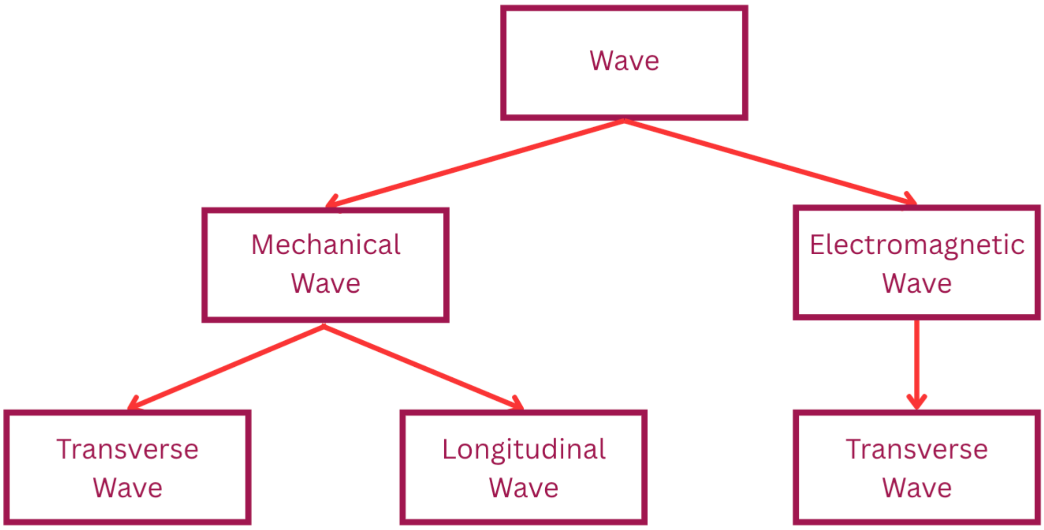
# U4 AOS 1 Summary - Wave Particle Duality and Special Relativity

## Wave Basics

**Definition:** A wave is a transfer of energy without any net transfer of matter.

**Types of Waves:**




- Mechanical waves require a medium, so they cannot travel in a vacuum, whereas electromagnetic waves do not require a medium.
- Transverse waves involve particles moving perpendicular to the direction of the wave (eg. light wave) whereas longitudinal waves involve particles moving parallel to the direction of the wave (eg. sound wave).

**Wave Equation:**

- The speed of a wave is constant if the medium is constant, since the wave speed is entirely dependent on the medium.

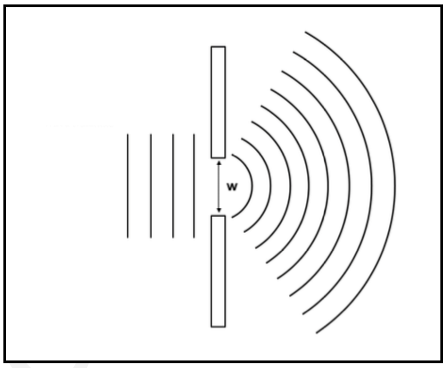
$$\boxed{v = \frac{\lambda}{T} \text{ OR } v = f\lambda}$$

**Diffraction:**

- Diffraction refers to the spreading of a wave around a gap or an obstacle.

$$\boxed{Diffraction \propto \frac{\lambda}{w}}$$

- As a wave spreads, its energy spreads across an increasing area. This means that the intensity (amplitude) of a wave is reduced with higher diffraction.
- Diffraction is significant when $\frac{\lambda}{w} \geq 1$. However, diffraction will still occur if this ratio is less than 1 as long as the $\lambda$ and $w$ are of the same order of magnitude.

