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On Bubbles in Cryptocurrency Prices

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Abstract

This paper develops a tractable model for the cryptocurrency prices based on the classical framework for rational bubbles. In the baseline equilibrium, investors hold cryptocurrency to sell them to future users. In a bubble equilibrium, investors hold cryptocurrency because they expect its price to appreciate due to future investment inflows. We establish the mathematical relationship between net investment flows and the nominal return on a cryptocurrency's exchange rate. The net investment flows required to sustain a bubble equilibrium increase in new coin issuance, the required return and the level of transactional demand, and temporarily decrease when transactional demand expands.

Keywords: Asset pricing, Bitcoin, crypto assets, exchange rates, rational bubble

JEL Classification Numbers: E41, F31, G12.

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I. Introduction

Prominent voices have expressed notably diverse opinions to describe cryptocurrencies like Bitcoin. Skeptics have called Bitcoin *the mother of all bubbles* (Roubini, 2018) and *a Ponzi-scheme* (Welch, 2017; Carstens, 2018). Enthusiasts have called it *the flagship of a new asset class* (Harvey et al., 2021) and *digital gold* (Popper, 2016; Fink, 2024). The recent surge in the Bitcoin price to record-breaking levels—reaching over $100,000—has ignited the bubble debate about cryptocurrencies once again.

The differences between cryptocurrencies and traditional securities present significant challenges in determining the correct stance for financial analysts. Asset pricing theory establishes a clear conceptual framework to identify overvaluation for traditional securities: Compare the current price to the fundamental value, where the latter is defined as the sum of discounted cash flows. However, this approach does not seem particularly helpful when considering a cryptocurrency that does not pay any cash dividends, or, at least, not in terms of fiat currency.[1] Another important distinction is that cryptocurrencies experience transactional demand whereas financial securities generally do not (imagine the practical difficulties of sending a remittance by transferring the ownership of a fractional share). At the same time, cryptocurrencies are also different from fiat currencies in that cryptocurrencies are not used as a unit of account. How much cryptocurrency settles the bill is usually determined by the amount due in dollars or euros and the cryptocurrency's latest exchange rate.

[1]Some cryptocurrencies offer additional tokens to their holders, for example, as staking rewards. Staking rewards are more akin to a stock dividend than a cash dividend in that they provide the holder with more units of the same asset rather than a declared sum of fiat money.

Perspectives on cryptocurrencies are frequently founded on informal reasoning. This paper takes a formal approach to analyze what type of underlying beliefs could justify the perspectives on both sides of the bubble debate. The paper offers two key contributions. First, the paper develops a minimalist model for cryptocurrency prices based on the classical approach to rational bubbles of Blanchard (1979) and Blanchard and Watson (1982). This approach permits bubble equilibria where an asset appreciates solely because of a widely-held belief that the price will continue to increase.[2] The present paper tailors this classical approach to rational bubble to analyze cryptocurrencies that face transactional demand but are not used as a unit of account. Second, the paper establishes the mathematical relationship between net investment flows and the nominal return on a cryptocurrency's exchange rate. This mathematical relationship is implied by market-clearing rather than behavioral assumptions. The paper then utilizes the relationship to examine the aggregate payoffs to investors across different equilibria, in order to investigate the validity of the alleged parallels between cryptocurrencies and Ponzi schemes.

The model analysis starts by determining the exchange rate path that we will refer to as the *baseline equilibrium* (Proposition 1). The baseline exchange rate is defined as the lowest possible equilibrium price, conditional upon current and future user demand. The exchange rate in the baseline equilibrium can be driven by user demand, investor demand, or both. Investors in the baseline equilibrium may hold coins with the objective of selling them at

[2] The concept of rational bubbles in asset prices has been described as fundamentally flawed because such bubbles are said to eventually outgrow all other wealth in the world. This critique does not fully appreciate all theoretical results from the rational bubble literature. Theory predicts that the expected return of an asset of which the price contains a rational bubble component equals the required return in equilibrium, as is the case for all other financial assets. Suppose all financial assets were to face the same required return and a rational bubble were present in the price of a single asset. Then the value of that bubble asset as a share of total financial wealth would be constant in expectation (abstracting from capital distributions, new issuance, the emergence of new financial assets, etc). Moreover, if the bubble asset faced a below-average required return, for example, because investors perceive the asset as an insurance for bad states of the world, then its share in total financial wealth would be diminishing in expectation.

a profit to future users. Investors choose to do so only if they anticipate sufficiently high growth in user demand and sufficiently low growth in the number of coins. The exchange rate in the baseline equilibrium will depend on the view of investors regarding the future peak level of the discounted user demand per coin.

