



Mathematical Methods

Question and Answer Book

SAC 2a (Tech-Free)

- Reading time is **5 minutes**
- Writing time is **45 minutes**

Approved materials

- Writing equipment (students are **not** permitted to bring any technology or notes of any kind)
- The VCAA formula sheet

Materials supplied

- Question and Answer Book of 8 pages

Instructions

Students are not permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

6 questions (30 marks)

pages

2 – 8

Instructions

Answer **all** questions in the spaces provided.

Write your responses in English.

In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

- a. Let $y = -\log_e(\cos(x))$. Find $\frac{dy}{dx}$. (1 mark)

- b. Let $f(x) = e^{-3x} \sin(x)$.

Find $f'\left(\frac{\pi}{3}\right)$. (2 marks)

Question 2 (3 marks)

Consider the polynomial $P(x) = 2x^3 - x^4$.

- a. Show that $y = P(x)$ has exactly two x – *intercepts*. (1 mark)

- b. Let the x -intercepts of $y = P(x)$ be a and b , with $a < b$.

Find the average value of $P(x)$ on $[a, b]$. (2 marks)

Question 3 (4 marks)

Let $g: \mathbb{R} \setminus \{-2\} \rightarrow \mathbb{R}$ be a function such that $g(0) = 1$ and

$$g'(x) = \frac{-3}{(x+2)^4}.$$

- a. What is the equation of the normal to the graph of $y = g(x)$ at $x = 0$? (2 marks)

- b. Find $g(1)$. (2 marks)

Question 4 (6 marks)

Consider the function $f(x) = \sqrt{x} + \sqrt{4-x}$.

- a. What is the implied domain of $f(x)$? (1 mark)

- b. Find the maximum of $f(x)$. (3 marks)

- c. Using the trapezium rule with four trapezia, estimate

$$\int_0^4 f(x) \, dx.$$

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Question 5 (7 marks)

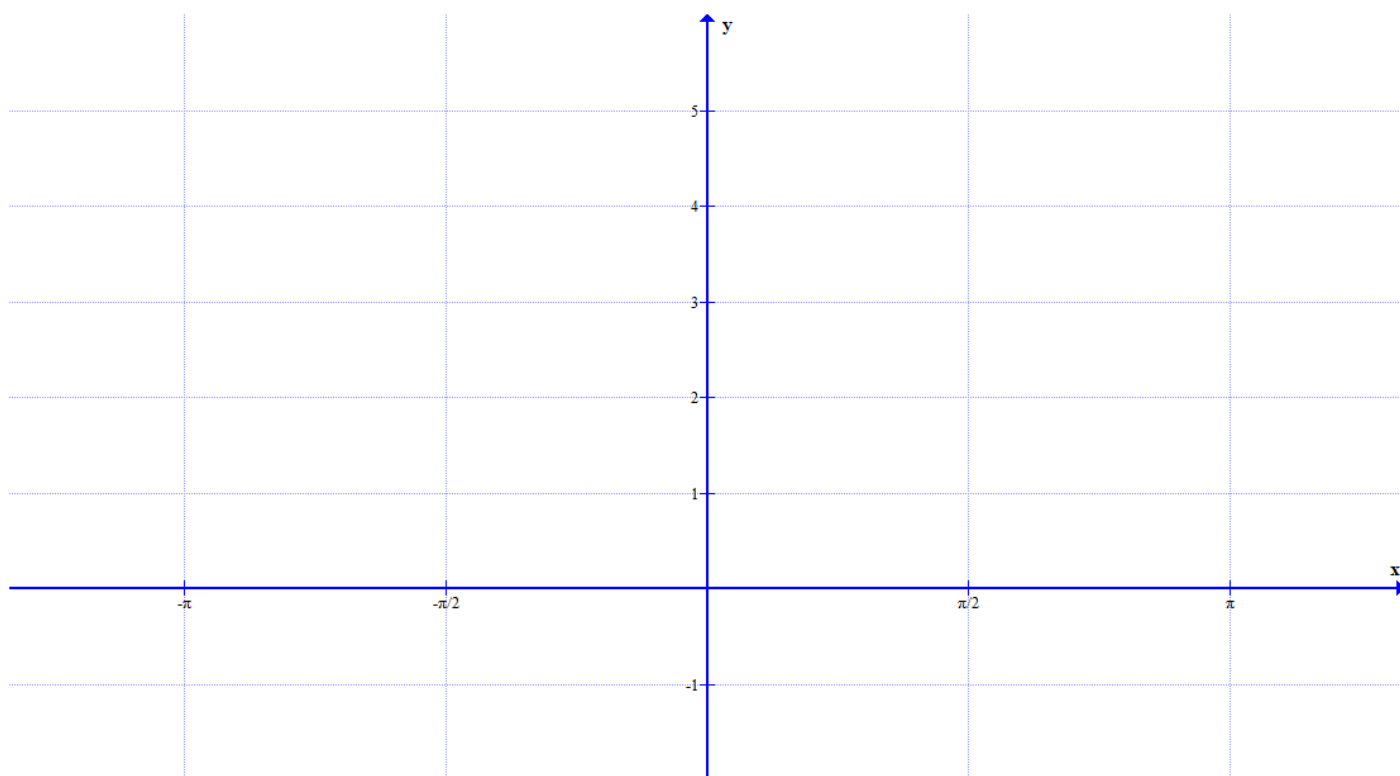
Consider the function $f: [-\pi, \pi] \rightarrow \mathbb{R}$, $f(x) = a \cos(x) + b$ (where a, b are real numbers).

- a. It is known that f has a point of inflection at $(\frac{\pi}{2}, 2)$ and that the y -intercept of f is $(0, 4)$.
- i. Show that $0, 4$ is a stationary point of $y = f(x)$. (1 mark)

- ii. Explain why it is a local maximum of $y = f(x)$. (1 mark)

- b. What are the values of a and b ? (2 marks)

- c. Sketch $y = f(x)$ on the axes below. Label all axis intercepts, turning points and points of inflection with their coordinates. (3 marks)



Question 6 (7 marks)

Given a real number $a > 1$, consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = xa^x$.

- a. Show that $y = x$ is tangent to the graph of $y = f(x)$ at $x = 0$. (2 marks)

- b. Show that the tangent at $x = 1$ to the graph of $y = f(x)$ has a negative y –intercept. (3 marks)

- c. Over what interval is f strictly increasing? (2 marks)

END OF QUESTION AND ANSWER BOOK

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