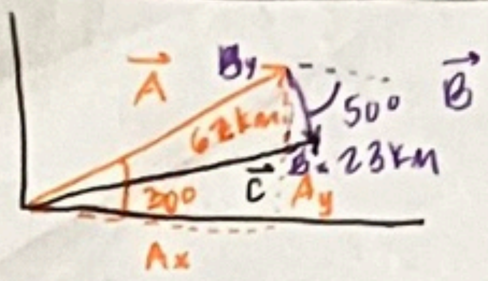
PHYSICS 103 Exam 1 Notesheet S 2024 CARLIE OSTROM



$A_y = 62\,km \sin 30° = 31\,km$
$A_x = 62\,km \cos 30° = 53.7\,km$
$B_y = -23\,km \sin 50° = -17.6\,km$
$B_x = 23\,km \cos 50° = 14.8\,km$
$C_x = A_x + B_x = 68.5\,km$
$C_y = A_y + B_y = 13.4\,km$
$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(68.5\,km)^2 + (13.4\,km)^2} = 69.8\,km$
$\vec{C} = \vec{A} + \vec{B}$
$\theta = \tan^{-1}(C_y/C_x) = \tan^{-1}(13.4\,km / 68.5\,km) = 11.1°$

average velocity: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\text{net distance (displacement)}}{\text{time}} = \frac{x_f - x_i}{t_f - t_i}$ position $= x = x_0 + v_x t$

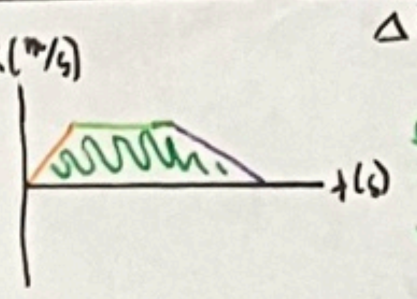
When swimmers meet, $x_A = x_B$, solve for t

average speed = $\frac{\text{total distance traveled}}{\Delta t}$

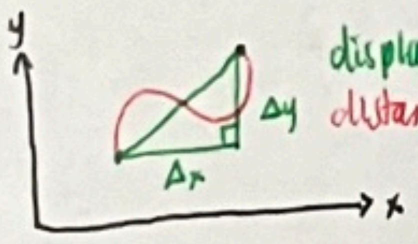
acceleration $\vec{a} = \frac{\Delta v_x}{\Delta t}$

speeds up: v & a same sign
slows down: v & a diff sign, nearing x axis

$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$

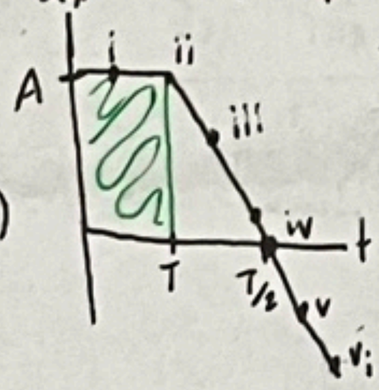


displacement $(x - x_0)$ = area under velocity vs time graph



displacement
distance

a_x velocity at point ii is zero.

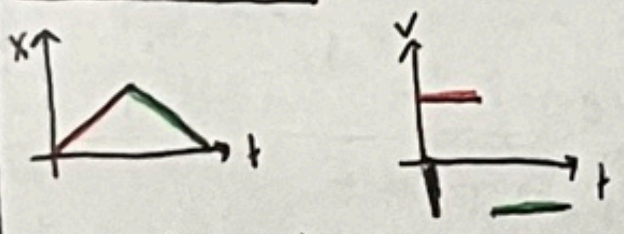


TRUE: velocity at point i is negative.
$A \cdot T = (v_f - v_0)$
$AT = (0 - v_0)$
$v_0 = -AT$ ← positive
positive
v_0 must be negative.

- instantaneous speed is magnitude of instantaneous velocity (though ave speed is not generally magnitude of ave. velocity)

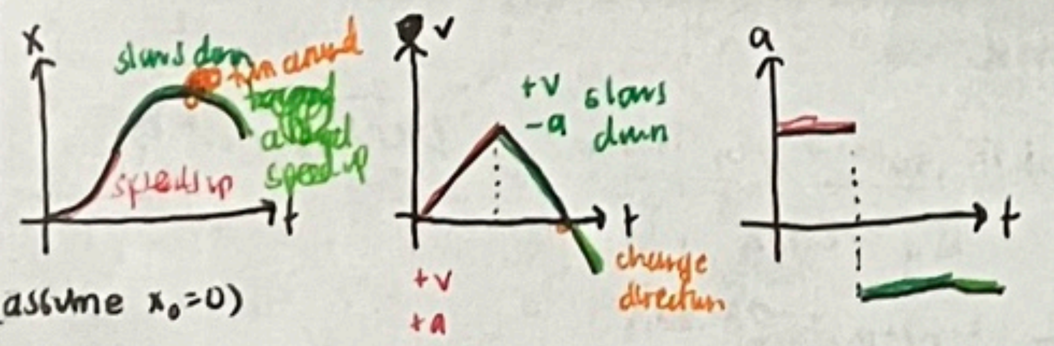
1D kinematic equations

constant velocity: $x = x_0 + vt$



constant acceleration

$v = v_{0x} + a_x t$ $\quad x = x_0 + v_{0x}t + \frac{1}{2}at^2$ $\quad v_x^2 = v_{0x}^2 + 2a(x - x_0)$

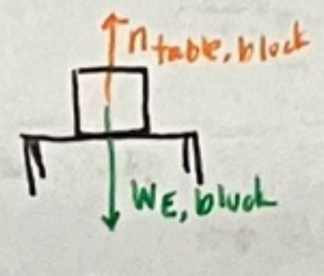


(assume $x_0 = 0$)