Some critics have claimed that bitcoins are worth zero. The baseline equilibrium indicates otherwise. It shows that a zero price cannot be an equilibrium outcome, provided that investors expect some nonzero user demand, either now or in the future. The only belief consistent with a zero price is the belief that the cryptocurrency does not face any user demand, nor that it will ever face any user demand in the future. For the major cryptocurrencies, such a belief seems inconsistent with real-world observations.[3]

The bottom line of the baseline equilibrium is that the current exchange rate can be explained by the future peak level of the discounted user demand per coin. If such an explanation is not possible, one might still claim that a cryptocurrency is a reasonable investment due to the anticipated future inflow of investors' funds. Such an outcome is indeed possible in the model, and is referred to as a *rational bubble equilibrium* (Propositions 2 and 3). The exchange rate of a cryptocurrency in a rational bubble equilibrium can be higher than that which can be explained by the future peak in transactional demand due to additional demand from investors. Investors in a rational bubble equilibrium choose to hold the cryptocurrency, not with the intend of selling it to future users, but because they expect it to appreciate as a consequence of a widely-held belief among investors that the price will continue to increase.

[3]For some examples of transactional demand for major cryptocurrencies, see the statistics provided by crypto payment processors that enable merchants to accept crypto payments (e.g. Bitpay, 2024), the empirical work by Von Luckner et al. (2023), or the fees paid for operating smart contracts on public blockchains (e.g., Coingecko, 2025).

A critical difference between the baseline equilibrium and a rational bubble equilibrium is the evolution in the share of coins held by investors. In the baseline equilibrium, the share of coins held by investors tends to decrease over time. Investors sell their coins to users as the user demand per coin approaches its discounted peak level. The share of coins held by investors converges to zero once the cryptocurrency reaches that peak. By contrast, in a rational bubble equilibrium, the share of coins held by investors tends to increase over time as the bubble persists. The reason is that users need progressively smaller amounts of coins to facilitate the same dollar amount of payments if investors trade the cryptocurrency at increasingly high prices. The reduction in the number of coins held by users implies an ongoing transfer to investors that persists as long as the bubble continues to grow.

After exploring the possible equilibrium price paths, we analyze the net investment flows that are implied by those price paths. We first establish the mathematical relationship between net investment flows and the nominal return on a cryptocurrency's exchange rate (Theorem 1). Based on this relationship, we show that sustaining a rational bubble equilibrium tends to require an ongoing net investment inflow to fund the ongoing shift in coin ownership from users to investors (Proposition 4).

The level of the net investment inflow necessary to sustain a bubble equilibrium depends on various factors. The anticipated coin purchases by investors in a bubble equilibrium must be sufficiently large to ensure that the cryptocurrency is expected to appreciate at the investors' required rate of return. A higher issuance of new cryptocurrency units increases the net investment inflow required to sustain a bubble equilibrium. The required net inflow of investors' funds will be temporarily lower, or may even turn negative, whenever there is growth in transactional demand. However, the required inflow to sustain a bubble equilibrium increases in the *level* of transactional demand. This holds true because a higher level of transactional demand is associated with a larger supply of cryptocurrency from users who need fewer units for transactional purposes when the cryptocurrency's price appreciates. Finally, the required net investment inflows to sustain a bubble equilibrium will be smaller if the required rate of return is lower (potentially because investors perceive the cryptocurrency as an insurance for bad states or "digital gold").

Some in the "bubble" camp have labeled cryptocurrencies such as Bitcoin a Ponzi-scheme. A Ponzi-scheme is an operation where the organizers pay returns to earlier investors from funds put into the scheme by later investors, who are then paid from funds contributed by even later investors, while the organizers skim off the scheme (SEC, 2024). A Ponzi-scheme is distinct from a cryptocurrency in the literal sense in that investors can choose to buy or sell them at the prevailing market price, but the payoffs could potentially display similarities in that the prolongation of a Ponzi-scheme requires a sustained inflow of aggregate investors' funds. To study the equivalence of payoffs of investors in a cryptocurrency and a Ponzi-scheme, we postulate a condition that we refer to as a Ponzi-scheme equivalence condition (Condition 1), which requires the future *aggregate* cash flows of investors from investing in the cryptocurrency to have a negative present value (even though it may still be a rational investment for an individual). We then verify which equilibria satisfy the Ponzi-scheme equivalence condition.

