

Beyond Assumptions: Applying Estimation Models to Predict Real-World Outcomes

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Abel J. Stephen is an accomplished Executive Manager specializing in program and project management, with over two decades of experience delivering enterprise solutions. His expertise lies in strategic planning, risk management, and advanced analytics, with a focus on creating datadriven frameworks to address complex challenges. Abel has designed and implemented enterprise-wide frameworks, such as Project Management Offices and operational strategies, that deliver lasting organizational impact. He holds a Bachelor of Science in Biology from Brooklyn College (CUNY), a Project Management Certificate from UCLA, and a Supply Chain Analytics Certificate from MITx. Guided by a commitment to blending faith and science, his work reflects precision and innovation, applying foundational principles to solve real-world challenges. Colleagues recognize Abel for his strategic leadership and ability to drive impactful, collaborative solutions.

Abstract

This paper presents a novel methodology for enhancing project duration estimation by sequentially integrating the Program Evaluation and Review Technique (PERT), beta distributions, and smoothed Monte Carlo simulation. Starting with PERT's three-point estimates, the approach models task durations using beta distributions, which are then sampled through smoothed Monte Carlo simulations to achieve a consistent 90th-percentile confidence level for robust predictions. Implemented in accessible tools like Google Sheets, the methodology ensures practical application across diverse project management contexts. The approach demonstrates the potential for improved accuracy and reliability in duration estimates, strengthening risk management and decision-making for project execution.

Enhancing Estimation Techniques Across Disciplines

Estimation is a challenge that transcends disciplines. Whether predicting costs, timelines, or resource requirements in fields like budgeting, construction, engineering, healthcare, or event planning, the challenge of providing reliable forecasts with incomplete information is universal. Estimation is not just a project management tool—it's a critical aspect of decision-making across various domains, influencing how organizations allocate resources, manage risks, and develop strategies.

Traditional estimation methods often rely on subjective "best guesses," based on experience or intuition. While this approach can sometimes guide decisionmaking, it leaves much to be desired in terms of accuracy. Even the most seasoned professionals can misjudge outcomes, leading to project delays, budget overruns, and unmet objectives. In situations where outcomes are inherently uncertain, structured approaches that predict a range of possibilities are essential for more reliable planning and risk mitigation.

This paper explores how two established mathematical techniques, called Program Evaluation and Review Technique (PERT) and Monte Carlo simulation, can come together in a novel way to provide a robust framework for handling the inherent uncertainties in estimation. Initially developed to better manage and forecast project timelines, these tools are adaptable and can improve predictions in any field where balancing optimistic, pessimistic, and most-likely outcomes is necessary. To illustrate their versatility, we first highlight practical applications across various disciplines, demonstrating how these methods address real-world estimation challenges.

Practical Applications Across Disciplines

The power of PERT and Monte Carlo simulations lies in their ability to provide clear, probabilistic insights across a wide range of fields, transforming estimation from guesswork to precision. Here are specific scenarios where these methods shine:

• Budgeting and Finance: A small business owner planning an annual budget might estimate marketing costs with an optimistic figure of \$10,000, a most likely cost of \$15,000, and a pessimistic cost of \$25,000. Using PERT and Monte Carlo,

they can determine there's an 80% chance costs will stay below \$17,500, enabling them to allocate funds confidently and reserve a buffer for unexpected expenses.

- Healthcare and Medical Research: A hospital administrator estimating patient recovery times after surgery might use PERT to model recovery durations (e.g., 5 days optimistic, 7 days most likely, 12 days pessimistic). Monte Carlo simulations could reveal a 90% probability that 95% of patients recover within 10 days, helping schedule follow-up care and optimize bed availability.
- **Supply Chain Management:** A logistics manager for an e-commerce company estimating delivery times across a national network might face delays due to weather or traffic. By inputting optimistic (2 days), most likely (3 days), and pessimistic (5 days) estimates into a Monte Carlo simulation, they find a 95% chance of delivering within 4 days, allowing them to set realistic customer expectations and plan contingency routes.
- **Construction and Engineering:** A construction manager overseeing a bridge project might estimate foundation work to take 20 days (optimistic), 30 days (most likely), or 45 days (pessimistic). Monte Carlo simulations show a 70% chance of completion within 35 days, enabling the manager to coordinate sub-contractors, secure funding, and communicate reliable timelines to stakeholders.