$t = \frac{v}{a}$ Ex $t_1 = \frac{2v_{0y}}{g}$

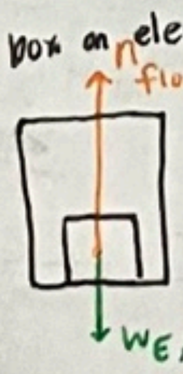
Newton's 2nd Law

$\vec{F}_{net} = \sum \vec{F}_{ext} = m\vec{a}$
$|\vec{W}_{earth,object}| = mg$

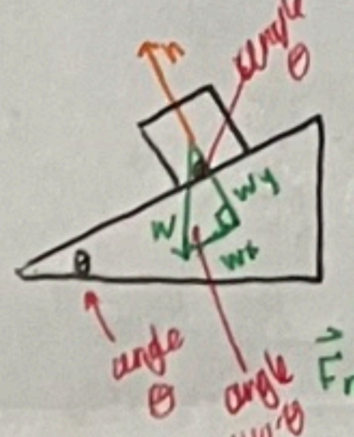


$\vec{F}_{net} = 0$
$\vec{a} = 0$
stationary or constant $\vec{v}$

box on elevator upward acceleration

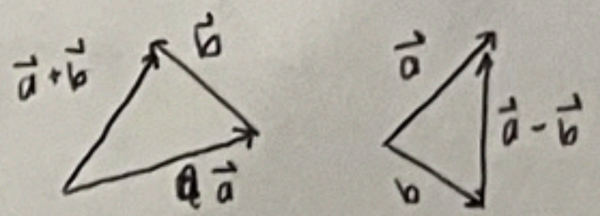


$\vec{F}_{net}$: ↑ up
$\vec{a}$ ↑ up
$F_{net} = n - W = ma$
$n = W + ma$

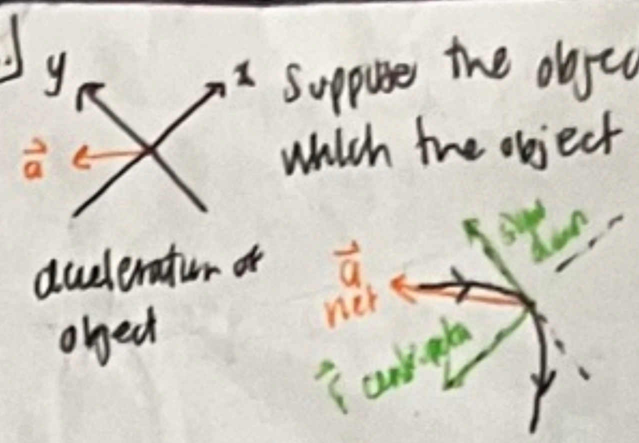


$W_y = W\cos\theta$
$W_x = W\sin\theta$
where $W = mg$
$\vec{F}_{net} = W_x = mg\sin\theta$
$a_x = \frac{F_{net,x}}{m} = g\sin\theta$

Vectors

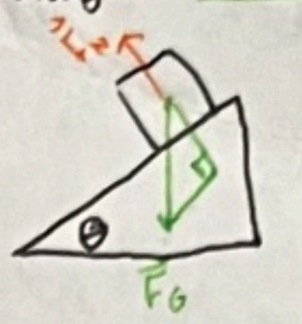


1) Suppose the object is slowing down. Match best trajectory on which the object is traveling.



$a_{centripetal} = \frac{v^2}{radius}$ larger radius, smaller acceleration

2) Largest net force? Largest acceleration?



case 1: 5 kg, θ=15°
$\vec{F}_G = 5kg \cdot 9.8\,m/s^2 = -49\,N$
$F_{Gx} = 49\sin 15° = -12.7\,N$ = net force since normal force & weight balance out (vertical has no net force)
$F/m = a = \frac{-12.7\,N}{5\,kg} = -2.54\,m/s^2$

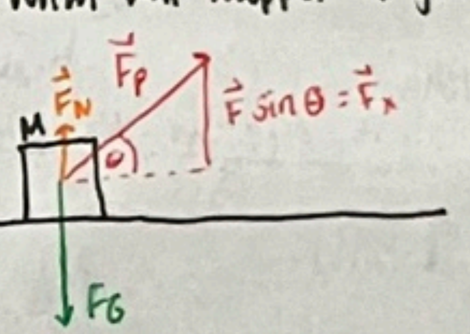
3) Toddler has 10 kg mass.

A. Elevator is stationary
$\vec{v} = 0\,m/s$
$\vec{a} = 0\,m/s^2$
$W: 10kg \cdot -9.8\,m/s^2 = -98\,N$
$F_{net} = n - W = ma$
$n - W = 0$
$n = -W = +98\,N$

B. Elevator accelerates
$\vec{a} = 2\,m/s^2$ ↑
$W = -98\,N$
$\vec{F}_{net} = 10kg \cdot 2\,m/s^2 = 20\,N$ ↑
$\vec{F}_{net} = n - W = ma$
$n - 98N = 10kg \cdot 2\,m/s^2$
$n = 20N + 98N$
$n = 118\,N$

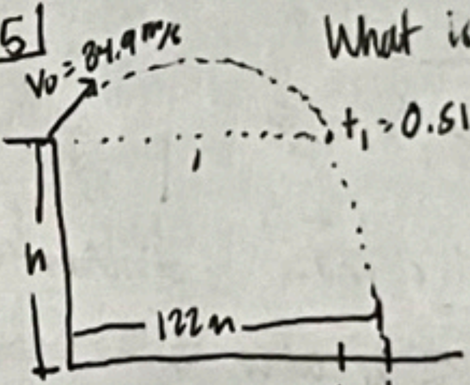
C. Elevator decelerates
$\vec{a} = -2\,m/s^2$ ↓
$W = -98\,N$
$\vec{F}_{net} = 10kg \cdot -2\,m/s^2 = -20\,N$
$n = ma + g$
$n = 10kg \cdot -2\,m/s^2 + 98\,N$
$n = 78\,N$

4) What will happen if you decrease θ but keep magnitude of pulling force the same?



- $\vec{F}_x$ increases
- acceleration of block will increase

5) What is h?



① solve for v_{0y}
$v = v_{0y} + a_y t_1$
$0 = v_{0y} - g(t_1/2)$
$v_{0y} = \frac{gt_1}{2} = \frac{-9.8\,m/s^2 \cdot 0.51s}{2}$
$\boxed{v_{0y} = 2.5\,m/s}$

② solve for v_{0x}
$v_0^2 = v_{0y}^2 + v_{0x}^2$
$v_{0x} = \sqrt{v_0^2 - v_{0y}^2} = \sqrt{(24.9\,m/s)^2 - (2.5\,m/s)^2}$
$\boxed{v_{0x} = 24.8\,m/s}$

③ solve for t_2
$122 = v_{0x} \cdot t_2$
$\frac{122s}{34.8\,m/s}$ $\boxed{t_2 = 3.5s}$

④ solve for h
$y = y_0 + v_{0y}t_2 + \frac{1}{2}a_y t^2$
$-h = 0 + 2.5\,m/s \cdot 3.5s + \frac{1}{2}(-9.8\,m/s^2)(3.5s)^2$
$\boxed{h = 51.3}$

6) elevator moves ↓ at constant speed (2 m/s) but decelerates to a stop along 4s. During when a scale reads 1960 N. What is mass of box?

$a = \frac{\Delta v}{\Delta t} = \frac{0 - (-2\,m/s)}{4s} = 0.5\,m/s^2$

$F_{net} = F_{scale} - W = ma$
$F_{scale} - mg = ma$
$F_{scale} = m(g + a)$
$m = \frac{F_{scale}}{g + a} = \frac{1960\,N}{9.8\,m/s^2 + 0.5\,m/s^2}$
$\boxed{m = 190\,kg}$

7) Tortoise & Hare start at same time to complete race of length D over time Δt.
Tortoise: constant V_T Hare: for ½Δt, acceleration A. Next ½Δt, constant velocity.

