real GDP Growth

Consider real GDP growth as measured in Definition 1. Appendix C shows that real GDP growth, Δy_t, admits the representation,

$$\Delta y_t = \boldsymbol{D}'\Delta \boldsymbol{z}_t + \boldsymbol{D}'\boldsymbol{\gamma}_d \boldsymbol{\alpha}_d \, \Delta \boldsymbol{k}_t, \tag{36}$$

where $\boldsymbol{D}$ is a vector of constant sectoral Domar weights. The first term recovers precisely the conventional version of Hulten's theorem, applied within each period. The contemporaneous effect of changes in sectoral productivity growth on GDP growth is captured by Domar weights. The second term, however, now captures a dynamic propagation channel: past innovations to sectoral TFP growth that affected investment decisions influence current GDP growth through the cumulative flow of capital services, weighted by Domar weights and adjusted for capital intensity. Equation (36), therefore, emerges as the structural version of Corollary 3. In the period following a productivity shock, the response of GDP growth arises by way of changes in sectoral capital stocks.

As in Hulten (1978), at the time a shock takes place, details of the production structure of the economy are unnecessary to assess its contemporaneous effect on GDP growth. Only Domar weights are relevant. However, in subsequent periods, the propagation of productivity shocks on future real GDP growth is intrinsically related to the economy's intermediate input network through the Leontief inverse, $\boldsymbol{\mathcal{L}}$, and investment network, Ω. Thus, combining equations (34) and (36) yields a *law of motion for real GDP growth* as a function of the entire history of productivity changes in all sectors,

$$\Delta y_t = \boldsymbol{D}'\Delta \boldsymbol{z}_t + \sum_{h=0}^{\infty} \boldsymbol{D}'\boldsymbol{\gamma}_d \boldsymbol{\alpha}_d \, \boldsymbol{\mathcal{J}}^h \, \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}} \, \Delta \boldsymbol{z}_{t-1-h}. \tag{37}$$

The weights on past sectoral productivity changes continue to include Domar terms, but now also include sectoral capital shares, the transition matrix $\boldsymbol{\mathcal{J}}$, the investment network $\boldsymbol{\Omega}$, and the Leontief inverse $\boldsymbol{\mathcal{L}}$. This is the applied counterpart of Theorem 1, as it arises in the canonical multisector growth model. In contrast, an approximation that uses only the first term in equation (37), $\Delta y_t \approx \boldsymbol{D}'\Delta \boldsymbol{z}_t$, corresponds to a purely contemporaneous application of Hulten's theorem in every period.

The propagation kernel in equation (37) is, in turn, the structural counterpart of the propagation component of Domar weights, D^x_{it}, in equation (17). By Corollary 3, the entire response of real GDP to a sectoral productivity shock in subsequent periods flows through the change in real aggregate capital services, $d \log K_t^s$. The Cobb–Douglas first-order conditions in the structural model further imply that the share-weighted change, $S_{kt}\, d \log K_t^s$, equals $\boldsymbol{D}' \gamma_d \boldsymbol{\alpha}_d\, d\boldsymbol{k}_t$. Iterating the law of motion for capital in equation (30) then traces out the geometric kernel in (37), with Appendix C.4 providing the derivation.

Impulse-Response Functions. Equation (37) implies that the response of real GDP growth at horizon h to a unit TFP growth innovation in sector-i is,

$$\frac{d\,\Delta y_{t+h}}{d\,\Delta z_{it}} = \begin{cases} D_i & \text{if } h = 0, \\ \boldsymbol{D}' \gamma_d \boldsymbol{\alpha}_d\, \boldsymbol{\mathcal{J}}^{h-1}\, \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\, \boldsymbol{e}_i & \text{if } h > 0, \end{cases} \tag{38}$$

where $\boldsymbol{e}_i$ is the ith standard basis vector. At $h = 0$, this response is simply D_i. At $h > 0$, the response of real GDP growth depends primarily on sector i's position in the economy's production networks and the capital intensity of its downstream users.

The Anatomy of Sectoral Productivity Shock Propagation. The impulse response of real GDP growth following a sectoral productivity shock admits a particularly revealing expression:

$$\frac{d\,\Delta y_{t+1}}{d\,\Delta z_{it}} = \sum_{j=1}^{N} \underbrace{(\zeta_j \gamma_j \alpha_j)}_{\substack{\text{production attributes} \\ \text{of downstream} \\ \text{sectors}}} \quad \underbrace{D_j}_{\substack{\text{relative size} \\ \text{of downstream} \\ \text{sectors}}} \quad \underbrace{[\Omega' \mathcal{L}]_{ji}}_{\substack{\text{dynamic} \\ \text{network effects}}} \quad . \tag{39}$$

In other words, the aggregate effect of sectoral productivity shocks at $h = 1$ is a weighted sum of *all* sectors' Domar weights, irrespective of which sector experienced a productivity change. Put another way, while sector i's Domar weight, D_i, determines its contemporaneous aggregate effect, how that effect unfolds over time is in part determined by the Domar weights of all its downstream users, D_j. Each share of the Domar sum decomposes into three multiplicative components. The first is a *dynamic network effect*, ($[\Omega' \mathcal{L}]_{ji}$), stemming from intermediate inputs. The matrix $\boldsymbol{\mathcal{L}}$ captures, for sector i, downstream sector j's total gross-output exposure to a productivity innovation in sector i, including all rounds of intermediate-input propagation, while the matrix $\boldsymbol{\Omega}'$ captures the extent to which sector i's output is used as investment in downstream sector j. The second is a *size effect* (D_j): the Domar weight of downstream sectors. The third term, ($\zeta_j \gamma_j \alpha_j$), scales the dynamic network and size effects by the investment adjustment costs, the value-added share of gross output, and the capital intensity of downstream sectors. If, for example, $\alpha_j = 0$ for some sector j (that sector uses no capital), then that sector plays no role in the propagation effects of sector i' shocks. More generally, a sector, i, whose output flows disproportionately into capital formation, that is towards capital-intensive sectors, either directly or through intermediate linkages, will have lasting aggregate effects, even if its own Domar weight is modest.

While potentially large differences in contemporaneous aggregate responses to sectoral shocks can arise from differences in Domar weights, impulse-responses in subsequent periods involve a (quasi) averaging of all sectors' Domar weights, weighted according to Equation (39). We can measure the persistence of sector-i's aggregate effect as the ratio,

$$\frac{d\,\Delta y_{t+1}/d\,\Delta z_{it}}{d\,\Delta y_t/d\,\Delta z_{it}} = \frac{\sum_j \left(\zeta_j \gamma_j \alpha_j\right) D_j \,[\Omega' \mathcal{L}]_{ji}}{D_i}. \tag{40}$$

In other words, all else equal, sectors with large Domar weights tend to have smaller persistence ratios and vice versa. This result suggests that, at medium horizons, the effects of sectoral productivity changes on real GDP growth tend to balance out, with small sectors catching up even if initial responses are tilted towards the larger sectors. It also suggests that these aggregate effects become less correlated with Domar weights. In Section 4, we explore these predictions in the data directly by way of local projection of real GDP growth on observed sectoral TFP growth.

3.4 Domar Weight Decomposition in the Structural Model

The structural model provides explicit expressions for the impact and propagation components of Domar weights introduced in Corollary 2. Sector i's Domar weight is

$$D_i = \frac{\boldsymbol{\mathcal{L}}'(\boldsymbol{\theta} + \beta\,\boldsymbol{\Omega}\boldsymbol{\zeta}_d \boldsymbol{B}_k)\,\boldsymbol{e}_i}{1 + \beta\,\boldsymbol{\zeta}'\boldsymbol{B}_k}, \tag{41}$$

where $\boldsymbol{B}_k' = \boldsymbol{\theta}'\boldsymbol{\mathcal{L}}\,\boldsymbol{\alpha}_d \gamma_d\,\left(\boldsymbol{I} - \beta\,\boldsymbol{\mathcal{J}}\right)^{-1}$. We can decompose this expression as $D_i = D_i^c + D_i^x$, with

$$D_i^c = S_c \cdot \boldsymbol{\theta}'\boldsymbol{\mathcal{L}}\,\boldsymbol{e}_i, \tag{42}$$

$$D_i^x = S_c \cdot \beta\,\boldsymbol{B}_k' \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\,\boldsymbol{e}_i, \tag{43}$$

where $S_c \equiv \widetilde{C}_t/\widetilde{Y}_t = 1/(1+\beta\,\boldsymbol{\zeta}'\boldsymbol{B}_k)$ is the aggregate consumption share of GDP.

The impact component, D_i^c, depends only on the intermediate-input network through the Leontief inverse, $\boldsymbol{\mathcal{L}}$, and the consumption shares, $\boldsymbol{\theta}$. The expression, $\boldsymbol{\theta}'\boldsymbol{\mathcal{L}}$, in equation (42) is precisely the influence vector discussed in Acemoglu et al. (2012), now weighted by the aggregate consumption share, S_c. The propagation component, D_i^x, depends on the investment network $\boldsymbol{\Omega}$, the Leontief inverse, $\boldsymbol{\mathcal{L}}$, and the vector, $\boldsymbol{B}_k$. Recall from the value function in (28) that $\boldsymbol{B}_k$ captures the entire forward-looking effect of capital on lifetime utility.[5] In addition, the sum, $D_i^c + D_i^x = D_i$, admits a welfare interpretation that is a special case of Proposition 1. Because $\mathcal{V}_t = \sum_{\tau \geq t} \beta^{\tau - t} \log C_\tau$ and S_c is constant in the structural model, we have that $\boldsymbol{D} = S_c\,\partial \mathcal{V}_t/\partial \boldsymbol{z}_t$, confirming that Domar weights also represent the marginal effect of sectoral productivity on lifetime utility, weighted by the consumption share of GDP.

Sectors with large ratios, D_i^x/D_i, such as Construction or Durable Goods as we show below,

[5] From the Bellman equation, $\boldsymbol{B}_k' = \sum_{\tau \geq t+1} \beta^{\tau-(t+1)}\,\partial c_\tau/\partial \boldsymbol{k}_{t+1}$, where $c_\tau = \boldsymbol{\theta}'\boldsymbol{c}_\tau$ is log aggregate consumption. See Appendix C for the closed-form expression.

are precisely the sectors for which TFP growth is informative with respect to future real GDP growth. Sectors with small ratios, D_i^x/D_i, such as Food & Accommodation, are those whose influence on GDP growth are negligible beyond the current period. The model's solution suggests that the horizon at which a sector's aggregate importance is largest depends on the split between contemporaneous and propagation components of Domar weights. Sectors dominated by D_i^c have their effects concentrated on impact, while sectors dominated by D_i^x have theirs at spread out over longer horizons.

3.5 Special Cases: the Role of Network Architecture

To build intuition for how network structure governs propagation, we study four special cases of a three-sector economy with symmetric parameters: $\alpha_i = \alpha$, $\gamma_i = \gamma$, and $\zeta_i = \zeta$ for all i. Three sectors are sufficient to isolate the role of intermediate inputs and investment linkages separately while also including a 'catch-all' sector whose role can be primarily confined to final demand. The cases we describe then differ in the network architecture of intermediate-inputs and investment. Under these symmetry restrictions, Domar weights sum to $1/\gamma$.

Case 1: Sector-Specific Input Networks. When every sector produces its own intermediate inputs and investment goods ($\boldsymbol{\Phi} = \boldsymbol{\Omega} = \boldsymbol{I}$), the response on impact and one-period-ahead response of GDP growth to a shock in any sector are, respectively,

$$\frac{d\,\Delta y_t}{d\,\Delta z_t} = \begin{pmatrix} 1/3\gamma & 1/3\gamma & 1/3\gamma \end{pmatrix},$$
$$\frac{d\,\Delta y_{t+1}}{d\,\Delta z_t} = \begin{pmatrix} \alpha\zeta/3\gamma & \alpha\zeta/3\gamma & \alpha\zeta/3\gamma \end{pmatrix}.$$

The response is *identical across sectors*: after impact, all shocks propagate identically because each sector's investment stays within its own sector. The sectoral ranking that prevails on impact (governed by Domar weights, $1/3\gamma$) is immediately replaced by a uniform response.

Case 2: Dense Networks. When every sector is equally important as a supplier of intermediate inputs and investment to every other sector, ($\phi_{ij} = \omega_{ij} = 1/3$ for all j), the response on impact and one-period-ahead response are again uniform,

$$\frac{d\,\Delta y_t}{d\,\Delta z_t} = \begin{pmatrix} 1/3\gamma & 1/3\gamma & 1/3\gamma \end{pmatrix},$$
$$\frac{d\,\Delta y_{t+1}}{d\,\Delta z_t} = \begin{pmatrix} \alpha\zeta/3\gamma & \alpha\zeta/3\gamma & \alpha\zeta/3\gamma \end{pmatrix}.$$

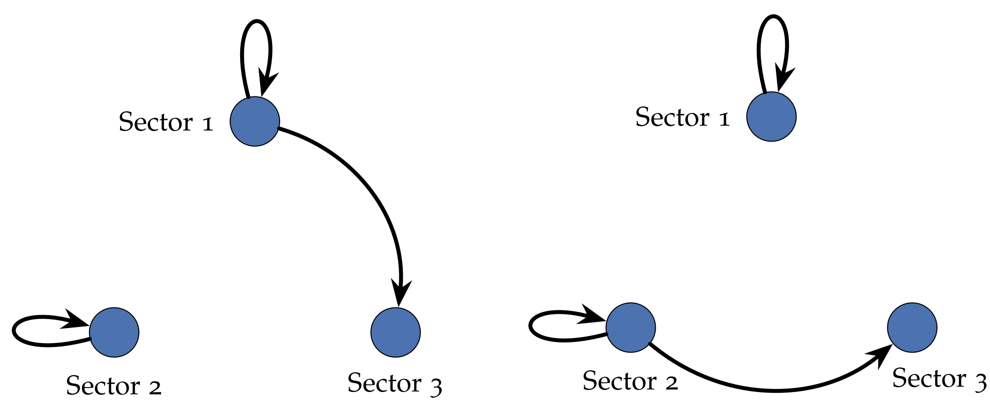
A dense symmetric network produces the same uniform propagation as one characterized by sector-specific inputs. In this case, symmetry ensures that every shock reaches every sector's capital stock with equal weight.