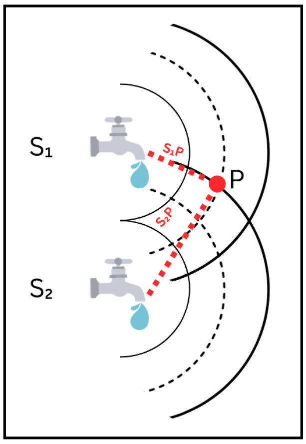


*Diffraction through a gap (slit)*

**Interference:**

- Interference is a phenomenon that occurs when two waves intersect at a point, experiencing *superposition.*
- This involves the addition of displacements of the two waves at the intersection point.
- If constructive interference occurs, then $p.d. = n\lambda$. If destructive interference occurs, then $p.d. = (n + \frac{1}{2})\lambda$. Note: n = 0, 1, 2, 3...

| Path Difference Formula | Uses |
|---|---|
| $p.d. = \lvert S_2P - S_1P \rvert$ | • This is the *geometric* definition of p.d.<br>• Use this equation if the geometric distances to an intersection point are given |
| $p.d. = n\lambda$<br>OR<br>$p.d. = (n + \frac{1}{2})\lambda$ | • This is the *formula definition* of p.d.<br>• Use either of these equations if the position of a node or an antinode can be determined. The further the node/antinode from the centre, the higher the value of 'n'. |



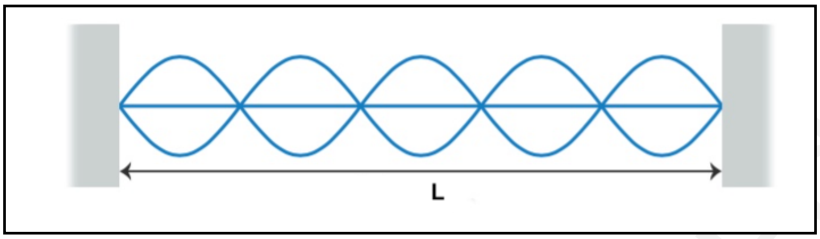
*Two water waves meeting at point P. Destructive interference occurs here.*

**Standing Waves:**

- Standing waves are a special type of interference pattern that contains fixed nodes (points of destructive interference). Points of constructive interference are called antinodes.

<u>Conditions to form a standing wave:</u>

- The two initial waves must be travelling in opposite directions and reflected at the ends.
- The two initial waves must be coherent (have same wavelength and are in-phase)
- The amplitude of the two initial waves must be the same
- The wavelength of the initial waves must be: $\lambda = \frac{2L}{n}$, $n = 1, 2, \ldots$



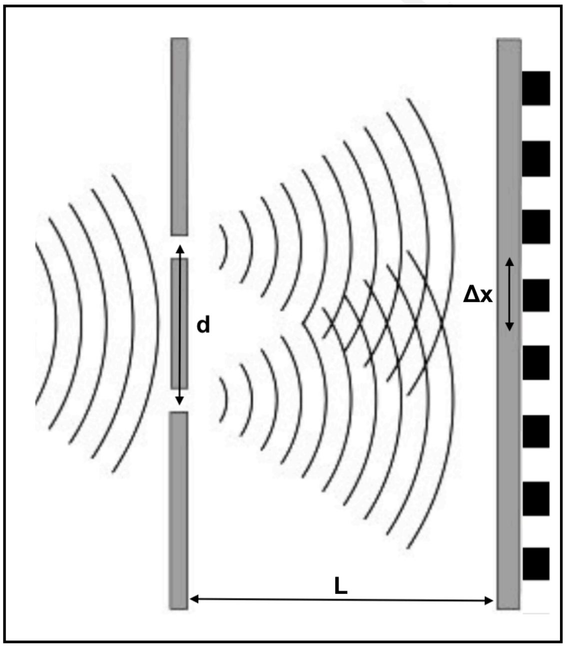

*An example of a standing wave. NOTE: the harmonic number, n, represents the number of 'half wavelengths' that fit in the length of a medium (ie. number of antinodes). So, in the example, n = 5.*

**<u>Light as a Wave:</u>**

**Young's Double Slit Experiment:**

- The Young's double slit experiment involved shooting a monochromatic (single frequency) light beam towards two slits, resulting in an interference pattern shown on a screen. These results support the *wave model,* since interference is a wave phenomenon.

$$\boxed{\Delta x = \frac{\lambda L}{d}}$$




*An illustration of Young's double slit experiment.*

*The position of the bands centre correspond to the value of 'n' used in the p.d. formula. So, n = 1 for the second dark band from the centre.*