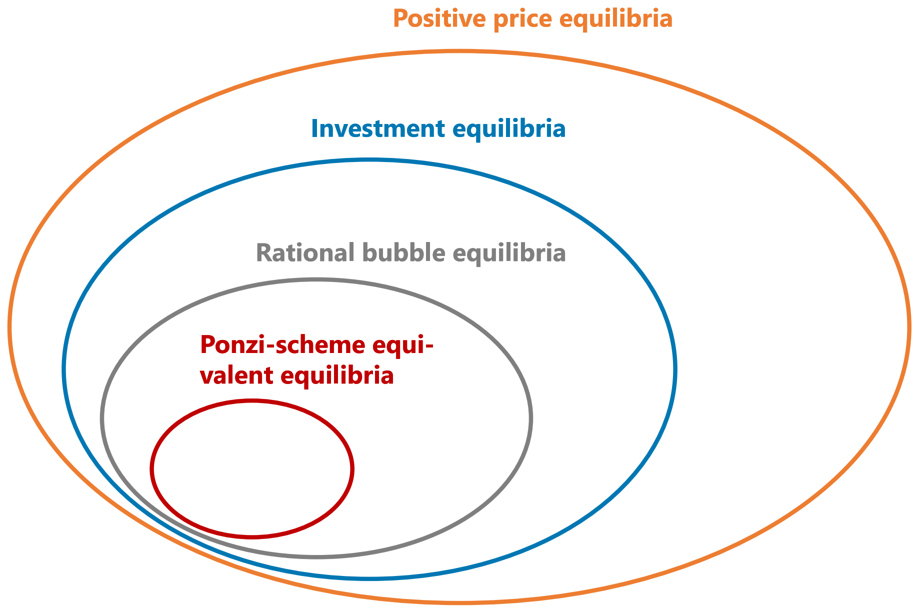
Cryptocurrencies do not satisfy the Ponzi-scheme equivalence condition if the exchange rate path follows the baseline equilibrium. The cash outflows that investors experience when they acquire cryptocurrency in the baseline equilibrium are balanced by expected cash inflows from selling cryptocurrency to future users. Investors benefit in expectation from holding the cryptocurrency, both individually and in the aggregate.

Investors in a cryptocurrency with a price path that follows a bubble equilibrium may experience aggregate cash flows that are equivalent to those from investing in a Ponzi-scheme (Proposition 5). This is the case if the following conditions jointly hold true: (1) the cryptocurrency has non-negative issuance, (2) the return required by investors is non-negative, and (3) the growth rate of user demand will be permanently lower than the required return at some point in the future. Conditions (2) and (3) are relatively standard, but condition (1) depends on the design of the supply of the cryptocurrency. Many cryptocurrencies satisfy this condition because of their fixed or increasing supply.

For cryptocurrencies with *negative money growth*, the relationship between bubble equilibrium paths and the Ponzi-scheme equivalance condition is more nuanced. Cryptocurrencies may face negative money growth, for example, because transaction fees that are paid with the cryptocurrency are partly burned (e.g., Ethereum). We show that, depending on the parameters, cryptocurrencies with negative money growth may follow a bubble equilibrium price path without satisfying the Ponzi-scheme equivalence condition (Propositions 6 and 7). The inflow of funds to support the price in such bubble equilibria comes from users who replenish their balances. A cryptocurrency with negative money growth is less likely to satisfy the Ponzi-scheme equivalence condition on a bubble equilibrium path if it experiences higher user growth and if the money growth is more negative.