① solve for d_1 then A (to D_1)
$d_1 = \frac{1}{2}At^2 = \frac{1}{2}A(\frac{1}{2}\Delta t)^2 = \boxed{\frac{1}{8}A(\Delta t)^2 = d_1}$
$\boxed{A = \frac{8d_1}{\Delta t^2}}$

② solve for d_2 (to D_2)
$v = at = \frac{8d_1}{(\Delta t)^2} \cdot \left(\frac{\Delta t}{2}\right) = \boxed{\frac{4d_1}{\Delta t} = v}$
$d_2 = vt = \frac{4d_1}{\Delta t} \cdot \left(\frac{\Delta t}{2}\right) = \boxed{2d_1 = d_2}$

If race length doubles, how change mag of A?
$D = d_1 + d_2 = d_1 + 2d_1 = 3d_1$
$V_T = D/\Delta t$; $\Delta t = D/V_T$

recall
$A = \frac{8d_1}{\Delta t^2} = \frac{8(\frac{1}{3}D)}{(D/V_T)^2} = \frac{8V_T^2}{3D}$ so if D → 2D

acceleration halves.

Oscillations

$F_{spring} = -kx$

$U_{spring} = 1/2 kx^2$

$E_{tot\ spring} = 1/2 kx^2 + 1/2 mv^2$

$E_{tot\ pend} = 1/2 g x^2 + 1/2 \ell v^2$

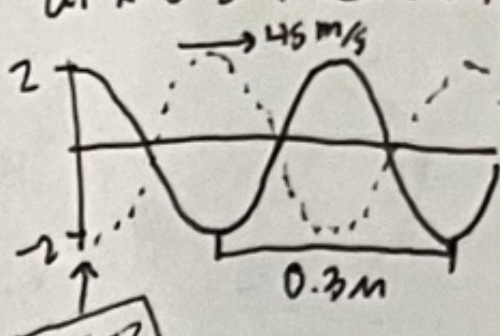
$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$ — ω: angular frequency; T: period; f: frequency

for a spring-mass system $\boxed{\omega = \sqrt{\frac{k}{m}}}$ $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$ $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

for a pendulum $\boxed{\omega = \sqrt{\frac{g}{\ell}}}$ $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}}$ $\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$

if $x = A$ at $t = 0$ $\boxed{x = A\cos(\omega t)}$ $\boxed{v = -A\omega\sin(\omega t)}$ $\boxed{a = -A\omega^2\cos(\omega t)}$

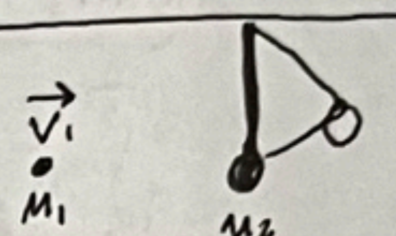
Figure is a snapshot at $t=0s$ of the displacement wave along a stretched string. What is displacement, y, at $x=0$ & $t = 10^{-2} s$?



$45 m/s \cdot 10^{-2} s = 0.45 m$ to the right, its 1.5 wavelengths.

$\boxed{y = -2}$ at $x = 0$, $t = 10^{-2} s$

What is the total work done by the string



as the pendulum ascends to its max vertical rise

- 0J. The tension is always perpendicular to the path of the pendulum so it does no work.

Waves

$\boxed{f = \frac{1}{T}}$

$\boxed{v_{wave} = f\lambda}$

$\boxed{v_{string} = \sqrt{\frac{F_{tension}}{\mu}}}$

$\boxed{k = \frac{2\pi}{\lambda}}$

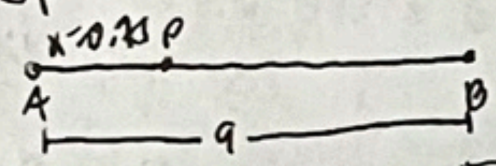
$\boxed{\omega = 2\pi f}$

$\boxed{y(x,t) = A\cos(kx - \omega t + \phi)}$ — A: amplitude

constructive interference
$\Delta p.l. = m\lambda$

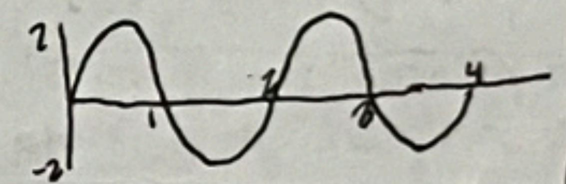
destructive interference
$\Delta p.l. = (n + 1/2)\lambda$ } $n = 0, 1, 2 \ldots$

$\Delta p.l.$ = path length $= D_2 - D_1$



$\Delta p.l. = 8.25 - 0.75 = \boxed{7.5m}$

$y(x=0, t=1) = 2cm\cos(\pi m^{-1})x - (\pi/2 s^{-1})t = 2cm\cos(0 - \pi/2) = 0$



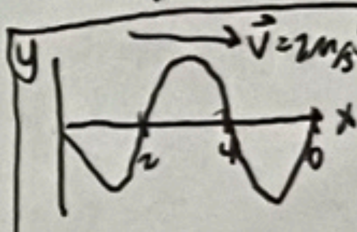
Standing waves

waves on a string $\lambda_n = \frac{2L}{n}$ $f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$ $n = 1, 2, 3 \ldots$

pipe open at both ends $\lambda_n = \frac{2L}{n}$ $f_n = \frac{n}{2L} \cdot v_{sound}$ $n = 1, 2, 3 \ldots$

pipe open at one end $\lambda_n = \frac{4L}{n}$ $f_n = \frac{n}{4L} \cdot v_{sound}$ $n = 1, 3, 5 \ldots$

$v = 2m/s$ snapshot of transverse wave. What is velocity direction at $x = 4$?



↑ up As the wave passes through you, you'd fly upwards.

Doppler Effect, Sound

$f_{listener} = \left|\frac{v_{sound} \pm v_{listener}}{v_{sound} \mp v_{source}}\right| \cdot f_{source}$

such that f↑ if listener & source are nearing each other; f↓ if going apart

rate of energy capture = power of sound wave $= \boxed{P = IA}$ — P: power (W); I: intensity of sound wave depends on distance from source (W/m^2); A: surface area of source (m^2)

4 nearby printers emitting 40 dB

$\boxed{\beta_4 = 10dB\log_{10}(10^{40/10} + 10^{40/10} + 10^{40/10} + 10^{40/10})} = 10\log_{10}(4 \cdot 10^4) = 46dB$

beat frequency

$\boxed{f_{beats} = |f_2 - f_1|}$

frequency of oscillation of the total wave

$\boxed{f = 1/2(f_1 + f_2)}$

Intensity

$\boxed{I = \frac{P_0}{4\pi r^2}}$ — I: intensity (W/m^2); P_0: power (W); r: distance (m)

$\boxed{I = 1/2 \rho v_{sound} \omega^2 A^2}$ — ρ: density of medium; v_{sound}: speed of sound waves in medium; ω: angular frequency of wave $= 2\pi$ resonance frequency; A: displacement amplitude

$\boxed{I = \frac{(p_{max})^2}{2\rho v}}$ — p_{max}: pressure (Pa); ρ: density of medium; v: velocity of sound in medium

- objects in the same fluid at the same distance from the top surface will have the same pressure.

