

# U2 AOS 1 Summary – Motion

## Scalars vs Vectors

- **Scalars**

Scalars are properties which can be measured and described using only a magnitude (i.e. number) and a unit

- **Vectors**

Vectors are properties which can be measured and described using a magnitude, unit and direction.

## Examples of Scalars and Vectors

- **Distance (d)**

How far something has travelled. Measured in metres (m).

- **Displacement (s)**

The change in position of an object, including direction. Measured in metres (m).

- **Speed (v)**

The rate of change of **distance**. Measured in metres-per-second ( $\text{m s}^{-1}$ )

$$\text{Average Speed} = \frac{d}{\Delta t}$$

- **Velocity (v)**

The rate of change of **displacement**. Includes direction. Measured in metres-per-second ( $\text{m s}^{-1}$ )

$$\text{Average velocity} = \frac{s}{\Delta t}$$

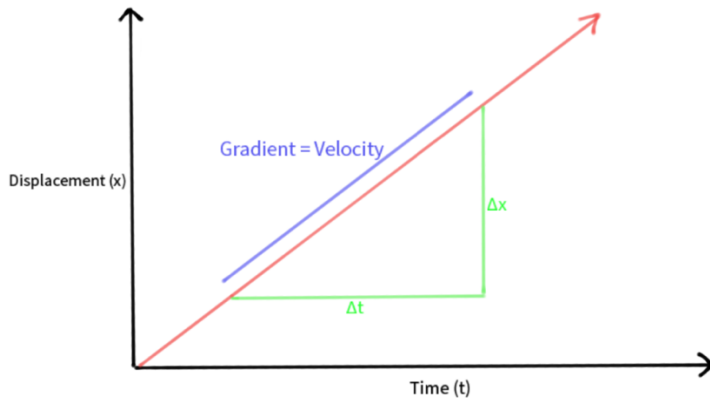
- **Acceleration (a)**

The rate of change of velocity. Typically includes direction unless otherwise stated. Measured in metres-per-second-per-second ( $\text{m s}^{-2}$ )

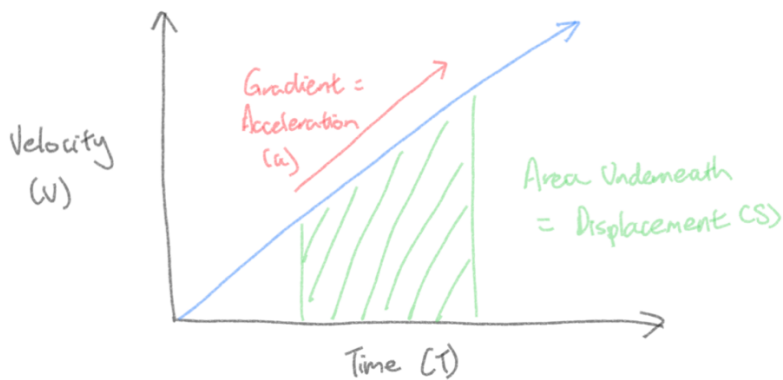
$$\text{Average acceleration} = \frac{v}{\Delta t}$$

## Graphs of motion

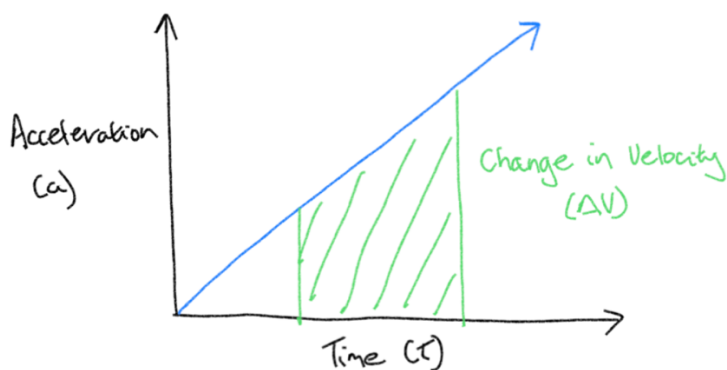
- **Displacement-time**



- **Velocity-time**



- **Acceleration-time**



One can convert between each graph by mapping the **rate of change** to move down this list. For example, a linearly increasing velocity-time graph will result in a constant, flat displacement time graph.