The Euler diagram in Figure 1 summarizes the possible equilibrium price paths in the model. Depending on the properties of the cryptocurrency, there may be an equilibrium where exclusively users choose to hold cryptocurrency as well as investment equilibria where

Figure 1: Possible Equilibria for the Exchange Rates of Cryptocurrencies



Note: This Euler diagram shows the nested relationships among various sets of equilibrium price trajectories for cryptocurrencies. The areas are not meant to accurately reflect the prevalence of equilibria in practice.

some agents hold the cryptocurrency purely for financial gain. In the investment equilibrium that correspond to the baseline equilibrium, investors hold tokens to sell them at a profit to future users. The other investment equilibria are rational bubble equilibria. Such bubble equilibria can, but do not have to, exhibit a payoff equivalence to Ponzi-schemes in that the prolongation of the bubble will require a sustained inflow of aggregate investors' funds.

The remainder of this paper is organized as follows. Section II discusses related literature. Section III introduces the model. Sections IV and V characterize, respectively, the baseline equilibrium and the bubble equilibria. Section VI shows the mathematical relationship between net investment flows and cryptocurrency returns. Based on this result, the section analyze in the different equilibria relate to the Ponzi-scheme equivalence condition. Section VII generalizes the results regarding investment flows in bubble equilibria and their relationship to investors' payoffs in Ponzi-schemes to a model where user demand responds to the expected return on holding the cryptocurrency. Section VIII provides concluding remarks. Proofs are in the appendix.

II. Related Literature

The present paper relates to a fast-growing theoretical literature on the exchange rates of cryptocurrencies (Athey et al., 2016; Bakos and Hałaburda, 2022; Biais et al., 2023; Chiu and Koeppl, 2022; Cong et al., 2021, 2022; Garratt and Van Oordt, 2023; Gryglewicz et al., 2021; Lee and Parlour, 2022; Pagnotta, 2022; Sockin and Xiong, 2023a,b). User demand in those models is driven by users who derive either transactional or utility benefits from holding the cryptocurrency. Some models for cryptocurrencies also explicitly incorporate the demand by forward-looking investors to analyze the dynamics of cryptocurrency prices (Bolt and Van Oordt, 2020; Canidio, 2023; Garratt and Van Oordt, 2022, 2024; Karau and Moench, 2023; Kogan et al., 2024; Malinova and Park, 2023; Prat et al., forthcoming; Wei and Dukes, 2021). The theoretical environments in those papers, with two exceptions, do not allow for the classical rational bubble equilibria driven by expectations of investor as explored by, among others, Blanchard (1979), Blanchard and Watson (1982) and Tirole (1982, 1985).[4]

The aforementioned exceptions that allow for rational bubble equilibria are the noteworthy studies by Wei and Dukes (2021) and Canidio (2023). Wei and Dukes (2021) focus on the impact of rational bubble equilibria on user adoption in an environment where the required return equals zero. They find that bubble equilibria may accelerate adoption of a cryptocurrency by regular users. Canidio (2023) shows that rational bubble equilibria can raise the revenue of issuing tokens beyond the level of selling products directly using fiat

[4]See Brunnermeier and Oehmke (2013) and Miao (2014) for excellent surveys of the bubble literature.

currency even in an environment with exogenous product demand. Differently from those studies, we focus on the investor inflows that are necessary to a sustain rational bubble equilibria. Our environment also permits for nonzero required returns, which is instrumental when comparing the relationship between the present value of investors net inflows for various potential price equilibria to Ponzi-schemes. The comparison reveals among others that bubble equilibria may exhibit payoffs to investors that are, in the aggregate, equivalent to Ponzi-schemes, but do not necessarily have to.

Brunnermeier et al. (2020) study bubbles on government bonds in a general equilibrium model that also includes an intrinsically useless alternative asset, which they refer to as a cryptocoin (but which could be any intrinsically useless asset). The alternative asset in their model faces no user demand and, hence, differs from that in the aforementioned papers which model the cryptocurrency price as a function of user demand stemming from transactional or utility benefits. In the present paper, the baseline equilibrium price of such a zero user demand asset would be zero and, hence, any positive price of such an asset would fully reflect a bubble component, which is in line with terminology used by Brunnermeier et al. (2020).

