Does a Quantum Apple Fall?

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Abstract

We study, quantum mechanically, the behavior of a particle in a constant external field and point out the extra care needed to reach a valid result.

The dynamics and motion of a particle in a constant external field are one of the first problems we discuss in introductory, classical, physics. We know that, in earth's gravitational field, an apple will fall with acceleration g and its behavior can be described by $z = z_0 - (1/2)gt^2$. The quantum behavior of a mass particle in a constant external field is less discussed but can be exactly solved and is well known. [1] For a particle in one dimension the solute wave function can be expressed in terms of Airy's function. [2] From the equation

$$\frac{d^2\Psi}{dz^2} + 2m(E + Fz)\Psi = 0 \tag{1}$$

Introducing variable $\xi = (z + E/F)(2mF)^{1/2}$, the equation has the simple form

$$\Psi'' + \xi\Psi = 0 \tag{2}$$

and can be solved in terms of Airy's function [2]

$$Ai(\xi) = \int_0^\infty [\cos(\frac{u^3}{3}) + u\xi]du \tag{3}$$

Noting that $Ai(\xi)$ is real, *i.e.* $Ai(\xi) = Ai^*(\xi)$, we see the current as determined by $j \propto [\psi^*(d/dz)\psi - \psi(d/dz)\psi^*]$ is exactly zero, which shows that, quantum mechanically, not only does an apple not fall with acceleration, it does not fall at all. As a final remark, we note that our discussion has been in a thermal–free environment from which we know the motion of the particle is time reversal invariant, *i.e.,* which demands not falling. Once we introduce environment effects [3] which will break the time reversal symmetry, it may well be possible to reach results more consistent with our expectations. From this reason, we note that much of the numerical work in microelectronics which mostly done without thermal environment may well have been inaccurate.

References

[1] See, *e.g.*, L. D. Landau and E. M. Lifshitz, *Quantum mechanics*, Pergamon Press, New York (1964). p. 24.

[2] N. N. Lebedev, *Special functions and their applications*, Dover, New York (1972). p. 136.

[3] A. J. Leggett, S. Chakravarty, A. J. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Modn. Phys. **59**, 1 (1987).