**Formulas of constant acceleration (SUVAT)**

|          |                         |
|----------|-------------------------|
| <b>S</b> | <b>Displacement</b>     |
| <b>U</b> | <b>Initial velocity</b> |
| <b>V</b> | <b>Velocity</b>         |
| <b>A</b> | <b>Acceleration</b>     |
| <b>T</b> | <b>Time</b>             |

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

It's important to remember that these formulas can only work in two dimensions – positive or negative. They do not work for two-dimensional movement, thus often requiring splitting motion into components.

**Momentum (p)**

An object's resistance to changes in motion. Measured in kilogram-metres-per-second ( $\text{kg m s}^{-1}$ ).

$$p = mv$$

Where:

$m$  = Mass (kg)

$v$  = Velocity ( $\text{m s}^{-1}$ )

Momentum is a vector quantity, thus also needing a direction.

**Law of conservation of momentum**

Total momentum in a collision is always conserved before and after.

$$\Sigma P(\text{before}) = \Sigma P(\text{after})$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

### Impulse (I)

The change in momentum. Also measured in kilogram-metres-per-second ( $\text{kg m s}^{-1}$ ) but can also be measured in Newton-seconds (Ns). Like momentum it is also a **vector**.

$$I = \Delta p = F_{\text{net}} \Delta t$$

Where:

$F_{\text{net}}$  = Average net force during collision (N)

$t$  = Change in time (s)

### Weight

Refers to the gravitational force on a body of mass

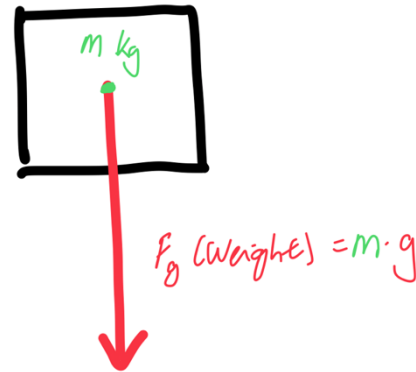
$$F_g = m g$$

Where:

$m$  = mass of object (kg)

$g$  = gravitational field strength ( $\text{N kg}^{-1}$ )

At the surface of the earth,  $g = 9.81 \text{ (N kg}^{-1}\text{)}$



### Newton's Laws of motion

- **Newton's first law (inertia)**

An object at rest will stay at rest, and an object in motion will stay in motion unless acted on by a net external force.

- **Newton's second law**

The acceleration of a body experiencing an unbalanced force is directly proportional to the net force and inversely proportional to the mass of the body.

$$\Sigma F = m a$$

- **Newton's third law**

For every action there is an equal and opposite reaction.

$$F_{\text{on } A \text{ by } B} = - F_{\text{on } B \text{ by } A}$$

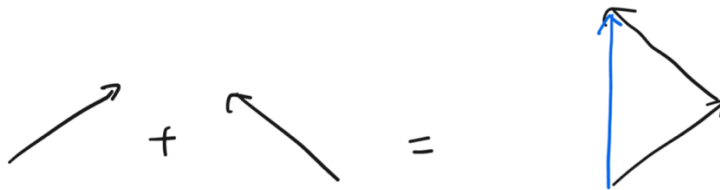
### Vector manipulation

Vectors are visually represented as arrows. The length of the arrow indicates the magnitude of the vector. The arrow shows the direction of the vector.