The volatile exchange rates of cryptocurrencies also have been a popular subject in the empirical literature (Yermack, 2015). The search query for "Bitcoin AND Bubble" on Google Scholar returns no less than 27,000 results, with many of those papers analyzing the empirical price trajectories of cryptocurrencies. Cheah and Fry (2015) and Cheung et al. (2015) apply the methodology of Phillips et al. (2015) to detect explosive paths on Bitcoin prices. Later studies vary in terms of statistical methods and data. Chaim and Laurini (2019), Geuder et al. (2019) and Cretarola and Figà-Talamanca (2020) apply alternative statistical methods on the price series of Bitcoin and Ethereum. Hafner (2020) considers a larger set of cryptocurrencies while accounting for time-varying volatility. Li et al. (2022) find media attention to be associated with higher future returns when bitcoin prices follow a bubbly price trajectory. Enoksen et al. (2020) relate trading and transaction volume to explosive price trajectories. Corbet et al. (2018) assesses the explosiveness of non-price series such as the block size and mining power. Lambrecht et al. (forthcoming) collect experimental evidence on whether the limited new issuance can fuel price bubbles in proof-of-work cryptocurrencies. The present paper offers various theoretical insights into testable factors that may render cryptocurrencies more or less susceptible to explosive price paths in experiments or empirical data. These are discussed in greater detail in the concluding remarks.

III. Model

The model is in the tradition of the classical approach to rational bubbles by Blanchard (1979) and Blanchard and Watson (1982). This partial equilibrium approach builds off from two main attributes: A rational expectations asset market model, which determines when investors are willing to hold an asset, and a market-clearing condition. We tailor this approach to build a tractable model for bubbles in cryptocurrency prices.

Time is discrete and denoted by $t = 0, 1, 2, ...$ We denote the exchange rate of the cryptocurrency in terms of dollars at time t by S_t. The exogenous number of units of the cryptocurrency at time t is denoted by $M_t > 0$. The demand for a cryptocurrency consists of investment demand and user demand.

The aggregate number of cryptocurrency units held for investment purposes is denoted by $Z_t \geq 0$. We impose a non-negativity constraint on the investment position reflecting the idea that investors in the aggregate cannot bring additional cryptocurrency units into existence, even though individual investors could maintain long or short positions. Let $r > 0$ denote the required nominal return on capital for investment holdings in the cryptocurrency. Investment demand is governed by the following simple rational expectations asset market model.

Assumption 1 (Rational expectations market model) *Investors adjust their investment holdings such that, for any t where $Z_t > 0$,*

$$\mathbb{E}_t\left(S_{t+1}\right) = (1+r)S_t. \tag{1}$$

Moreover, investors do not hold the asset, so that $Z_t = 0$, whenever $\mathbb{E}_t\left(S_{t+1}\right) < (1+r)S_t$.

The $\mathbb{E}_t$-notation is used as short-hand for the expectation conditional upon the information set available at time t, which is assumed to be common to all agents.[5]

Assumption 1 reflects the idea that risk-neutral investors would purchase additional units of a cryptocurrency if the expected return were higher than the required return. Buying pressure from investors would drive up the current exchange rate until (1) holds true, after which there are no incentives for investors to further adjust their holdings. If the expected return were less than the required return, then selling pressure from rational investors would put downward pressure on the current exchange rate. The investors would continue to sell until either (1) holds true or $Z_t = 0$.

The user demand in terms of dollars is denoted by $X_t^{\$} \geq 0$. The model is agnostic regarding the precise source of user demand. The primary interpretation of is the transactional

[5]The information set Ω_t contains at least the sequences $(M_t, M_{t+1}, M_{t+2}, ...)$ and $(X_t^{\$}, X_{t+1}^{\$}, X_{t+2}^{\$}, ...)$ as well as the current and past values of the exchange rate S_t and the past information set Ω_{t-1}. For more rigorous notation; see, e.g., Blanchard (1979) and Tirole (1982).

demand for a cryptocurrency that is used as a means of payments but not as a unit of account (Bolt and Van Oordt, 2020; Prat et al., forthcoming). If individuals use a cryptocurrency to make payments for a total dollar-amount of $T_t^\$$ dollar, and if the cryptocurrency units that are used to making payments have an average velocity of V_t^*, then this implies a transactional demand of $X_t^\$ = T_t^\$/V_t^*$ dollar.[6] In some situations, a cryptocurrency is used as the exclusive means of payment to transact on a particular platform, for example to execute smart contracts on a blockchain network, in which case the platform's performance would be an important driver of the transactional demand (Cong et al., 2021; Gryglewicz et al., 2021). The path of the user demand in terms of dollars is assumed to be exogenous, in line with the exogenous cash dividend path in the classical literature on rational bubbles in securities prices. We relax this assumption in Section VII.[7]

The market-clearing condition requires the total value of all units to reflect the combined value of units held to satisfy user demand and the units held purely for investment purposes.