These examples illustrate how PERT and Monte Carlo simulations reduce uncertainty and foster data-driven decisions, making them invaluable tools for professionals across disciplines.

My Journey to Rigorous Estimation in Project Management

One area of my professional experience and expertise is program and project management, where I, like many others, have faced the challenge of making accurate, data-driven decisions with incomplete information. My initial interest in structured estimation arose from my experience in project management, where timelines shift, risks arise, and resources fluctuate. Estimating project timelines felt like aiming at a moving target, one that could change unpredictably with every new variable or unforeseen obstacle.

As I searched for ways to improve my approach to project planning, I discov-

ered techniques like PERT and Monte Carlo simulations, which we'll describe in more detail below. These methods provided a structured way to handle uncertainty, giving project managers like me a solid foundation upon which to build predictions. The structure they offered helped take the guesswork out of estimation and allowed for projections based on data, making my planning more accurate and realistic. However, these were generally standalone techniques, and I wanted to find a way bridge these two simply and effectively.

But before diving into the technical details, I'd like to share the personal journey that brought me to this intersection of faith and science—a journey that continues to shape how I approach every task, big or small.

Faith as a Foundation

One of the guiding scriptures in my life is Hebrews 11:3: "By faith we understand that the universe was formed at God's command, so that what is seen was not made out of what was visible." This verse speaks to the reality that the visible world we interact with is built upon invisible, foundational truths. Just as God's command brought the universe into existence, we rely on unseen principles—whether they are physical laws or mathematical models—to navigate the complexities of life and work.

My fascination with foundational truths has always driven my work, as I strive to glorify God in all that I do. Reinforcing this perspective is Colossians 1:16 "For in him all things were created: things in heaven and on earth, visible and invisible, whether thrones or powers or rulers or authorities; all things have been created through him and for him." Even in the context of estimating project durations, I see a divine order at play. Each calculation, each prediction, is a glimpse into the wisdom of how God designed the world. This wisdom, I believe, is available to help us manage the tasks before us more effectively.

For about a year, I prayed for fellowship with believers who shared this understanding—those who see science, math, and work as sacred, sanctified acts of worship. My prayer was answered when I attended the Math3ma Symposium in 2024, organized by The Master's University. During the event, Dr. Tai-Danae Bradley opened with the words, "We have prayed for all of you (in attendance) by name!" This moment affirmed that I wasn't alone in my desire to glorify God through my professional and academic pursuits. During a break, I met Daniel Tsang, with whom I shared my ideas about applying PERT and Monte Carlo simulations. To my surprise, he said he was a graduate statistician and immediately offered to review my work, providing the expertise and encouragement I needed. This experience confirmed for me that when we step out in faith, God provides the guidance and support we need. This experience reminded me of Psalm 32:8: "I will instruct you and teach you in the way you should go; I will counsel you with my loving eye on you."

These moments laid the foundation for my exploration into estimation techniques, blending my faith with my professional pursuits. In the next sections, we'll take a closer look at these mathematical tools.

The Universal Challenge of Estimation

Imagine you're a healthcare manager estimating patient recovery times, a logistics planner predicting delivery schedules, or a finance professional forecasting an annual budget. In each case, you face a common problem: you must predict future outcomes with limited information. Traditional estimation methods are often based on rough guesses about best-case, worst-case, and most likely scenarios and they frequently fall short of capturing the full range of possibilities.

For instance, let's say you are tasked with estimating the cost of a project, forecasting the duration of an event setup, or predicting demand for a product. Ideally, you would consider a range of outcomes rather than a single number, reflecting both optimistic and pessimistic scenarios while emphasizing the most probable result. This range-based perspective is precisely what PERT and Monte Carlo simulations can provide. By offering a clearer view of potential outcomes and their associated probabilities, these methods offer a powerful alternative to traditional estimation approaches, transforming vague assumptions into data-informed projections. For example, traditional estimation approaches, such as expert judgment and analogous estimation, often rely on subjective assessments or comparisons to past projects, resulting in educated guesses prone to optimism or oversight. By structuring these inputs with PERT's three-point estimates and enhancing them with Monte Carlo simulations, our methods offer a powerful alternative, transforming vague assumptions into data-informed projections with clear probabilities of potential outcomes.

