2026

Mathematical Methods

Question and Answer Book

SAC 1b (Tech-active)

- Reading time is **10 minutes**
- Writing time is **80 minutes**

Approved materials

- One scientific calculator and one approved CAS calculator
- Bound reference
- Writing equipment
- The VCAA formula sheet

Materials supplied

- Question and Answer Book of 15 pages

Instructions

Students are not permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

SECTION A – Multiple-choice questions

Instructions for Section A
Answer **all** questions by circling the correct answer or clearly indicating it
Choose the response that is **correct** or that **best answers** the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

What are the amplitude and period of $f(x) = -2\sin(6\pi x + 3)$?

A. Amplitude -2, period $1/3$

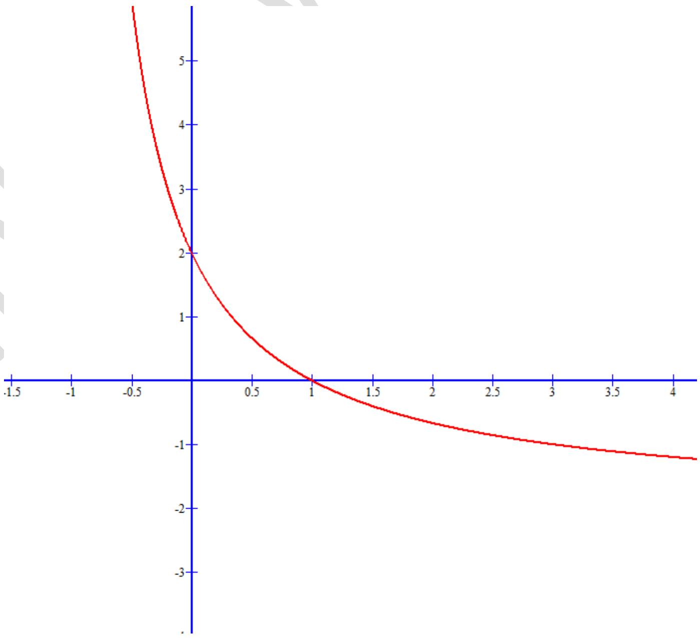
B. Amplitude 2, period $1/3$

C. Amplitude -2, period $1/6$

D. Amplitude -2, period $1/6$

Question 2

The graph below is for $y = \frac{A}{x+1} + B$. The values of A and B are:



A. $A = 2, B = -1$

B. $A = 2, B = 0$

C. $A = 4, B = -2$

D. $A = 4, B = 2$

Question 3

Consider the function $f: [0,4\pi] \setminus \{\pi, 3\pi\} \to R$, $f(x) = \tan^2(x/2)$. Which of the following statements is false?

A. The graph of $y = f(x)$ has an asymptote at $x = \pi$

B. There is no inverse function for f

C. The function f is strictly increasing on $[0, \pi) \cup [2\pi, 3\pi)$

D. f is periodic

Question 4

The minimum value of the function $f: [0,2] \to R$, $f(x) = 2x^3 - 6x^2 + 5x$ occurs at

A. $x = 0$

B. $x = 1$

C. $x = 1 - \sqrt{6}/6$

D. $x = 1 + \sqrt{6}/6$

Question 5

Functions $f: R \to R$ and $g: R \to R$ are given such that $f(0) = 2, g(0) = 2, f'(0) = 6, g'(0) = 4$. The derivative of $f(x)/g(x)$, evaluated at $x = 0$, is

A. 0

B. 1

C. 2

D. 3

Question 6

Consider the following system of equations, where $k \in R$ is a parameter.

$$kx + y = 3$$

$$6x + (k+1)y = 9$$

The values of k for which this system has infinitely many solutions are

A. $k = 3$

B. $k \in R \setminus \{-3, 2\}$

C. $k = 2$

D. $k \in \{-3, 2\}$

Question 7

For how many real numbers a in the interval $[-1,1]$ does the equation $(2\sin(x) - 1)(\cos(x) - a)$ have exactly two solutions in the interval $0 \leq x \leq \pi$?

A. 1

B. 2

C. 3

D. 4

Question 8

A function f has domain $[-1,5]$. After applying a dilation of factor $a > 0$ from the y-axis and a translation by $b > 0$ units in the positive direction of the x-axis to the graph of $y = f(x)$, the result has a domain of $[-1,8]$. The value of b is

A. 0

B. 3

C. $3/2$

D. $1/2$

Question 9

To two decimal places, the positive real number a for which the graphs of $y = \sqrt[3]{x + a}$ and $y = x^3 - a$ are tangent is

A. 0.37

B. 0.38

C. 0.39

D. 0.40

Question 10

Let $r < 0$ be a real number. The value(s) of c for which the function $f: (1, \infty) \to R, f(x) = \log_e(x^r + c)$ is well-defined are

A. $c \in [0, \infty)$

B. $c \in [1, \infty)$

C. $c \in R$

D. $c \in (-\infty, -1]$

END OF SECTION A

TURN OVER

SECTION B

Instructions for Section B
Answer **all** questions in the spaces provided.
Write your responses in English.
In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

An alien spy has come to Earth, and just so happened to land in Australia. It needs to learn English in order to blend in; unfortunately, the only word it bothered to learn before coming to Earth was “hello”.