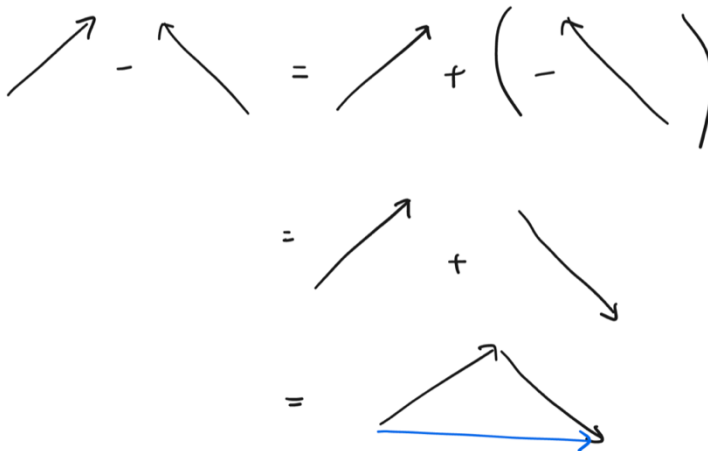
- Vector addition**

Vectors can be added using the “head-to-tail” method



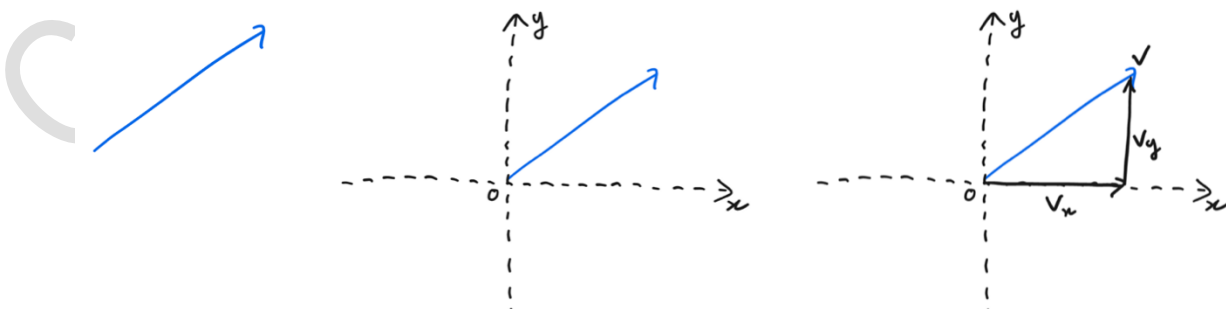
- Vector subtraction**

Similar to vector addition, but swap the direction of the vector to subtract.



- Vector components**

Vectors can be broken down into components.



### Torque

Turning force. A vector quantity measured in Newton-metres (Nm).

$$\tau = r_{\perp} F$$

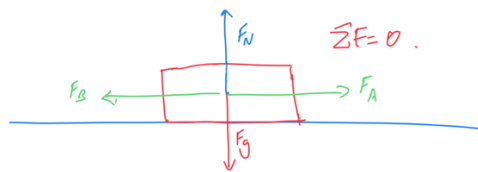
Where:

$r$  = distance from pivot to the point of Force application (m)

$F$  = Force applied to that point (N)

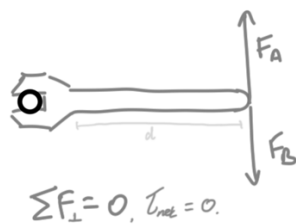
- **Translational equilibrium**

Translational equilibrium occurs when the sum of all forces acting on an object adds to give a net zero force.



- **Rotational equilibrium**

Rotational equilibrium occurs about a set reference point when the sum of all torques acting on the clockwise direction and the anti-clockwise direction sums to give a net zero torque.



### Work

Work is equal to force multiplied by the displacement which it acted. Positive work done on object: object gains energy via force. Negative work done on object: object loses energy via force.

$$W = Fs$$

Where:

$F$  = constant force (N)

$s$  = displacement (m)

### Hooke's Law

Hooke's Law states that the *constant* force applied by a spring is proportional to the length of spring extension/compression, but opposite in direction.

$$F = -k\Delta x$$

Where:

$k$  = spring constant (Unit =  $\text{N m}^{-1}$ )

$\Delta x$  = length of extension/compression of spring (Unit = m)

**Energy:**

- **Gravitational Potential Energy (GPE)**

The gravitational potential energy of an object due to work done against the gravitational field is given by:

$$E_g = m g \Delta h$$

Where  $E_g$  = gravitational potential energy (J)

$m$  = mass of object (kg)

$G$  = gravitational field strength (9.81 N kg<sup>-1</sup> on Earth)

$\Delta h$  = change in height of object (m)

- **Elastic potential energy (EPE)**

$$E_s = \frac{1}{2} k \Delta x^2$$

Where:

$E_s$  = elastic potential energy stored in the extension/compression (J)

$k$  = spring constant (N m<sup>-1</sup>)

$\Delta h$  = length of extension/compression of ideal spring (m)

- **Kinetic energy (KE)**

The kinetic energy of an object is equal to half of its mass times its velocity squared, given by:

$$E_k = \frac{1}{2} m v^2$$

Where:

$E_k$  = kinetic energy of the object (J)

$m$  = mass of object (kg)

$v$  = velocity of object (ms<sup>-1</sup>)

- **Power**

Power is the rate of energy transfer

$$P = \frac{E}{t}$$

Where:

$P$  = power (W) = (J s<sup>-1</sup>)

$E$  = energy (J)

$t$  = time (s)

- **Efficiency**

$$\eta = \frac{\text{useful energy out}}{\text{total energy in}}$$