Case 3: Key Supplier (Star) Networks. When a single sector (say sector 1) is the sole supplier of both intermediate inputs and investment goods, $\phi_{1j} = \omega_{1j} = 1$ for all j, real GDP growth responds in the period following a productivity shock only if it hits the key supplier sector. In the limit where $\beta \to 1$ and $\zeta \to 1$ (to simplify notation, as in Case 4 below), the aggregate consumption share of GDP is $S_c = 1 - \alpha$, and

$$\begin{aligned}\frac{d\,\Delta y_t}{d\,\Delta z_t} &= \Big(\big(3 - 2\gamma(1-\alpha)\big)/3\gamma \quad (1-\alpha)/3 \quad (1-\alpha)/3 \Big),\\ \frac{d\,\Delta y_{t+1}}{d\,\Delta z_t} &= \Big(\alpha/\gamma \quad 0 \quad 0 \Big).\end{aligned}$$

In this case, only sector 1's output is used as capital in other sectors and, therefore, only that sector will have lasting real aggregate effects. In addition, relative to the consumption influence vector, $\boldsymbol{\theta}'\boldsymbol{\mathcal{L}} = \big((3-2\gamma)/3\gamma,\ 1/3,\ 1/3\big)$, the forward-looking value of investment reallocates Domar weight toward the key supplier of investment goods.

Figure 2: **Asymmetric Key Supplier Networks**



(a) **Capital Input Flows, Ω** (b) **Intermediate Input Flows, Φ**

Case 4: Asymmetric Key Supplier Networks (i.e., Construction vs. Wholesale & Retail vs. Food & Accommodation). It turns out that to gain insight into the data, the most instructive case is one that separates the intermediate-input and investment networks in a way that reflects an asymmetry between them. Thus, as shown in Figure 2, we assign sector 1 the role of a key supplier of investment goods (akin below to the Construction sector), sector 2 the role of a key supplier of intermediate inputs (akin to Wholesale & Retail Trade), and sector 3 the role of a downstream user with no meaningful contributions to either network (akin to Food & Accommodation). Under this network architecture, the intermediate-input and investment networks are

$$\boldsymbol{\Phi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Omega} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Sector 2 supplies intermediate inputs to itself and to sector 3; sector 1 supplies investment goods to itself and to sector 3. In the limit where $\beta \to 1$ and $\zeta \to 1$ (to simplify notation), the sectoral decompositions of Domar weights becomes

$$\boldsymbol{D}^c = \begin{pmatrix} 0 \\ (1-\alpha)(1-\gamma)/\gamma \\ 1-\alpha \end{pmatrix} \quad \text{and} \quad \boldsymbol{D}^x = \begin{pmatrix} \alpha \\ \alpha(1-\gamma)/\gamma \\ 0 \end{pmatrix}.$$

When $\alpha = 1/2$ and $\gamma = 1/2$, approximately the U.S. sectoral averages for capital shares of value added and intermediate input shares of gross output, sector 1's Domar weight is entirely determined by its propagation component, $D = D_1^x = 1/2$. In contrast, sector 2's Domar weight is largest, $D_2 = 1 > D_1 = D_3 = 1/2$, and divided evenly between its impact and propagation components, $D_2^c = D_2^x = 1/2$. Finally, sector 3's Domar weight is identical to that of sector 1 but, in contrast to that sector, is entirely determined by its impact component, $D = D^c = 1/2$. In this case, the impulse responses at horizons, $h = 0, 1$, are[6]

$$\frac{d\,\Delta y_t}{d\,\Delta \boldsymbol{z}_t} = \begin{pmatrix} \alpha & (1-\gamma)/\gamma & 1-\alpha \end{pmatrix},$$
$$\frac{d\,\Delta y_{t+1}}{d\,\Delta \boldsymbol{z}_t} = \begin{pmatrix} \alpha & \alpha(1-\gamma)/\gamma & 0 \end{pmatrix}.$$

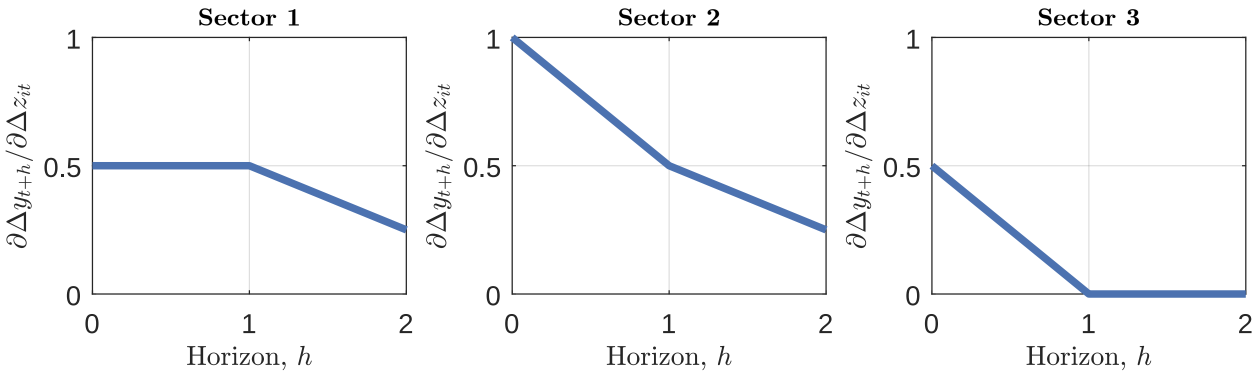
Therefore, sector 1 (the key supplier of investment) is characterized by a persistence ratio of $\frac{d\,\Delta y_{t+1}/d\,\Delta z_{it}}{d\,\Delta y_t/d\,\Delta z_{it}} = \alpha/\alpha = 1$. Put another way, the contemporaneous aggregate effect of a productivity shock in that sector is reproduced in full in the next period. By comparison, sector 2 (the key supplier of intermediate inputs) is characterized by a persistence ratio of $\frac{d\,\Delta y_{t+1}/d\,\Delta z_{it}}{d\,\Delta y_t/d\,\Delta z_{it}} = \alpha = 1/2$; the aggregate effect of a productivity shock in sector 2 in the period following the shock is 1/2 as large as its contemporaneous effect, but this contemporaneous effect is largest among all three sectors since $D_2 > D_1 = D_3$. Finally, the aggregate effects of shocks to productivity in sector 3 are similar to those of sector 1 on impact but they have no persistence and vanish entirely after impact, $\frac{d\,\Delta y_{t+1}/d\,\Delta z_{it}}{d\,\Delta y_t/d\,\Delta z_{it}} = 0$. We show in the next sections that these analytical impulse responses of GDP growth to sectoral shocks, shown in Figure 3, end up mirroring both the structural model's predictions and the empirical contrast between Construction, Wholesale & Retail, and Food & Accommodation.

3.6 Summary

The structural model delivered three main insghts. First, real GDP growth could be expressed in closed form as the sum of two components i) a conventional Hulten (1978)-like term reflecting the contemporaneous effects of sectoral shocks, and ii) a propagation term summarizing the entire

[6] While this section focuses on the contrast between $\frac{d\,\Delta y_t}{d\,\Delta z_t}$ and $\frac{d\,\Delta y_{t+1}}{d\,\Delta z_t}$, analytical expressions can also be obtained for $\frac{d\,\Delta y_{t+2}}{d\,\Delta z_t} = \boldsymbol{D}'\gamma_d \boldsymbol{\alpha}_d\, \mathcal{J}\, \zeta_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\, \boldsymbol{e}_i$ from equation (38). See the **online Technical Apppendix**

Figure 3: Asymmetric Key Supplier Impulse Responses



Notes: Each subplot tracks the impulse response of aggregate GDP growth to a unit innovation in the TFP growth of each sector in the asymmetric key supplier network where $\alpha = \gamma = 1/2$, $\beta \to 1$ and $\zeta \to 1$. Sector 1 plays the role of a key supplier of investment goods, Sector 2 the role of a key supplier of intermediate inputs, and sector 3 the role of a downstream user that produces all final consumption. Compare with Figure 7.

history of aggregate effects resulting from past sectoral shocks. These components are primarily shaped by three observable primitives of technology: the Leontief inverse, $\boldsymbol{\mathcal{L}}$ (summarizing the network effects of intermediate-input linkages), the investment-share matrix, $\boldsymbol{\Omega}$ (summarizing the effects of investment linkages), and the vector of sectoral capital shares $\boldsymbol{\alpha}$. Second, the decomposition of Domar weights, underscored in Corollary 2, can look dramatically different depending on their contributions to the economy's production networks and final demand. In particular, two sectors with identical Domar weights, sectors 1 and 3 above, can have starkly contrasting effects on the persistence of real GDP growth. In Figure 3, sector 1's Domar weight is entirely made up of its propagation component, $D^x = \alpha$, which creates fully persistent aggregate effects in the year following the disturbance, while sector 3's Domar weight consists entirely of its impact component, $D^c = 1 - \alpha$, in which case aggregate effects vanish entirely within a year of materializing. Third, the special cases of network architecture discussed herein indicate that the asymmetry between the intermediate input and investment networks is central in explaining the relationship between the aggregate effects of sectoral shocks at different horizons and Domar weights. In the next ection, we work out the quantitative implications of our analysis for the U.S. economy.

4 Quantitative Analysis

This section quantifies the structural model introduced in the previous section. We first compare the analytical insights from that section to the predictions from the full structural model - incorporating both network and capital accumulation effects emphasized in Section 3. We then compare these structural predictions against the data more directly by way of model-free local projections.

4.1 Data

Annual data on intermediate input use and sources of final demand are obtained from the Bureau of Economic Analysis (BEA)'s Input-Output tables. Data on investment flows are constructed and made available by vom Lehn and Winberry (2022), which we extend from 2018 to 2024. Annual industry productivity data are jointly provided by the Bureau of Economic Analysis (BEA) and the Bureau of Labor Statistics (BLS) through their Integrated Industry-Level Production Accounts (KLEMS). Finally, data pertaining to sectoral capital are obtained from the BEA Fixed Asset Tables (FATs). Our datasets cover the period 1947 - 2024.[7]

We construct sectoral total factor productivity (TFP) series for eleven production sectors of the U.S. private economy: Mining & Utilities, Construction, Durable Goods, Nondurable Goods, Wholesale & Retail Trade, Information, Finance & Insurance, Real Estate, Professional & Business Services, Education & Health, and Food & Accommodation. We choose this industry breakdown in part to underline the different roles played by individual U.S. production sectors while keeping the analysis relatively tractable.[8] We define TFP growth as annual log changes in sectoral gross-output TFP (times 100), and GDP growth as annual log changes in real GDP.[9]

4.2 Attributes of Sectoral Production in the U.S. Economy

Intermediate Inputs and Investment Networks The intermediate-input network, $\mathbf{\Phi}$, and investment network, $\mathbf{\Omega}$, are shown in Figure 4. Their structures differ sharply. In the intermediate-input network (Figure 4a), Construction is a negligible intermediate-input supplier to virtually all sectors. The investment network (Figure 4b) tells a different story. Construction is a major supplier of investment goods, particularly to Real Estate (77.1 percent). In contrast, Wholesale & Retail plays a notable role in providing intermediate inputs to the goods sectors, as well as itself, but not so investment goods. Finally, a sector like Food & Accommodation contributes very little to either production network. Appendix D.1 provides details on the construction of these networks.

Other Parameters Sectoral capital shares, α_i, and value-added shares in gross output, γ_i, are obtained from the BEA industry accounts. The investment elasticity, ζ_i, is estimated so as to match the relative persistence of local projections of GDP on sectoral productivity, presented below, to the relative persistence of model-based impulse responses. The subjective discount factor, β, is set to 0.96. Consumption shares, θ_i, are obtained from the BEA Use tables, and are equivalent to NIPA personal consumption expenditure data. Along with the intermediate

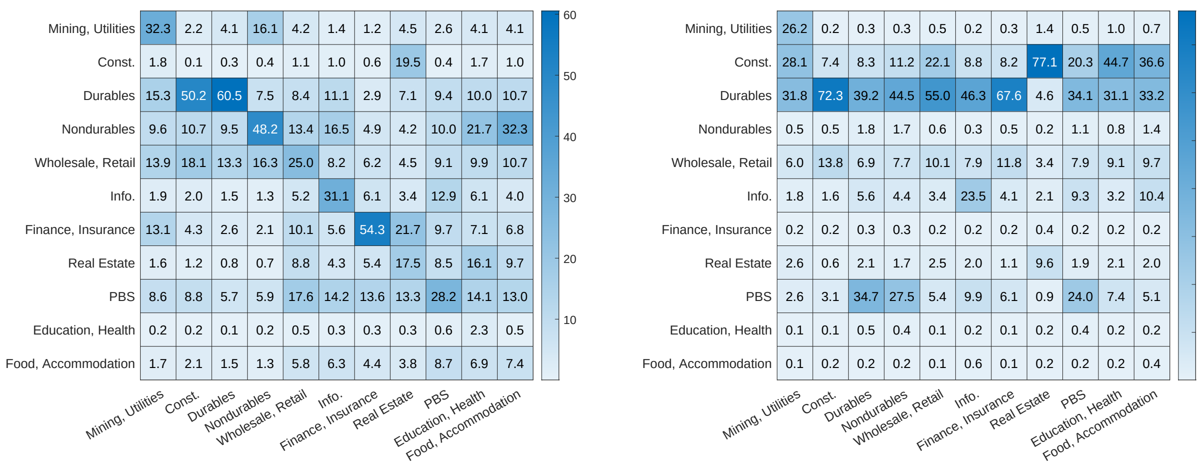
[7] Appendix D provides the exact data vintage, industry-to-sector concordance, and details on the construction of all parameter matrices.

[8] In addition, increasingly higher levels of disaggregation can lead to significant idiosyncratic noise that masks pertinent sectoral information, see Foerster et al. (2011).

[9] Sectoral gross-output TFP is computed as Solow residuals and aggregated using Domar-weighted aggregation. Details on data construction, including industry-to-sector mappings and robustness checks, are provided in Appendix A.

Figure 4: **Intermediate-Input and Investment Expenditure Shares**

(a) **Intermediate-Input Expenditure Shares Φ (%)** (b) **Investment Expenditure Shares Ω (%)**



Notes: Entry ϕ_{ji} is the share of sector i's intermediate-input expenditure purchased from sector j. Entry ω_{ji} is the share of sector i's investment expenditure on goods from sector j. Columns sum to 1 up to rounding. Source: BEA input–output accounts (intermediate inputs) and BEA capital flow tables (investment).

Table 1: **Decomposition of sectoral weights (%)**

Sector	S_i^{data}	S_i	D_i^{data}	D_i	D_i^c	D_i^x
Mining & Utilities	4.42	5.19	8.03	9.03	5.93	3.10
Construction	5.39	6.17	11.93	13.46	0.88	12.58
Durable Goods	13.89	18.52	33.88	38.59	11.89	26.69
Nondurable Goods	9.51	8.79	30.62	27.41	21.98	5.43
Wholesale & Retail	17.76	17.98	29.46	27.92	20.02	7.90
Information	5.14	4.96	8.61	8.01	4.43	3.58
Finance & Insurance	7.71	6.58	13.73	11.37	8.99	2.37
Real Estate	12.99	10.95	17.11	14.30	11.76	2.54
Professional & Business Services	9.63	9.55	15.04	14.85	6.37	8.48
Education & Health	6.81	5.43	10.74	8.47	8.29	0.19
Food & Accommodation	6.75	5.89	12.00	10.15	9.02	1.13

Notes: S_i^{data} and D_i^{data} are value-added shares and Domar weights, respectively, computed directly from the data. S_i, D_i, D_i^c, and D_i^x are model-implied. $D_i = D_i^c + D_i^x$ up to rounding.

input and investment networks, we use these parameters to construct and decompose model-implied Domar weights for each sector.. Table 1 reports and compares these weights with their values as computed directly from the data.[10] Similar to the special 'asymmetric network' case studied in Section 3, Construction is small in size, at 5 percent of GDP, comparable to Food & Accommodation at around 6 percent. In contrast, Wholesale & Retail trade is one of the largest sectors, at almost 20 percent of GDP.

