RCL Circuitry in Quantum Electrical Engineering

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Abstract

The study of the X-ray edge and infrared catastrophe may help the understanding of the AC and transient behavior of a nano–scopic electronic device which, because of these, cannot be modeled by a simple RCL circuit element with constant R, C, and L in general. Our discussions illustrate that a direct and simple application of the standard approaches does not provide valid results, with potentially useful implications in quantum electrical engineering. [1]

In our earlier studies we discussed the AC response of nanoscopic electronic systems. [2] We have shown that, in addition to the real part of the conductance there is an additional, imaginative part in the conductance, a *quantum* inductance, as observed experimentally [3] and discussed theoretically. [2, 4, 5] Much of the behaviors of small, quantum mechanical systems have considered single body systems. We have noted that qualitatively different behaviors could be expected in the time–dependent behaviors of many body quantum systems. To appreciate fully the subtlety and complication of quantum transient response, we discuss here the problem related to an exactly solved problem, the X-ray edge problem. [6, 7]

Consider the absorption of an X-ray photon in state $|q>$ by an electron in a localized core level of energy E_i excited to an extended level of energy E_f above the Fermi level E_F. The golden rule gives:

$$\frac{d\sigma}{d\omega} \propto \Sigma| < f|H|i, q > |^2 \delta(E_i + \omega_q - E_f) \tag{1}$$

Initially there is a photon and the electrons are in an N body state $|i>$ with energy E_i, having N-1 electrons in the Fermi sea and one in the core. Following the excitation the photon is gone and the electrons are in a state $|f>$ with energy E_f with the core electron moved from the core to the state $|k>$ in a formally excited state so that the initial Hamiltonian:

$$H = \Sigma e_p a_p^+ a_p + \lambda \Sigma a_p^+ a_p d^+ d \tag{2}$$

is different from the final Hamiltonian:

$$H = \Sigma e_p a_p^+ a_p \tag{3}$$

Note that $\Sigma a_p^+ a_p = \psi^+(0)\psi(0)$ in a single scatterer model. We are essentially considering the response of a noninteracting electron gas to a sudden change in Hamiltonians. The sudden appearance of a local scatterer:

$$H' = -\int d^3x j(x) A(x) \tag{4}$$

where $A \propto e_q e^{-iqx}$ and:

$$j = \Sigma a_n^+ a_m < n|j(x)|m > \tag{5}$$

has a_m removing a core electron and a_n^+ creating an electron in state n. The matrix element $M =< k|j(x)|core >$ is assumed to be a constant. Therefore

$$\frac{d^2\sigma}{d\omega dE} \propto \sum | < f|d|i > |^2 B \tag{6}$$

with

$$B = \exp[it(E_i + \omega_q - E_f - \epsilon_k)] = \int \frac{dt}{2\pi} e^{it\Omega} g(t) \tag{7}$$

where $\Omega = \omega_q - \epsilon_k + \epsilon_{core}$ is the energy transferred to the N-1 electrons. Thus we have:

$$g(t) \equiv < i|d^+(t)d(0)|0 > \tag{8}$$

with $d^+(t) \equiv e^{iHt}d^+e^{-iHt}$ and

$$g(t) =< i|\exp[-i\int_0^t dtH(t)]|t > \tag{9}$$

with $H = \lambda \sum a_p^+ a_p'$. Expanding H we have

$$\exp(-i\int Hdt) = B < j|\frac{i^2\lambda^2}{2}|j > \tag{10}$$

with

$$\int_0^t d\tau_1 \int_0^t d\tau_2 H(\tau_1)H(\tau_2)|i > \equiv -C(t) \tag{11}$$

and $C(t)$ equals

$$\frac{\lambda^2}{2}\int_0^t d\tau_1 \int_0^t d\tau_2 e^{i(\epsilon-\epsilon')(\tau_1-\tau_2)} f(\epsilon)[1-f(\epsilon')] = \lambda^2 N^2 \ln(i\Omega t) \tag{12}$$

thus

$$g(t) = e^{-C(t)} = (i\Omega t)^{-\lambda^2 N^2} \tag{13}$$

We see then that the decay is highly non-exponential. The exponent $\lambda^2 N^2$ depends on the details of the interaction between the holes and the electrons. In comparison with classical behavior or with quantum decay in single body systems, the quantum decay of a many body system is most often non-exponential in that it may involve the overlap of a very large number of wave functions. When a system has many low energy excited states, it is possible to excite many of them simultaneously with little energy cost. The overlap among those states goes down and the transient response becomes singular: the well known infrared or orthogonal "catastrophe." [8] We can also relate to this to the quantum watchdog effect, [9] that repeated interrogations of a system will prohibit scatterings of the decayed state to go back into the parental state, making in this way the decay exponential. We note that in general discussions of quantum decay we usually start with the system prepared in an unstable state and consider the outcome of the decay product. This is not what we do in quantum transport where, typically, given that an experiment will produce a certain result we ask how to prepare the system's initial state that might produce the result. Finally we note that while there have been significant amounts of work showing non-exponential quantum decay, a power law result such as $g(t)$ is new. Today's study of transport in electronic systems often uses models with static scatterers, the dynamic scatterers are considered only while for providing thermal agitations. We see quantum mechanically, the time dependent behavior of a many body system may differ qualitatively from that of a few–body system. From the above we may expect to see highly nontrivial time dependent behavior of the system which is another example of the subtle and nontrivial behaviors of nanoscopic systems demanding more careful studies. No simple, classical RCL circuitry system has likewise behavior, and we see that quantum electrical engineering offers new opportunities for new applications.

