

U3 AOS 1 Summary – Motion

Newton's Three Laws

1. An object at rest or moving at a constant velocity will continue to do so unless acted on by an unbalanced force.
2. $F_{net} = ma$. The acceleration of a body experiencing an unbalanced force is directly proportional to the net force and inversely proportional to the mass of the body.
3. $F_{B \text{ on } A} = -F_{A \text{ on } B}$. For every action force there is an equal and opposite reaction force. These forces act on **different objects** and are of the same type.

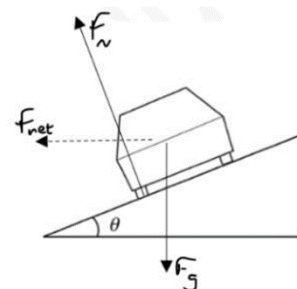
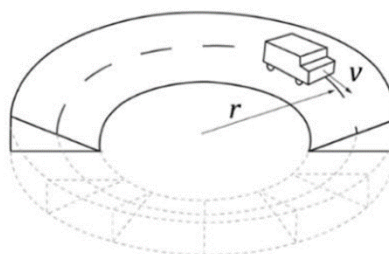
Circular motion

- **Uniform circular motion:** F_{net} (centripetal force) is constant & perpendicular to object's velocity

Centripetal force $F_{net} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$	Centripetal acceleration $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$
Velocity (circular motion) $v = \frac{2\pi r}{T}, \text{ where } T = \frac{1}{f}$	

Banked tracks

- $\tan(\theta) = \frac{v^2}{rg}$
- The horizontal component of the normal force provides the centripetal force, reducing the need for a sideways frictional force (car can travel at greater speeds around corners).
- $v > \sqrt{rg \tan(\theta)}$: car slides up the slope (friction acts down the slope).
- $v < \sqrt{rg \tan(\theta)}$: car slides down the slope (friction acts up the slope).



Vertical circles

Scenario 1: highest point (outside circle)	Scenario 2: highest point (inside circle)	Scenario 3: lowest point (inside circle)
$F_{net} = F_g - F_N$ ($F_N < F_g \therefore$ feel lighter)	$F_{net} = F_g + F_N$	$F_{net} = F_N - F_g$ ($F_N > F_g \therefore$ feel heavier)

Minimum speed (scenario 2), or **maximum speed** (scenario 1) to remain in contact with track:

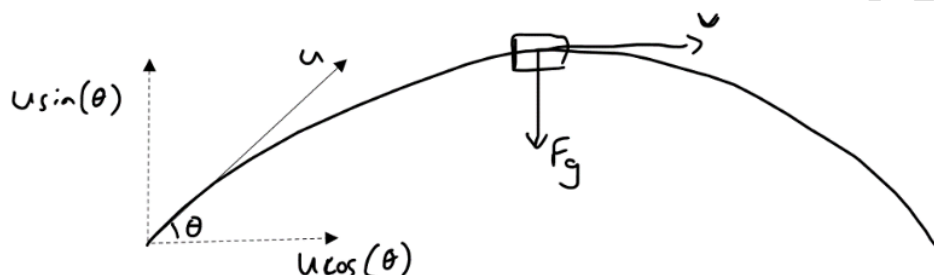
$$v = \sqrt{rg}$$

Projectile motion

Horizontally: $s = ut$ ($a = 0$)

Vertically ($a = 9.81 \text{ m s}^{-2}$ downwards):

- $v = u + at$
- $v^2 = u^2 + 2as$
- $s = \frac{1}{2}(u + v)t$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

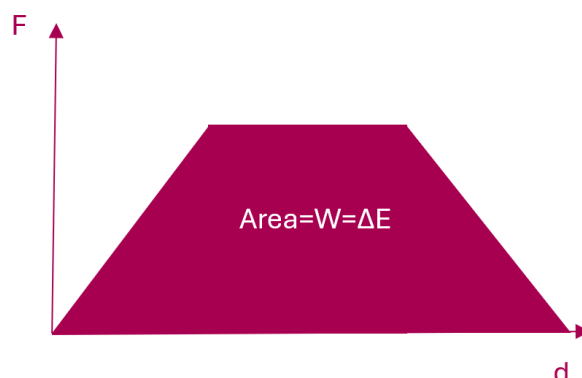
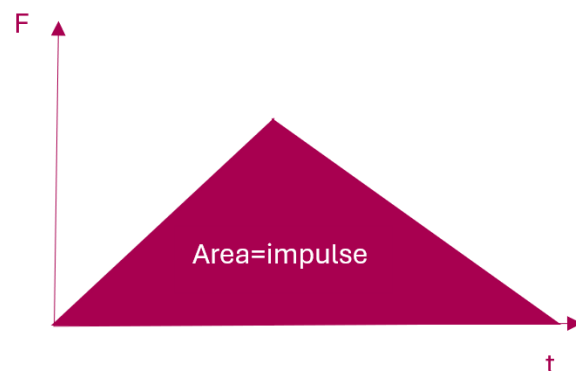