[10]Appendix D reports all parameter values and their sources.

Table 2: **Sectoral TFP Growth Summary Statistics**

	Gross-output TFP growth (%)				
Sector	**Mean**	**Std. Dev.**	**Min**	**Max**	**Autocorr.**
Mining and Utilities	0.32	2.85	-6.41	10.51	0.15
Construction	0.01	1.55	-4.79	3.44	0.45
Durable Goods	0.96	1.56	-3.67	5.05	0.21
Nondurable Goods	0.48	1.16	-2.57	3.48	0.09
Wholesale & Retail	1.37	1.64	-2.34	5.65	0.05
Information	1.31	2.09	-4.50	6.35	-0.01
Finance and Insurance	-0.16	2.34	-4.73	10.65	-0.07
Real Estate	0.63	1.40	-2.69	3.99	0.12
Professional & Business Services	0.74	1.55	-3.87	4.21	0.25
Education and Health	0.41	1.29	-2.08	4.78	0.13
Food & Accommodation	0.08	1.27	-3.55	4.11	-0.10

Notes: "Autocorr." is the first-order autocorrelation of annual sectoral TFP growth.

Sectoral TFP Growth Figure 5 displays gross output TFP growth by sector over the sample period. Motivated by the evidence in Appendix A.3, TFP growth is modeled as an independent process across sectors. To accommodate moderate serial correlation in sectoral TFP growth where appropriate, for example in Construction, we modify equation (27) to allow for an AR(1) component in sector-specific TFP growth,

$$\Delta z_{it} = (1 - \rho_i^z) g_i^z + \rho_i^z \Delta z_{i,t-1} + \varepsilon_{it}, \tag{44}$$

where ρ_i^z is consistent with the first-order correlations shown in Table 2. This extension naturally preserves certainty equivalence and continues to admit closed-form policy functions and impulse responses.[11] Observe that sectoral TFP growth generally exhibits very little serial correlation. Therefore, the dynamic aggregate effects of changes in sectoral productivity in the structural model will be mostly governed its internal propagation mechanisms. Below, we refer to the model with $\rho^z \neq 0$ as the specification 'with shock dynamics' and the version with $\rho^z = 0$ as the specification 'without shock dynamics.'

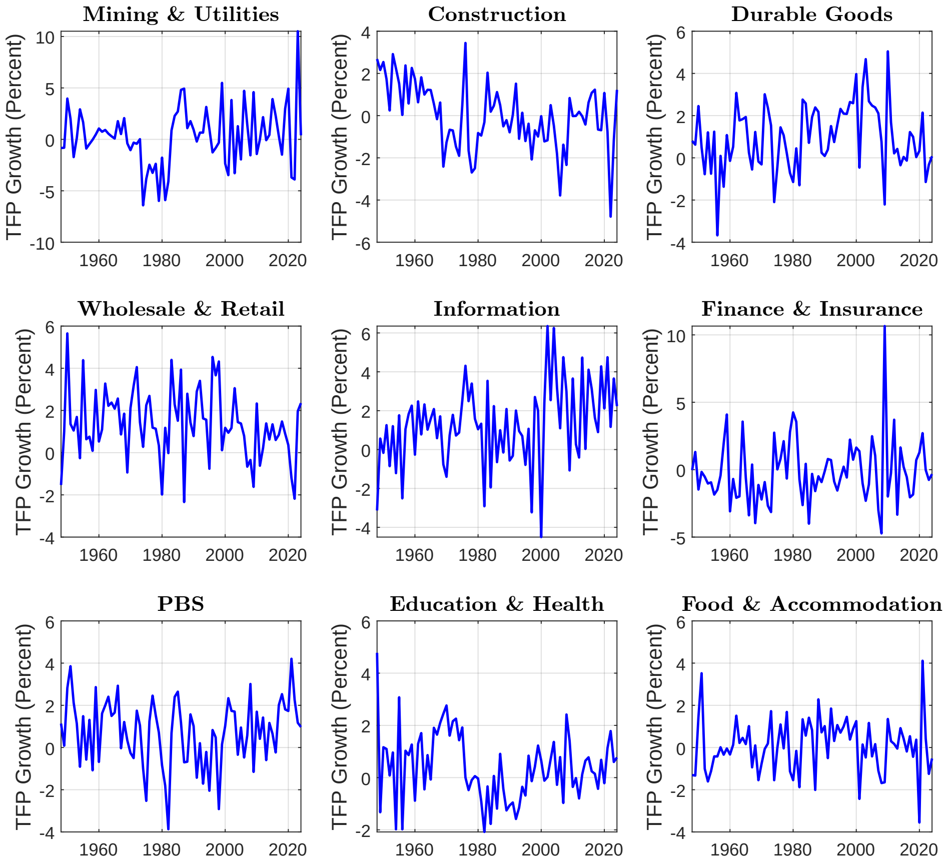
4.3 Impulse Response Functions: How Network Propagation Flattens Out the Aggregate Ranking of Sectoral Shocks

Figure 6 plots the impulse response of real GDP growth to a unit productivity shock in each of the eleven sectors under the benchmark model with shock dynamics (i.e., with the AR(1) component of TFP growth in dotted blue), and without shock dynamics (i.e., with $\rho_i^z = 0\ \forall i$ in solid blue).

On impact (at $h = 0$), Construction generates a small response (13.5 percent) relative to sectors such as, Wholesale & Retail (27.9 percent), Durable Goods (38.6 percent) or Nondurable Goods

[11] The extended process augments the state vector but does not alter the linearity of the value function. Appendix D provides the modified expressions.

Figure 5: **Sectoral TFP growth**

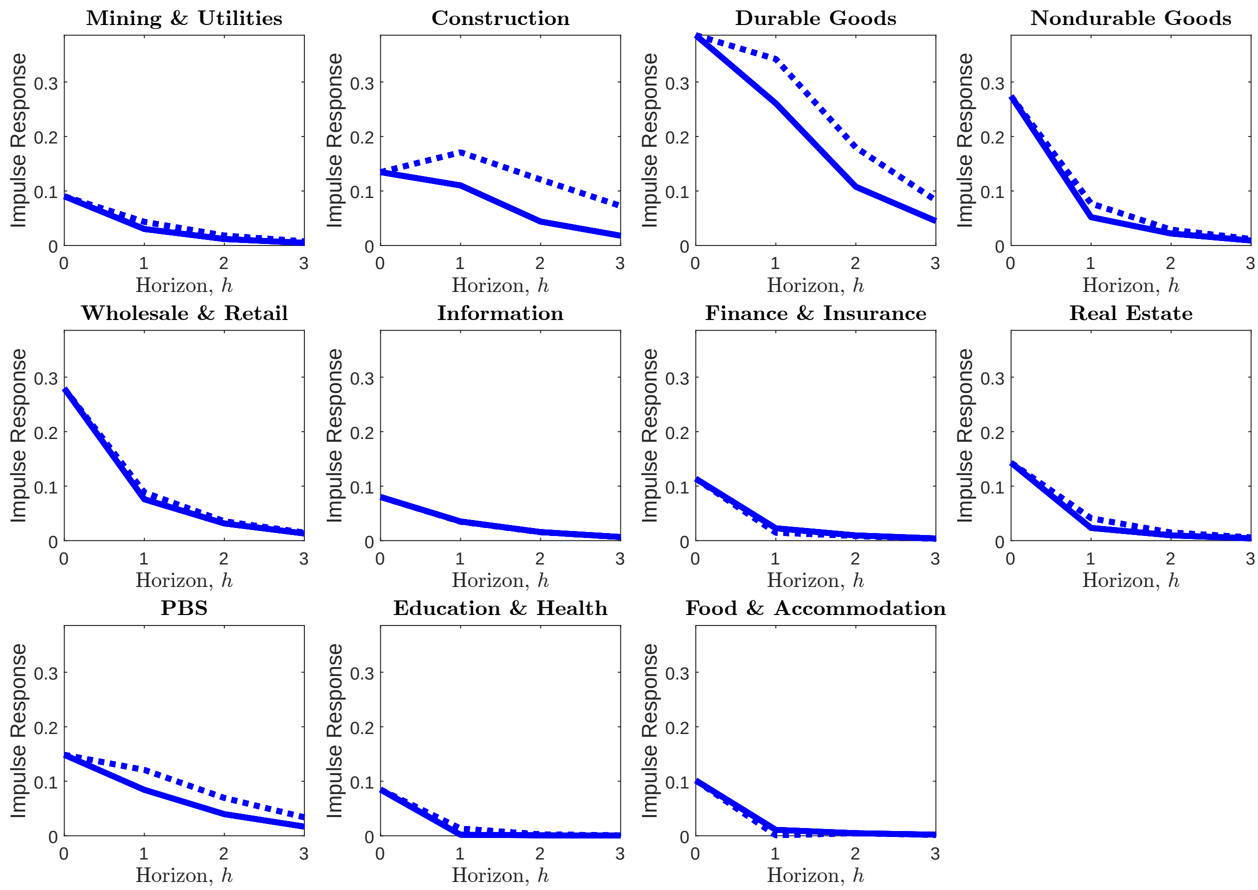


Notes: Annual gross-output TFP growth for each of the eleven sectors over the sample period. See Appendix A for data sources and construction.

(27.4 percent). This is Hulten (1978) in action. At $h = 1$ and beyond, this initial hierarchy of sectoral ranking changes notably. Even without shock dynamics, Construction's response in the year following the shock, at $h = 1$, is 82 percent of its contemporaneous response. With shock dynamics, Construction features a hump-shaped impulse response that is actually *larger* at $h = 1$ than at $h = 0$. In other words, productivity in Construction *leads* GDP growth, a feature that has made it stand out historically and replicated here in the structural model. Wholesale & Retail's response, by contrast, falls more quickly after its much larger initial impact. In Food & Accommodation, the impulse response is not only small initially, comparable to that of Construction, but also essentially zero in the subsequent period and thereafter, unlike Construction. Shock dynamics make the pattern more pronounced in Figure 6 but are not a fundamental source of the findings.

The aggregate impulse responses to unit innovations in Construction, Wholesale & Retail, and Food & Accommodation, depicted in Figure 7, display a particularly revealing resemblance to those of the asymmetric key supplier network worked out in Figure 3. Similar to sector 1 in that network architecture, Construction is a key supplier of investment goods to every sector

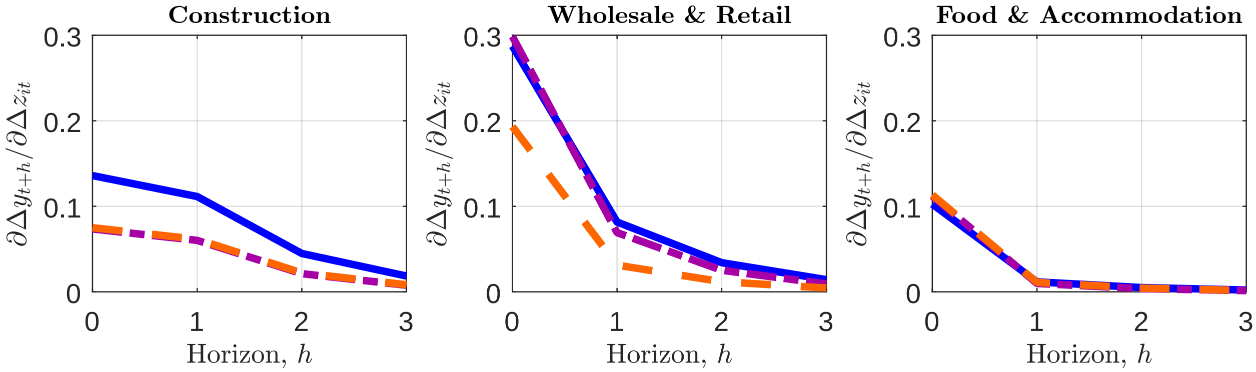
Figure 6: **Aggregate Impulse Responses to Unit Sectoral Productivity Shocks**



Notes: Each panel shows the response of real GDP growth at horizon h to a unit shock in the indicated sector's TFP growth. Solid: benchmark model without shock dynamics ($\rho_i^z = 0$ for all i). Dashed: model with shock dynamics.

while, similar to sector 2, Wholesale & Retail is a notable supplier of intermediate inputs. Food & Accommodation is akin to sector 3, mostly a downstream user with no meaningful contributions to either network. That said, the economy's production networks shape only in part the aggregate dynamics implied by sectoral shocks. The production attributes of downstream sectors also play an important role. For example, in Figure 7, the purple dashed line depicts a counterfactual scenario in which the capital share of the Real Estate sector is reduced from its actual value of 94 percent to that of Construction, 14 percent. Not surprisingly, value added in Real Estate consists almost exclusively of structures purchased from the Construction sector. However, that single parameter change essentially halves the aggregate effects of a productivity shock in Construction at all horizons. More generally, capital shares of downstream sectors play a critical role ($\alpha_i = 0 \; \forall i$ creates a static environment) but so do the production needs of downstream sectors. A world in which Wholesale & Retail Trade provide no inputs to the goods sectors, namely Mining & Utilities, Construction, Durable Goods, and Nondurable Goods, noticeably reduces its aggregate effects at all horizons (shown in dashed orange in Figure 7).

Figure 7: **Aggregate Impulse Responses to a Shock in Construction, Wholesale & Retail or Food & Accommodation**



Notes: Response of GDP output growth to a one-standard-deviation shock to Construction, Wholesale & Retail, or Food & Accommodation TFP growth. Purple: small Real Estate capital share. Orange: small Real Estate capital share and zero Wholesale & Retail intermediates inputs share for goods-producing sectors (Mining & Utilities, Construction, Durable Goods, Nondurable Goods).

