CAMBRIDGE

Sixth Term Examination Paper (STEP)

Mathematics 2 (9470)

2025

Examiners' report and mark scheme

STEP Mathematics 2 examiners' report

Paper 2 overview

As is commonly the case, the vast majority of candidates focused on the Pure questions in Section A of the paper, with a good number of attempts made on all of those questions. Candidates that attempted the Mechanics questions in Section B generally answered both questions. More candidates attempted Question 11 in Section C than either Mechanics question, but very few attempted Question 12 in that section.

There were a large number of good responses seen for all the questions, but a significant number of responses lacked sufficient detail in the presentation, particularly when asked to prove a given result or provide an explanation.

Candidates who did well on this paper generally:	Candidates who did less well on this paper generally:
• gave careful explanations of each step within their solutions • indicated all points of interest on graphs and other diagrams clearly • made clear comments about the approach that needed to be taken, particularly when having to explore a number of cases as part of the solution to a question • used mathematical terminology accurately within their solutions.	• made errors with basic algebraic manipulation, such as incorrect processing of indices • produced sketches of graphs in which significant points were difficult to see clearly because of the chosen scale • skipped important lines within lengthy sections of algebraic reasoning.

Section A: Pure Mathematics overview

The Pure questions were considerably more popular than the questions in the other sections of the paper. A significant number of attempts at each of the questions were seen and there were some very good solutions for each of the questions.

Question 1

1 The function Min is defined as

$$\text{Min}(a, b) = \begin{cases} a & \text{if } a \leqslant b \\ b & \text{if } a > b\,. \end{cases}$$

(i) Sketch the graph $y = \text{Min}(x^2, 2x)$.

(ii) Solve the equation $2\,\text{Min}(x^2, 2x) = 5x - 3$.

(iii) Solve the equation $\text{Min}(x^2, 2x) + \text{Min}(x^3, 4x) = mx$ in the cases $m = 2$ and $m = 6$.

(iv) Show that $(1, -3)$ is a local maximum point on the curve $y = 2\,\text{Min}(x^2, x^3) - 5x$ and find the other three local maxima and minima on this curve.

Sketch the curve.

This was a popular question and there were many very good responses seen, with a small number of candidates scoring full marks. Almost all responses included attempts at all parts of the question.

Part (i) was generally answered well, although many candidates did not make clear that the gradient of the quadratic section was zero at the origin. Additionally, while most sketches showed two straight line sections for the parts that should show the line $y = 2x$, it was not always clear that these two straight line sections were parts of the same straight line.

Part (ii) was well answered, but many candidates omitted the coefficient of 2 when solving the equation and therefore were not able to reach the correct points. Additionally, many solutions did not show sufficient evidence of checking that the solutions fell within the required ranges. There was a small, but significant, number of candidates that struggled to factorise their correct quadratic equation.

Part (iii) was well answered, with most candidates able to identify the correct function for each of the ranges and solve the corresponding equations. A common error, however, was to solve the equation $6x = 6x$ either as $x = 0$ or as $x = 0 \text{ or } 1$, rather than noting that it is valid for any value of x within the relevant range. Some candidates did not combine all of their results from the different ranges correctly but were awarded the marks provided that all of the correct values were seen somewhere within the solution.

Many candidates struggled with the explanation that $(1, -3)$ is a local maximum of the curve, although there were some very good explanations seen. Candidates were generally good at identifying the other maxima and minima on the curve, although numerical errors, particularly in the simplification of the y coordinates, were common. When sketching the graphs, many candidates were able to draw the quadratic and cubic sections, although there were several examples where the symmetry of the curves was not evident. Many candidates tried to smooth the graph around the point where the two sections join, rather than having a clear change of gradient at that point. There were also several cases where points of significance were not marked on the graph. Almost no candidates attempted to justify the relative positioning of the two minimum points on the graph.

Question 2

2 **(i)** **(a)** Show that if the complex number z satisfies the equation

$$z^2 + |z + b| = a,$$

where a and b are real numbers, then z must be either purely real or purely imaginary.