Note:

- Specify which direction (up or down) is positive for vertical motion.
- Max height: $v_y = 0$
- To find **final velocity**: find vertical and horizontal components and use vector triangle to find magnitude and angle above horizontal.
- To find **final speed**: vector triangle OR conservation of energy
- **Range formula**: $R = \frac{u^2 \sin(2\theta)}{g}$ (if trajectory symmetrical)
- Effect of **air resistance**: reduces maximum height and range

Momentum and impulse

- **Momentum** (vector): $p = mv$ (specify which direction is positive)
- **Law of conservation of momentum**: $\sum p_i = \sum p_f$
- **Change in momentum**: $\Delta p = p_f - p_i = mv - mu = m\Delta v$
- **Impulse** (vector): $I = F_{av}\Delta t = \Delta p = m\Delta v$
OR area under force-time graph
- Safety features: increase time of interaction (Δt) → reduce average force (F_{av}) since impulse is constant.



Work done

- **Work done**: $W = Fs$ OR area under force-distance graph
- If F and s not in same direction, use $W = Fscos(\theta)$

- Work done **on an object**: increase total energy (F and s in same direction)
- Work done **against an object**: reduce total energy (F and s in opposite directions)

Energy

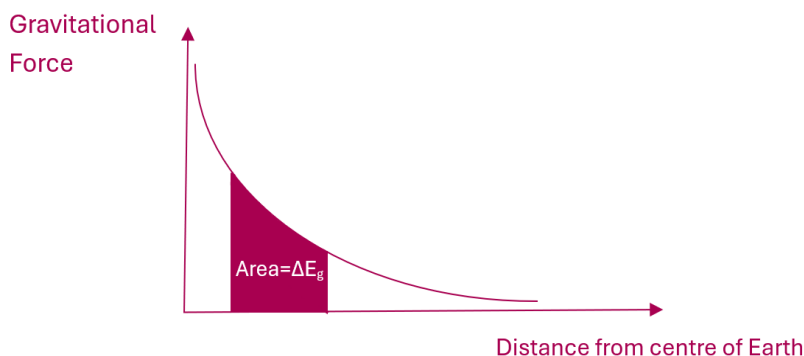
- **Law of conservation of energy**: energy cannot be created or destroyed, only transformed
- Energy can be dissipated to the environment (considered as a combination of heat, sound and deformation of material).

Kinetic energy: $E_k = \frac{1}{2}mv^2$

- **Elastic collision**:
 - $\sum E_{k(i)} = \sum E_{k(f)}$
- **Inelastic collision**:
 - $\sum E_{k(i)} > \sum E_{k(f)}$

Gravitational potential energy:

$E_g = mg\Delta h$ OR area under force-distance graph



Springs

Elastic potential energy:

- $E_s = \frac{1}{2}k\Delta x^2$ OR area under force-extension (or force-compression) graph

Hooke's Law: $F_s = -k\Delta x$

Vertical springs:

- **Total energy:** $E_T = E_g + E_s + E_k$
- F_s varies, F_g constant
- **Above midpoint:** $F_g > F_s$
(F_{net} /acceleration downwards)
- **At midpoint:** $F_g = F_s$
($F_{net} = a = 0$, max speed/ E_k)
- **Below midpoint:** $F_s > F_g$
(F_{net} /acceleration downwards)
- At lowest point of oscillation:
 $\Delta E_g = \Delta E_s$
- Note: $\Delta E_s = E_{s(f)} - E_{s(i)}$
- When the spring-mass system reaches **equilibrium** (stops oscillating or is gently lowered until at rest), this will be at the point where $F_g = F_s$ (at rest, $F_{net} = 0$). Energy has not been conserved, as the kinetic energy has been transformed into other forms such as heat/sound.