The Structured Approach: PERT and Beta Distributions

Program Evaluation and Review Technique (PERT) has its origins in the 1950s when it was developed by the U.S. Navy for the Polaris missile project, one of the most complex projects of its time. The Polaris project involved coordinating thousands of contractors and subcontractors across various disciplines to develop the first submarine-launched ballistic missile system. It required integrating advanced technologies in propulsion, guidance systems, and underwater launch capabilities-all under a tight schedule driven by Cold War pressures. The scale, technological innovation, and strategic importance of the project made traditional management techniques inadequate, prompting the need for a more sophisticated scheduling and risk assessment tool like PERT-a project management tool designed to handle uncertainty in scheduling. It helps break large, complex projects into smaller tasks and makes it easier to estimate how long each one will take. Instead of relying on a single guess for each task's duration, PERT uses three estimates: an optimistic time (if everything goes smoothly), a pessimistic time (if things go wrong), and a most likely time (based on past experience or expert judgment). These three points are used to build a simple statistical model that gives a more realistic picture of how long the project might actually take, especially when dealing with unpredictable or high-stakes work.

At the heart of PERT is the concept of a weighted average. Unlike traditional estimations that might rely on a single-point estimate, PERT incorporates the range of possible outcomes to provide a more balanced perspective. It calculates the expected time for a task by weighing the "most likely" outcome four times more than the optimistic and pessimistic estimates Concretely, if a project can be completed within an estimated optimistic time duration *O*, a most-likely time duration *M*, and a pessimistic time duration *P*, then PERT defines the expected time for the task or project to be (O + 4M + P)/6. As an example, if a task could most likely be completed in 5 days, or within 7 days in a worst-case scenario and within 2 days in a best-case scenario, then PERT predicts the task will take (2 + 4(5) + 7)/6 = 4.83 days to complete, which is a refinement of the initial "guess" that the project would probably take 5 days. The weighting used in the formula reflects a realistic skew toward the most likely scenario, while still acknowledging potential best- and worst-case extremes. The PERT formula weights the most likely outcome four times more

than the optimistic and pessimistic outcomes, a heuristic selected for its practical effectiveness in approximating the mean of a beta distribution [MRCF59]. While this 4:1:1 ratio is not mathematically derived and alternative weightings can be applied based on project requirements, it has proven reliable for managing uncertainty in project estimates.

But PERT is more than just a number, and the formula that computes an average is just the beginning of a larger technique often used by project managers. One typically computes the time estimates for each task in a project, which can then be arranged in a kind of flowchart that shows the sequence of tasks and how they depend on one another. (For instance, some cannot begin until others are completed.) This visual layout reveals how the entire project fits together. With the help of a few additional calculations, PERT then determines the total expected project duration and highlights the most critical tasks — those that, if delayed, would delay the entire project. These tasks make up what is known as the "critical path," and identifying them allows project managers to focus attention where it is most needed to keep the project on schedule.

Adding Flexibility with the Beta Distribution

Although PERT refers to the full technique just described, let's return our attention back to the average computed by the formula (O + 4M + P)/6. It turns out that this number is just a "snapshot" of a particular curve that shows the spread of possible task durations and how likely each one is. That curve is related to what's known as the beta distribution, a cousin of the familiar bell curve. Many people are familiar with the bell curve-a smooth, hill-shaped graph that shows most outcomes happening near the average, with fewer happening at the extremes. This shape works well when things tend to go as expected. But in real-life projects, things don't always follow such a neat pattern. Sometimes outcomes are more likely to be closer to the best case, or closer to the worst case, rather than right in the middle. That's why PERT is closely related to a different kind of curve, called the beta distribution, which can take on many shapes. Imagine the graph of a curve, where the x-axis represents different possible durations, and the y-axis shows how likely each one is. It can lean to one side, be flat, or even look like a bell curve-depending on how uncertain or skewed the estimates are. This flexibility makes it a natural fit for project planning, where it's important to consider a range of possibilities, not just the average outcome.

The beta distribution helps explain the reasoning behind the PERT formula, by modeling how task durations might be distributed. And unlike PERT's single number, the beta distribution provides a probability distribution that indicates the likelihood of various completion times, durations, or costs. This curve enables planners to visualize the "shape" of uncertainty, providing insight into how likely each estimate is. For example, in a project where most tasks are expected to finish close to the "most likely" time, the beta distribution might show a steep curve that peaks near this value. In projects with more unpredictable variables, the curve might be wider, indicating a greater spread of possible outcomes.

