Unit 4 AOS 1 Calculus – Cheat Sheet

Differentiation

Function type	Function	Derivative
First principles	$f(x)$	$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
Power function	Ax^n	nAx^{n-1}
Exponentials	e^{kx}	ke^{kx}
Logarithms	$log_e(ax+b)$	$\frac{a}{ax+b}$
Sine	$\sin(kx)$	$kcos(kx)$
Cosine	$\cos(kx)$	$-ksin(kx)$
Tangent	$\tan(kx)$	$ksec^2(kx)$
Chain rule	$f(g(x))$	$f'\big(g(x)\big)g'(x)$ OR: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Product rule	$f(x)g(x)$	$f(x)g'(x) + g(x)f'(x)$ OR: $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$\frac{f(x)}{g(x)}$	$\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ OR: $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

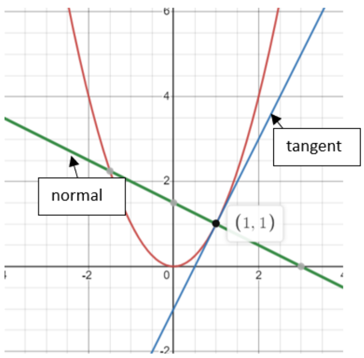
Derivative graphs

Original graph $y = f(x)$	**Derivative graph $y = f'(x)$**
Stationary point	x-intercept
Negative gradient	Below the x-axis
Positive gradient	Above the x-axis
Point of inflection	Turning point
Endpoints (included or excluded)	Excluded endpoints **(derivative UNDEFINED at endpoints)**
Cusp or sharp point (not smooth)	Does not exist **(derivative UNDEFINED at points which are not smooth)**

Limits, Continuity and Differentiability

- **Limits:**
  - $\lim_{x\to a} f(x) = p$ means that the limit of $f(x)$, as x approaches a, is p.
  - Basic limits: substitute the value that x approaches into $f(x)$.
  - **For functions that are not defined at the value that x approaches:** may need to simplify (eg. by cancelling out brackets) in order to evaluate
  - The limit as x approaches a **exists only if** both the limit from the **left** $(x \to a^-)$ and the limit from the **right** $(x \to a^+)$ exist and are equal:
    - If $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = p$ then $\lim_{x\to a} f(x) = p$.
- **Continuity:**
  - A function with rule f(x) is said to be **continuous** at x=a if:
    - f(x) is defined at $x = a$.
    - $\lim_{x\to a} f(x) = f(a)$ (check limits from left and right).
- **Differentiability:**
  - A function is differentiable if:
    - f(x) is **continuous** at $x = a$
    - f(x) is **smooth** at $x = a$. The gradients directly to the left and right of $x = a$ must be equal, i.e. $\lim_{x\to a^+} f'(x) = \lim_{x\to a^-} f'(x)$.
  - Note that graphs will not be differentiable at **endpoints, points of discontinuity and points where the graph is not smooth**. These should be OPEN CIRCLES in derivative graphs, since the derivative is undefined at these points.

Tangents, normals and rates of change



- To find the tangent or normal to a curve $f(x)$ at the point $(a, f(a))$:
  - **Tangent:** $y - f(a) = f'(a)(x - a)$
  - **Normal:** $y - f(a) = -\frac{1}{f'(a)}(x - a)$
- **Average rate of change** $= \frac{f(b)-f(a)}{b-a}$
- **Instantaneous rate of change** $= f'(a)$

Stationary Points

- A point $(a, f(a))$ on a curve $y = f(x)$ is said to be a **stationary point** if $f'(a) = 0$.

