2026

Mathematical Methods

Question and Answer Book

SAC 2b (Tech-active)

- Reading time is **5 minutes**
- Writing time is **55 minutes**

Approved materials

- One scientific calculator and one approved CAS calculator
- Bound reference
- Writing equipment
- The VCAA formula sheet

Materials supplied

- Question and Answer Book of 15 pages

Instructions

Students are not permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

SECTION A – Multiple-choice questions

Instructions for Section A
Answer **all** questions by circling the correct answer or clearly indicating it
Choose the response that is **correct** or that **best answers** the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = xe^{x+2}$. Then $f'(x) = -1$ at

A. $x = -2$

B. $x = -1$

C. $x = 0$

D. $x = 1$

E. $x = 2$

Question 2

The integral

$$\int_{-\pi/2}^{\pi/2} 24\,x^2 + a\cos(x)\ \mathrm{d}x$$

is equal to zero. The value of a is

A. 0

B. π^3

C. $-\pi^2$

D. $-\pi^3$

E. $2\pi^3$

Question 3

The average value of the derivative of $x^2 \log_e(ax^2 + 1)$ (where a is a real number) from $x = -1$ to $x = 1$ is

A. -2

B. 0

C. 1

D. 3

E. 2

Question 4

The tangent to the graph of $y = f(x)$ at $x = 2$ has the equation $y = x + 5$. The tangent to the graph of $y = 2f(1 - x)$ at $x = -1$ has equation

A. $y = x + 5$

B. $y = 6 - 2x$

C. $y = 12 - 2x$

D. $y = 8 - 5x$

E. $y = 2x + 10$

Question 5

Let $f(x) = cx^3$. The area bounded between the graphs of $y = f(x)$ and $y = f^{-1}(x)$ is equal to 5. The value of c is

A. 0.10

B. 0.15

C. 0.20

D. 0.25

E. 0.50

Question 6

The function $f: R \to R$ satisfies $f(0) = 1$, and $f'(x) = e^{x^2} - 1$ for all x. At the point $(0,1)$, the graph of $y = f(x)$ has

A. An x-intercept

B. A local maximum

C. A local minimum

D. A point of inflection

E. A discontinuity

END OF SECTION A

TURN OVER

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Write your responses in English.

In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (12 marks)

Lucy is trying to build a roller coaster. Her initial plan starts with a section of the graph of the function $f: R \to R, f(x) = x^3 - 5x^2 + 7x$.

a. Her plan is for the segment to start at $x = 0$ and end at a local minimum $x = a$ of the graph of $y = f(x)$. Show that the segment ends at the point $\left(\frac{7}{3}, \frac{49}{27}\right)$. (2 marks)

b. What is the steepest negative slope on the roller coaster? (2 marks)

c. Lucy intends to build some supporting beams for the roller coaster segment. For one of them, she selects the normal to the graph of $y = f(x)$ at $x = 2$, drawn from the point $(2,2)$ to the point at which it intersects the roller coaster segment again.

i. Find the equation of this normal. (1 mark)

ii. Find (to two decimal places) the coordinates of the point at which the beam meets the roller coaster segment again. (1 mark)

iii. How long (to two decimal places) is the beam? (1 mark)

d. She decides to include some space for advertising material in the design. For this purpose, she reserves the space **above** the supporting beam in part c., between the beam and the track. To two decimal places, what is the area of this advertising space? (1 mark)

e. For the second segment of the roller coaster, she decides to add another drop. To do this, she decides to make the second segment the graph of $y = g(x)$, where $g: \left[\frac{7}{3}, \frac{49}{27}\right] \to R$ is a cubic. She wants the following two conditions to hold:

- The graphs of $y = f(x)$ and $y = g(x)$ are joined smoothly at $x = \frac{7}{3}$ so that the ride is not bumpy;
- The graph of $y = g(x)$ joins smoothly to the x-axis at $x = 4$ so that the ride can end there.

i. By considering the derivative of $g(x)$, show that

$$g(x) = a\left(x^3 - \frac{19}{2}x^2 + 28x\right) + b$$

for some real numbers a and b. (2 marks)

ii. To two decimal places, what are the values of a and b? (2 marks)

Question 2 (8 marks)

A farmer is draining his rainwater tank because maintenance is required. After t hours, the amount of water that has left the tank is modelled by $V(t)$, where

$$\frac{dV}{dt} = e^{-t}(2-t)^2$$

and V is measured in millions of litres. He keeps the tap open until all of the water has been emptied out of the tank and water stops flowing out, at which point the model ends.

a. What is the implied domain of $V(t)$? (1 mark)

b. How much water is in the tank? (1 mark)

c. In litres per hour, how quickly does water exit the rainwater tank when he first opens the tap? (1 mark)

d. To the nearest minute, find the time at which the water is draining from the tank at one hundred thousand litres per hour. (1 mark)

e. To the nearest minute, how long does it take for half of the water to have emptied from the tank? (2 marks)

f. Draw a graph of $V(t)$ over its implied domain. Include the axis intercepts with their coordinates. (2 marks)


y
2.5
2
1.5
1
0.5
-0.5
x
-0.5
0.5
1
1.5
2
2.5

Question 3 (10 marks)

For each positive integer n, consider the function $f_n(x) = \sqrt{2 - x^{2n}}$.

a. If f_m has an implied domain of $\left[-\sqrt{2}, \sqrt{2}\right]$, show that $m = 1$.

b. Show that for any $a \in \left(-\sqrt{2}, \sqrt{2}\right)$, the normal to the graph of $y = f_1(x)$ at $x = a$ passes through the origin. (3 marks)

c. Calculate

$$\int_{-\sqrt{2}}^{\sqrt{2}} f_1(x)\, \mathrm{d}x.$$

(1 mark)

d. Jonathan is trying to approximate the value found in part c.

i. Suppose he uses the trapezium rule with two equal-width trapeziums. What is the resulting estimate? (1 mark)

ii. Jonathan decides to use two trapeziums, but he is going to try to cheat by making them different widths. Suppose his first trapezium is from $x = -\sqrt{2}$ to $x = a$, and his second from $x = a$ to $x = \sqrt{2}$, where a is some real number. Determine, in terms of a, the resulting estimate. (1 mark)

iii. For which value of a is Jonathan's cheated estimate closest to the true value? (1 mark)

e. For each positive integer n, consider the area of the region formed by the graph of $y = f_n(x)$ and the x-axis. What is the largest real number A such that this area is always greater than A, for all positive integer values of n? (1 mark)

f. Consider the function g defined by

$$g(x) = \begin{cases} f_n(x) & 0 \leq x \leq 1 \\ f_n^{-1}(x) & 1 < x \leq \sqrt{2} \end{cases}$$

and its graph $y = g(x)$. Its graph, together with the x and y axes, bounds a region.

There are real numbers a and b such that the area of this region can be expressed in the form

$$a + b\int_0^1 f_n\,(x)\,\mathrm{d}x$$

regardless of the value of n.

Find the values of a and b. (1 mark)

END OF QUESTION AND ANSWER BOOK