Now we consider the decay of a quantum system in general. We assume that at $t = 0$ the system is in state $|\Psi >$ and calculate the "not yet decayed" amplitude that it remains in state $|\Psi >$ at time t, $P(t) = |A(t)|^2$, with

$$A(t) =< \Psi|e^{-iHt}|\Psi > \tag{14}$$

Let:

$$e^{-iHt}|\Psi >= A|\Psi > +|\Phi(t) > \tag{15}$$

where $\Phi(t)$ is the decayed part and $\Psi(t)$ the remaining part orthogonal to $\Phi(t)$ with

$$< \Phi(t)|\Psi(t) >= 0 \tag{16}$$

We have then:

$$< \Psi|e^{-iHt'}e^{-iHt}|\Psi >= A(t)A(t') \tag{17}$$

At the same time, it equals

$$A(t+t')+ < \Psi|\Phi(t+t') > \tag{18}$$

Since $< \Psi|\Phi > \neq 0$ in general, we have

$$A(t+t') \neq A(t)A(t') \tag{19}$$

and we see that $A(t)$ cannot be a simple exponential of the form $A(t) \propto e^{-\lambda t}$, [10, 11] consistent with the behavior of the example we illustrated above. Fundamentally this is because, unlike the decay we discuss in statistical mechanics, a quantum decay is not a one-way process in that quantum mechanics is time reversibly invariant, that there is always certain, possibly small but still nonzero amplitude for part of the decayed state to "decay back" into the parental state, making therefore a quantum mechanical decay different from a thermodynamic decay. The quantum decay of an N-body system is generally non–exponential as it typically involves the overlap of a large number of wave functions. A mathematical treatment with similar conclusion has been given in. [12] Most of the discussions about decaying process in elementary quantum mechanics and much of the present experimental work such as the observation of the decay of an atom from an excited level, however, have not been able to detect decays with departures from a simple, exponential decay rate. That quantum decay is often non-exponential is known since the work by Ersak, [11] who gave an argument similar to ours, and was also discussed by Williams, [13] Fonda *et al.* [14] and Sinha [15], among others. A good qualitative discussion was given by Hopfield. [16] Khafin [17] and Schwinger [18] have also shown that quantum decay is exponential during intermediate times only; at both short and long times it is proportional to either $t^{-\alpha}$ or $\exp(-t^{\beta})$, with β not necessarily equalling to 1. It is also known that in a thermal environment the quantum decay may become exponential [19] which is not unexpected as a thermal environment will break the time reversal symmetry and suppress the decaying back amplitude, making thus the decay process exponential, for reasons discussed above. The problem of quantum mechanical process under the influence of an environment has been studied in great detail by Leggett and others. [20] Related to this is the quantum watchdog effect, [9] that repeated interrogations of a system prohibit scatterings of the decayed state going back into the parental state, making thus the decay exponential. This is the dephasing effect in mesoscopics context. Today's study of transport in mesoscopic systems has mostly considered with dynamic scatterers in dephasing or inelastic scatterings discussion only, that the system may lose coherent quantum interference, the behavior of the system can still be exponential. It is useful to point out that non-static scatterings have more effects beyond what is generally considered in that they can affect the AC response of the system. [2] Given the nontrivial frequency dependency of the quantum conductance, the transient behavior of a quantum device can be very complicated. We are not aware of any experimental report of the subject to be compared with our result given here. While discussing a system's conductance, a simple, constant scattering time τ is generally used. Although that may be appropriate in discussing DC transport, we see that to describe a scattering process faithfully and for time dependent applications, it is necessary to take into account the orthogonality or infrared catastrophe as discussed above. For that purpose it may well be insufficient to merely replace a constant scattering time τ by a certain frequency dependent $\tau(\omega)$. We note, finally, that our discussion is incomplete in that, while it does point out the non–exponential nature of quantum decay in general, it has not provided a way to quantitatively characterize the "non–exponentialness" of a decay process to allow direct, convenient, and useful experimental determination of this feature.

To summarize, we have shown that quantum mechanically, a nanoscopic device cannot be modeled accurately by a simple RC circuit element with simple, constant, and frequency independent R and C in general. Our study and the observation of the AC response of a mesoscopic electronic device may provide additional ways to study the time dependent quantum behaviors of systems in general as well as to useful applications in microelectronics. We understand that much of the current discussion of the quantum watchdog effect has been in the context of quantum cosmology [21] and hope that our discussion may provide new and potentially more convenient possibilities to observe and understand the effect. There has been discussion in literature [22] about the time-dependent quantum behaviors and the tunneling. We hope our discussion could prove useful in these studies as well.

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