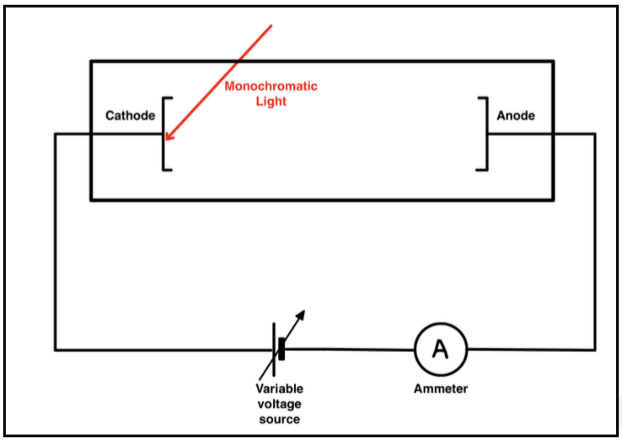
*The fringe spacing, $\Delta x$, is the distance between two consecutive light bands (antinodes) or dark bands (nodes)*

## Light as a Particle:

### Photoelectric Effect Experiment:

- Einstein's photoelectric effect experiment provided *three* pieces of evidence supporting the particle model of light.

- When conducting this experiment, one of the following can be the independent variable: frequency of light, intensity (brightness) of light, voltage.




*An illustration of the experimental setup involving a series circuit.*

*Light of a single frequency is shone on a metal plate, cathode, resulting in the absorption of light energy by electrons.*

*If electrons receive enough energy to overcome the work function, they will be ejected and move towards the anode, possibly resulting in a photocurrent.*

| Component | Function |
|---|---|
| Photoelectric cell | • Contains the cathode and anode metal plates |
| Variable voltage source | • Can provide a *forward voltage* or a *reverse voltage* by making the cathode and anode plates charged<br>• If the reverse voltage is high enough, it is called the *stopping voltage* ($V_s$), which repels even most energetic electron. |
| Ammeter | • Measures the current |

### Energy Formulas:

$$E_{photon} = hf = \frac{hc}{\lambda}$$

*Energy of a single photon formula. h is the Planck's constant and is provided in the VCAA formula sheet.*

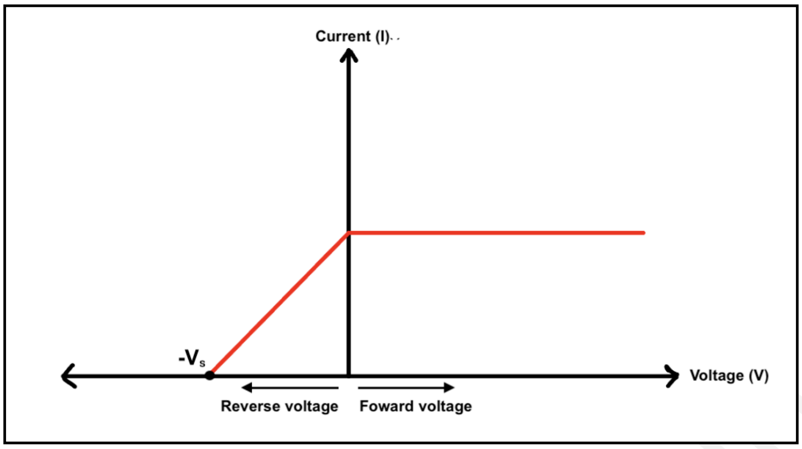
$$KE_{max} = hf - \phi$$

*Maximum kinetic energy of an ejected electron in the photoelectric cell.*

$$\phi = hf_0$$

*Work function of a metal. This is the minimum energy required by light in order to just barely eject an electron. Note, if the energy of a photon is equal the work function, the KE of the ejected electron is 0*

Current - Voltage Graph:

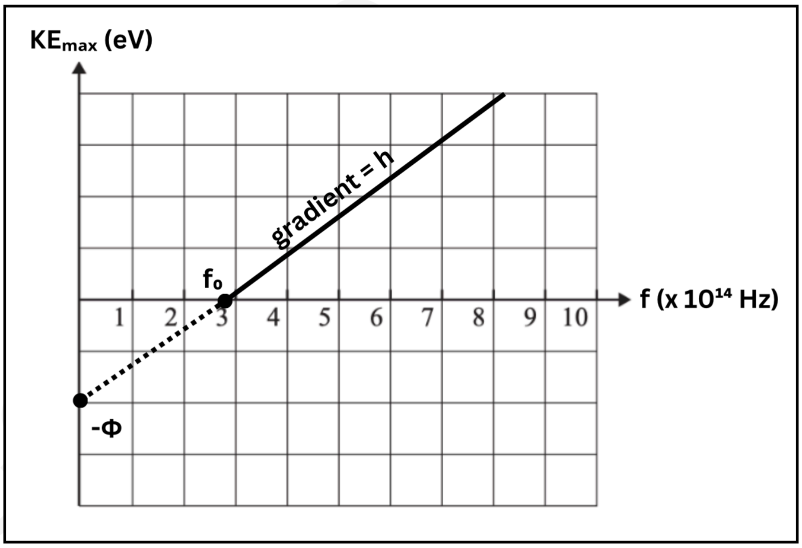



*The I-V graph. The independent variable here is the voltage.*

*The horizontal intercept is -$V_s$. The stopping voltage can be used to determine the maximum kinetic energy of the electrons, since $KE_{max} = eV_s$ (in J) and $KE_{max} = V_s$ (in eV).*

- If the intensity of light increases, the maximum photocurrent increases however the stopping voltage remains the same.
- If the frequency of light increases, the maximum photocurrent stays the same however the stopping voltage increases.
- NOTE: If you are given a question which states that the frequency of light increases however the light output power stays the same, then it must mean that the intensity (number of photons per second) of light decreases. This would result in a smaller photocurrent.

Kinetic Energy (max) - Frequency Graph:




*The $KE_{max} - f$ graph. The units of the kinetic energy can be in J or eV, which would affect the units of 'h'.*

*NOTE: some graphs will have the stopping voltage $V_s$ as the y-value. This does not make a difference in our analysis since $V_s = KE_{max}$ (in eV)*

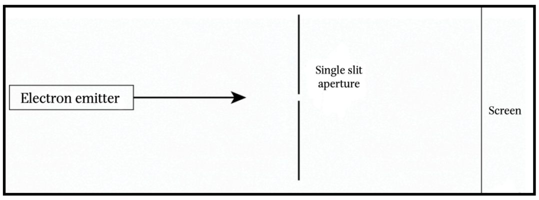
Evidence for the Particle Model:

| Evidence | Explanation |
|---|---|
| Existence of a threshold frequency | • If the frequency of light decreases below the threshold frequency, $f_0$, then no photocurrent is detected.<br>• This supports the particle model as it shows that light comes in discrete packets of energy called photons, with energy dependent on frequency (E=hf). Since the energy transferred must exceed the work function for photoemission, the frequency must exceed a certain threshold frequency.<br>• This goes against the wave model which predicts that the measure of a wave's energy is its amplitude, not frequency, so light of all frequencies should be able to cause photoemission (if intensity is high enough). |
| Intensity corresponds to photocurrent | • If the intensity of light is increased, the photocurrent also increases.<br>• This supports the particle model which predicts that as light intensity increases, the number of emitted photons per second increases (while the energy of each remains unchanged), so photoelectron emission will increase.<br>• This goes against the wave model, which predicts that as intensity increases, the energy transferred per unit time will increase. Thus, it would increase the maximum kinetic energy of photoelectrons and thus stopping voltage instead of the photocurrent. |
| Lack of time delay | • When light is shone on the cathode, a photocurrent is detected immediately without any time delay.<br>• This supports the particle model as it shows that light is comprised of discrete photons (E=hf) that should immediately transfer energy to electrons in a one-to-one interaction.<br>• This goes against the wave model which predicts that light is a continuous transfer of energy which should take time to eject electrons when they have accumulated sufficient energy to overcome the work function. |