4.4 Local Projections

Given the BEA and BLS Integrated Industry-Level Production Accounts (KLEMS), sectoral TFP growth is directly observable as Solow residuals, shown in Figure 5, and so is real GDP growth. Therefore, whatever the structural model, one can always compare its impulse responses against estimates of local projections of h-year-ahead GDP growth on each sector's current TFP growth,

$$\Delta \mathrm{GDP}_{t+h} \;=\; a_{i,h} \;+\; b_{i,h}\, \Delta \mathrm{TFP}_{it} \;+\; \mathbf{w}_t' \gamma_{i,h} \;+\; \varepsilon_{it,h}, \qquad h = 0, 1, 2, 3, \tag{45}$$

separately for each sector i and horizon h, where $\mathbf{w}_t$ is a vector of controls.[12] The estimated coefficient, $\hat{b}_{i,h}$, measures the response of h-year-ahead GDP growth to a unit innovation in sector i's current TFP growth.

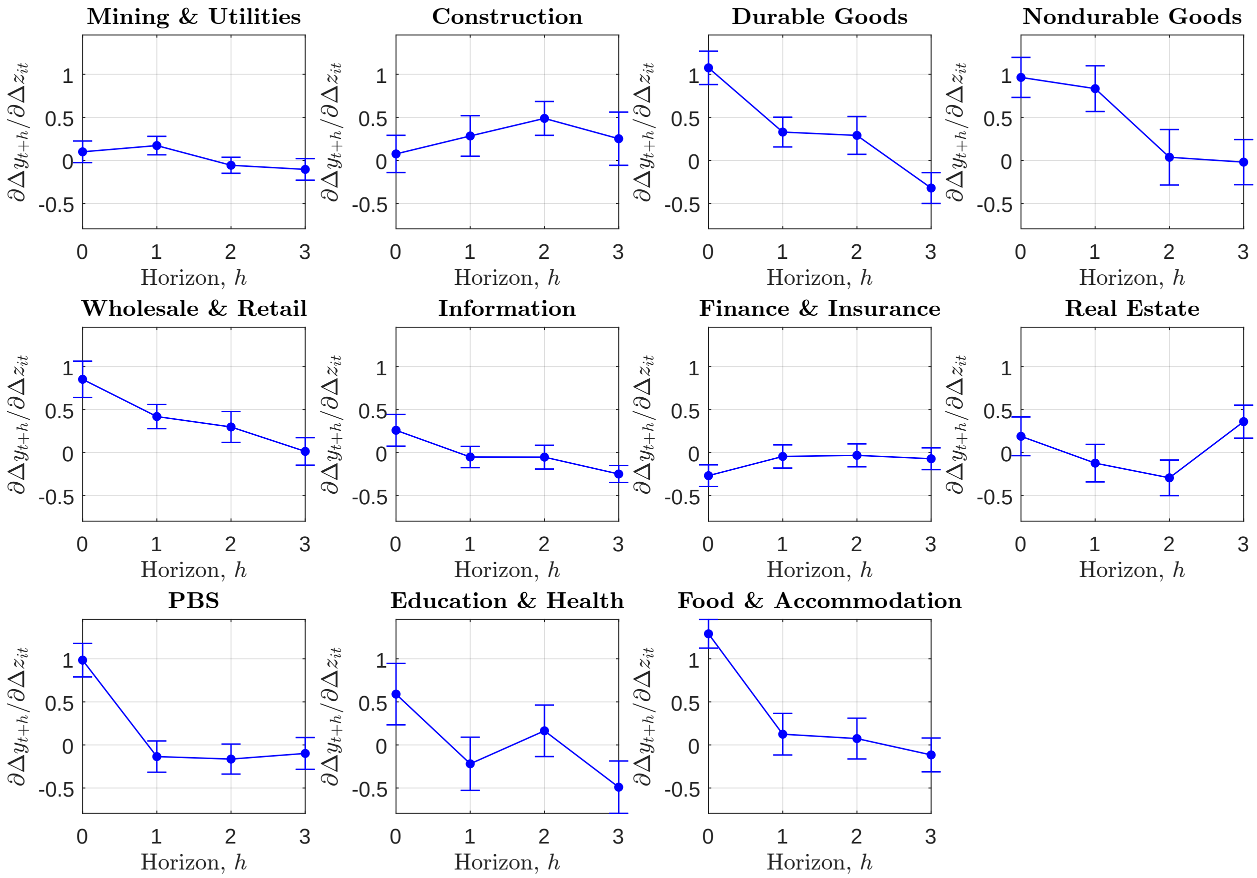
Under Hulten (1978)'s static benchmark, $\hat{b}_{i,0}$ should approximate sector i's Domar weight and, for $h > 0$, $\hat{b}_{i,h}$ should be close to zero. In the absence of an internal propagation mechanism, and given that sectoral TFP growth is nearly *i.i.d.*, current productivity innovations have no effect on real GDP growth beyond their contemporaneous effect. Figure 8 plots the estimated coefficients by sector.

Despite being model-free, the local projection estimates line up generally well with the static benchmark. Changes in productivity growth in sectors with larger Domar weights have predominantly larger contemporaneous effects on real GDP growth.[13] Observe that in the Construction sector, the estimated responses of real GDP growth to TFP growth are larger at $h = 1$ and 2 than

[12]Our baseline specification controls for lags of GDP growth and lags of sectoral TFP growth, and uses heteroskedasticity- and autocorrelation-robust standard errors. Appendix A reports the exact specification, standard-error construction, and robustness exercises including alternative control sets.

[13]vom Lehn and Winberry (2022) point out that deviations from Domar weights will emerge even at $h = 0$ when labor supply is elastic.

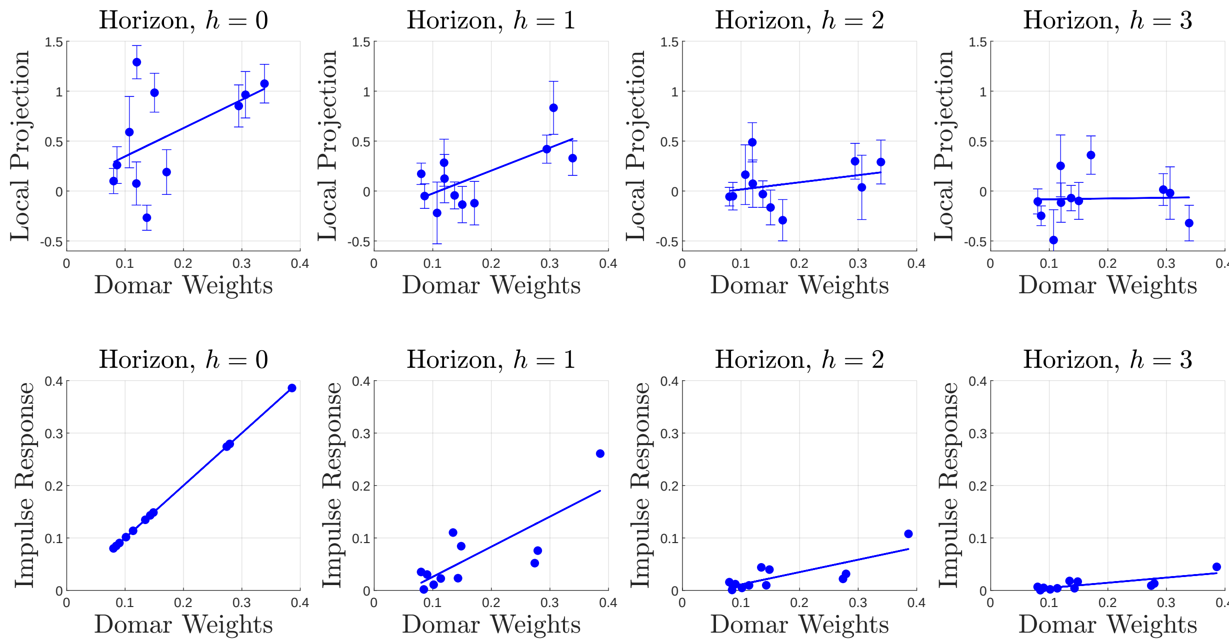
Figure 8: **Local projections of Current and Future Real GDP Growth on Current Sectoral TFP Growth**



Notes: Each panel plots the local-projection coefficient $\hat{b}_{i,h}$ from equation (45) against the horizon h. Newey–West confidence intervals, joint tests, and cross-sector contrasts are reported in Appendix A.

at $h = 0$, and larger than its Domar weight. In other words, productivity changes in Construction *lead* real GDP growth by an even wider margin than suggested by our structural model (the dotted line in the corresponding panel of Figure 6). This suggests internal propagation mechanisms in addition to those we consider, for example in the form of time-to-build as explored in Leng et al. (2025) and Taschereau-Dumouchel and Schaal (2025). The initial aggregate effects of a productivity shock in Wholesale & Retail trade are larger than in Construction but decay more quickly. In other service-producing sectors, including Information, Finance & Insurance, or Food & Accommodation, the local projection estimates are relatively small initially and statistically indistinguishable from zero in the year following the shock and thereafter, as suggested by the structural model in Figure 6. The patterns documented here thus bring us back full circle to the theoretical question at the heart of this paper.

Figure 9: **Local Projections vs Dynamic Model Impulse Responses**



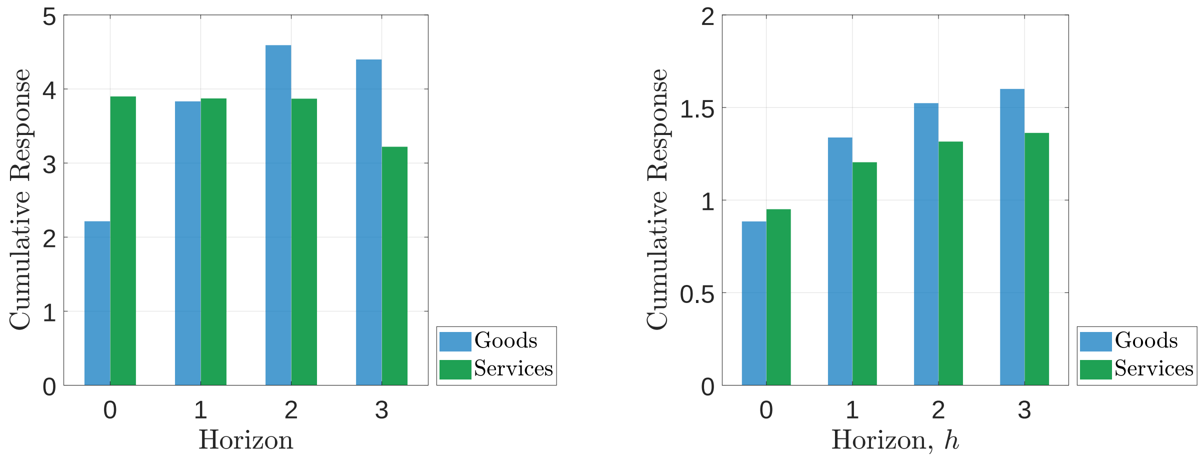
Notes: Each column plots local-projection coefficients $\hat{b}_{i,h}$ and impulse responses $d\Delta y_{t+h}/d\Delta z_{it}$ against Domar weights at horizon h. Top: data (Figure 8). Bottom: model impulse responses.

4.5 Medium-Run Aggregate Impact: Data vs. Model

Figure 9 compares the local-projection coefficients shown in Figure 8 to the corresponding impulse responses from the structural model in Figure 6, plotted against Domar weights for different horizons, $h = 0, 1, 2, 3$. In other words, we compare the exercise in equation (45) to the impulse responses in equation (38).

In both the structural model and the local projections, the initial aggregate responses of GDP growth to sectoral shocks clearly increases with Domar weights. At $h = 0$, the model-implied impulse responses lie exactly on the 45 degree line for each sector, consistent with the conventional version of Hulten's theorem. As the horizon increases, from $h = 2$ to $h = 4$, the time profile associated with the aggregate effects of sectoral shocks gradually rotates clockwise, or gradually flattens out, in a way that is nearly identical across both the structural model-implied impulse responses and the local projection coefficient estimates. Put another way, both the structural model and the data indicate that Domar weights become gradually less informative with respect to the aggregate effects of sectoral shocks at about the same pace. In contrast, in a static environment, the flattening out of dynamic aggregate responses shown in Figure 9 would be immediate at $h = 1$ and thereafter.

Figure 10: **Cumulative Aggregate Responses: Goods vs Services**



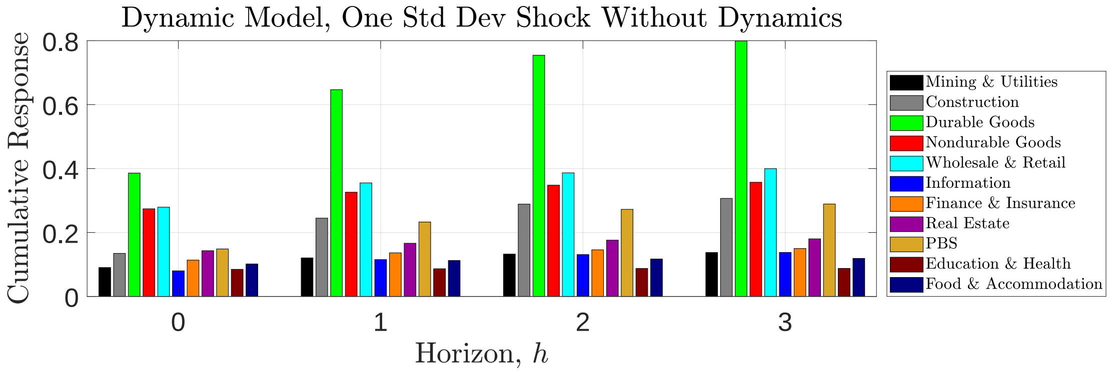
Notes: Goods: Mining & Utilities, Construction, Durable Goods, Nondurable Goods. Services: Information, Finance & Insurance, Real Estate, Professional & Business Services, Education & Health, Food & Accommodation. The left sub-figure plots the cumulative local projection of GDP on sectoral TFP growth in each group, while the right sub-figure plots the cumulative aggregate impulse response to a one-standard-deviation shock in each group.

4.6 Cumulative Aggregate Response: Data vs. Model

Should we heed the aggregate effects of smaller sectors? Figure 10 groups sectors into goods producers (Mining & Utilities, Construction, Durable Goods, Nondurable Goods) and service producers (Information, Finance & Insurance, Real Estate, Professional & Business Services, Education & Health, Food & Accommodation) and plots the cumulative aggregate effect of a productivity shock to each group over a 3-year period. The left panel illustrates the cumulative response in the data (as measured by the cumulative corresponding aggregate local projections' estimates), while the right panel illustrates the cumulative response implied in the structural model (as measured by the cumulative impulse response to a unit productivity shock in each group). On impact, service-sector shocks dominate, consistent with the larger Domar weights of service-producing industries. However, as the horizon increases, the aggregate influence of goods sectors catches up since these sectors generally produce more persistent aggregate effects. At $h = 3$, the cumulative aggregate effects of shocks to the goods-producing sectors actually exceed those of service-producing sectors, despite goods-producing sectors accounting for less than $1/3$ of GDP.