(b) Show that the equation

$$z^2 + \left|z + \tfrac{5}{2}\right| = \tfrac{7}{2}$$

has no purely imaginary roots.

(c) Show that the equation

$$z^2 + \left|z + \tfrac{7}{2}\right| = \tfrac{5}{2}$$

has no purely real roots.

(d) Show that, when $\frac{1}{2} < b < \frac{3}{4}$, the equation

$$z^2 + |z + b| = \tfrac{1}{2}$$

will have at least one purely imaginary root and at least one purely real root.

(ii) Solve the equation

$$z^3 + |z + 2|^2 = 4.$$

This was a popular question and there was a wide variety in the quality of responses seen, with a small proportion of candidates producing perfect, or close to perfect, solutions.

Part (i) (a) was generally completed well, with most candidates explaining the reasoning carefully in their responses. Part (i) (b) was also completed well by many candidates, with numerical errors being the main area where marks were lost. In part (i) (c), however, a large number of candidates did not realise there were two cases to be considered and instead only analysed one case. In part (i) (d) there was a lot of variation in the quality of responses seen. Those who had successfully completed part (i) (b) were often able to demonstrate that there was a purely imaginary root. Those who had successfully completed part (i) (c) were often able to make good progress in showing that there must be a purely real root, although many stopped at the point of showing that the discriminant was positive and did not show that at least one of the roots of the quadratic equation lay within the appropriate range. As with part (i) (c), there were a large number of candidates who did not recognise that there were two cases to be explored in this part of the question.

Part (ii) was completed well by many candidates, including those who had lost marks in previous parts of the question. Several candidates lost marks through numerical errors or by not explaining clearly enough that all of the solutions had been found in each case.

Question 3

3 **(i)** Sketch a graph of $y = \dfrac{\ln x}{x}$ for $x > 0$.

(ii) Use your graph to show the following.

(a) $3^\pi > \pi^3$

(b) $\left(\frac{9}{4}\right)^{\sqrt{5}} > \sqrt{5}^{\,\frac{9}{4}}$

(iii) Given that $1 < x < 2$, decide, with justification, which is the larger of x^{x+2} or $(x+2)^x$.

(iv) Show that the inequalities $9^{\sqrt{2}} > \sqrt{2}^9$ and $3^{2\sqrt{2}} > \left(2\sqrt{2}\right)^3$ are equivalent. Given that $e^2 < 8$, decide, with justification, which is the larger of $9^{\sqrt{2}}$ and $\sqrt{2}^9$.

(v) Decide, with justification, which is the larger of $8^{\sqrt[3]{3}}$ and $\sqrt[3]{3}^{\,8}$.

This was a popular question with many candidates able to make good progress through most parts of the question.

Part (i) was answered well by the vast majority of candidates, with the maximum point clearly labelled in most cases. In a small number of cases the asymptotes were not sufficiently clear, although in a small number of cases the behaviour near $x = 0$ was not correct.

Part (ii) (a) was generally completed well, with most candidates recognising the relationship between the graph and the required results. In some cases, solutions were presented showing that the given result implied the correct ordering of the numbers, but did not present the logic correctly to show that the ordering of the numbers implies the given result. In part (ii) (b) a significant number of candidates did not justify the order of the three numbers within their solution.

Many candidates were able to produce good solutions to part (iii). Some chose to consider a translation of the curve from part (i) and looked for the point of intersection between $y = \frac{\ln x}{x}$ and $y = \frac{\ln(x+2)}{x+2}$ as the method to justify the inequality.

Part (iv) was well answered by most candidates who attempted it. Most were able to show the equivalence of the two inequalities, but some only showed the logic in one direction. A large proportion of candidates were then able to see how to apply the equivalence between the two inequalities to determine which of the given values was the larger.

Part (v) was found to be very challenging, with some candidates making no written attempt. Those candidates that made progress deduced that there was a need to find an equivalent inequality to allow a similar process to part (iv) to be carried out. Several candidates made mistakes with their manipulation of indices within this part of the question. A good proportion of those who identified the equivalent inequality were then able to recognise that an approach similar to part (iii) was required to reach the final answer.