The number of English words it knows d days after arriving in Australia can be modelled as

$$W(d) = 1 + cd^n,$$

where c and n are parameters that depend on how good the alien is at learning words.

a. By day 4, the only new word the alien has learnt is “goodbye”. But by day 20 the alien knows exactly one word for each letter of the alphabet! (That is, 26 words.) Show that $n = 2$ and $c = 1/16$. (2 marks)

b. Online estimates suggest that a vocabulary of 8000 words is sufficient to read the newspaper, a key skill for any spy who wants to understand Earth-dwelling society. To the nearest day, how long will this take the alien? (1 mark)

c. On day X, the alien's computer sends a congratulatory notification, remarking that the alien has managed to learn an average of two words per day since landing in Australia. What is the value of X? (1 mark)

d. At the one-year mark (i.e. after 365 days), how many words per day would the alien be learning? (1 mark)

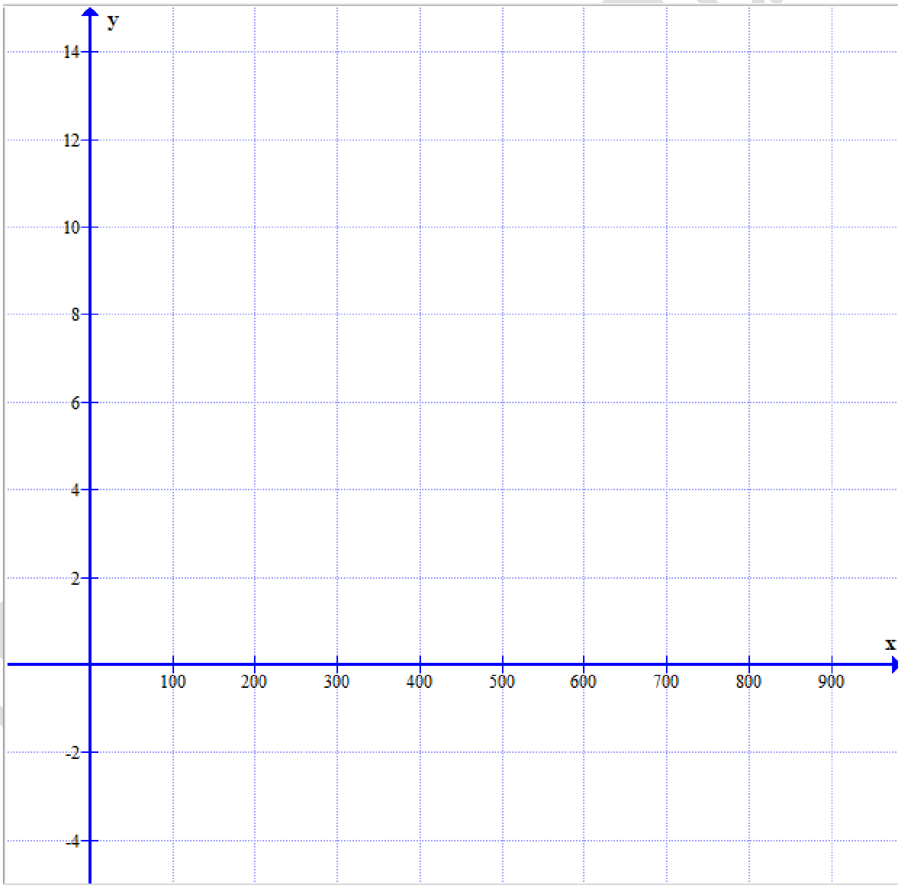
The alien decides to get some practice by going out and conversing with its neighbours, but quickly finds that this is quite difficult due to a lack of confidence. It goes back home and finds out that a scientifically-backed measure of language confidence is the base 2 logarithm of the number of words known, i.e. $\log_2(W(d))$.

e. Find the real number t such that from the point of time t in days, it takes 60 days to improve the alien's confidence level by 1. (2 marks)

Unfortunately, the bad experience was so demotivating that the alien slowed down in language learning ability. The alien's backup team's modelling suggests that this resulted in a change to $W(d)$, such that it is better modelled by the function $W_1(d) = e^{-bd/1000}W(d)$, for some $b > 0$.

f. The initial estimate is $b = 15$. What is the largest that the alien's vocabulary can possibly get? (2 marks)

g. Draw a graph of the alien's confidence as a function of d when $b = 15$, labelling all axis intercepts and the turning point with their coordinates. (2 marks)



h. To two decimal places, find the smallest value of b such that the alien will never reach level 13 confidence, which the computer system says is the benchmark at which it should try reading a newspaper.