Consider a marketing team estimating the duration of a campaign. Using the beta distribution, they can visualize not only the most likely duration but also the probability of finishing sooner or later than expected. This model helps in setting realistic expectations and preparing for a variety of scenarios, improving both planning and communication with stakeholders.

The adaptability of the beta distribution allows for a more nuanced view of potential outcomes, making it invaluable for disciplines that demand precision. By showing the full range of possible results, the beta distribution provides decision-makers with a more informed perspective, allowing them to plan for contingencies and allocate resources effectively.

While the beta distribution provides a useful way to model uncertainty in individual task durations, as seen in the PERT method, when many such uncertain tasks are combined in a single project, it becomes harder to predict the outcome using just a formula. This is where our next topic—Monte Carlo simulation—offers a powerful alternative.

From Theory to Real-World Simulation: Monte Carlo Simulations

Monte Carlo simulations turn abstract math into something you can experiment with, thanks to the pioneering work of scientist Stanislaw Ulam in the 1940s. While working on complex probability problems during the development of nuclear technology, Ulam realized he could simulate thousands of random scenarios to see what kinds of outcomes might happen. This approach was later formalized and named "Monte Carlo" by Ulam and his colleague John von Neumann, inspired by the method's reliance on probability and its application to high-stakes decisionmaking.

In practice, Monte Carlo simulations involve running an experiment thousands of times, each under slightly different conditions, to capture many potential outcomes. By applying this method, we can approximate changes in the real-world and see the likelihood of various scenarios. Each simulation represents a subtle change in the environment, building a distribution that better reflects the uncertainty and complexity of real-life events.

Imagine you're a logistics planner responsible for estimating delivery times across multiple locations. Using Monte Carlo simulations, you could simulate thousands of delivery schedules, factoring in variables like traffic, weather, and staff availability. The result is a probability distribution that provides not just a single estimate but a comprehensive view of what's possible. This empowers users to make well-informed decisions with a clear understanding of potential risks and probabilities.

Understanding Monte Carlo Simulation: Simulating Thousands of Festivals

Introducing Markov Chains: Modeling Dependencies

Before diving into the details of Monte Carlo simulation, it's helpful to explore a related concept: Markov chains. Markov chains are powerful tools for modeling systems where each event depends on the outcome of the previous one. For example, in weather prediction, if it rains today, there may be a higher probability that it will rain tomorrow, creating a dependency between the weather on consecutive days. This makes Markov chains ideal for scenarios where outcomes are sequentially linked.

Imagine predicting the weather: on any given day, there is a 30% chance of rain and a 70% chance of sun. Using this probability distribution, you could generate random samples for each day of the week to simulate possible weather patterns, such as three sunny days on Monday through Wednesday and four rainy days for the rest of the week. This process is like Monte Carlo simulation, where each day's weather is sampled independently from the probability distribution. However, if you wanted to incorporate dependencies—such as the likelihood of rain tomorrow increasing to 60% if it rains today—you would use a Markov chain. Markov chains model these dependencies by adjusting probabilities based on the current state.

Independence in Project Estimation: Why Monte Carlo Fits Best

In our project estimation, we are assuming that the cost or duration of one task does not directly influence another in the same way. Instead, tasks and costs are treated as independent variables. This independence makes Monte Carlo simulation the more appropriate method. Unlike Markov chains, Monte Carlo simulation is designed to handle independent variables by repeatedly sampling from a defined probability distribution—such as the Beta distribution derived from PERT estimates—to estimate a range of possible outcomes.

Starting with Markov chains, we gain a broader understanding of how probabilities can be used to model different types of systems. While Markov chains are valuable for systems with dependencies, Monte Carlo simulation is better suited to project estimation because it simplifies the process, avoids unnecessary complexity, and focuses directly on capturing variability and uncertainty.