Question 4

4 Let $\lfloor x \rfloor$ denote the largest integer that satisfies $\lfloor x \rfloor \leqslant x$.
For example, if $x = -4.2$, then $\lfloor x \rfloor = -5$.

(i) Show that, if n is an integer, then $\lfloor x + n \rfloor = \lfloor x \rfloor + n$.

(ii) Let n be a positive integer and define function f_n by

$$\mathrm{f}_n(x) = \lfloor x \rfloor + \left\lfloor x + \frac{1}{n} \right\rfloor + \left\lfloor x + \frac{2}{n} \right\rfloor + \ldots + \left\lfloor x + \frac{n-1}{n} \right\rfloor - \lfloor nx \rfloor$$

(a) Show that $\mathrm{f}_n\left(x + \dfrac{1}{n}\right) = \mathrm{f}_n(x)$.

(b) Evaluate $\mathrm{f}_n(t)$ for $0 \leqslant t < \dfrac{1}{n}$.

(c) Hence show that $\mathrm{f}_n(x) \equiv 0$.

(iii) **(a)** Show that $\left\lfloor \dfrac{x}{2} \right\rfloor + \left\lfloor \dfrac{x+1}{2} \right\rfloor = \lfloor x \rfloor$.

(b) Hence, or otherwise, simplify

$$\left\lfloor \frac{x+1}{2} \right\rfloor + \left\lfloor \frac{x+2}{2^2} \right\rfloor + \ldots + \left\lfloor \frac{x+2^k}{2^{k+1}} \right\rfloor + \ldots .$$

This was a popular question, but candidates often struggled to explain their reasoning with sufficient clarity in many parts.

Almost all candidates seemed to understand why the result in part (i) is true, but many were not precise enough in their explanation. The most common approach was to split x as the sum of two values, one of which was an integer, but many candidates did not state that the other part of the number was greater than or equal to 0 and strictly less than 1.

Part (ii) (a) was answered well in general, with almost all candidates realising that part (i) could be applied here. Solutions were often very well presented for this part. Part (ii) (b) also had many good responses, but in some cases the justification that one or more of the terms must be equal to 0 was missing. Many candidates realised that they could combine the results from the previous two parts to obtain this result, but many arguments were incomplete. In particular, some only showed that the result applied for $x \geq 0$.

Part (iii) (a) was answered well, with most candidates choosing to split the argument into two cases. A significant number realised that this is also a special case of the result in part (ii) (c) and obtained the result from the fact that $\mathrm{f}_2\left(\frac{x}{2}\right) = 0$. Many candidates realised in part (iii) (b) that the result from (iii) (a) could be applied so that the sum could be expressed in a form where most terms cancelled. A common mistake was simply to claim that all but the first term in the sum would cancel and ignore the final term of the partial sum. A few candidates successfully managed to find the complete solution by considering the cases for the different signs of x.

Question 5

5 *You need not consider the convergence of the improper integrals in this question.*

(i) Use the substitution $x = u^{-1}$ to show that

$$\int_0^\infty \frac{\sqrt{x}-1}{\sqrt{x(x^3+1)}}\,dx = 0\,.$$

(ii) Use the substitution $x = u^{-2}$ to show that

$$\int_0^\infty \frac{1}{\sqrt{x^3+1}}\,dx = 2\int_0^\infty \frac{1}{\sqrt{x^6+1}}\,dx\,.$$

(iii) Find, in terms of p and s, a value of r for which

$$\int_0^\infty \frac{x^r-1}{\sqrt{x^s(x^p+1)}}\,dx = 0,$$

given that p and s are fixed values for which the required integrals converge.

(iv) Show that, for any positive value of k, it is possible to find values of p and q for which

$$\int_0^\infty \frac{1}{\sqrt{x^p+1}}\,dx = k\int_0^\infty \frac{1}{\sqrt{x^q+1}}\,dx\,.$$

This was the second most popular question after Question 1 and was attempted by the vast majority of candidates. In general, solutions were very good, particularly for the first two parts.