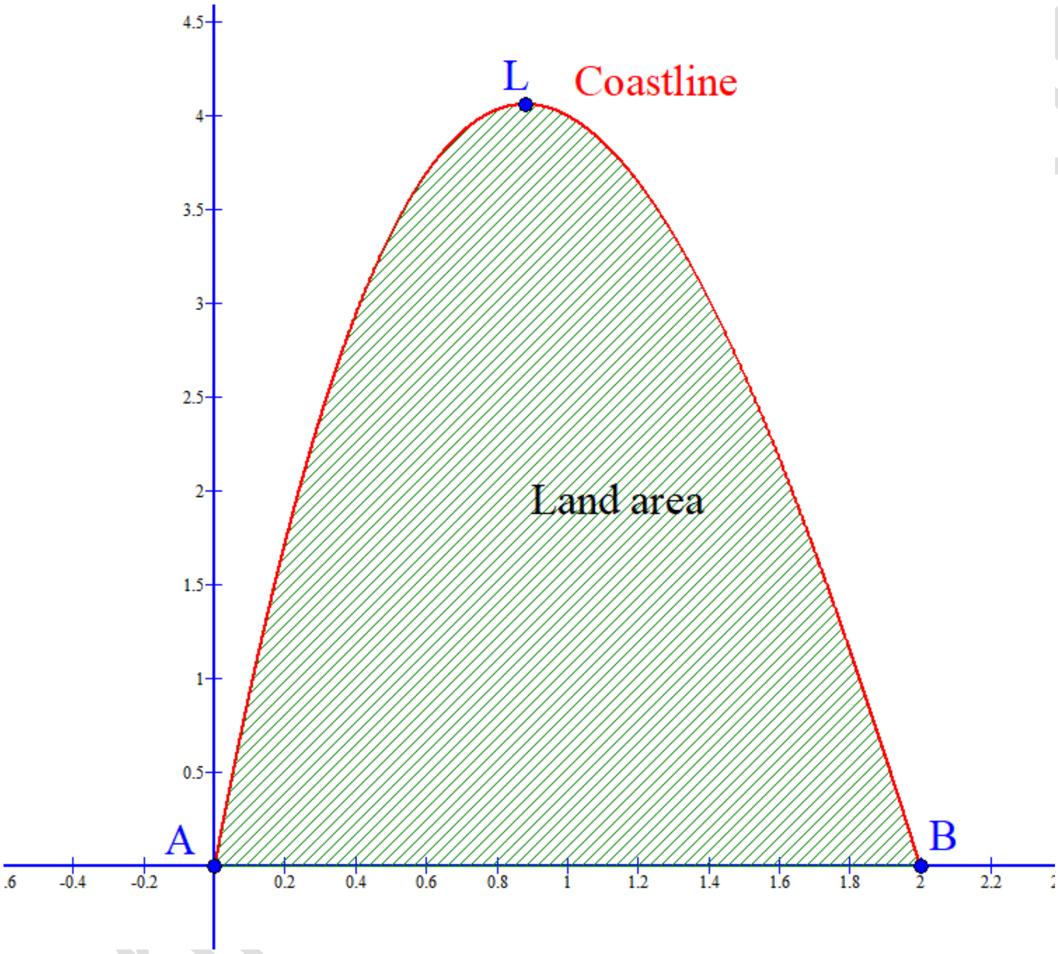
(2 marks)

Question 2 (9 marks)

Councillors are debating over how best to zone a certain block of land next to the coastline.

The coastline snakes along the plane, and its path (neighbouring the land under consideration) can be modelled by the function $f: [0,2] \rightarrow R, f(x) = x(x-2)(x-5)$, as below.