Figure 11 disaggregates these results by sector. On impact, the ranking mirrors that of Domar weights: Durable Goods, Nondurable Goods, and Wholesale & Retail have the largest contemporaneous effects. As the horizon unfolds, however, Construction's cumulative effect grows rapidly and overtakes those of several service sectors. At $h = 3$, the cumulative effect of a shock to TFP growth in Construction is comparable to that in Wholesale & Retail Trade, despite Construction's Domar weight being less than $1/2$ that of Wholesale & Retail.

Figure 11: **Cumulative Aggregate Impulse Responses, All Sectors**



Notes: Each panel plots $\sum_{s=0}^{h} d\Delta y_{t+s}/d\Delta z_{it}$ for the indicated sector i, as a function of the cumulation horizon h.

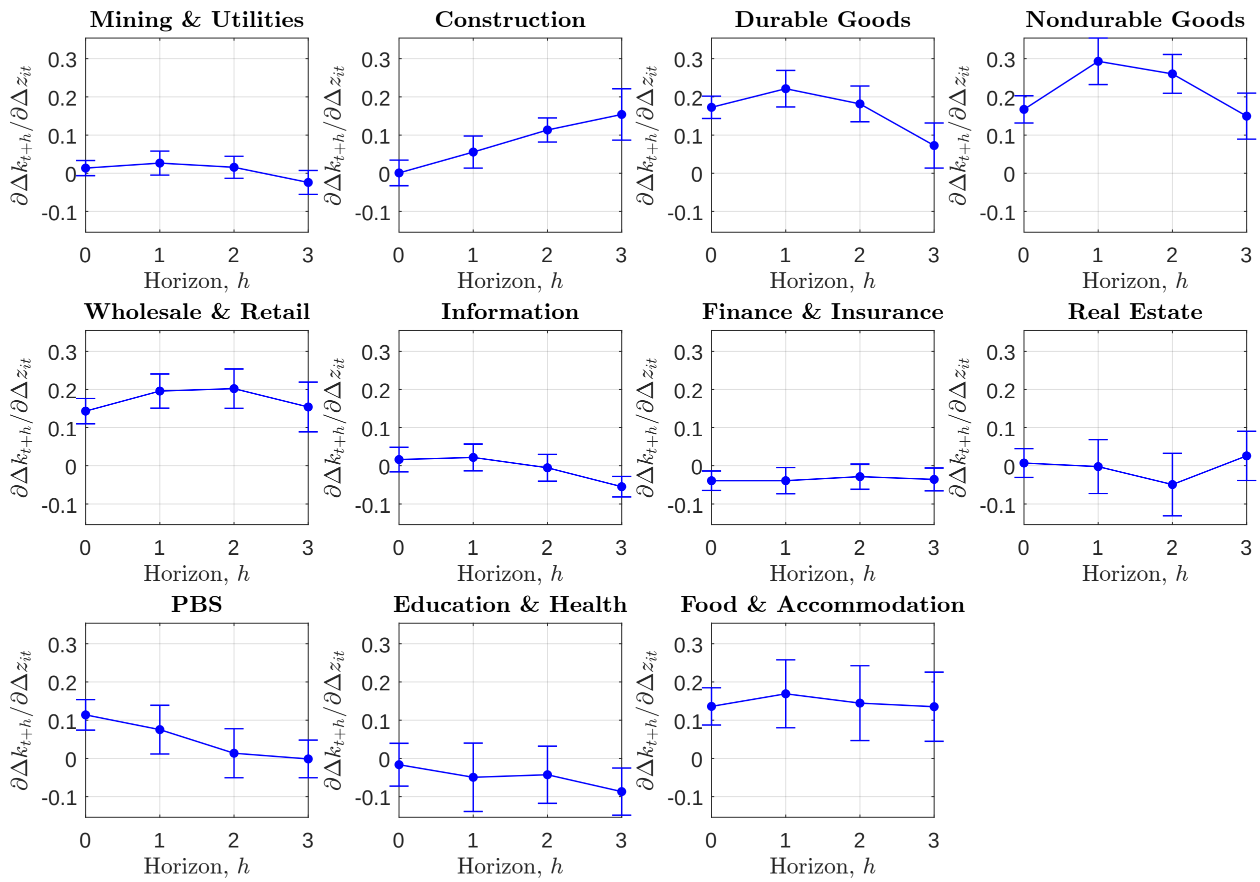
4.7 The Role of Capital

Figures 12 and 13 illustrate how one of our linchpin theoretical results, Corollary 3, plays out in the data directly, without reference to our structural model. According to Corollary 3, a sectoral productivity shock that materializes today affects future real GDP through the resulting change in the aggregate flow of capital services. These changes are constructed as described in equation (8) in Section 2.1 using data on nominal payments to capital and real capital stocks from the BEA Fixed Asset Tables. Since the corollary identifies capital accumulation as the sole channel through which sectoral shocks generate persistent aggregate effects, the exercise in this section directly assesses whether the mechanism at the core of the paper is operative in U.S. data. Thus, Figure 12 shows estimates from local projections of changes in aggregate capital services onto sectoral TFP growth for every sector at different horizons. As expected, in the majority of sectors, changes in aggregate capital services are either statistically insignificant or small at $h = 0$ and gradually build as the horizon unfolds. Figure 13 then illustrates the relationship between the local projections coefficients of GDP growth and those of changes in aggregate capital services on sectoral TFP growth for $h > 0$ and each sector. As suggested by Corollary 3, the relationship is clearly increasing in every sector.

4.8 Discussion

The results in this section, from both the structural model and model-free local projections, give a clear quantitative answer to the question we started out with: do disturbances in sectoral productivity today meaningfully inform the behavior of GDP growth in the next two to four years, and if so how? Sectoral TFP shocks can indeed give rise to aggregate effects that persist over several years. In particular, because goods-producing sectors have effects that persist over time, by virtue of their contributions to inputs or capital used by other sectors, their aggregate effects catch up and overtake those of service-producing sectors despite being initially smaller.

Figure 12: **Local Projections of Changes in Current and Future Real Aggregate Capital Services on Current Sectoral TFP Growth**

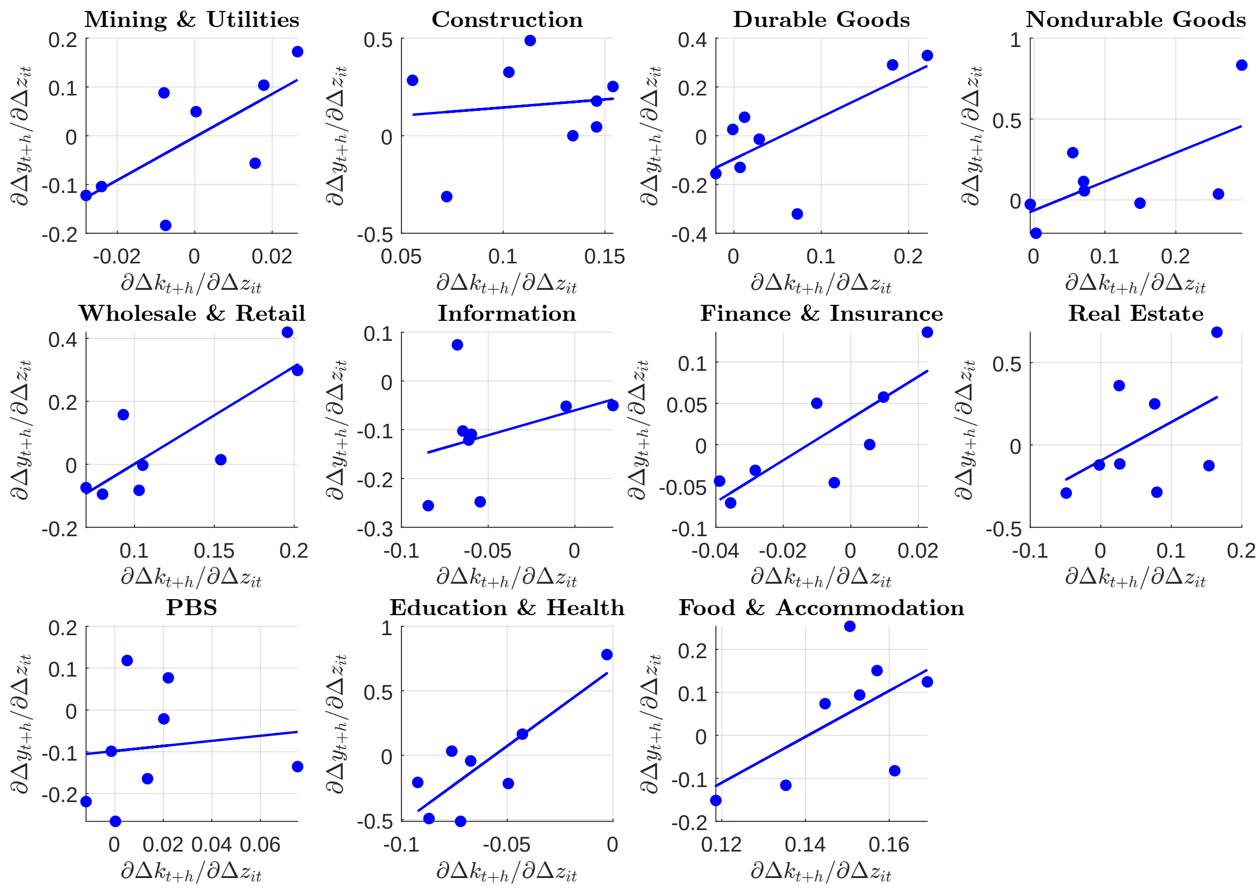


Notes: Each panel plots the local-projection coefficient $\hat{b}_{i,h}$ from equation (45), with aggregate capital stock growth ΔK_{t+h} as the dependent variable, against the horizon h. Newey–West confidence intervals, joint tests, and cross-sector contrasts are reported in Appendix A.

Several mechanisms interact to produce this finding. First, the *investment network* channels the output of sectors such as Construction or Durable Goods as new capital in downstream sectors. Second, the *capital intensity* of downstream users amplifies the effect of changes in investment on future production. This is especially true in Real Estate whose capital share is almost 1 and whose capital consists mainly of structures purchased from the Construction sector (77 percent in Figure 4b). Third, the *intermediate-input network* ensures that productivity disturbances in goods-producing sectors reach other sectors' gross output, which in turn feeds into their investment demand. The Leontief inverse, $\boldsymbol{\mathcal{L}}$, captures this intermediate-input propagation, while the investment network, $\boldsymbol{\Omega}$, maps it into capital accumulation.

Given the characteristics of U.S. sectoral production, the findings in this section directly inform the empirical evidence in Section 4.4 as well as the theoretical results in Sections 2 and 3. The horizon-dependent relationship between sectoral TFP growth and future GDP growth arises because sectors differ in their Domar weight composition. Sectors whose Domar weights are

Figure 13: **Local Projections of Changes in Real Aggregate Capital Services vs. Local Projections of Real GDP Growth**



Notes: Each panel plots the local-projection coefficients $\hat{b}_{i,h}$ from equation (45), with GDP growth ΔGDP_{t+h} as the dependent variable, against the local projection coefficient for aggregate capital stock growth ΔK_{t+h} at the same horizon h. Newey–West confidence intervals, joint tests, and cross-sector contrasts are reported in Appendix A.

largely tilted towards their impact component generate dynamic effects that dissipate relatively quickly. In contrast, sectors whose Domar weights are mostly tilted towards their propagation component generate aggregate effects that persist over several years. Thus, Construction and Food & Accommodation which have closely matched Domar weights end up shaping aggregate outcomes very differently.

5 Concluding Remarks

This paper studies how sectoral productivity disturbances propagate across sectors and over time to shape aggregate dynamics at different horizons. We combine the insight in Hulten (1978), mapping productivity shocks to real GDP through sales-based (Domar) exposure, with that of Hulten (1979), highlighting the effects of intertemporal propagation through endogenous capital accumulation. A central finding is that the aggregate response to sectoral productivity shocks differs markedly across sectors at medium horizons. Put another way, in the medium run, Hulten (1979) eclipses Hulten (1978).

Three key findings emerge. First, each sector's Domar weight can be decomposed into an impact component, reflecting that sector's contemporaneous effect on GDP through static input-output linkages, and a propagation component, reflecting its aggregate effects over time through investment and capital accumulation. This decomposition follows from production efficiency alone, without reference to preferences or demand-side optimality. Second, a sector's ability to generate persistent aggregate effects depends primarily on four observable primitives of sectoral production, its position in the Leontief inverse (capturing the effects of intermediate-input linkages), its contribution to the investment-share matrix (capturing the effects of the investment network), the size of its downstream sectors through their Domar weights, and the capital shares of its downstream sectors. In a quantitative application reflecting the characteristics of U.S. production sectors, these primitives generate notable cross-sector heterogeneity. Nearly all of Construction's Domar weight reflects its propagation component, while those of service-based sectors such as Food & Accommodation consist almost entirely of their impact components. Third, a quantitative model-free application making use of detailed sectoral information regarding U.S. production reproduces the horizon-dependent empirical patterns that motivate the paper, namely the comovement between current sectoral TFP growth and future real GDP growth.

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Appendix Table of Contents

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A TFP and GDP Growth Data

A.1 Data sources

We calculate industry-level gross-output total factor productivity (TFP) using data from the Bureau of Economic Analysis (BEA) industry accounts and the Bureau of Labor Statistics (BLS) multifactor productivity program.[14] In particular, we construct a Solow residual using nominal and quantity indices for capital, labor, and gross output. Real GDP growth is constructed from quantity indices of sectoral value added, and is essentially identical to that from the National Income and Product Accounts (NIPA). The sample is at an annual frequency and covers 1947 through 2024.

A.2 Sector definitions and concordance

Growth rates for the real quantity indices associated with the eleven broad sectors used throughout the paper are defined by aggregating detailed BEA industries using Divisia aggregation. We focus on Private Nonfarm sectors, hence omitting the Government and Agriculture & Fishing sectors. Table A1 summarizes the constituent industries in each of the eleven sectors we consider.

A.3 Principal-component and factor diagnostics

Figure 1 employs two distinct methodologies to establish that sectoral TFP growth rates exhibit negligible comovement. The upper panel illustrates the scree plot associated with sectoral productivity, which shows a sharp drop and leveling off in the eigenvalues. In the lower panel, we apply the IC_{p1} information criterion of Bai and Ng (2002) and find that the information criterion is minimized at 11, coinciding with the number of industries we consider and further underscoring the lack of factor structure.[15]

A.4 Local projection specification

The baseline local projection estimated in Section 4.4 takes the form

$$\Delta \text{GDP}_{t+h} \;=\; a_{i,h} \;+\; b_{i,h}\,\Delta \text{TFP}_{it} \;+\; \sum_{\ell=1}^{p} \gamma_{\ell,i,h}\,\Delta \text{GDP}_{t-\ell} \;+\; \sum_{\ell=1}^{q} \delta_{\ell,i,h}\,\Delta \text{TFP}_{i,t-\ell} \;+\; \varepsilon_{it,h}.$$

We compute White standard errors, following Montiel Olea and Plagborg-Møller (2021).