Parts (i) and (ii) were answered well, with candidates generally showing a good level of proficiency with completing the given substitutions. In part (i) a small number of candidates did not give enough detail in their method when dealing with the limits of the integral following the substitution. Most recognised that the substitution could be used to show that $I = -I$ and produced clear explanations of this. Solutions to part (ii) were often fully correct.

Those candidates who were able to identify the correct substitution were often successful in solving part (iii), although in some cases errors were made with the indices when simplifying the expression. Some candidates attempted substitutions which did not allow them to make any significant progress on solving this part of the question.

Part (iv) was generally answered more successfully than part (iii) with most candidates able to identify the correct substitution to be made. Some candidates started with a more general substitution, from which the form that was needed was deduced. The substitution was again completed successfully by most candidates who reached this part, and the most complete responses noted that the change to the limits would be valid for any of the appropriate values for k. Having completed the substitution, many were able to identify a possible pair of values for p and q. Those who tried to argue that such a pair must exist often did not explain their reasoning clearly enough.

Question 6

6 **(i)** The circle $x^2 + (y-a)^2 = r^2$ touches the parabola $2ky = x^2$, where $k > 0$, tangentially at two points. Show that $r^2 = k(2a-k)$.

Show further that if $r^2 = k(2a-k)$ and $a > k > 0$, then the circle $x^2 + (y-a)^2 = r^2$ touches the parabola $2ky = x^2$ tangentially at two points.

(ii) The lines $y = c \pm x$ are tangents to the circle $x^2 + (y-a)^2 = r^2$. Find r^2, and the coordinates of the points of contact, in terms of a and c.

(iii) C_1 and C_2 are circles with equations $x^2 + (y-a_1)^2 = r_1^2$ and $x^2 + (y-a_2)^2 = r_2^2$ respectively, where $a_1 \neq a_2$ and $r_1 \neq r_2$.

Each circle touches the parabola $2ky = x^2$ tangentially at two points and the lines $y = c \pm x$ are tangents to both circles.

(a) Show that $a_1 + a_2 = 2c + 4k$ and that $a_1^2 + a_2^2 = 2c^2 + 16kc + 12k^2$.

(b) The circle $x^2 + (y-d)^2 = p^2$ passes through the four points of tangency of the lines $y = c \pm x$ to the two circles, C_1 and C_2. Find d and p^2 in terms of k and c.

(c) Show that the circle $x^2 + (y-d)^2 = p^2$ also touches the parabola $2ky = x^2$ tangentially at two points.

This was one of the less popular questions from the Pure section of the paper, but still received many attempts. This question was found to be challenging and few candidates gained many of the marks.

In part (i) almost all solutions attempted to use a calculus or discriminant argument. When arguing based on the discriminant, candidates often did not explain sufficiently clearly how the value of the discriminant related to tangency. Many candidates only solved this part of the question in one direction, not realising that the converse required a separate argument.

Part (ii) was generally completed more successfully than part (i), but responses frequently included algebraic mistakes or did not appreciate the number of solutions that needed to be found.

Very few candidates attempted part (iii). Part (iii) (a) was done well by many of those who attempted it. In part (iii) (b) there were again a number of algebraic mistakes seen. Most candidates who attempted part (iii) (c) related it back to part (i), but most did not make any justification beyond the algebraic manipulation.

Question 7

7 The differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 2x\frac{\mathrm{d}x}{\mathrm{d}t}$$

describes the motion of a particle with position $x(t)$ at time t. At $t = 0$, $x = a$, where $a > 0$.

(i) Solve the differential equation in the case where $\frac{\mathrm{d}x}{\mathrm{d}t} = a^2$ when $t = 0$.

What happens to the particle as t increases from 0?

(ii) Solve the differential equation in the case where $\frac{\mathrm{d}x}{\mathrm{d}t} = a^2 + p^2$ when $t = 0$, where $p > 0$.

What happens to the particle as t increases from 0?