**Wave-Particle Duality:**

- Various experiments have shown us that particles also behave as waves, displaying behaviours such as diffraction and interference.
- The extent of wave-like behaviour exhibited by a particle is measured by the de Broglie's wavelength:

$$\boxed{\lambda = \frac{h}{p}}$$




*An illustration of the electron single slit experiment, which showed an electron diffraction pattern*

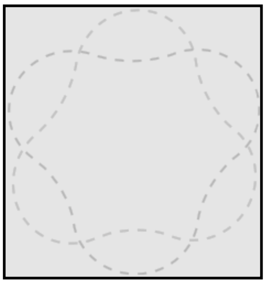
- The above formula can also be rearranged to find the momentum of photons: $p = \frac{h}{\lambda} = \frac{h}{mv}$.
  Keep in mind that 'h' must be in J s when using this formula.
- NOTE: X-ray photons and electrons will produce similar diffraction patterns if they have the same wavelength (given the crystal spacing is the same). Thus, they must have the same momentum, however they will **not** have the same energy (wavelength is directly related to momentum, not energy).

Energy Formulas:

| | Energy Formula(s) |
|---|---|
| **Particles** | $E_k = \frac{1}{2}mv^2$ <br> $E_k = \frac{p^2}{2m}$ |
| **Light** | $E_{photon} = hf$ <br> $E_{photon} = pc$ |

Electron Standing Waves:

- The energy levels (shells) of electrons are quantised (discrete), meaning that electrons can only exist in a finite number of energy levels.
- This is explained by the electron standing wave model in which the electron standing waves can only exist at certain radial distances from the nucleus such that $n\lambda = 2\pi r$, else destructive interference occurs.
- NOTE: Electron standing waves are not explicitly covered in the VCAA Physics Study Design anymore however it may be indirectly assessed in SACs still.



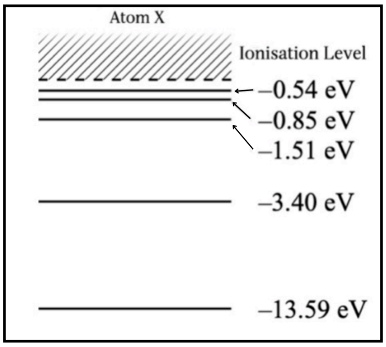
*An illustration of an electron standing wave at the 3rd electron shell from the nucleus (n = 3)*

Energy Level Diagrams:

- Energy level diagrams represent the energies that electrons have at particular electron shells. The electron shell number is 'n'.
- If an electron moves from a higher to a lower energy level then it emits light of energy equivalent to the energy level difference, $\Delta E$. If an electron moves from a lower to a higher energy level, then must have absorbed light of energy $\Delta E$.

$$\boxed{\Delta E = E_{photon}}$$

- The energy level difference can be calculated using: $\Delta E = E_n - E_{n-1}$. E.g. for the example on the right diagram, $\Delta E = E_3 - E_1 = -1.5 - (-13.6) = 12.1\ eV$.




*'Negative energy' values in a diagram represent the required energy for an electron to be ionised. The last energy level at 0 eV represents the ionisation level.*

Single photon and electron double slit experiments:

- The single photon double slit experiment involved shining light at a double slit and reducing the intensity such that only a single photon was passing through the slits at a time. This ensured that there would be no interactions between photons.
- Over time, an interference pattern was formed with alternating regions of high and low intensity, indicating that individual photons exhibit wave-like behaviour and can interfere with themselves.
- However, if the photons are observed as they pass through the slits (by detecting which slit the photon passed through), two distinct bands are produced rather than an interference pattern, which is particle-like behaviour.
- The electron double-slit experiment was similar to the single photon experiment, except that it involved firing electrons at a double slit.
- Overall, these experiments demonstrated the wave-particle duality of light and matter.

## Special Relativity:

### Michelson-Morley Experiment:

- The Michelson–Morley experiment aimed to determine the speed of light in various spatial directions with the goal of identifying the Earth's motion through the presumed *ether*.

- The experimental setup involved dividing a light beam into two perpendicular paths and subsequently reuniting them to examine potential interference patterns. The hypothesis was that if the Earth moved through the ether, the speed of light would vary between the two perpendicular directions, with one being faster than the other. If this was the case, there would be a shift in the interference pattern if the apparatus was rotated 90 degrees.

