Calculus Diagnostic Quiz

Calculator Free:

Q1) The derivative of $y = \frac{1}{\sqrt{f(x)}}$ with respect to x is:

A. $\frac{-1}{2\sqrt{(f(x))^3}}$
B. $\frac{2f(x)}{f'(x)}$
C. $-\frac{f'(x)}{2\sqrt[3]{(f(x))^2}}$
D. $-\frac{1}{2\sqrt[3]{(f(x))^2}}$
E. $-\frac{f'(x)}{2\sqrt{(f(x))^3}}$

$y = (f(x))^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}(f(x))^{-\frac{3}{2}} \times f'(x)$ (chain rule)

$\frac{dy}{dx} = \frac{-f'(x)}{2\sqrt{(f(x))^3}}$

Q2) Let $f: [3, \infty) \to R, f(x) = 2x^2 - 12x + 21$ and $f(5) = 11$. The function g is the inverse function of f. $g'(11)$ is equal to:

A. 5
B. $\frac{1}{8}$
C. 32
D. 8
E. $\frac{1}{32}$

$f'(x) = 4x - 12$

$f'(5) = 8$

The point (5,11) on f corresponds to the point (11,5) on g. By symmetry, the gradient of f at (5,11) must be the **reciprocal** of the gradient of g at (11,5).

Therefore, $g'(11) = \frac{1}{8}$

Q3) Which of the following is an antiderivative of $cos(2x) + \frac{1}{3x}$?

A. $\sin(2x) - \frac{1}{3x^2}$
B. $\frac{1}{2}\sin(2x) + \frac{1}{3}log_e(5x)$
C. $-2\sin(2x) + \frac{1}{3}log_e(x)$
D. $-\frac{1}{2}\sin(2x) + \frac{1}{3}log_e(x)$
E. $-\sin(2x) - \frac{1}{3x^2}$

$\int cos(2x) + \frac{1}{3x}\,dx = \frac{1}{2}sin(2x) + \frac{1}{3}log_e(x) + c$

Option B is the only possible antiderivative, where $c = \frac{1}{3}log_e(5)$.

Q4) A curve has the gradient $\frac{dy}{dx} = cos(3kx) + 1$ where k is a constant and passes through the stationary point $(\frac{\pi}{2}, 3)$. A possible equation of the curve is:

A. $y = -\frac{1}{2}\sin(2x) + x + 3 - \frac{\pi}{2}$
B. $y = \frac{1}{2}\sin(2x) + x + 3 - \frac{\pi}{2}$
C. $y = -\frac{1}{2}\sin(2x) + x + 3 + \frac{\pi}{2}$
D. $y = \sin(x) + x + 3 - \frac{\pi}{2}$
E. $y = 2\sin(2x) + x + 3 - \frac{\pi}{2}$

Sub $x = \frac{\pi}{2}, \frac{dy}{dx} = 0$ into derivative:

$0 = \cos\left(3k\left(\frac{\pi}{2}\right)\right) + 1$
$\cos\left(\frac{3k\pi}{2}\right) = -1$
$\frac{3k\pi}{2} = \pi$
$k = \frac{2}{3}$

Sub into derivative and antidifferentiate:

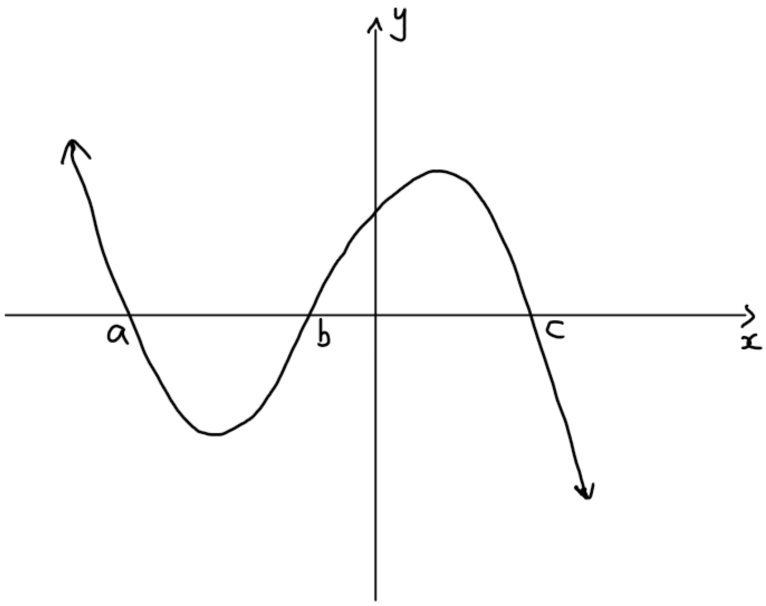
$y = \int \cos(2x) + 1\,dx = \frac{1}{2}\sin(2x) + x + c$