Assumption 2 (Market-clearing condition) *For any t, we have that*

$$M_t S_t = X_t^\$ + Z_t S_t. \tag{2}$$

The final assumption reflects the idea that users can always dispose of cryptocurrency without incurring any costs, a process that is sometimes referred to as *burning* cryptocurrency.

[6]See Bolt and Van Oordt (2020) for more details; here, we intentionally use the same notation.

[7]Section VII relaxes the assumption on the exogenous user demand by allowing the user demand to respond to the expected rate of appreciation. The exogenous user demand helps in terms of tractability by creating a dichotomy between user demand and investment demand. Relaxing the assumption is inconsequential for what are arguably the most novel results in the present paper (Theorem 1, Propositions 5, 6 and 7 and Corollary 1).

Assumption 3 (Free disposal) *Individuals can dispose of cryptocurrency units at no cost so that $S_t \geq 0$ for any t.*

The model setup is closed with the following equilibrium definition.

Definition 1 *An equilibrium path for the exchange rate is defined as any path $\mathbb{E}_0(S_0), \mathbb{E}_0(S_1), \mathbb{E}_0(S_2), ...$ that satisfies Assumptions 1-3 for given sequences $(M_0, M_1, M_2, ...)$ and $(X_0^\$, X_1^\$, X_2^\$, ...)$.*

An attentive reader may have noticed that the framework does not impose a transversality condition on the exchange rate. There no requirement that the present value of the future exchange rate converges to zero as the horizon extends to infinity, and, hence, the model permits outcomes where $\lim_{t\to\infty} \mathbb{E}_0(S_t)/(1+r)^t \neq 0$. This is on purpose. Otherwise, rational bubble equilibria would be ruled out a priori, which would prevent us form studying the characteristics of such equilibria (as noticed by, e.g., Miao, 2014). Importantly, we will also see that there are specific circumstances under which all equilibria, even those without rational bubbles, will be ruled out if a transversality condition were imposed on the exchange rate.

IV. Baseline Equilibrium

Traditional rational bubble models for asset prices typically derive an equilibrium with a fundamental value that reflects the present value of expected cash flows. The fundamental value in such models reflects the floor for the equilibrium value of the asset if negative price bubbles are ruled out (in our model, negative price bubbles are ruled out as a consequence of Assumption 3). A fundamental value in the sense of the present value of discounted cash flows does not apply directly to cryptocurrencies that do not pay cash dividends. However, we can still derive a floor for the equilibrium exchange rate path of cryptocurrencies. We refer to this path as the baseline equilibrium path.

It will be convenient to rewrite the market-clearing condition in the form of an equation for the exchange rate.

Lemma 1 *At any time t where $Z_t < M_t$,*

$$S_t = \frac{X_t^{\$}}{M_t - Z_t}. \tag{3}$$

The equation reflects the intuitive notion that a higher exchange rate may follow from higher user demand (i.e., transactional demand or utility demand), a lower number of issued tokens, or a higher investment demand. The lemma also implies a hypothetical reference level that corresponds to the lowest possible level of the exchange rates since $Z_t \geq 0$. The lowest possible level of the exchange rate is that where $Z_t = 0$ and equals $X_t^{\$}/M_t$. This reference level does not necessarily correspond to an equilibrium. Investors may have incentives to purchase units of the cryptocurrency if they anticipate a sufficiently strong appreciation of the exchange rate. Nonzero investment demand would raise the exchange rate above the hypothetical reference level.[8]

[8] It is worth noting that, if one were to have empirical data on the number of tokens that are not held to satisfy user demand, then one could calculate the value of the hypothetical reference level from the actual exchange rate and the total number of coins as $X_t^{\$}/M_t = S_t(M_t - Z_t)/M_t$. In practice, precise empirical data on Z_t are not available since it requires knowledge of motives for token ownership. Proxies for Z_t based on blockchain data do exist, such as the share of coins held in *dormant* addresses (i.e., that do not transact for an extended period) or the amount of coins held in the largest addresses. Such proxies suggest that a large share of the holdings of major cryptocurrencies do not involve user demand (Garratt and Van Oordt, 2023, Figure 2). The hypothetical reference level for those cryptocurrencies must be substantially below their current exchange rates.