$\boxed{\beta = (10dB)\log_{10}\left(\frac{I}{I_0}\right)}$ Each 10 dB added is another 10x in intensity.

$\boxed{I_0 = 10^{-12} W/m^2}$

$\boxed{\beta_{rel} = 10dB\log_{10}\left(\frac{I_2}{I_1}\right)}$ — I_2: more distant listener; I_1: relative to less distant listener

$\boxed{\rho_{water} = 1000 kg/m^3}$

$P_{atm} = 101{,}000 Pa = 760mmHg$

Fluid Statics

$\boxed{P = \frac{F}{A}}$ — P: pressure (N/m^2) (Pa); F: force (N); A: area (m^2)

$\boxed{\vec{F}_\perp = \frac{\Delta\vec{p}}{\Delta t}}$ — $\vec{p}$: momentum

$\boxed{F_B = \rho_{fluid} V_{displaced}\, g}$

fraction submerged $= \frac{V_{submerged}}{V_{object}} = \frac{V_{fluid\ displaced}}{V_{object}} = \frac{\text{density object}}{\text{density fluid}}$

$\boxed{\rho = \frac{m}{V}}$ — ρ: density (kg/m^3) (g/cm^3); m: mass; V: volume

$\boxed{p = p_0 + \rho dg}$ — p: pressure at 2nd, lower point in fluid; p_0: pressure at known, higher pt in fluid

$\boxed{F_{tot} = W - F_B}$

$F_{tot} > 0$, sinks

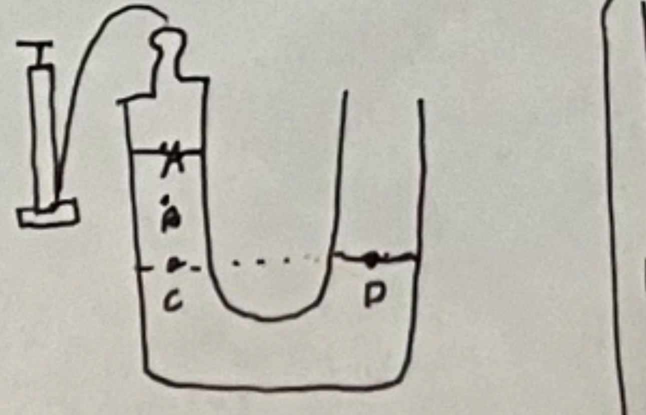
$F_{tot} < 0$, floats

$\boxed{P_{gauge} = P_{inside} - P_{outside}}$

$P_{below} = P_{above} + \rho dg$

$\boxed{P_{gauge} = P_{tot} - P_0 = \rho dg}$

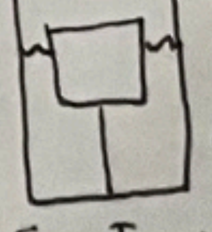
$P_{gauge} = \rho dg$



$P_A < P_B < P_C = P_D$

- pressure increases w depth
- points at same depth have same pressure in static fluid.

What is the tension in the cord? $V_{block} = 1m^3$, 80% submerged, $\rho_{block} = 650 kg/m^3$ $\rho_w = 1000 kg/m^3$



$F_B = T + W$ $m_b = \rho_b \cdot V_b$

$T = F_B - W$

$T = \rho_w \cdot 0.8V_b \cdot g - m_b \cdot g$

$T = \rho_w \cdot 0.8V_b \cdot g - \rho_b \cdot V_b g$

$T = 1000kg/m^3 \cdot 0.8 \cdot 1m^3 \cdot 9.8m/s^2 - 650kg/m^3 \cdot 1m^3 \cdot 9.8m/s^2$

$\boxed{T = 1470N}$

Fluid dynamics

Bernalli's equation: no viscosity + **Equation of continuity**

$$\boxed{P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \underbrace{\tfrac{1}{2}\rho v_2^2}_{\text{kinetic energy}} + \underbrace{\rho g y_2}_{\text{gravitational PE}}}$$

~~Bernoulli~~ assumptions:
1. incompressible, laminar, steady
2. inviscid. No friction within fluid or w pipe.

Equation of continuity

$$\boxed{A_1 V_1 = A_2 V_2 = Q}$$

A_1: cross-sectional area (m^2); V_1: flow speed (m/s); Q: flow rate volume/s (m^3/s)

- if fluid is at rest, $V_1 = V_2 = 0$

$$\boxed{P_1 = P_2 + \rho g (y_2 - y_0)}$$

pressure gradient $\rightarrow \Delta P = Q R \leftarrow$ resistance

$$\boxed{\Delta P = Q\left(\frac{8\eta l}{\pi r^4}\right)}$$

Q: flow; l: length of tube; r: radius of tube

$$\boxed{Q = A \cdot v}$$

Q: flow (m^3/s); A: area (m^2); v: velocity (m/s)

diffusion

$$\boxed{X_{RMS} = \sqrt{2D\Delta t}}$$

D: diffusion coefficient

$$\boxed{\text{diffusion rate} = \frac{\Delta M}{\Delta t} = DA\left(\frac{\Delta C}{L}\right)}$$

M: mass; A: ~~cross-sectional~~ area; $\frac{\Delta C}{L}$: concentration gradient; L: distance (m)

vertical free fall motion

$$\boxed{V_f^2 = V_i^2 + 2g\Delta y}$$

$$\boxed{\eta = \frac{Fd}{Av}}$$

η: viscosity; F: force (N); d: spacing b/w plates (m); A: area of each plate (m^2); v: speed of one plate relative to other (m/s)

small object moving through a fluid

$$\boxed{Re_1 = \frac{\rho l v}{\eta}}$$

ρ: (kg/m³); l: length of object (m); η: viscosity ($\frac{Ns}{m^2}$); v: velocity (m/s)

$$\boxed{\eta = \frac{Fd}{Av}}$$

η: viscosity (Ns/m^2) (Pa·s); F: force (N); d: distance b/w plates (m); A: area of plates; v: velocity (m/s)

fluid moving through tube

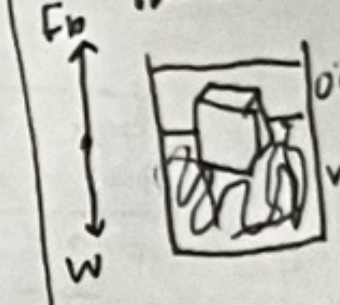
$$\boxed{Re_2 = \frac{\rho D v}{\eta}}$$

ρ: (kg/m³); D: diameter of tube (m); v: velocity (m/s); η: viscosity (Ns/m^2)

Re: 0 — very viscous, laminar — 2100 — transition — 4000 — not viscous, turbulent

"viscosity is more important"

Wooden box $\rho = 700\,kg/m^3$, $V = 1m^3$ floating tank of water $\rho = 1000\,kg/m^3$ and oil $\rho = 600\,kg/m^3$. How much of the box is below the surface of the water?