In our baseline, we take $p = q = 2$. As robustness checks, we also consider $(p,q) \in \{(1,1),(1,2)\}$ or controlling for other sectors' TFP growth. In addition, we consider dropping the

[14] We use the Experimental BEA/BLS Integrated Production Accounts (KLEMS) for the years 1947-1962 (44 industries) and 1963-2016 (63 industries) and the Official BEA/BLS Integrated Production accounts for 1987-2021 (63 industries) and 1997-2024 (63 industries).

[15] The results are indistinguishable with the alternative IC_{p2} or IC_{p3} information criteria in Bai and Ng (2002).

Table A1: Industry-to-sector concordance

Sector	Constituent BEA industries
Mining & Utilities	Oil and gas extraction
	Mining, except oil and gas
	Support activities for mining
	Utilities
Construction	Construction
Durable Goods	Wood products
	Nonmetallic mineral products
	Primary metals
	Fabricated metal products
	Machinery
	Computer and electronic products
	Electrical equipment, appliances, & components
	Motor vehicles, bodies and trailers, & parts
	Other transportation equipment
	Furniture and related products
	Misc. manufacturing
Nondurable Goods	Food and beverage and tobacco products
	Paper products
	Textile mills and textile product mills
	Apparel and leather and allied products
	Printing and related support activities
	Petroleum and coal products
	Chemical products
	Plastics and rubber products
Wholesale & Retail Trade & Support Activities	Wholesale trade
	Retail trade
	Transportation & warehousing
Information	Information
Finance & Insurance	Finance & insurance
	Rental and leasing services and lessors of intangible assets
Real Estate	Real estate
Professional & Business Services	Professional, scientific, & technical services
	Management of companies and enterprises
	Administrative and waste management services
Education, Health Care, & Social Assistance	Educational services
	Healthcare & social assistance
Food & Accommodation	Arts, entertainment, & recreation
	Accommodation
	Food services and drinking places
	Other services except government

Notes: We consolidate Farms and Forestry, Fishing, & Related Activities into an Agriculture sector, and consolidate Federal and State & Local Government into a Government sector. We omit these from our analysis.

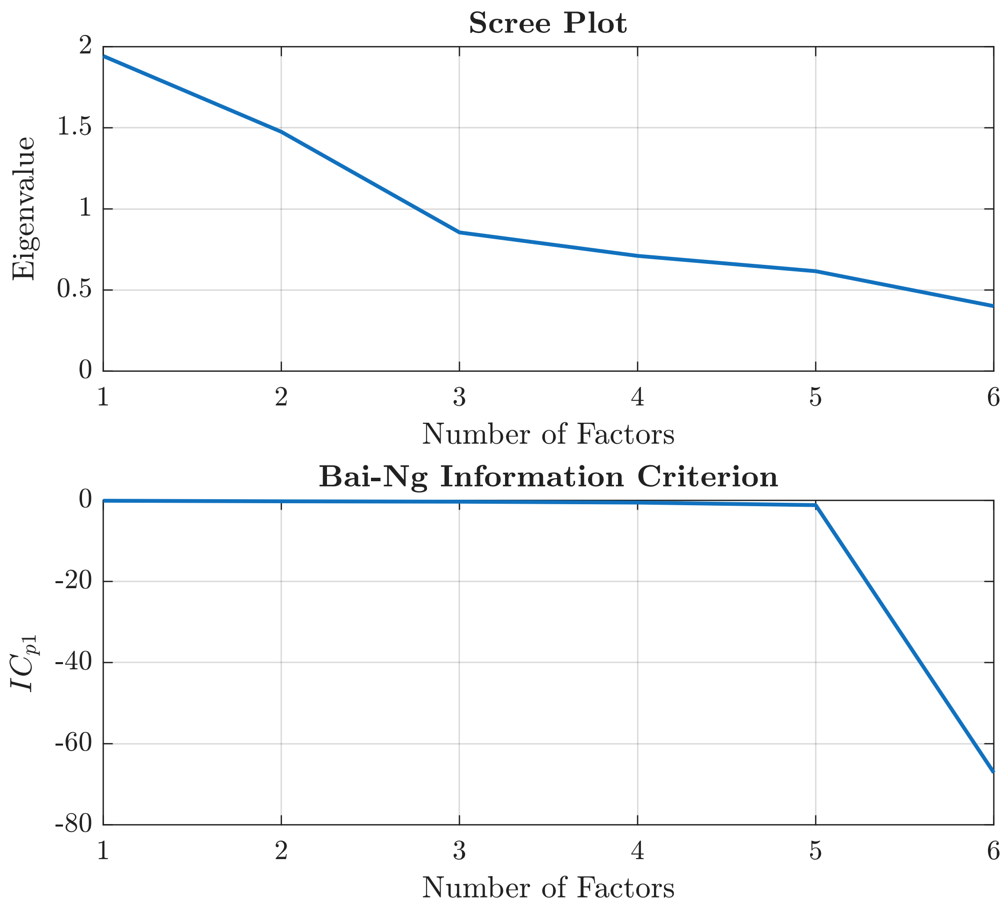


Figure A1: Factor diagnostics

COVID-19 observations (2020 and 2021) and starting the sample after 1985 post great-moderation. In each case, our main findings from Figure 9 continue to hold. First, the best fit line for the local projection coefficients rotates clockwise and flattens out as the horizon increases. Second, the best fit lines from the dynamic model closely match their empirical counterparts.

B Theory Appendix

This appendix provides derivations and proofs for the results stated in Section 2.

B.1 Derivation of the GDP decomposition

This section addresses equation (11) in the main text. Using the CRS property of $\mathcal{K}_i$ and the first-order conditions for investment firms, nominal investment satisfies

$$\widetilde{X}_t = \sum_{j,i} P_{jt} X_{jit} = \sum_i P^K_{it+1} K_{it+1} - \sum_i (P^K_{it} - R_{it}) K_{it}. \tag{46}$$

Substituting $P^K_{it} = (1 + r_{t-1})P^K_{it}$ and defining $\widetilde{K}^s_t \equiv \sum_i R_{it}K_{it}$ and $\widetilde{K}_t \equiv \sum_i P^K_{it}K_{it}$, we obtain

$$\widetilde{X}_t = \widetilde{K}^s_t + \widetilde{K}_{t+1} - (1 + r_{t-1})\,\widetilde{K}_t.$$

Combined with $\widetilde{Y}_t = \widetilde{C}_t + \widetilde{X}_t$ and the definitions of $d\log C_t$, $d\log K^s_t$, and $d\log K_t$, yields equation (11).

B.2 Proof of Hulten's theorem (static, two approaches)

First proof - a static production possibilities frontier (PPF). Define the single-period aggregate value-added function

$$\mathcal{T}(\check{\boldsymbol{P}}_t; \boldsymbol{K}_t, L_t; \boldsymbol{Z}_t) = \max_{\{L_{it}, \boldsymbol{M}_t\}} \sum_i \check{P}_{it}\big(Z_{it}\mathcal{F}_i(L_{it}, K_{it}, \boldsymbol{M}_{it}) - \textstyle\sum_j M_{ijt}\big) \quad \text{s.t.} \quad \textstyle\sum_j L_{jt} \le L_t.$$

The production possibilities function is $\mathcal{Z}(\boldsymbol{Y}_t; \boldsymbol{K}_t, L_t; \boldsymbol{Z}_t) \equiv \sup_{\check{\boldsymbol{P}}_t} \check{\boldsymbol{P}}_t \cdot \boldsymbol{Y}_t - \mathcal{T}(\check{\boldsymbol{P}}_t; \boldsymbol{K}_t, L_t; \boldsymbol{Z}_t)$. The envelope conditions give $\partial\mathcal{Z}_t/\partial Z_{it} = -P_{it}Q_{it}/Z_{it}$ and $\partial\mathcal{Z}_t/\partial Y_{it} = P_{it}$. Differentiating $\mathcal{Z}_t = 0$ with respect to Z_{it}, holding $\boldsymbol{K}_t$ fixed, yields

$$\frac{d\log Y_t}{d\log Z_{it}} = \frac{P_{it}Q_{it}}{\widetilde{Y}_t} = D_{it}.$$

Second proof - a single-period dynamic PPF. Define the function capturing the total value of consumption and end-of-period capital:

$$\mathcal{T}^{(1)}\big(\check{\boldsymbol{P}}_t, \check{\boldsymbol{P}}^K_{t+1}; \boldsymbol{K}_t, L_t; \boldsymbol{Z}_t\big) = \max_{\{\boldsymbol{Y}_t, \boldsymbol{X}_t\}} \sum_i \check{P}_{it}\big(Y_{it} - \textstyle\sum_j X_{ijt}\big) + \textstyle\sum_i \check{P}^K_{it+1}\mathcal{K}_i(K_{it}, \boldsymbol{X}_{it})$$
$$\text{s.t.} \quad \mathcal{Z}(\boldsymbol{Y}_t; \boldsymbol{K}_t, L_t; \boldsymbol{Z}_t) \le 0.$$

The single-period dynamic PPF is $\mathcal{Z}^{(1)}(\boldsymbol{C}_t, \boldsymbol{K}_{t+1}; \boldsymbol{K}_t, L_t; \boldsymbol{Z}_t) = 0$, defined analogously. The envelope conditions for $\mathcal{Z}^{(1)}$ satisfy $\partial\mathcal{Z}^{(1)}/\partial Z_{it} = -P_{it}Q_{it}/Z_{it}$, $\partial\mathcal{Z}^{(1)}/\partial C_{it} = P_{it}$, and $\partial\mathcal{Z}^{(1)}/\partial K_{it+1} = P^K_{it+1}$. Differentiating $\mathcal{Z}^{(1)} = 0$ with respect to Z_{it} and using $d\log K_{jt} = 0$ in equation (11) yields the same result $d\log Y_t/d\log Z_{it} = D_{it}$.

We highlight this second proof because it serves as the bridge to our main theorem in an environment with an arbitrary number of periods: it works with both consumption and capital as outputs of the production possibilities set, rather than restricting attention to value added alone.

B.3 Proof of Theorem 1 (Hulten's Theorem in a dynamic economy)

Extend the single-period function $\mathcal{T}^{(1)}$ to $T+1$ periods:

$$\begin{aligned}
&\mathcal{T}^{(T)}\big(\widehat{\boldsymbol{P}}_t, \ldots, \widehat{\boldsymbol{P}}_{t+T}, \widehat{\boldsymbol{P}}^K_{t+T+1}; \boldsymbol{K}_t, \boldsymbol{L}, \boldsymbol{Z}\big) \\
&= \max \sum_{\tau=t}^{t+T} \sum_i \widehat{P}_{i\tau}\big(Y_{i\tau} - \textstyle\sum_j X_{ij\tau}\big) + \textstyle\sum_i \widehat{P}^K_{it+T+1} \mathcal{K}_i(K_{it+T}, \boldsymbol{X}_{it+T}) \\
&\quad \text{s.t.} \quad \mathcal{Z}(\boldsymbol{Y}_\tau; \boldsymbol{K}_\tau, L_\tau; \boldsymbol{Z}_\tau) \leq 0, \quad K_{i\tau+1} = \mathcal{K}_i(K_{i\tau}, \boldsymbol{X}_{i\tau}), \quad t \leq \tau \leq t+T.
\end{aligned}$$

The corresponding dynamic production possibilities frontier is

$$\mathcal{Z}^{(T)}(\boldsymbol{C}_t, \ldots, \boldsymbol{C}_{t+T}, \boldsymbol{K}_{t+T+1}; \boldsymbol{K}_t, \boldsymbol{L}, \boldsymbol{Z}) = 0.$$

Comparing the first-order conditions of $\mathcal{T}^{(T)}$ with competitive-equilibrium conditions establishes that the shadow prices are proportional to discounted spot prices, $\widehat{P}_{it'} = \Lambda_{t'} P_{it'}$, where $\Lambda_{t'}/\Lambda_{t'-1} = (1 + r_{t'-1})^{-1}$. Since shadow prices are defined only up to a common scale, we normalize $\Lambda_\tau = 1$ at the date the shock materializes, so that

$$\Lambda_{t'} = \beta_{\tau,t'}, \tag{47}$$

with $\beta_{\tau,t'}$ as defined in Theorem 1. The envelope conditions for $\mathcal{Z}^{(T)}$ then yield

$$\frac{\partial \mathcal{Z}^{(T)}}{\partial Z_{i\tau}} = -\frac{P_{i\tau} Q_{i\tau}}{Z_{i\tau}}, \qquad \frac{\partial \mathcal{Z}^{(T)}}{\partial C_{it'}} = \Lambda_{t'} P_{it'}, \qquad \frac{\partial \mathcal{Z}^{(T)}}{\partial K_{it+T+1}} = \Lambda_{t+T} P^K_{it+T+1}.$$

Differentiating $\mathcal{Z}^{(T)} = 0$ with respect to $Z_{i\tau}$, substituting these envelope conditions, and using the aggregate growth rates in Definition 1 gives

$$\frac{P_{i\tau} Q_{i\tau}}{\widetilde{Y}_\tau} = \sum_{t'=t}^{t+T} \beta_{\tau,t'} \frac{\widetilde{C}_{t'}}{\widetilde{Y}_\tau} \frac{d \log C_{t'}}{d \log Z_{i\tau}} \; + \; \beta_{\tau,t+T} \frac{\widetilde{K}_{t+T+1}}{\widetilde{Y}_\tau} \frac{d \log K_{t+T+1}}{d \log Z_{i\tau}},$$

which, upon writing $\widetilde{C}_{t'} = S_{c,t'} \widetilde{Y}_{t'}$ and $\widetilde{K}_{t+T+1} = S^+_{k,t+T} \widetilde{Y}_{t+T}$, is equation (13).