To find the floor for the *equilibrium* exchange rate of a cryptocurrency, we first define $\tau(1)$ as the time at which the discounted value of the hypothetical reference level without investors, $X_t^{\$}/M_t$, is maximized, i.e.,

$$\tau(1) =: \inf \operatorname*{argmax}_{t\in\mathbb{N}} \frac{X_t^{\$}/M_t}{(1+r)^t}. \tag{4}$$

The $\mathbb{N}$ corresponds to all positive integers including zero. If there are multiple $t \in \mathbb{N}$ that maximize the argument, then the infimum-operator ensures that $\tau(1)$ corresponds to the time at which the maximum occurs first. At $t = \tau(1)$, we repeat the same procedure to define $\tau(2)$ as the next point in time that maximizes the discounted value of the reference level for the exchange rate. Repeating this procedure yields the sequence $(\tau(1), \tau(2), \tau(3), ...)$ that corresponds to the points in time where the future discounted value of $X_t^{\$}/M_t$ is maximized from the perspective of, respectively, time $t = (\tau(0), \tau(1), \tau(2), ...)$ where $\tau(0) \triangleq -1$. Formally, the values of $\tau(n)$ for $n \in \mathbb{N}$ are defined as

$$\tau(n) =: \inf \operatorname*{argmax}_{t\in\mathbb{N}^{>\tau(n-1)}} \frac{X_t^{\$}/M_t}{(1+r)^t}. \tag{5}$$

Given the sequence $\tau(n)$, we obtain the path for the exchange rate under the baseline equilibrium in the following proposition.

Proposition 1 (Baseline equilibrium) *The lowest possible level of the exchange rate on an equilibrium path for any t such that $\tau(n-1) < t \leq \tau(n)$ is*

$$S_t^* = \frac{X_{\tau(n)}^{\$}/M_{\tau(n)}}{(1+r)^{\tau(n)-t}}.$$

Proof. See Appendix A. ■

The solid green line in the upper panel of Figure 2 provides an illustration of the exchange rate path under the baseline equilibrium where the baseline equilibrium corresponds to an investment equilibrium. The dashed black line reflects the exogenous evolution of the hypothetical reference level without investors.[9] The initial exchange rate under the baseline equilibrium is higher than the reference level, because investors hold the tokens in expectation of higher future demand by users.

