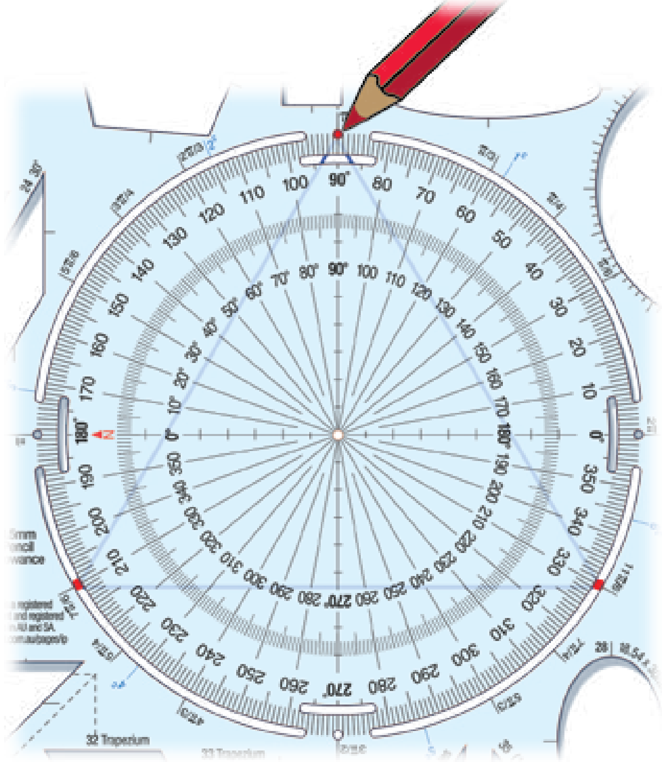
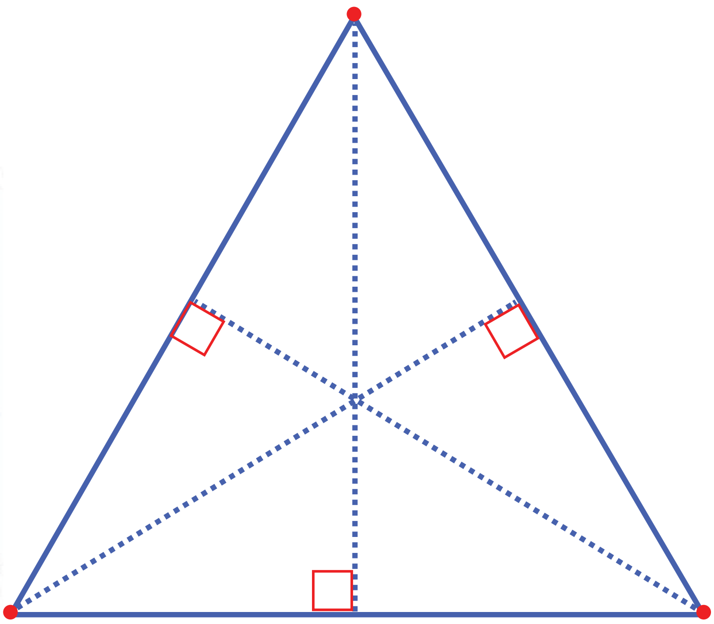
Ratios and fractions with shapes

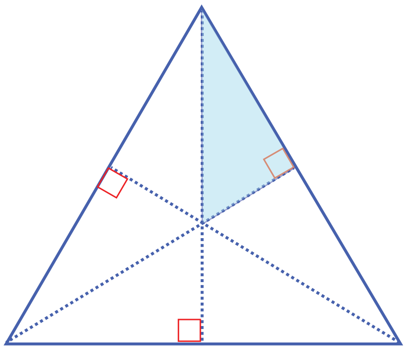
Exploring composite shapes with the equilateral triangle



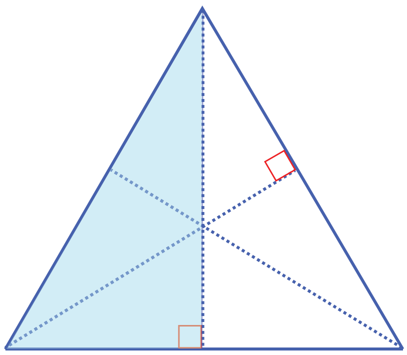


There are many ways to draw larger triangles with Mathomat. This method uses the protractor to mark off points at 90°, 210° and 330° (use the outer hole at 90°'s point).

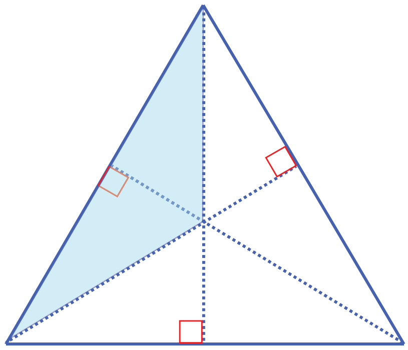
Connect the points to make an equilateral triangle with 94mm sides. Draw perpendicular bisectors with the guides on Mathomat to bisect each angle.



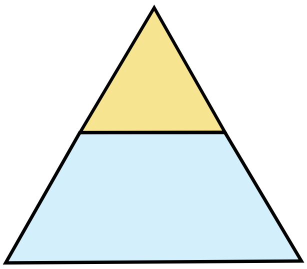
This scalene right-angled triangle is 1/6 of the area of the original triangle.