(iii) Solve the differential equation in the case where $\frac{\mathrm{d}x}{\mathrm{d}t} = a^2 - q^2$ when $t = 0$, where $q > 0$.

What happens to the particle as t increases from 0? Give conditions on a and q for the different cases which arise.

While there were several very good responses to this question, there were also a significant number of candidates who did not recognise that the first differential equation could easily be turned into a first-order differential equation. Where the correct solution method was not identified, responses often did not make any further progress with the question, and many responses were very brief before the candidate opted to move on to a different question.

Those who recognised the method that was needed were often able to solve part (i) well although many candidates appeared to assume that the information $\frac{dx}{dt} = a^2$ at $t = 0$ would mean that $\frac{dx}{dt} = x^2$ for all t.

In part (ii) many candidates appeared to be familiar with the form of the required integral and so were able to reach a solution of the differential equation, although many struggled to explain the initial motion sufficiently clearly.

Part (iii) was generally answered well by those that attempted it, but in many cases the absolute value signs within the integrals was not dealt with sufficiently clearly within the solution. Many responses to this part of the question did not consider all of the possible cases, with the case where $a = q$ being the most commonly omitted.

Throughout the question, responses often attempted to explain the motion of the particle as $t \to \infty$, rather than the motion as t increases from 0.

Question 8

8 If we split a set S of integers into two subsets A and B whose intersection is empty and whose union is the whole of S, and such that

- the sum of the elements of A is equal to the sum of the elements of B
- and the sum of the squares of the elements of A is equal to the sum of the squares of the elements of B,

then we say that we have found a *balanced partition* of S into two subsets.

(i) Find a balanced partition of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ into two subsets A and B, each of size 4.

(ii) Given that $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_m$ are sequences with

$$\sum_{k=1}^{m} a_k = \sum_{k=1}^{m} b_k \quad \text{and} \quad \sum_{k=1}^{m} a_k^2 = \sum_{k=1}^{m} b_k^2,$$

show that

$$\sum_{k=1}^{m} a_k^3 + \sum_{k=1}^{m} (c + b_k)^3 = \sum_{k=1}^{m} b_k^3 + \sum_{k=1}^{m} (c + a_k)^3$$

for any real number c.

(iii) Find, with justification, a balanced partition of the set $\{1, 2, 3, \dots, 16\}$ into two subsets A and B, each of size 8, which also has the property that

- the sum of the cubes of the elements of A is equal to the sum of the cubes of the elements of B.

(iv) You are given that the sets $A = \{1, 3, 4, 5, 9, 11\}$ and $B = \{2, 6, 7, 8, 10\}$ form a balanced partition of the set $\{1, 2, 3, \dots, 11\}$.

Let $S = \{n^2, (n+1)^2, (n+2)^2, \dots, (n+11)^2\}$, where n is any positive integer. Find, with justification, two subsets C and D of S whose intersection is empty and whose union is the whole of S, and such that

- the sum of the elements of C is equal to the sum of the elements of D.

Of the Pure questions, this attracted a relatively small number of responses, although many good solutions were seen.

Many candidates were able to write down the partition of the set as their answer to part (i) without much supporting working and this was awarded full marks.

Part (ii) was answered well by most candidates, but many responses did not give sufficiently clear explanations. In particular, some simply produced the two binomial expansions and then claimed that the result would be true.

A small number of candidates attempted to solve part (iii) without using the result from part (ii). Such attempts were rarely successful. Of those who applied the result from part (ii), many did not show that the properties of a balanced partition would be satisfied by their solution. Some candidates simply showed that these were true by calculating the values for the specific case rather than showing a more general result.

In part (iv) many candidates recognised the need to include 0 in the set and then deduced a correct partition. However, in many cases there was insufficient justification that the two sets would have the required property.

Section B: Mechanics overview

Only a small proportion of candidates attempted questions from this section of the paper, although many of those candidates attempted both Mechanics questions.

Question 9

9 Points A and B are at the same height and a distance $\sqrt{2}\,r$ apart. Two small, spherical particles of equal mass, P and Q, are suspended from A and B, respectively, by light inextensible strings of length r. Each particle individually may move freely around and inside a circle centred at the point of suspension.