$\rho V g = 700\,kg/m^3 \cdot 1m^3 \cdot 9.8\,m/s^2 = 6860\,N$

$6860\,N = \rho_w V_{w\,disp}\, g + \rho_o V_{o\,disp}\, g$

$6860\,N = 1000\,kg/m^3 \cdot x \cdot 10\,m/s^2 + 600\,kg/m^3 \cdot (1-x) \cdot 10\,m/s^2$

$6860\,N = 9800x + 5880 - 5880x$

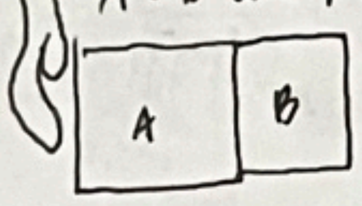
$0.25 = x$

25% below surface of water.

$\beta_{rel} = 10\,dB \log_{10}\left(\frac{I/4^2}{I}\right) = -12\,dB$.

The distance b/w a point source and a detector ↑ by a factor of 4. By how many decibels will the sound intensity level be decreased?

A & B are pushed on a frictionless surface. $F_{hand} = 20N$, $M_B = 3kg$, $F_{B,A} = 12N$. What is mass of block A?



$|F_{A,B}| = |F_{B,A}| = 12N$

$F_{net,A} = F_{hand} - F_{B,A} = 20N - 12N = 8N$

$F_{net,B} = F_{A,B} = M_B a$

$a = \frac{F_{A,B}}{m_B} = \frac{12N}{3kg} = 4\,m/s^2$

$F_{net,A} = m_A \cdot a$

$\boxed{\frac{8N}{4\,m/s^2} = a} = 2kg$

$f_1 = \frac{1}{2L}\sqrt{\frac{F_T}{\mu}} = \frac{1}{2L}\sqrt{\frac{mg}{\mu}}$

$\frac{f^E}{f^B} = \frac{\frac{1}{2L}\sqrt{\frac{m_E g}{\mu}}}{\frac{1}{2L}\sqrt{\frac{m_B g}{\mu}}} = \frac{\sqrt{M_E}}{\sqrt{M_B}}$

$\left(\frac{f_E}{f_B}\right)^2 = \frac{M_E}{M_B}$

$\left(\frac{330\,Hz}{247\,Hz}\right)^2 \cdot 1.1kg = M_E = 1.96\,kg$

↑ You suspend a 1.1 kg mass from a string to get a harmonic ($n=1$) $f = 247$ Hz, an open B string on a guitar. In order to tune the first harmonic to an E (330 Hz), what mass should you suspend from the string?

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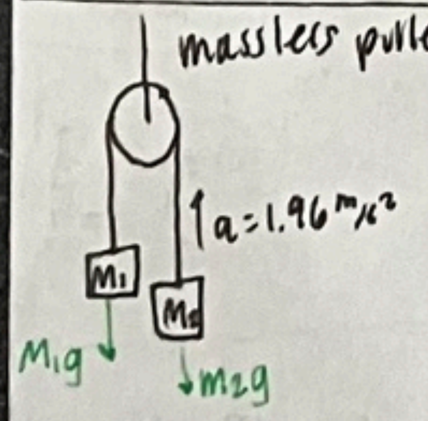
massless pulley, no friction



$F_{net,1} = M_1 a = M_1 g - T$; $T = M_1(g - a)$

$F_{net,2} = M_2 a = T - M_2 g$; $T = M_2(a + g)$

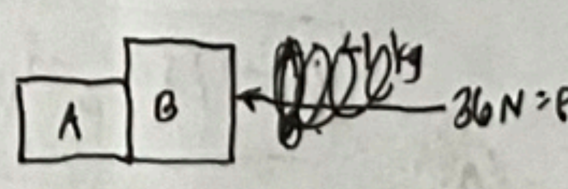
$m_1(g - a) = m_2(g + a)$

$m_1(9.8\,m/s^2 - 1.96\,m/s^2) = M_2(9.8\,m/s^2 + 1.96\,m/s^2)$

$M_1 \cdot 7.84 = M_2 \cdot 11.76$

$\frac{M_1}{M_2} = \frac{11.76}{7.84} = 1.5$ "M_1 is 1.5x M_2"

What is the magnitude of the ~~force~~ normal force of A on B? Frictionless surface.



a: 4 kg
b: 20 kg

$\Sigma F_x^{ext} = Ma_x = -36N$

$a_x = \frac{-36N}{(20kg + 4kg)} = -1.5\,m/s^2$

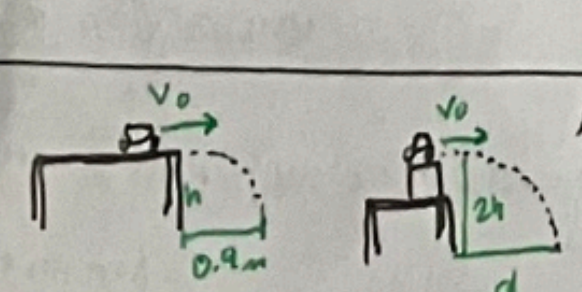
$\Sigma F_b = M_b a_x = -F + N_{a,b}$

$20kg \cdot -1.5\,m/s^2 = -36N + N_{a,b}$

$-30N + 36N = N_{a,b}$

$\boxed{6N = N_{a,b}}$

At what distance d from edge of table does the car land when h is doubled?



$x = x_0 + V_{0x} t$

$\Delta x = V_{0x} t$

$0.9m = V_{0x} t$

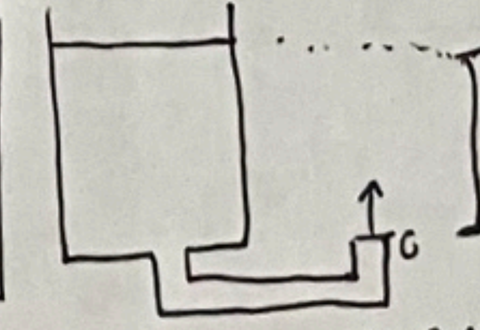
$y = y_0 + \cancel{V_{y0} t} + \frac{1}{2}at^2$

$\Delta y = \frac{1}{2}gt^2 = h$

$t = \sqrt{\frac{2h}{g}}$ doubling h increases t by $\sqrt{2}$.

$d = V_0 t\sqrt{2} = 0.9\sqrt{2} = \boxed{1.27\,m}$

What is the volumetric flow rate of fluid at point C?