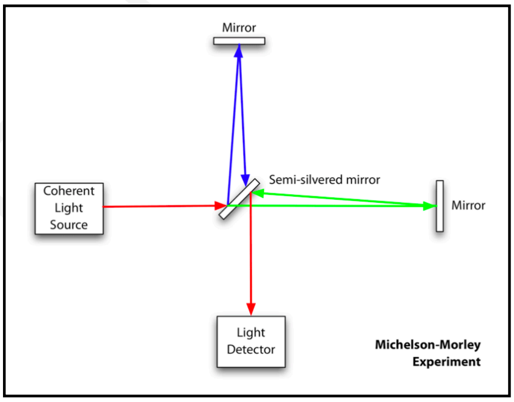



*Illustration of experimental setup for Michelson-Morley experiment. You do not need to know the exact details of the setup.*

- However, there was no shift in the interference pattern, suggesting that the speed of light was the same in both perpendicular directions, which was a null result. This was the basis of Einstein's 2nd postulate.

### Einstein's Postulates:

1. The laws of physics are the same in all inertial (non-accelerating) frames of reference.

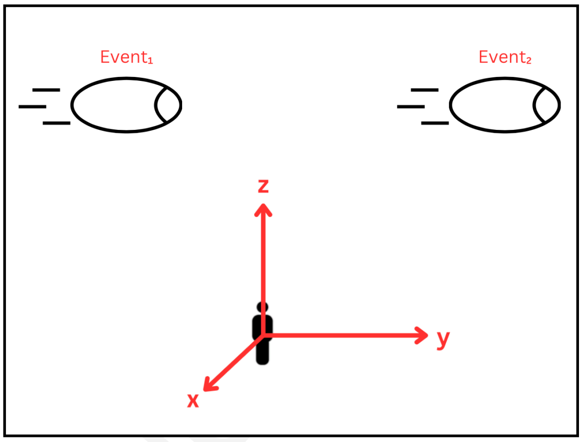
2. The speed of light in a vacuum has a constant value for all observers in inertial frames of reference regardless of their motion or the motion of the source.

**Time Dilation:**

- A frame of reference for which $event_1$ and $event_2$ occur in the same point in space measures proper time, $t_0$, whereas a frame of reference for which $event_1$ and $event_2$ occur in different points in space measures dilated time, $t$.

$$\boxed{t = \gamma t_0}$$

$$\boxed{\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}}$$




*A frame of reference can be thought of as a set of x, y, z axes extending from an observer.*

*Here an astronaut is observing a space pod move from one point to another point, referred to as $event_1$ and $event_2$. Logically, the coordinates of these to events would be different from the astronauts frame of reference, hence he will measure dilated time.*

**Length Contraction:**

- A frame of reference for which a length is stationary measures proper length, $L_0$, whereas a frame of reference for which a length is moving measures contracted length, $L$.

$$\boxed{L = \frac{L_0}{\gamma}}$$

**Understanding Muons:**

- Muons are fast moving unstable particles formed in the atmosphere. Special relativity can explain how these particles are able to reach the ground despite their short half-life. Either time dilation or length contraction can be used to explain this.

| Phenomenon | Explanation |
|---|---|
| Time Dilation | • According to the frame of reference of an observer on the ground, the half life of the muons is dilated (ie. they appear to last longer).<br>• Hence the observer witnesses a greater number of muons reaching the ground before decaying compared to the prediction made by classical physics. |
| Length Contraction | • According to the frame of reference of the muons, the length from the atmosphere to the surface is contracted.<br>• As the muons have to travel a shorter distance, a greater number of muons are able to reach the ground before decaying compared to the predication made by classical physics. |

**Mass-Energy Equivalence:**

- According to the principle of mass-energy equivalence, mass is a form of energy known as rest energy, which can be calculated using: $E_0 = mc^2$.
- An object in motion has relativistic kinetic energy given by: $E_k = (\gamma - 1)mc^2$.

$$\boxed{E_{tot} = E_k + E_0 = \gamma mc^2}$$

- The total mass-energy of a system is always conserved, so $E_i = E_f$.