B.4 Proof of Corollary 3

Consider equation (13) from the standpoint of periods $T-1$ and T, and equate the right-hand sides. For $\tau < t+T$, this yields

$$S^+_{k,t+T-1} \frac{d \log K_{t+T}}{d \log Z_{i\tau}} = \frac{\widetilde{Y}_{t+T}}{(1 + r_{t+T-1}) \widetilde{Y}_{t+T-1}} \Big(S_{c,t+T} \frac{d \log C_{t+T}}{d \log Z_{i\tau}} + S^+_{k,t+T} \frac{d \log K_{t+T+1}}{d \log Z_{i\tau}} \Big).$$

Substituting this expression into equation (11) for a date $t > \tau$ (so that $d \log Z_{it} / d \log Z_{i\tau} = 0$ and the direct productivity effect is absent), and noting that beginning-of-period capital is predetermined, gives

$$\frac{d\log Y_t}{d\log Z_{i\tau}} = S_{kt}\frac{d\log K_t^s}{d\log Z_{i\tau}}, \qquad \tau < t.$$

B.5 Proof of Proposition 1 (Welfare interpretation)

The representative household solves

$$\mathcal{V}_t(\boldsymbol{K}_t; L^{(t)}; \boldsymbol{Z}^{(t)}) = \max_{\boldsymbol{C}_t, \boldsymbol{K}_{t+1}} L_t\,\mathcal{U}\left(\frac{\boldsymbol{C}_t}{L_t}\right) + \beta\,\mathcal{V}_{t+1}(\boldsymbol{K}_{t+1}; L^{(t+1)}; \boldsymbol{Z}^{(t+1)}) \quad \text{s.t.} \quad \mathcal{Z}^{(1)} \le 0.$$

The first-order condition with respect to C_{it} gives $\partial_i \mathcal{U}_t = \xi_t P_{it}$, where ξ_t is the Lagrange multiplier. The static expenditure function $\mathcal{E}(\mathcal{U}_t; \boldsymbol{P}_t)$ satisfies $1/\xi_t = (\widetilde{C}_t/L_t)/(\mathcal{U}_t/\varepsilon_t)$. The envelope condition then gives

$$\frac{\partial \mathcal{V}_t}{\partial Z_{it}} = \xi_t \frac{\partial \mathcal{Z}_t^{(1)}}{\partial Z_{it}} = \xi_t \frac{P_{it} Q_{it}}{Z_{it}}.$$

Rearranging yields equation (20). Under $\mathcal{U}(\boldsymbol{C}) = \log \mathcal{C}(\boldsymbol{C})$ with $\mathcal{C}$ homogeneous of degree one, $\varepsilon_t = \mathcal{U}_t$ and the expression simplifies to equation (21).

C Structural Model Derivations

This appendix provides derivations for the results stated in Section 3.

C.1 The social planner's problem and first-order conditions

The social planner maximizes $\sum_{\tau=t}^{\infty} \beta^{\tau-t} \boldsymbol{\theta}' \boldsymbol{c}_\tau$ subject to the production technologies (23) -(24), the capital accumulation equation (25), the TFP process (44), as well as market clearing, where all variables are in logs.[16] We conjecture that the value function takes the linear form $\mathcal{V}(\boldsymbol{k}_t, \boldsymbol{z}_t, \boldsymbol{z}_{t-1}) = \boldsymbol{B}_k' \boldsymbol{k}_t + \boldsymbol{B}_z' \boldsymbol{z}_t + \boldsymbol{B}_z^{-\prime} \boldsymbol{z}_{t-1} + B$. Given the process for TFP, $E_t(\boldsymbol{z}_{t+1}) = \left(\boldsymbol{I} + \boldsymbol{\rho}_d^z\right) \boldsymbol{z}_t + \left(\boldsymbol{I} - \boldsymbol{\rho}_d^z\right) \boldsymbol{g} - \boldsymbol{\rho}_d^z \boldsymbol{z}_{t-1}$ and, since next period's capital is known once chosen this period, the Bellman equation becomes

$$\boldsymbol{B}_k' \boldsymbol{k}_t + \boldsymbol{B}_z' \boldsymbol{z}_t + \boldsymbol{B}_z^{-\prime} \boldsymbol{z}_{t-1} + B = \max\left\{\boldsymbol{\theta}' \boldsymbol{c}_t + \beta\left(\boldsymbol{B}_k' \boldsymbol{k}_{t+1} + \boldsymbol{B}_z' E_t(\boldsymbol{z}_{t+1}) + \boldsymbol{B}_z^{-\prime} \boldsymbol{z}_t + B\right)\right\},$$

subject to the technology and market-clearing constraints.

The first-order conditions (FOCs) with respect to q_{it}, y_{it}, c_{it}, k_{it+1}, m_{jit}, x_{jit}, and ℓ_{it} are:

$$\psi_i = \mu_{it} Q_{it}, \qquad \psi_i \gamma_i = \nu_{it}, \qquad \theta_i = \mu_{it} C_{it},$$
$$\beta B_{ki} = \eta_i, \qquad \psi_i (1-\gamma_i)\phi_{ji} = \mu_{jt} M_{jit}, \qquad \eta_i \zeta_i \omega_{ji} = \mu_{jt} X_{jit},$$
$$\nu_{it}(1-\alpha_i) = \xi L_{it}.$$

[16] Note that the TFP process described in (27) represents a special case of (44), where $\rho_i^z = 0$, $\forall i$.

The Lagrange multipliers $\boldsymbol{\psi}$, $\boldsymbol{\eta}$, and ξ are constant over time, confirming the conjectured linear value function.

Substituting these expressions into the resource constraint and solving gives

$$\boldsymbol{\psi} = \boldsymbol{\mathcal{L}}' \big(\boldsymbol{\theta} + \beta\, \boldsymbol{\Omega} \boldsymbol{\zeta}_d \boldsymbol{B}_k\big),$$

where $\boldsymbol{\mathcal{L}} = (\boldsymbol{I} - (\boldsymbol{I} - \gamma_d)\boldsymbol{\Phi}')^{-1}$ is the Leontief inverse. From the production functions and FOCs, $\log \boldsymbol{\mu}_t = -\boldsymbol{\mathcal{L}} \boldsymbol{z}_t - \boldsymbol{\mathcal{L}} \boldsymbol{\alpha}_d \gamma_d \boldsymbol{k}_t + \boldsymbol{A}_\mu$ for a constant vector $\boldsymbol{A}_\mu$. The policy functions for consumption and capital then follow from $c_{it} = \log \theta_i - \log \mu_{it}$ and the capital accumulation equation, which then immediately give equations (29) - (30) in the main text.

C.2 Solving the Bellman equation

Matching coefficients on $\boldsymbol{k}_t$ in the Bellman equation gives

$$\boldsymbol{B}_k' = \boldsymbol{\theta}' \boldsymbol{\mathcal{L}}\, \boldsymbol{\alpha}_d \gamma_d + \beta\, \boldsymbol{B}_k' \big(\boldsymbol{I} - \boldsymbol{\zeta}_d + \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\, \boldsymbol{\alpha}_d \gamma_d\big),$$

which solves to

$$\boldsymbol{B}_k' = \boldsymbol{\theta}' \boldsymbol{\mathcal{L}}\, \boldsymbol{\alpha}_d \gamma_d \big[\boldsymbol{I} - \beta(\boldsymbol{I} - \boldsymbol{\zeta}_d + \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\, \boldsymbol{\alpha}_d \gamma_d)\big]^{-1}. \tag{48}$$

An alternative representation uses the policy functions to show that

$$\boldsymbol{B}_k' = \sum_{\tau \geq t+1} \beta^{\tau-(t+1)} \frac{\partial c_\tau}{\partial \boldsymbol{k}_{t+1}},$$

confirming that $\boldsymbol{B}_k$ captures the discounted effect of capital on the entire future consumption path. Matching coefficients on $\boldsymbol{z}_t$ and $\boldsymbol{z}_{t-1}$, and solving the resulting system, gives

$$\boldsymbol{B}_z' = \big[\boldsymbol{\theta}' \boldsymbol{\mathcal{L}} + \beta \boldsymbol{B}_k' \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\big] \big[\boldsymbol{I} - \beta(\boldsymbol{I} + \boldsymbol{\rho}_d^z) + \beta^2 \boldsymbol{\rho}_d^z\big]^{-1}$$

$$\boldsymbol{B}_z^{-\prime} = -\beta \boldsymbol{B}_z' \boldsymbol{\rho}_d^z.$$

Matching the constants terms determines B. Setting $\boldsymbol{\rho}_d^z = \boldsymbol{0}_{N\times N}$ recovers the solution for the case where sectoral TFP follows a random walk:

$$\boldsymbol{B}_z' = \big[\boldsymbol{\theta}' \boldsymbol{\mathcal{L}} + \beta \boldsymbol{B}_k' \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\big] \big[\boldsymbol{I} - \beta \boldsymbol{I}\big]^{-1}$$

$$\boldsymbol{B}_z^{-\prime} = \boldsymbol{0}_{1\times N}$$

C.3 Derivation of real GDP growth

Real GDP growth is $\Delta y_t = \Delta \log(\widetilde{C}_t + \widetilde{X}_t) - \sum_i S_{it}\, \Delta p_{it}^Y$, where p_{it}^Y is the log value-added deflator. In the structural model, nominal GDP equals $\widetilde{Y}_t = (1/\Lambda_t)(1 + \beta\, \boldsymbol{\zeta}' \boldsymbol{B}_k)$, where $\Lambda_t = 1/C_t$ is the marginal utility of the consumption aggregate. Therefore $\Delta \log(\widetilde{C}_t + \widetilde{X}_t) = \boldsymbol{\theta}' \Delta \boldsymbol{c}_t$.

Value-added shares are constant:

$$S_i = \frac{\left[\gamma_d \boldsymbol{\mathcal{L}}'(\boldsymbol{\theta} + \beta\,\boldsymbol{\Omega}\boldsymbol{\zeta}_d \boldsymbol{B}_k)\right]_i}{1 + \beta\,\boldsymbol{\zeta}'\boldsymbol{B}_k}.$$

Value-added price changes satisfy $\Delta \boldsymbol{p}_t^Y = \gamma_d^{-1} \boldsymbol{\mathcal{L}}^{-1}(\mathbf{1}\boldsymbol{\theta}' - \boldsymbol{I})\boldsymbol{\mathcal{L}}\,(\Delta \boldsymbol{z}_t + \boldsymbol{\alpha}_d \gamma_d\,\Delta \boldsymbol{k}_t)$. Combining and simplifying yields equation (36):

$$\Delta y_t = \boldsymbol{D}'\Delta \boldsymbol{z}_t + \boldsymbol{D}'\gamma_d \boldsymbol{\alpha}_d\,\Delta \boldsymbol{k}_t,$$

where $\boldsymbol{D} = \boldsymbol{\mathcal{L}}'(\boldsymbol{\theta} + \beta\,\boldsymbol{\Omega}\boldsymbol{\zeta}_d \boldsymbol{B}_k)/(1 + \beta\,\boldsymbol{\zeta}'\boldsymbol{B}_k)$. Domar weights and value-added shares are closely related since $\boldsymbol{D} = \gamma_d^{-1}\boldsymbol{S}$.

C.4 Propagation kernel via Corollary 3

The Cobb–Douglas first-order conditions imply $R_{it}K_{it} = \alpha_i\gamma_i\,P_{it}Q_{it} = \alpha_i\gamma_i D_{it}\widetilde{Y}_t$. Aggregating, $\widetilde{K}_t^s = \widetilde{Y}_t \sum_j \alpha_j\gamma_j D_{jt}$ and $S_{kt} = \sum_j \alpha_j\gamma_j D_{jt}$, so that

$$S_{kt}\,d\log K_t^s = \boldsymbol{D}'\gamma_d \boldsymbol{\alpha}_d\,d\boldsymbol{k}_t. \tag{49}$$

Combining (49) with Corollary 3 gives

$$\frac{d\log Y_{t'}}{d\log Z_{it}} = \boldsymbol{D}'\gamma_d \boldsymbol{\alpha}_d \frac{d\boldsymbol{k}_{t'}}{d\log Z_{it}}, \qquad t' > t.$$

First-differencing, iterating on the law of motion for capital growth (34) from $d\Delta \boldsymbol{k}_t/d\Delta z_{it} = 0$ (capital is predetermined at the time a TFP shock materializes and so does not respond contemporaneously), and using the AR(1) structure of TFP growth (44), for $h \geq 1$, gives

$$\frac{d\Delta \boldsymbol{k}_{t+h}}{d\Delta z_{it}} = \sum_{j=0}^{h-1} (\boldsymbol{\rho}_d^z)^{h-1-j} \boldsymbol{\mathcal{J}}^j\,\boldsymbol{\zeta}_d \boldsymbol{\Omega}'\boldsymbol{\mathcal{L}}\,\boldsymbol{e}_i,$$

and

$$\frac{d\Delta \boldsymbol{y}_{t+h}}{d\Delta z_{it}} = \sum_{j=0}^{h-1} \boldsymbol{D}'\gamma_d \boldsymbol{\alpha}_d (\boldsymbol{\rho}_d^z)^{h-1-j} \boldsymbol{\mathcal{J}}^j\,\boldsymbol{\zeta}_d \boldsymbol{\Omega}'\boldsymbol{\mathcal{L}}\,\boldsymbol{e}_i.$$

Using the random-walk structure of TFP (27) and iterating on the level law of motion for capital (30) from $d\boldsymbol{k}_t/d\log Z_{it} = 0$ yields,

$$\frac{d\boldsymbol{k}_{t+h}}{d\log Z_{it}} = \sum_{j=0}^{h-1} \boldsymbol{\mathcal{J}}^j\,\boldsymbol{\zeta}_d \boldsymbol{\Omega}'\boldsymbol{\mathcal{L}}\,\boldsymbol{e}_i, \qquad h \geq 1.$$

First-differencing the GDP impulse response, $d\Delta y_{t+h}/d\Delta z_{it} = d\log Y_{t+h}/d\log Z_{it} - d\log Y_{t+h-1}/d\log Z_{it}$, telescopes the geometric series and gives

$$\frac{d\,\Delta y_{t+h}}{d\,\Delta z_{it}} = \boldsymbol{D}'\gamma_d \boldsymbol{\alpha}_d\, \boldsymbol{\mathcal{J}}^{h-1} \boldsymbol{\zeta}_d \boldsymbol{\Omega}' \boldsymbol{\mathcal{L}}\, \boldsymbol{e}_i, \qquad h \geq 1,$$

which is the IRF kernel of equation (38) for $h > 0$. Aggregating across past shocks recovers equation (37). The same kernel emerges directly by iterating the growth-rate law (34) into equation (36).

C.5 Domar weight decomposition

The Domar weight decomposes as $\boldsymbol{D} = \boldsymbol{D}^c + \boldsymbol{D}^x$ with

$$\boldsymbol{D}^{c'} = \frac{\boldsymbol{\theta}'\boldsymbol{\mathcal{L}}}{1+\beta\,\boldsymbol{\zeta}'\boldsymbol{B}_k} = S_c\,\boldsymbol{\theta}'\boldsymbol{\mathcal{L}}, \text{ and } \boldsymbol{D}^{x'} = \frac{\beta\,\boldsymbol{B}_k'\boldsymbol{\zeta}_d\boldsymbol{\Omega}'\boldsymbol{\mathcal{L}}}{1+\beta\,\boldsymbol{\zeta}'\boldsymbol{B}_k} = S_c\,\beta\,\boldsymbol{B}_k'\boldsymbol{\zeta}_d\boldsymbol{\Omega}'\boldsymbol{\mathcal{L}}.$$

To verify that this recovers Corollary 2: $D_i^c = S_c\,\partial c_t/\partial z_{it}$ captures the contemporaneous consumption effect, and $D_i^x = S_c \sum_{\tau>t} \beta^{\tau-t}\,\partial c_\tau/\partial z_{it}$ captures the discounted future consumption responses. Since the discount factor in this model satisfies $\beta_{t,t'}\,\widetilde{Y}_{t'}/\widetilde{Y}_t = \beta^{t'-t}$ (because $\Lambda_{t'}/\Lambda_t$ cancels with $\widetilde{Y}_{t'}/\widetilde{Y}_t$) and S_c is constant, the terminal capital term in Corollary 2 vanishes by the transversality condition, confirming the decomposition.