$A = 0.01m^2$
$\rho = 1000\,kg/m^3$

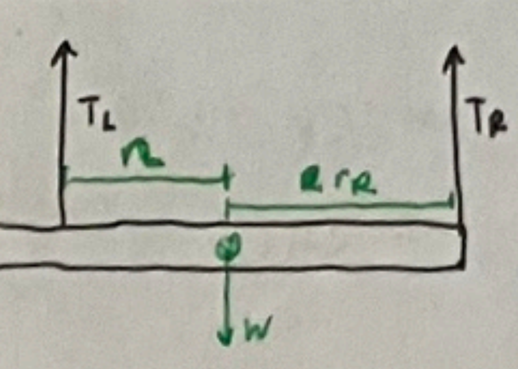
$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h = P_2 + \frac{1}{2}\rho V_2^2 + \rho g H$

$P_{atm} + 0 + \rho g h = P_{atm} + \frac{1}{2}\rho V_2^2 + 0$

$\sqrt{2gh} = V_2 = 4.43\,m/s$

$Q = Av = 0.01m^2 \cdot 4.43\,m/s = \boxed{0.0443\,m^3/s}$

What is the correct ranking of tensions T_L & T_R on the strings and the rod's weight W?



$T_L + T_R = W$, W is greatest.

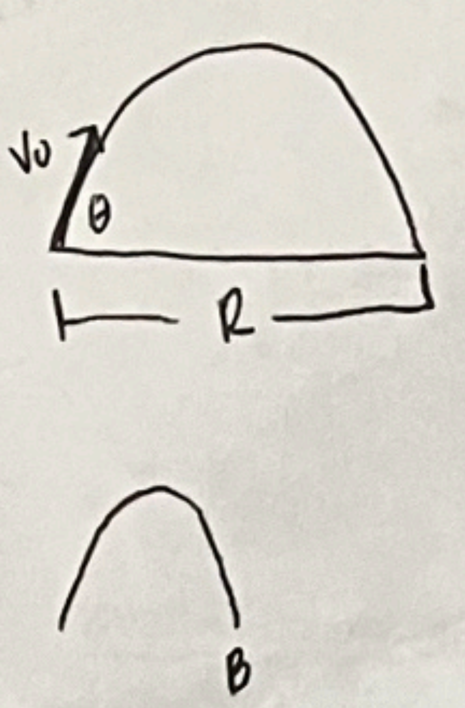
$|T_L| = |T_R|$

$r_L T_L = r_R T_R$

since $r_L < r_R$, $T_L > T_R$.

Range equation: use to solve for Δx when there isn't time given & you know θ, v_0

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$



UNIT 2

Action-reaction - Newton's 3rd Law

$$\vec{F}_{A,B} = -\vec{F}_{B,A}$$

Work & KE

$W = Fd\cos\theta$

$W_{net} = \Delta K = K_f - K_i$

Forms of energy

$K_{trans} = \frac{1}{2}mv^2$

$K_{rot} = \frac{1}{2}I\omega^2$ Not a conserved quantity

$U_{grav} = mgy$

$U_{spring} = \frac{1}{2}kx^2$

moment of inertia how difficult it is to create an angular acceleration of the object about axis of rotation

$I = \frac{L}{\omega}$ ← angular momentum / ← angular velocity

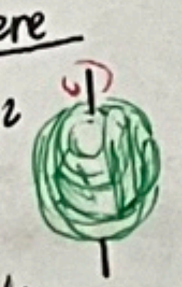
$L = I\omega = rp_\perp = rp\sin\theta$

particle
$I = MR^2$

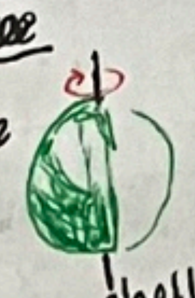
$I_{zi} = I_{zf}$

$I_{tot} = \Sigma MR^2$

solid sphere
$I = \frac{2}{5}MR^2$

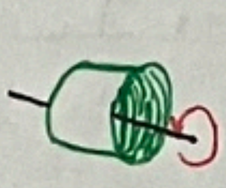


thin spherical shell / hollow ~~sphere~~
$I = \frac{2}{3}MR^2$

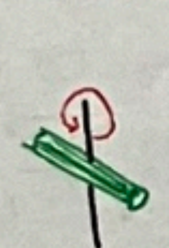


solid cylinder
$I = \frac{1}{2}MR^2$

thin hollow cylindrical ~~sphere~~ shell
$I = MR^2$



thin rod
$I = \frac{1}{12}ML^2$



thin rod
$I = \frac{1}{3}ML^2$

- generally, the further the mass is distributed from axis of rotation, the greater the inertia.
  - if most mass is near outside, moment of inertia will be larger. More energy will be required to rotate at a given speed. More K_{rot} means less K_{trans}.

rotational speed about axis through center of mass ω

$\omega = \frac{5 \text{ revolutions}}{\text{min}} \cdot \frac{1 \text{ min}}{60 s} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 0.524 \text{ rad/s}$

$\omega = \frac{\Delta\theta}{\Delta t}$

rolling without slipping

$v_{cm} = R\omega$ ← rotational speed
↑ translational speed, ↑ radius

torque

$\tau = rF\sin\theta = r_\perp F$

$\Sigma\tau_z = I\alpha_z$
↑ moment of inertia, ↑ angular acceleration

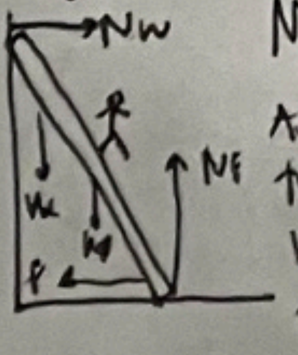
Energy

$\underbrace{U_i + K_i}_{E_i} + W_{ext} = \underbrace{U_f + K_f}_{E_f}$ ← friction is negative

$\Delta E = \Delta U + \Delta K = W_{ext}$ where $W_{ext} = f_k \cdot D$

$\Delta E = \Delta U + \Delta K = 0$ if $W_{ext} = 0$

Ladder, wall, person



$N_W = F$, $W_L + W_P = N_f$
As person moves up ladder the magnitude of the N_f ↑, bc torque from W_P ↑. And they must offset each other

Friction & centripetal

$0 < f_{static} \le \mu_s n = f_{s,max}$

$f_{kinetic} = \mu_k n$

$F_{centrip} = \frac{mv^2}{r} = ma_{centrip}$

Momentum, vector quantity

$\vec{p} = m\vec{v}$ $\qquad \Delta\vec{p} = \vec{p}_f - \vec{p}_i$

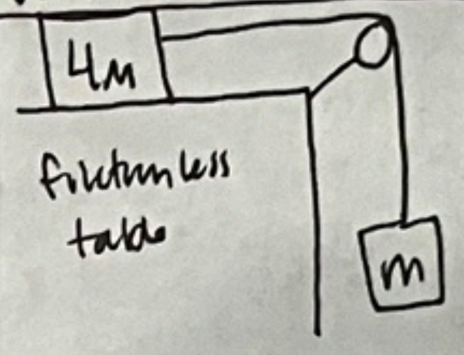
$\underbrace{\Delta p = 0 \text{ or } \vec{p}_f = \vec{p}_i}_{\text{momentum is conserved}}$ if no net external force acting on system