The particles are projected simultaneously from points which are a distance r vertically below their points of suspension, directly towards each other and each with speed u. When the particles collide, the coefficient of restitution in the collision is e.

(i) Show that, immediately after the collision, the horizontal component of each particle's velocity has magnitude $\frac{1}{2}ev\sqrt{2}$, where $v^2 = u^2 - gr\left(2 - \sqrt{2}\right)$ and write down the vertical component in terms of v.

(ii) Show that the strings will become taut again at a time t after the collision, where t is a non-zero root of the equation

$$(r - evt)^2 + \left(-r + vt - \tfrac{1}{2}\sqrt{2}gt^2\right)^2 = 2r^2.$$

(iii) Show that, in terms of the dimensionless variables

$$z = \frac{vt}{r} \quad \text{and} \quad c = \frac{\sqrt{2}\,v^2}{rg}$$

this equation becomes

$$\left(\frac{z}{c}\right)^3 - 2\left(\frac{z}{c}\right)^2 + \left(\frac{2}{c} + 1 + e^2\right)\left(\frac{z}{c}\right) - \frac{2}{c}(1+e) = 0.$$

(iv) Show that, if this equation has three equal non-zero roots, $e = \frac{1}{3}$ and $v^2 = \frac{9}{2}\sqrt{2}\,rg$. Explain briefly why, in this case, no energy is lost when the string becomes taut.

(v) In the case described in **(iv)**, the particles have speed U when they again reach the points of their motion vertically below their points of suspension. Find U^2 in terms of r and g.

Only a small number of candidates attempted this question and many were not able to set up the problem sufficiently well to make good progress. Those who recognised that conservation of energy could be used in part (i) were often able to reach the given results successfully, although many assumed that the vertical component of the velocity would also change during the collision.

Both parts (ii) and (iii) were well answered by those that attempted them, with errors in the algebra being the main cause of marks being lost.

Very few of those who attempted part (iv) were able to explain why no energy is lost when the string becomes taut again, but most were then able to produce good solutions to part (v).

Question 10

10 The lower end of a rigid uniform rod of mass m and length a rests at point M on rough horizontal ground. Each of two elastic strings, of natural length ℓ and modulus of elasticity λ, is attached at one end to the top of the rod. Their lower ends are attached to points A and B on the ground, which are a distance $2a$ apart. M is the midpoint of AB.

P is the point at the top of the rod and lies in the vertical plane through AMB.

Suppose that the rod is in equilibrium with angle $PMB = 2\theta$, where $\theta < 45°$ and ℓ is such that both strings are in tension.

(i) Show that angle APB is a right angle.

Show that that the force exerted on the rod by the elastic strings can be written as the sum of

- a force of magnitude $\dfrac{2a\lambda}{\ell}$ parallel to the rod
- and a force of magnitude $\sqrt{2}\,\lambda$ acting along the bisector of angle APB.

(ii) By taking moments about point M, or otherwise, show that $\cos\theta + \sin\theta = \dfrac{2\lambda}{mg}$.

Deduce that it is necessary that $\frac{1}{2}mg < \lambda < \frac{1}{2}\sqrt{2}\,mg$.

(iii) N and F are the magnitudes of the normal and frictional forces, respectively, exerted on the rod by the ground at M.

Show, by taking moments about an appropriate point, or otherwise, that

$$N - F\tan 2\theta = \tfrac{1}{2}mg.$$

As with Question 9, very few candidates attempted this question, and those that did were often unable to set up the problem sufficiently well to make much progress.

A significant number of candidates were able to produce a geometric argument to show that the angle is a right angle in part (i), but many then did not produce correct expressions for the tension in the two strings. Many candidates struggled with the concept of resolving forces in two non-orthogonal directions and so struggled to make any progress beyond this point.

The small number of candidates who were able to complete part (i) often managed to solve the remaining two parts of the question well.

Section C: Probability and Statistics overview

Like Section B, there were significantly fewer responses to the questions in this section than for Section A. Question 11 was the most popular from either Section B or C, while Question 12 was the least popular.