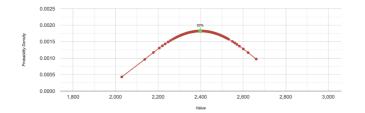
How Monte Carlo Simulation Works: From Beta to Real-World Variability

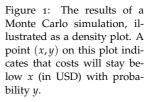
Monte Carlo simulation enhances estimation by using probabilities to predict a range of possible outcomes. Imagine Monte Carlo as simulating thousands of "possible festivals," where each festival represents a unique scenario for the total cost. The process begins by defining the key input variables — the optimistic cost (\$1,800, as an example), most likely cost (say, \$2,400), and pessimistic cost (for instance, \$3,000). These values, which were initially used to create the beta distribution, form the foundation of the simulation. The beta distribution is crucial because it provides a smooth probability curve, showing how likely costs are to cluster around the most likely value and taper off toward the extremes.

Insights from Monte Carlo Simulation: Confidence Levels and Practical Applications

Monte Carlo generates thousands of random "trials" using this beta curve. Each trial is a single simulated festival cost, determined by randomly sampling a value from the beta distribution. This sampling is done by generating a random number between 0 and 1, corresponding to a cumulative probability on the beta curve. That probability is then mapped back to a specific cost value within the range. For instance, a random number close to 0.5 would align with a cost near the peak (\$2,400), while numbers closer to 0 or 1 would correspond to values near \$1,800 or \$3,000. This process is repeated thousands of times, creating a large dataset of potential costs, each weighted by the probabilities defined by the beta curve.

After completing these trials, Monte Carlo builds a full probability distribution of costs. This distribution reveals not only the range of possible outcomes but also the frequency with which each cost occurs. In this example, most trials will cluster around \$2,400, reflecting the highest likelihood, while fewer trials will generate values near the extremes of \$1,800 or \$3,000. These results are often visualized as a density plot (Figure 1), with the *x*-axis representing costs and the *y*-axis showing probability density. For example, Monte Carlo calculates a 90% confidence level, indicating that there's a 90% chance costs will stay below \$2,772.17.





These values, derived from sorting and analyzing the simulation data, provide a clear boundary for planning, with the "approximate" value smoothed to reduce sensitivity to outliers.

Monte Carlo simulation transforms the beta distribution's theoretical probabilities into practical insights by simulating real-world variability. As already shown, this process begins with PERT, which provides a foundational estimate by balancing optimistic, most likely, and pessimistic scenarios. The beta distribution refines this further by creating a smooth probability curve that highlights the most likely outcomes while acknowledging variability. Monte Carlo builds on these methods, using the beta distribution to run thousands of simulated scenarios, offering planners a precise understanding of likely outcomes. Together, these methods empower decision-makers to allocate resources effectively, make informed plans, and prepare for uncertainties with confidence.

Making Advanced Techniques Accessible: The Google Sheets Plugin

While the mathematics of PERT, beta distributions, and Monte Carlo simulations may seem complex, their accessibility is enhanced through a user-friendly Google Sheets plugin, PMC Estimator, which enables individuals—regardless of mathematical background—to apply these methods to their projects [Ste].

All you need to do is enter your three estimates (optimistic, pessimistic, and most likely), and the plugin handles the rest. Within seconds, it runs the necessary calculations, generates the beta distribution, and performs Monte Carlo simulations, giving you a complete picture of the risks and probabilities involved in your project.

By democratizing these advanced techniques, we've made it possible for small business owners, project managers, and event organizers to plan with the same level of sophistication as large corporations. And because it is built into Google Sheets, it is easy to use and integrates seamlessly into existing workflows.

The Broader Impact: Transforming Project Management

The combination of PERT, beta distributions, and Monte Carlo simulations represents a shift in how we approach project management. These methods provide a level of precision and foresight that simply wasn't possible with traditional estimation techniques.

Imagine planning your next project with this kind of insight. Instead of guessing how long it will take or worrying about unexpected delays, you'd have a datadriven model that shows you the full range of possibilities. You'd be able to make informed decisions about where to allocate resources, how to communicate with stakeholders, and when to build in contingency plans.

Further Reading: Diving Deeper into the Math

For a deeper exploration of the mathematics behind PERT and Monte Carlo simulations, see my published work on statistically mitigating subjective duration estimates [Ste24] in the bibliography.

As technology continues to evolve, tools like PERT and Monte Carlo simulation will become even more accessible, empowering individuals and organizations to make data-driven decisions with confidence. By blending rigorous methods with user-friendly tools, we can transform estimation into an easy and accessible cornerstone of precision and innovation across disciplines.

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