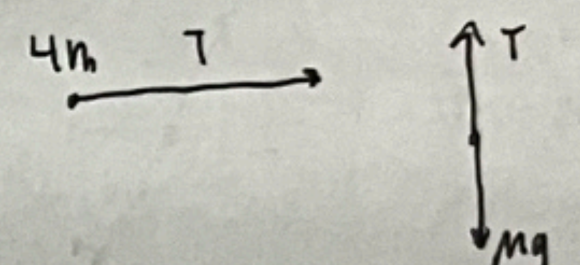
$\Delta p = \vec{F}_{avg}\Delta t$
↑ impulse = change in momentum

For an object in equilibrium:

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma\tau = 0$

Ex **What is acceleration of mass m?**



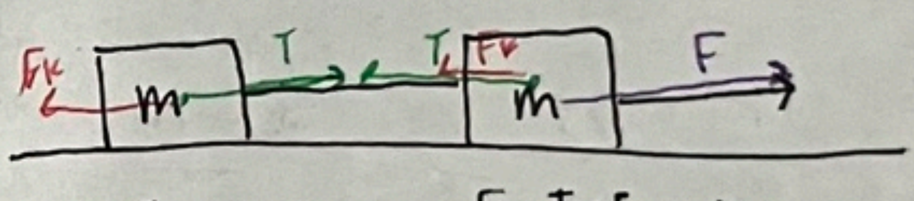


$T = (4m)a$

$F_{net} = mg - T = ma$
$= mg - (4m)a = ma$

$\boxed{g/5 = a}$

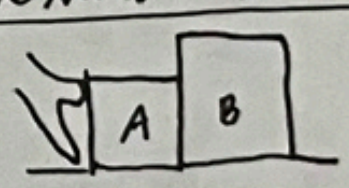
What is tension in string connecting the masses?



$T - F_k = ma \qquad F - T - F_k = ma$

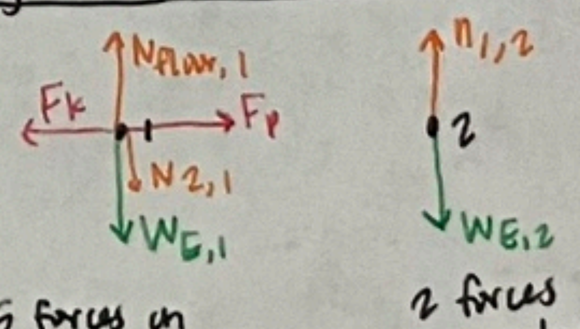
$T - F_k = F - T - F_k$
$2T = F$
$\boxed{T = F/2}$

Block B has twice the mass of Block A. Net force on Block A is 24N. What is mag of $F_{B,A}$?



$M_B = 2M_A$
$F_{net B} = 2F_{net A}$
$F_{net B} = 2(24N)$
$\boxed{F_{net B} = 48N}$

Person pushing on bottom crate



- force of Fs btw crate 1 & 2 is zero.
- Fk on Crate 1 = Fp on crate 1

Hockey rink is 61m long. μ_k: 0.05 M_{puck}: 0.15kg. What v_i is necessary for the puck to just barely travel the full length of rink?

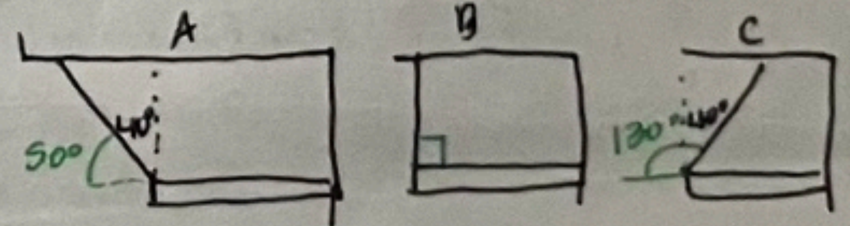
$W = \Delta K = 0 - \frac{1}{2}mv_i^2$

$W_{friction} = F_k \cdot d = -\mu_k nd = -\mu_k mgd$

$+\mu_k mgd = +\frac{1}{2}mv_i^2$

$\sqrt{2\mu_k gd} = v_i = \sqrt{2 \cdot 0.05 \cdot 9.8 m/s^2 \cdot 61 m/s} = 7.7 m/s$

Correctly Rank T of wire



$A = C > B \qquad T_{wire} = LT\sin\theta$

Rank magnitude of vertical component of hinge

$A = B = C$

$F_{hinge,y} = Mg - T\sin\theta$

Heat & Phase change

$\boxed{Q = mc\Delta T}$ ← $(T_H - T_0)$ ... cork

heat (J), mass (kg), specific heat (J/kg·K)

$Q > 0$: heat flows into object

$Q < 0$: heat flows out of object

$Q = 0$: no net flow of heat

1 calorie: 4.186 J

1000 calorie = 1 Calorie (food)

1 BTU = 1055 J

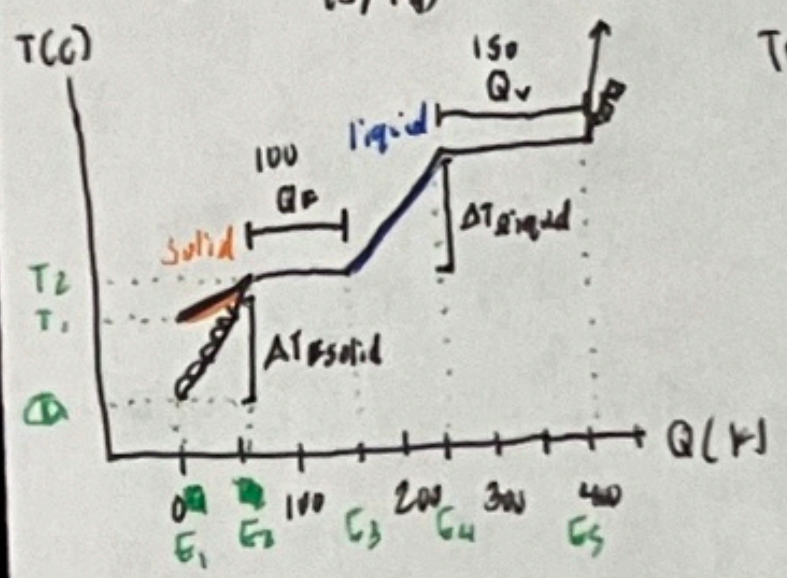
$\boxed{Q = \pm mL}$

mass (kg), latent heat (J/kg)