Question 11

11 **(i)** By considering the sum of a geometric series, or otherwise, show that

$$\sum_{r=1}^{\infty} r x^{r-1} = \frac{1}{(1-x)^2} \quad \text{for } |x| < 1.$$

(ii) Ali plays a game with a fair $2k$-sided die. He rolls the die until the first $2k$ appears. Ali wins if all the numbers he rolls are even.

(a) Find the probability that Ali wins the game.

If Ali wins the game, he earns £1 for each roll, including the final one. If he loses, he earns nothing.

(b) Find Ali's expected earnings from playing the game.

(iii) Find a simplified expression for

$$1 + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + \ldots + (n+1)x^n,$$

where n is a positive integer.

(iv) Zen plays a different game with a fair $2k$-sided die. She rolls the die until the first $2k$ appears, and wins if the numbers rolled are strictly increasing in size. For example, if $k = 3$, she wins if she rolls 2, 6 or 1, 4, 5, 6, but not if she rolls 1, 4, 2, 6 or 1, 3, 3, 6.

If Zen wins the game, she earns £1 for each roll, including the final one. If she loses, she earns nothing.

Find Zen's expected earnings from playing the game.

(v) Using the approximation

$$\left(1 + \frac{1}{n}\right)^n \approx \mathrm{e} \quad \text{for large } n,$$

show that, when k is large, Zen's expected earnings are a little over 35% more than Ali's expected earnings.

A large number of very good solutions to this question were seen.

Part (i) was completed well by the majority of candidates. However, many candidates did not identify the correct probabilities to use in the calculations for part (ii). Attempts at part (ii) (b) often applied a correct method for calculating the expected value, but used the incorrect value for the probability that had been used in part (ii) (a) and so gained the method mark for part (ii) (b).

Most candidates who attempted part (iii) were able to complete it successfully, usually by applying a similar approach to the one used in part (i).

Combinatorial errors were common in part (iv), with candidates often confusing $2k$ and $2k-1$ in their calculations or incorrectly accounting for the requirements of the order.

Part (v) was usually completed well by those candidates that had previously obtained the correct expressions for the expected values.

Question 12

12 Let X be a Poisson random variable with mean λ and let $p_r = \mathrm{P}(X = r)$, for $r = 0, 1, 2, \ldots$. Neither λ nor $\lambda + \frac{1}{2} + \sqrt{\lambda + \frac{1}{4}}$ is an integer.

(i) Show, by considering the sequence $d_r \equiv p_r - p_{r-1}$ for $r = 1, 2, \ldots$, that there is a unique integer m such that $\mathrm{P}(X = r) \leqslant \mathrm{P}(X = m)$ for all $r = 0, 1, 2, \ldots$, and that $\lambda - 1 < m < \lambda$.

(ii) Show that the minimum value of d_r occurs at $r = k$, where k is such that

$$k < \lambda + \tfrac{1}{2} + \sqrt{\lambda + \tfrac{1}{4}} < k + 1.$$

(iii) Show that the condition for the maximum value of d_r to occur at $r = 1$ is

$$1 < \lambda < 2 + \sqrt{2}.$$

(iv) In the case $\lambda = 3.36$, sketch a graph of p_r against r for $r = 0, 1, 2, \ldots, 6, 7$.

This was the least popular question on the paper. Most candidates were able to make good progress with parts (i) and (ii), but many then struggled with the remaining parts.

In part (i) many candidates did not justify their handling of the inequalities or to deal properly with the fact that λ was not an integer.

Part (ii) similarly involved a number of attempts that did not justify the handling of the inequalities sufficiently well. Additionally, some showed that a minimum would satisfy the given conditions if it exists, but did not show that there must be a minimum.

Most of the candidates who reached part (iii) were able to derive the bound $\lambda < 2 + \sqrt{2}$, but almost none were able to prove that $\lambda > 1$.

A number of good sketches of the graph were produced for part (iv), but a significant number sketched it as a continuous curve.