

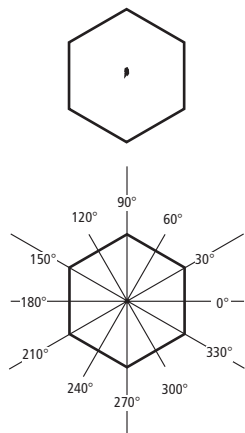
Snowflake geometry

Each snowflake that falls through the air has its own unique pattern of crystals. No two are exactly alike.

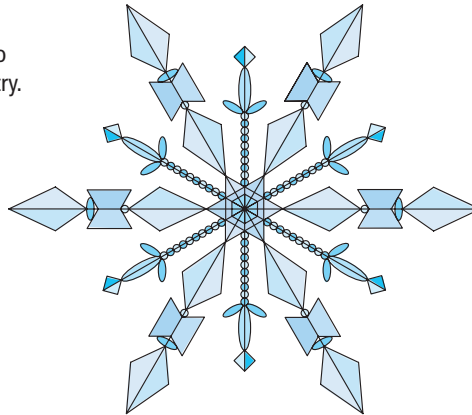
The shape of a regular hexagon forms the basis of most snowflakes. Mathomat can be used to create a snowflake design. Start with the hexagon (shape 4 or 7) and draw in lines of symmetry, using the protractor. Each shape you add to one arm must be repeated on all the others to maintain the symmetry. See 'Mr Symmetrical' (in the manual pages 32/33 and 5.3 on MAC).



Above: Each snowflake has a unique pattern of crystals. Below: A mathematical snowflake created on an isometric grid.



The protractor in your Mathomat can be used to mark the axes of symmetry.



Proving Pythagoras

Pythagoras' theorem

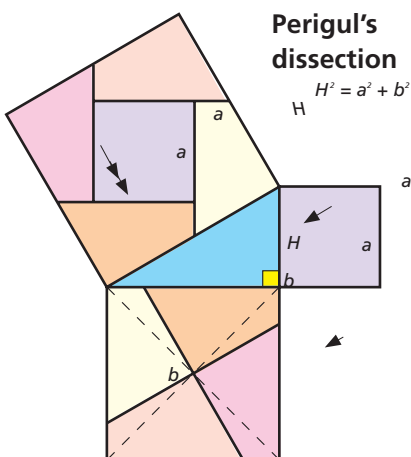
Pythagoras' theorem states that in any right-angled triangle, the area of the square of the hypotenuse (the longest side) is equal to the sum of the squares of the other two sides.

$$a^2 + b^2 = H^2$$

There are over 300 proofs of Pythagoras' theorem. With Mathomat you can draw many of them. Here are two (a further two are to be found in 'Pythagoras to the rescue').



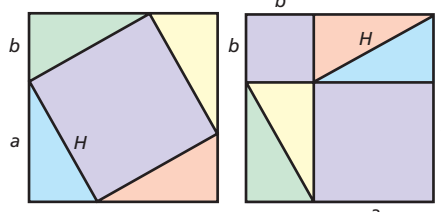
Pythagoras was born in Samos about 570 BC. The teaching of geometry was by word of mouth with drawings demonstrated in the sand. The theorems and proofs may have been lost if the mathematician Euclid had not collected them together in the 3rd century BC and written them down in a series of thirteen books called 'The Elements'.



Perigul's dissection

$$H^2 = a^2 + b^2$$

Another proof with shape 36
This illustrates that the area of the large square $(a + b)^2$ is the area of the purple square (H^2) plus the areas of the four triangles.



$$a^2 + 2ab + b^2 = H^2 + 2ab$$

$$a^2 + b^2 = H^2$$

Use shape 36, the right-angled triangle.

Pythagorean triads

To be a Pythagorean triad a set of three positive whole numbers must fit the rule: $a^2 + b^2 = H^2$. Here are some:

3	4	5	8	15	17
5	12	13	7	24	25
6	8	10	20	21	29

Fibonacci and the Golden Rectangle

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...