Type	$f'(x)$ to the left	$f'(x)$ at point	$f'(x)$ to the right
Local maximum	$f'(x) > 0$	$f'(x) = 0$	$f'(x) < 0$
Local minimum	$f'(x) < 0$	$f'(x) = 0$	$f'(x) > 0$
Stationary point of infection (increasing)	$f'(x) > 0$	$f'(x) = 0$	$f'(x) > 0$
Stationary point of infection (decreasing)	$f'(x) < 0$	$f'(x) = 0$	$f'(x) < 0$

Maximum and Minimum Problems

- The local maximum or minimum value: solve $\boldsymbol{f'(x) = 0}$ and find the y-coordinate (or use Function Minimum/Function Maximum on CAS)
- **Absolute maximum/minimum:** the actual maximum/minimum value of a function
- **Local maximum/minimum:** the y-coordinate of maximum/minimum turning point of a function
- Note: When a graph has a **restricted domain**, the maximum or minimum value may be at an endpoint instead of a stationary point:
    - To find the absolute maximum or minimum value of a function with a restricted domain, find the coordinates of the endpoints and stationary points (the max or min value **must** occur at a **stationary point or endpoint**)

Newton's Method

- The **iterative formula** for Newton's method is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, where n=0,1,2...

Antidifferentiation

Function type	Function	Antiderivative
Power function	x^n	$\frac{x^{n+1}}{n+1}+c$
Linear expression raised to a power (except -1)	$(ax+b)^r$, where $r \neq -1$	$\frac{(ax+b)^{r+1}}{a(r+1)}+c$
Linear expression raised to power -1	$\frac{m}{ax+b}$	$\frac{m}{a}log_e\lvert ax+b\rvert+c$
Exponentials	e^{kx}	$\frac{1}{k}e^{kx}+c$
Sine	$\sin(kx)$	$-\frac{1}{k}cos(kx)+c$
Cosine	$\cos(kx)$	$\frac{1}{k}sin(kx)+c$

Finding areas

Area	Rule
Trapezium rule (estimation)	$\frac{x_n-x_0}{2n}[f(x_0)+2f(x_1)+2f(x_2)\ldots+2f(x_{n-1})+f(x_n)]$ Note: $n =$ number of trapeziums If graph is concave down (gradient decreasing), area will be an underestimate If graph is concave up (gradient increasing), area will be an overestimate
Area between curve and x-axis (above x-axis)	$\int_a^b f(x)\,dx$
Area between curve and x-axis (below x-axis)	$-\int_a^b f(x)\,dx$
Area between two curves (where $f(x)$ is above $g(x)$)	$\int_a^b (f(x)-g(x))\,dx$
Area between curve and y-axis	$\int_a^b x\,dy$ Need to make x the subject of the equation. Note: the terminals are y-values.
Area between log function and x-axis	The area between a log and the x-axis is equal to the area between the inverse function (exponential) and the y-axis, by symmetry. The required area can be found by subtracting the area under the inverse function from the area of the rectangle.

Properties

Properties of antidifferentiation	Properties of definite integrals
$\int f(x) + g(x)\, dx = \int f(x)\, dx + \int g(x)\, dx$ $\int f(x) - g(x)\, dx = \int f(x)\, dx - \int g(x)\, dx$ $\int kf(x)\, dx = k \int f(x)\, dx$	$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$ $\int_a^a f(x)\, dx = 0$ $\int_a^b kf(x)\, dx = k \int_a^b f(x)\, dx$ $\int_a^b f(x) \pm g(x)\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$ $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

- Dilation factors from the y-axis will cause the **area to be multiplied by that factor** (given that the terminals are multiplied by that factor).
- **Translations in the x-direction** (left/right) won't affect the area (given that the terminals are also translated), but **translations in the y-direction** (up/down) will.

Antidifferentiation by recognition

- Based on the recognition that $\frac{d}{dx}(f(x)) = g(x)$ **implies** $\int g(x)\, dx = f(x) + c$
- First part of the question: **differentiate** a certain function (eg. $f(x) = xlog_e(x)$).
- The next part of the question: integrate a function (eg. $y = log_e(x)$) using your answer to the previous part. This can be achieved by antidifferentiating both sides of the derivative obtained in the first part, and rearranging to find the required integral.

Antiderivative graphs

Original graph $y = f(x)$	Antiderivative graph $y = F(x)$
x-intercept	Stationary point
Below the x-axis	Negative gradient
Above the x-axis	Positive gradient
Turning point	Point of inflection

Average value

- **Average value:** $\frac{1}{b-a}\int_a^b f(x)\, dx$
- The average value is equal to the **height of a rectangle** having the same area as the area under the graph for the interval [a,b].
- DO NOT CONFUSE this with **average rate of change**, which is the **average gradient**, and is given by $\frac{f(b)-f(a)}{b-a}$.