Sub $x = \frac{\pi}{2}, y = 3$:

$3 = \frac{1}{2}\sin(\pi) + \frac{\pi}{2} + c$
$c = 3 - \frac{\pi}{2}$

$y = \frac{1}{2}\sin(2x) + x + 3 - \frac{\pi}{2}$

Q5) Part of the graph of the function f is shown below:



The total area bounded by the curve $y = f(x)$ and the x-axis on the interval $[a, c]$ is given by:

A. $\int_a^c f(x)\,dx$
B. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$
C. $-\int_a^0 f(x)\,dx + \int_0^c f(x)\,dx$
D. $\int_b^a f(x)\,dx + \int_b^c f(x)\,dx$
E. $-\int_a^b f(x)\,dx + \int_c^b f(x)\,dx$

Areas under the graph have a negative signed area (need to be multiplied by -1 to make them positive), while areas above the x-axis have a positive signed area.

$Area = -\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

When there is a negative sign in front of an integral, the terminals can be switched:

$Area = \int_b^a f(x)\,dx + \int_b^c f(x)\,dx$

Calculator active:

Q6) Which of the following functions satisfies $f'(0) = 1$?

A. $f(x) = e^x + x$
B. $f(x) = \cos(x)$
C. $f(x) = log_e(x+1)$
D. $f(x) = (\tan(x))^2$
E. $f(x) = 1$

A: $f'(x) = e^x + 1, f'(0) = 2$

B: $f'(x) = -\sin(x), f'(0) = 0$

C: $f'(x) = \frac{1}{x+1}, f'(0) = 1$ (correct answer)

D: $f'(x) = 2\tan(x)sec^2(x), f'(0) = 0$

E: $f'(x) = 0, f'(0) = 0$

Q7) The function $f(x) = ax + be^x$ has a turning point at (2,3). The value of b is:

A. 3
B. $-\frac{3}{e^2}$
C. $\frac{3}{e}$
D. $log_e(3)$
E. $\frac{1}{3}$

Solve $f(2) = 3$ and $f'(2) = 0$ simultaneously on CAS for a and b:

$a = 3, b = -\frac{3}{e^2}$

Q8) Which of the following is tangent to the line $y = 2x + 5$ at $x = 0$?

A. $y = 2e^x + 5$
B. $y = 2log_e(x) + 7$
C. $y = 2\sin(x) + 5$
D. $y = 2\cos(x) + 3$

Use tanline/Tangent line on CAS for each curve to find tangent equation at $x = 0$:

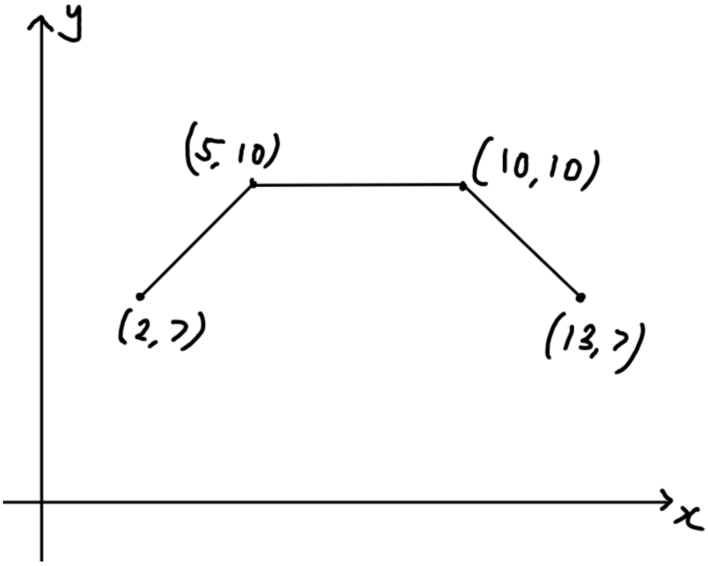
A: $y = 2x + 7$

B: undefined (no tangent at $x = 0$)

C: $y = 2x + 5$ (correct answer)

D: $y = 7$

Q9) The graph of h is shown.



The average value of h is closest to

A. 7.2
B. 7.5
C. 8.2
D. 8.5
E. 9.2

Average value is the area under the curve divided by the difference between the final and initial x-values:

$$Average\ value = \frac{\frac{1}{2}(11+5)\times 3 + 11\times 7}{13-2}$$

$$Average\ value = \frac{101}{11} \approx 9.2$$

Q10) How many real numbers $a \in [0, 5\pi]$ satisfy $\int_1^a sin(x)dx = 0$?

A. 2
B. 3
C. 4
D. 5
E. 6

Solve $\int_1^a sin(x)dx = 0$ on CAS for $a \in [0,5\pi]$:

$a = 1, 5.28, 11.57, 7.28, 13.57$