The northernmost point of the coastline has a lighthouse, at point L.



a. What are the coordinates of L? (1 mark)

One proposal is to designate a triangular area on the western side as a commercial area for shops and restaurants, with vertices $(0,0)$, $(a, 0)$ and $\left(a, f(a)\right)$ for some value $0 < a < 2$, and retain a triangular area on the eastern side as parks and nature reserves, with vertices $(a, 0)$, $\left(a, f(a)\right)$ and $(2,0)$.

b. i. Write the rule $D(a)$ for the area of the western triangular development. (1 mark)

ii. One councillor who works in the restaurant business suggests choosing a to maximise the area of the western development. To two decimal places, what is this maximum possible area? (2 marks)

c. A less money-driven councillor says it would be better if the eastern area was at least as big as the western area, to preserve nature for the residents nearby. For which values of a is this satisfied? (2 marks)

d. A third councillor notes that there is a lighthouse at the northernmost point anyway, and it would be visually appealing if the lighthouse was on the border between the western and eastern sides, because then there could be a nice walking trail in straight lines from A to L to B. To two decimal places, what is the length of this trail? (1 mark)

e. The watchperson at the lighthouse notices a ship in trouble at $(0, 3)$. To get to the ship the fastest, the coast guard needs to determine the closest point along the coastline to the ship. To two decimal places, what are the coordinates of this point? (2 marks)

Question 3 (13 marks)

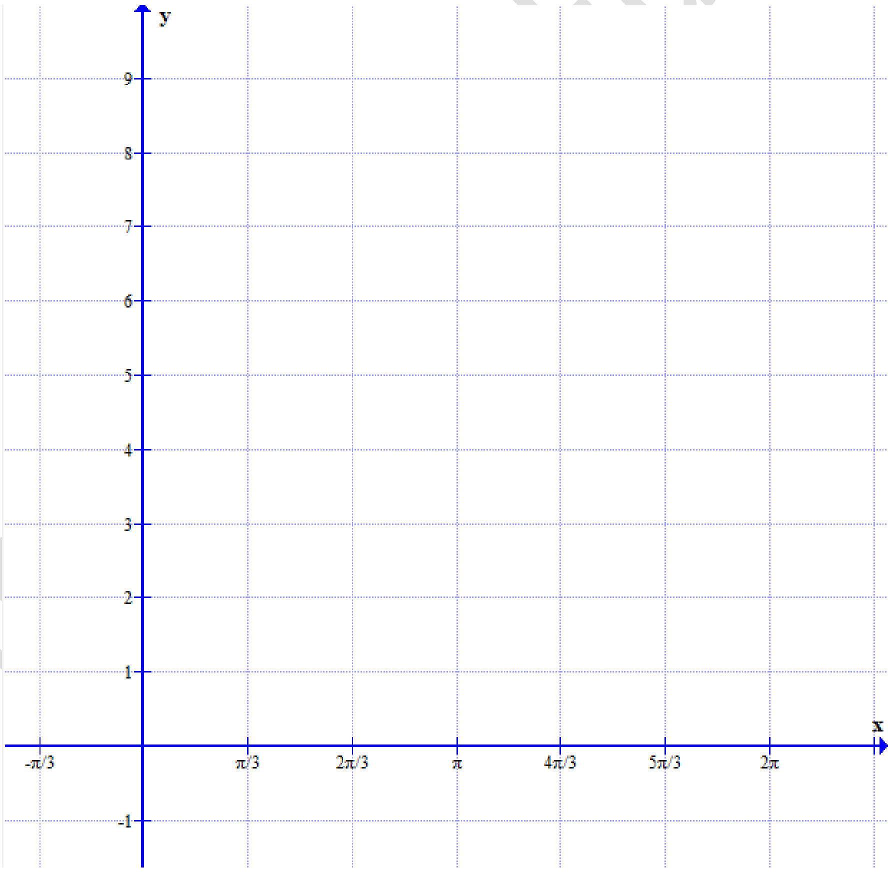
Consider the function $f: [0,2\pi] \to R$, $f(x) = x + \sin(x)$.

a. Show that f is an increasing function. (2 marks)

b. Suppose that the real number a has the property that the tangent at $x = a$ to the graph of $y = f(x)$ passes through the origin. Show that in fact $\tan(a) = a$. (3 marks)

c. i. Show that the graph of $y = f(x)$ has a point of inflection. (2 marks)

ii. Sketch $y = f(x)$ over its domain on the below axes, labelling endpoints and the inflection point with their coordinates. (2 marks)



Consider the set of functions $f_c: [0, 2\pi c] \to R$, where the graph of $y = f_c(x)$ is obtained by applying a dilation of factor $c > 0$ to the graph of $y = f(x)$.

d. For which values of c do $y = f_c(x)$ and $y = f_c^{-1}(x)$ intersect exactly once? Give your answer to two decimal places. (2 marks)

Consider the set of functions of the form $f_b: [0, 2\pi] \to R$, $f_b(x) = f(x) + b$, where b is a real number.

e. For which values of b do the graphs of $y = f_b(x)$ and $f_b^{-1}(x)$ have no points of intersection? (2 marks)

END OF QUESTION AND ANSWER BOOK