D Quantifying the Structural Model

This appendix describes the details of how we quantify the structural model in Section 3 up to the eleven-sector aggregation of the U.S. economy used in Section 4.

D.1 Network matrices

We briefly describe below the construction of the two network matrices using notation and methodology presented in Chapter 12 of Horowitz and Planting (2009).

The intermediate-input network, $\boldsymbol{\Phi}$, is constructed from the BEA annual Use table before definition, which we transform using the annual Make table before redefinition.[17] Let $D = Vq_d^{-1}$ be the market share matrix where V is industry production from the Make table and q_d is the diagonal matrix of the vector denoting total use of commodities. We then pre-multiply the industry use of commodities matrix from the Use table, U by the market share matrix to get a matrix that provides the industry sources of intermediate-input use: $\tilde{U} = DU$. We then get the objects g and v from the Use table, which denote the gross output by industry and value-added by industry respectively. The intermediate-input network matrix is then: $\boldsymbol{\Phi} = \tilde{U}(g-v)_d^{-1}$. For

[17] For each industry, the Use table documents the use of commodities and the Make table documents the production of commodities.

each pair of sectors (j, i), the entry ϕ_{ji} is then the share of sector i's total intermediate-input expenditure that is purchased from sector j. For further details, see the technical appendix of Foerster et al. (2022).

The investment network $\mathbf{\Omega}$ is constructed using data from vom Lehn and Winberry (2022), as well as the BEA Fixed Assets Tables (FAT) and final demand components from the Use table. To begin, we need to ensure that the total private investment in vom Lehn and Winberry (2022) matches the sum of private industry investment in the Use table. To that end, we scale the vom Lehn and Winberry (2022) investment shares to produce investment flows to match total investment in the FAT. We take the FAT and Use tables to be consistent with each other since the two have sums of all private industry investment that are within 1.5 percentage points of each other.

Next, consider the FAT-scaled investment flow data from vom Lehn and Winberry (2022), which has the commodity-by-industry structure of the Use table. Since the vom Lehn and Winberry (2022) data focuses on the purchase of new capital goods and ignore used capital goods transactions, their data misses the "scrap" and "non-comparables" commodity categories. Consequently, the column sums of the investment-flow matrices do not match the corresponding commodity sources of investment from the Use table. We modify the data such that their column sums equal the corresponding final demand component in the Use table while minimizing the change in relative commodity contributions to industry investment.

Subsequently, we pre-multiply the investment-flow table by the market share matrix D defined above: $\tilde{X} = DX$, where X is the adjusted commodity-by-industry investment flows matrix. To correct for a small number of negative investment flows we adopt the following procedure:

1. Set negative investment flow entries to zero.

2. Rescale each industry's investment demand so that it remains unchanged.

3. Redefine final demand for private and government investment as the sum of the corresponding columns of X.

4. Redefine the inventory investment plus net-exports category to make up the difference between the original and rescaled non-negative final demand for investment.

Finally, we define the investment network as: $\mathbf{\Omega} = \frac{\tilde{X}_{ij}}{\sum_k \tilde{X}_{kj}}$. For each pair (j, i), the entry ω_{ji} is then the share of sector i's total investment expenditure on goods from sector j.

All data are annual and span 1947 through 2024. We construct the networks for each year and average over the sample period.

D.2 Production parameters

Sectoral capital shares α_i are computed as the ratio of capital compensation to value added in each sector from the KLEMS data. Value-added shares in gross output γ_i are the ratio of sectoral value added to sectoral gross output (which are close to those from the BEA industry accounts). The consumption shares θ_i are calibrated to the corresponding shares in the BEA data.

The investment elasticities ζ_i are estimated so as to minimize the distance between the relative persistence of IRFs and that of local projections, i.e.,

$$\arg\min_{\zeta_d} \sum_i \sum_{h=0}^{8} \left(\frac{d\Delta y_{t+h}/d\Delta z_{it}}{d\Delta y_t/d\Delta z_{it}} - \frac{\widehat{b}_{i,h}}{\widehat{b}_{i,0}} \right)^2,$$

where $\left\{\widehat{b}_{i,h}\right\}$ are estimated under the specification

$$\Delta \text{GDP}_{t+h} = a_{i,h} + b_{i,h}\Delta \text{TFP}_{it} + \boldsymbol{w}_t'\gamma_{i,h} + \varepsilon_{it,h},$$

with control vector $\boldsymbol{w}_t$ including two lags of aggregate GDP growth and sectoral TFP growth.[18] To show that the similarities in the shapes of sectoral impulse responses and local projections are not simply a result of allowing for flexibility in the form of sector-specific ζ_i's, we provide an alternative estimation in which ζ is common across all sectors. Appendix D.4 reports the results obtained under this alternative calibration.

Table A2 reports the values of α_i, γ_i, θ_i, and ζ_i for each sector.

Table A2: Calibrated parameters

Sector	**Consumption Expenditure Share** (θ_i)	**Capital Share of Value Added** (α_i)	**Value Added Share of Gross Output** (γ_i)	**Capital Adjustment Costs** (ζ_i)
Mining & Utilities	2.64	65.40	57.51	0.990
Construction	0.00	13.66	45.84	0.990
Durable Goods	7.21	34.67	40.76	0.990
Nondurable Goods	16.82	46.83	32.06	0.990
Wholesale & Retail	21.52	26.48	64.39	0.935
Information	3.66	54.90	61.94	0.990
Finance & Insurance	6.58	47.53	57.86	0.990
Real Estate	14.94	94.14	76.53	0.767
PBS	1.55	32.03	64.32	0.990
Education & Health	13.26	16.47	64.07	0.705
Food & Accommodation	11.82	18.81	58.00	0.773

D.3 TFP process with shock dynamics

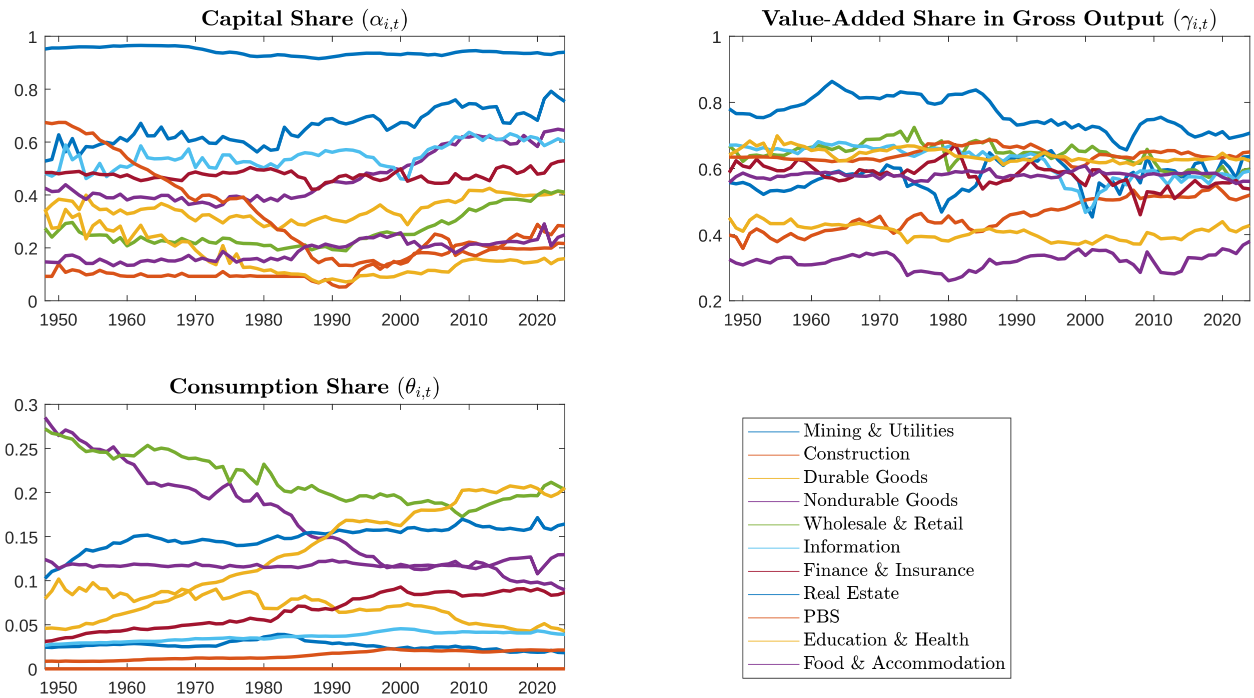
The extended TFP process in equation (44) allows for an AR(1) component in sectoral TFP growth. Under this extension, the state vector is augmented to include lagged TFP growth for each sector. The value function remains linear in the extended state, so the certainty equivalence property is preserved.

[18] In the steady state, $K_i = \chi_i^{\frac{1}{\zeta_i}} \prod_j (X_{ji}/\omega_{ji})^{\omega_{ji}}$, so that $\chi_i^{-\frac{1}{\zeta_i}}$ denotes the investment rate needed to maintain a constant capital stock in the long run. Since χ_i is a free parameter, we cannot calibrate ζ_i to the steady state capital stock, unlike in a model with linear depreciation.

D.4 Sensitivity and robustness

Figure A2 shows the values of α_i, γ_i, and θ_i computed year-by-year. While there is some visible variation over time, the key features of the data remain consistent over the sample. Importantly, the capital share for investment has remained close to one throughout the sample.

Figure A2: **Time Series of Ratios in Data**



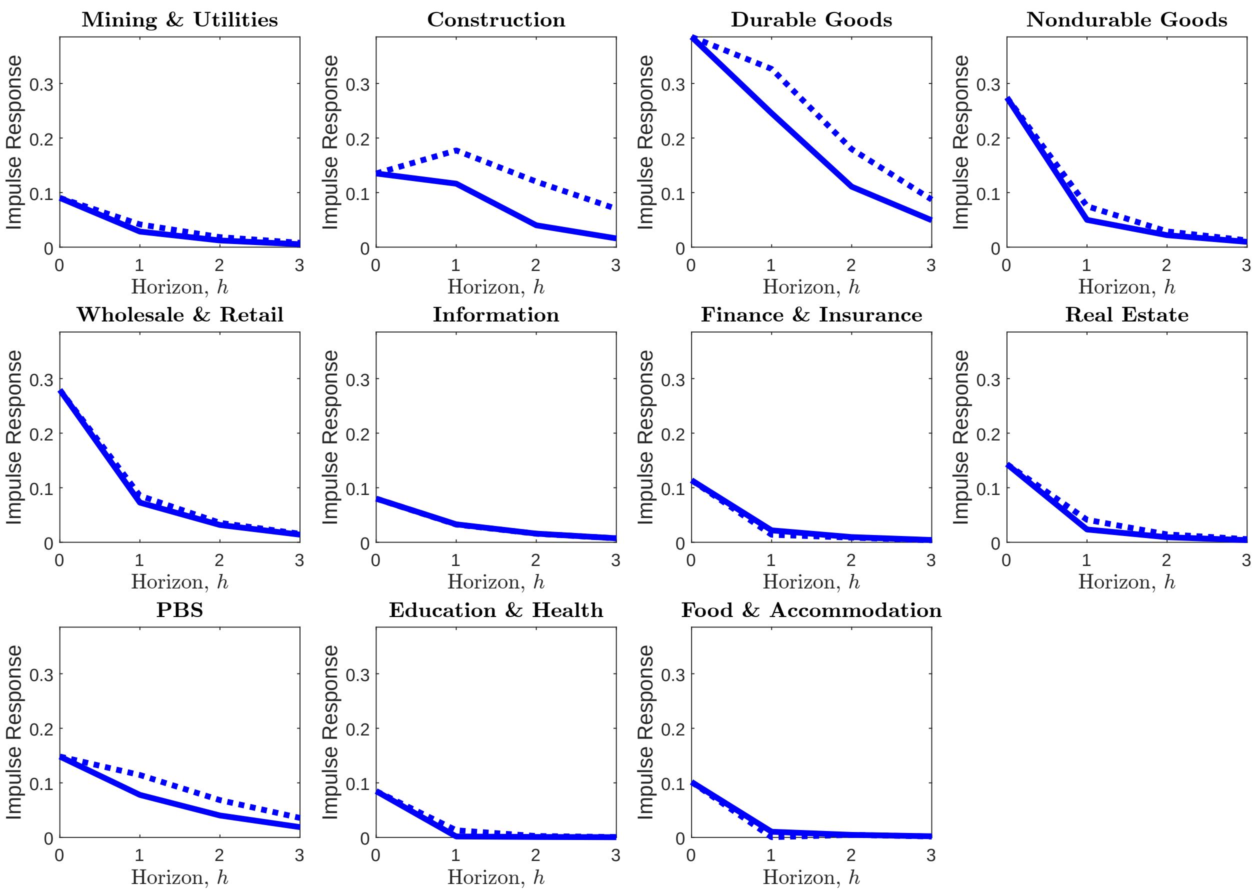
The production networks are similarly stable over time. To check this, we vectorize $\boldsymbol{\Phi}$ and $\boldsymbol{\Omega}$ in the first and second halves of the sample. The correlation between the two subsamples is 0.89 for $\boldsymbol{\Phi}$ and 0.92 for $\boldsymbol{\Omega}$.

To show that our results do not depend upon calibrating $\boldsymbol{\zeta}$ to match the persistence of impulse responses to those of local projections by sector, we alternatively calibrate a common ζ across all sectors. I.e.,

$$\arg\min_{\zeta} \sum_i \sum_{h=0}^{8} \left(\frac{d\Delta y_{t+h}/d\Delta z_{it}}{d\Delta y_t/d\Delta z_{it}} - \frac{\widehat{b}_{i,h}}{\widehat{b}_{i,0}} \right)^2$$
$$\text{s.t. } \boldsymbol{\zeta}_d = \zeta \boldsymbol{I}$$

We then compute impulse responses using the resultant $\zeta = 0.88$, which we show in Figure A3. The impulse responses appear near identical to those of Figure 6.

Figure A3: **Aggregate Impulse Responses to Unit Sectoral Productivity Shocks**



Notes: Each panel shows the response of real GDP growth at horizon h to a unit shock in the indicated sector's TFP growth. Solid: benchmark model without shock dynamics ($\rho_i^z = 0$ for all i). Dashed: model with shock dynamics.