L_F: latent heat of ~~vaporization~~ fusion

L_V: latent heat of vaporization

usually $L_F < L_V$



Total $Q = mc\Delta T + mL$

$c = \frac{Q}{m\Delta T}$

$c = \frac{E_2 - E_1}{m(T_2 - T_1)}$

$L_V > L_F$ since $(E_5 - E_4) > (E_3 - E_2)$

$c_F > c_V$ since ℓ slope > s slope

& $\frac{\Delta T}{\Delta t} \sim \frac{1}{c}$

radiation of object

$\boxed{P = \sigma \varepsilon A T^4}$

rate of energy radiated (J/s); Stefan Boltzmann constant $5.67 \cdot 10^{-8}\ W/m^2 \cdot K^4$; emissivity (0, 1) 0: bad radiator 1: great radiator; SA object (m^2); Temp (K)

net radiation

$\boxed{P = \frac{Q}{\Delta t} = \sigma \varepsilon A (T_E^4 - T_0^4)}$

Temp of environment (K); temp of object (K)

conduction

$\boxed{H = \frac{Q}{\Delta t} = \frac{kA}{L}(T_{Hot} - T_{cold})}$

heat transfer of conduction (J/s) (W); thermal conductivity of material (J/s·m·C); cross sectional SA (m^2); thickness through which heat flows (m); ΔT (K) hot/environment cold/object

Thermal Expansion

linear $\boxed{\Delta L = \alpha L \Delta T}$ (K)

coeff of linear expansion (K^{-1}); original length (m)

volumetric $\boxed{\Delta V = \beta V_0 \Delta T}$

What flows out of object A = what flows into object B.

$|Q_A| = |Q_B|$

$|m_A c_A \Delta T_A| = |m_B c_B \Delta T_B|$

$m_A c_A \Delta T_A = -m_B c_B \Delta T_B$

$m_A c_A \Delta T_A + m_B c_B \Delta T_B = 0$

$\boxed{K_{trans,avg} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT}$

$\boxed{v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}}$ $k = 1.38 \cdot 10^{-23}\ J/K$

single molecule mass (kg); molar mass (kg/mol)

$\boxed{PV = NkT = nRT}$

atoms; # mols; 8.314 J/mol·K

ideal gas law

$\boxed{PV = nRT = NkT}$

Thermodynamic Processes & The First Law

$\boxed{\Delta U = Q - W_{by}} = Q + W_{on}$ & $\boxed{W = P(V_f - V_i)}$ isobaric only

heat absorbed by system (J); work done by system (J) (N·m); work (J); pressure (Pa); volume (m^3)

1 atm = 101.3 kPa

monatomic

$\boxed{U = \frac{3}{2} Nk T \text{ or } U = \frac{3}{2} nRT}$

atoms; # mols

$\boxed{C_V = \frac{3}{2} R}$ $\boxed{\gamma = 5/3}$ $\boxed{C_p = \frac{5}{2} R}$

$\Delta U = \frac{3}{2}(P_f V_f - P_i V_i)$

Isochoric $\Delta V = 0$, $W_{by} = 0$ $\boxed{Q = nC_V \Delta T}$ $\boxed{\Delta U = Q}$

Isobaric $\Delta P = 0$, $W_{by} = P\Delta V$ $\boxed{W = P(V_f - V_i)}$ $\boxed{Q = nC_p \Delta T}$ $\boxed{\Delta U = Q - W_{by}}$ $\boxed{\Delta U = \frac{C_V P \Delta V}{R}}$

Isothermal $\Delta T = 0$, $W_{by} = Q$ $\boxed{W_{by} = nRT \ln\left(\frac{V_f}{V_i}\right)}$ $\boxed{\Delta U > 0}$

Adiabatic $Q = 0$ $\boxed{PV^{5/3} = \text{constant}}$ $\boxed{PV^{\gamma} = \text{constant}}$ $\boxed{W_{by} = P\Delta V}$ $\boxed{Q = nC_p \Delta T}$ $\boxed{\Delta U = -W_{by}}$

diatomic

$\boxed{U = \frac{5}{2} NkT = \frac{5}{2} nRT}$

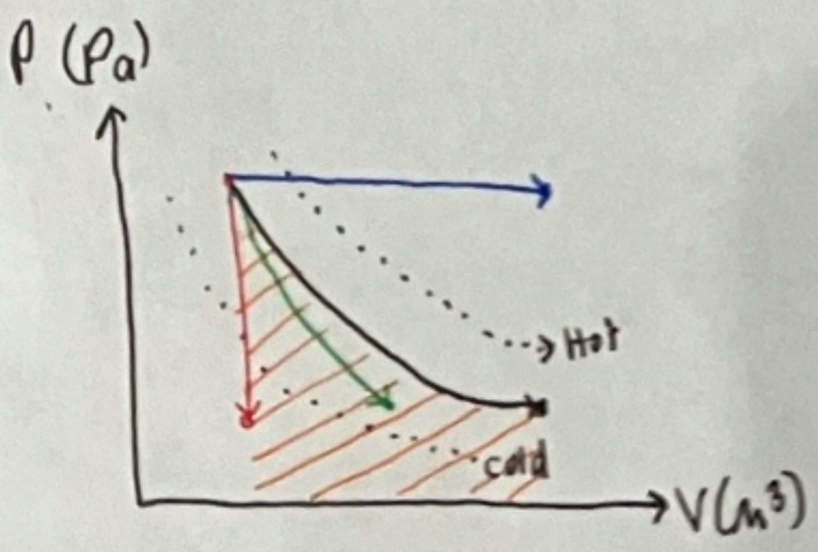
$\boxed{C_V = \frac{5}{2} R}$ $\boxed{\gamma = 7/5}$

$C_p = \frac{7}{2} R$

general

$\boxed{C_p = C_V + R}$

$\boxed{\gamma = \frac{C_p}{C_V}}$



W = area under curve

- if V decreases, W on gas is negative

- if a point goes from a hot to cold isothermal line, ΔU is negative. cold → hot, ΔU is positive.

Heat Engines

actual efficiency

$\boxed{e = \frac{W}{Q_H} = \frac{Q_H - |Q_C|}{Q_H} = 1 - \frac{|Q_C|}{Q_H}}$

max possible efficiency

$\boxed{e_{carnot} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}}$ $= \frac{W}{Q_H}$

If $e > e_C$, the heat engine is not possible.

$\boxed{COP_{ref} = \frac{Q_C}{W}} = \frac{\text{heat from cold}}{\text{work}} = \frac{\text{what we get out}}{\text{what we put in}}$

fridge { counterclockwise

$Q_H = Q_C + W$

$\boxed{COP = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}}$

heat ~~pump~~ engine clockwise

$\boxed{COP = \frac{Q_H}{W}}$