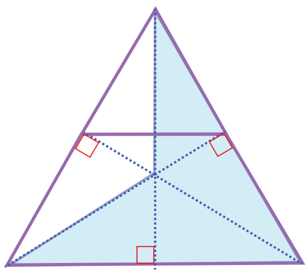
The scalene right-angled triangle here is 1/2 of the area of the original triangle.



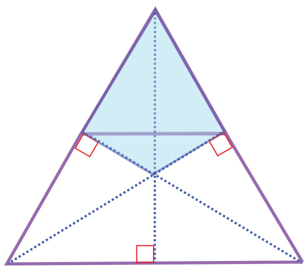
The isosceles triangle is 1/3 of the area of the original triangle.



Now draw the trapezium (shape 32) to see its relation to the equilateral triangle (shape 5) with Mathomat.



Using the perpendicular bisectors again to find the concave kite in the larger triangle. It is 2/3 of the area.

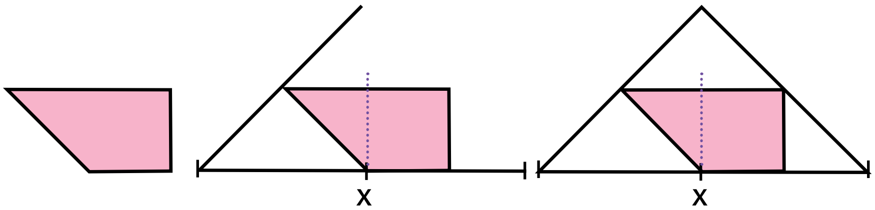


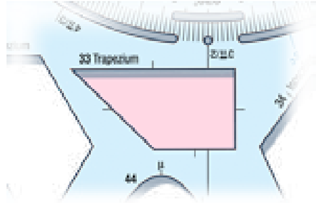
This convex kite is double the area of the scalene right-angled triangle. It is 1/3 of the total area.

The midpoint theorem

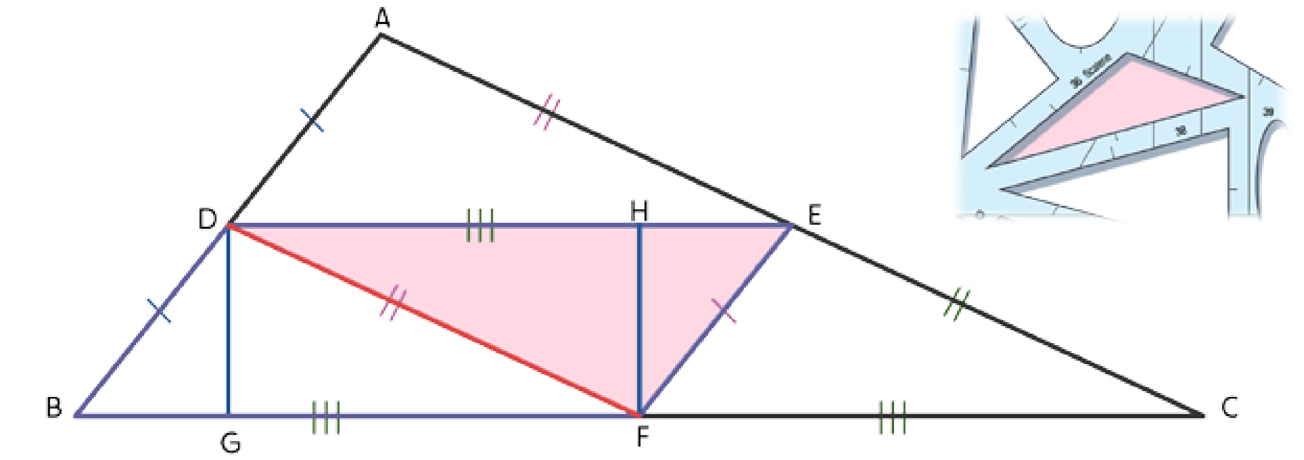
The midpoint theorem:
The line segment in a triangle joining the midpoint of any two sides of the triangle is said to be parallel to its third side and is also half the length of the third side.

We can use **Mathomat** to draw a parallel shape, **the trapezium** (shape 33), and enclose it in a triangle to illustrate this theorem. Draw a triangle's base line making point x its midpoint. The other two sides will go through the trapezium's vertices at their midpoints.





To present the diagram below to illustrate this theorem draw the diagram with the **scalene triangle** (shape 36), **tessellating** it four times as shown (BDF, FEC, DAE and DEF) (using translating and reflection). These become the unit triangles in this diagram.
F is at the midpoint of the line BC, and you have created the parallelogram DEFB.



Measure AB, BC and AC
Measure the angles of the unit triangles using **Mathomat**'s protractor.
Measure angles $\hat{B}$, $\hat{C}$ and $\hat{A}$ using **Mathomat**'s protractor.

Conclusions:

1. The large triangle's sides are exactly **DOUBLE** those of the unit triangle.
2. Each unit triangle's angles measure exactly the same magnitude as triangle ABC's angles (the large triangle.)
3. DE is exactly half of BC.

Their similarity enables proportionality or ratios to come into effect. This is seen by the ratio of the sides:
DA = ½ of AB   DE = ½ of BC   AE = ½ of AC

This illustrates that if a line is drawn from the midpoint of one side of a triangle (DE) to the midpoint of a second side of a triangle (AC) and it is parallel to the third side, then the line will be **HALF** of the third side.

This theorem is usually done with an exterior-drawn parallelogram and congruent triangles to complete the proof. This can also be proved with algebra.