The special series of numbers above are called the *Fibonacci numbers*. They are named after the 13th century Italian mathematician Leonardo Fibonacci who recognised their importance. Each new term in the series is obtained by summing the two previous terms. Examples of the Fibonacci numbers occur frequently in nature and in art. For example, the number of clockwise and anti-clockwise spirals in many natural objects, from shells to pineapples, are pairs of Fibonacci numbers.

The Golden Rectangle

Shape 17 on Mathomat is very special, it is known as the Golden Rectangle. The sides of this rectangle have a ratio of 1 : 1.618, this is known as the *Golden Ratio*. Pairs of Fibonacci numbers approach this same ratio when the number on the right is divided by that to the left. Use a calculator to explore.

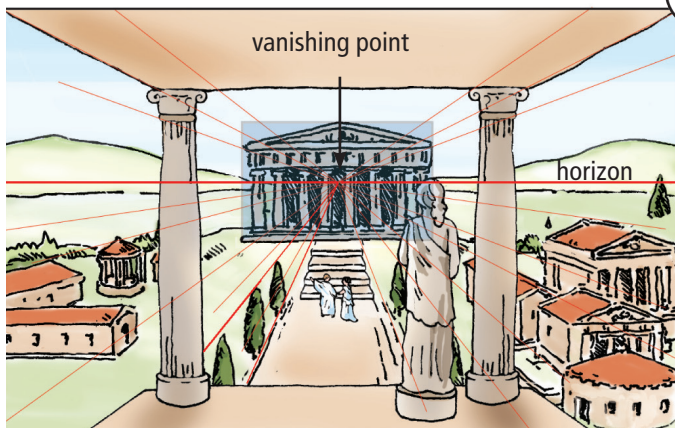
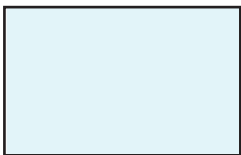
1/1	2/1	3/2	5/3	8/5	13/8	21/13	34/21	55/34	89/55	144/89	...
1.0	2.0	1.5									

The actual value of the Golden Ratio is a non-terminating decimal 1.618034... It is the positive root of the equation $x^2 - x - 1 = 0$, equal to $1 + \text{sq root of } 5 \text{ over } 2$.

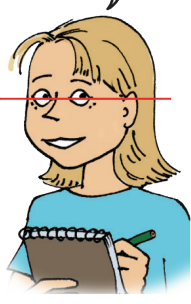
The Golden Rectangle and Art

The length of the Golden Rectangle is just $\frac{6}{10}$ longer than the width. This shape is considered to have the most pleasing of proportions, and many artists and architects have based their designs on it.

This is a 'one point' perspective study. Guide lines converge on a 'vanishing point' on the horizon line (or eye-level). Using these lines, objects and buildings can be drawn to give the illusion of depth. (In more complex drawings, there can be several vanishing points.)



In this sketch the Parthenon is framed by a Golden Rectangle (shape 17) to illustrate its proportions.



To 'scale' a rectangle draw a diagonal line through two opposite corners. Any rectangle (sharing the same starting corner) on that line will have the same proportions.

Leonardo Fibonacci
Italian mathematician, Leonardo Fibonacci (1175-1250) travelled widely and came into contact with Arab mathematics. In 1202 he published 'Liber Abaci' (Book of Calculation). This introduced the Hindu numerals (as used by the Arabs) to Western Europe. These are the numerals we use today. The book also contained a new mathematical problem about rabbits, the solving of which leads to generating the Fibonacci numbers.

Growing a Golden Rectangle
Using Mathomat shape 14 and pairs of Fibonacci numbers you can build a Golden Rectangle. Always add squares to the longest side. Do you see how the Fibonacci numbers make up each new rectangle?