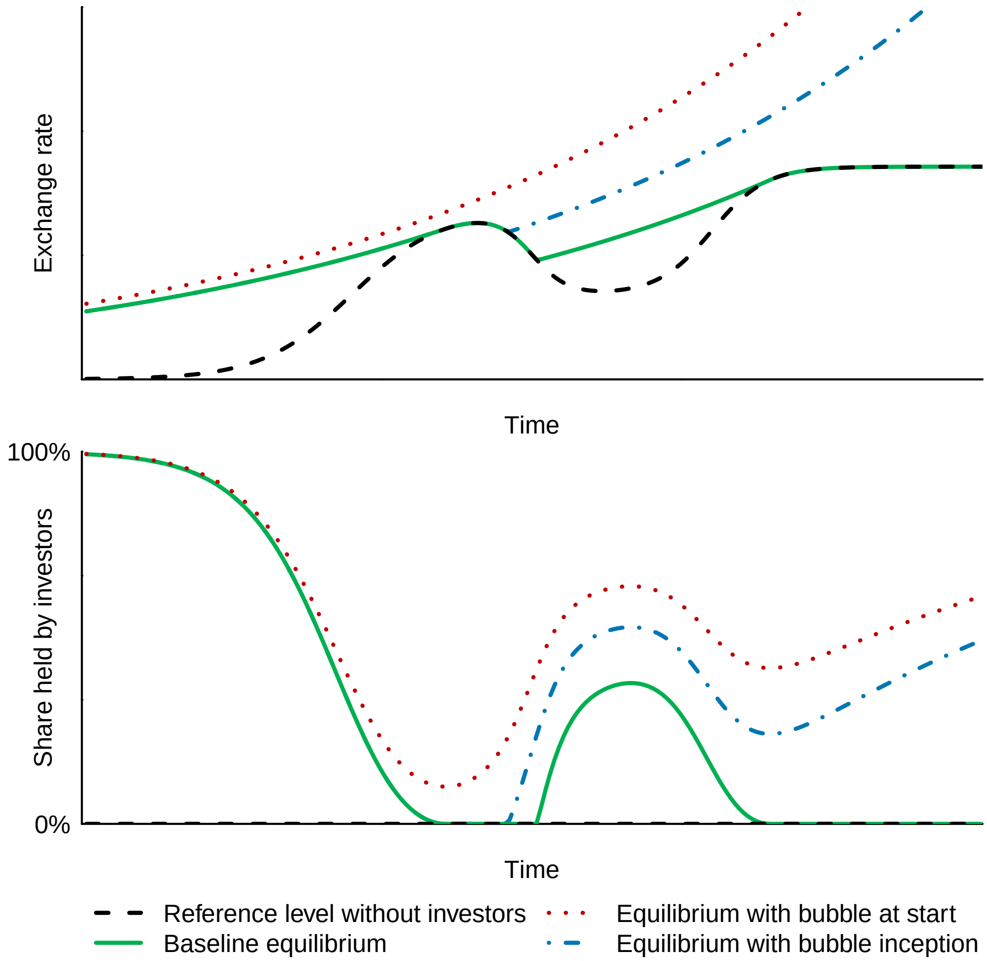
The exchange rate on the baseline equilibrium path equals to the reference level $X_t^{\$}/M_t$ at any $t = \tau(n)$. At such points in time, all cryptocurrency units are held by users and no units are held by investors. The equilibrium share of coins held by investors can be calculated from the equilibrium exchange rate as $Z_t/M_t = 1 - X_t^{\$}/(M_t S_t^*)$. Initially, investors hold almost all the coins as shown in the bottom panel of the figure. The share of coins held by investors converges to zero in the run-up to the next peak in the discounted level of the user demand per coin (i.e., when t approaches the next value in the sequence of $\tau(n)$).

If the peak value of the discounted user demand per coin occurs at $t = 0$ (i.e., $\tau(1) = 0$), then the baseline equilibrium is characterized by zero initial investment demand so that the equilibrium corresponds to a positive price equilibrium but not an investment equilibrium in Figure 1. Otherwise, some cryptocurrency units will be held by investors so that the baseline equilibrium corresponds to an investment equilibrium.

The exchange rate of a cryptocurrency in the baseline equilibrium and the fundamental price of a security have crucially different relationships to their underlying values. For a

[9] The double-hump shaped path for the reference level in the upper panel of Figure 2 illustrates a scenario with strong initial growth and then a temporary decline in user demand. The path is arbitrary and was generated using a combination of three logistic functions.

Figure 2: Exchange Rate and Investment Share in Equilibrium



security, the material underlying values are the discounted cash flows over the entire lifetime of the security. The fundamental price is calculated as the *sum* of the discounted cash flows of the security. For a cryptocurrency, the material underlying value is the discounted user demand per coin. The current exchange rate under the baseline equilibrium is determined by the *peak value* of the discounted user demand per coin. This peak value occurs at $t = \tau(1)$ and, hence, the initial exchange rate at $t = 0$ under the baseline equilibrium equals $(X^{\$}_{\tau(1)}/M_{\tau(1)})/(1 + r)^{\tau(1)}$.

Finally, there are direct implications of Proposition 1 for equilibrium existence. The baseline equilibrium requires $\tau(1)$ to be finite. In general, this will be true as long as the growth rate of user demand per coin stabilizes at a level that is less than the required return at some point in the future. If the baseline equilibrium—which was defined as the lowest possible equilibrium exchange rate path—cannot exist, then no equilibrium can exist. In other words, the existence of *any* equilibrium requires $\tau(1)$ to be finite.[10]

V. Bubble Equilibria

The exchange rate under the baseline equilibrium in Proposition 1 is based on one particular solution of the difference equation in Assumption 1. Also other solutions to the difference equation that reflect equilibria exist. In particular, at $t = 0$, the exchange rate may be higher due to the presence of a rational bubble component that is stacked on top of the baseline equilibrium. Such equilibria with a bubble component from the outset are characterized in the following proposition.

Proposition 2 (Rational bubble equilibrium) *Any path for the expected exchange rate that satisfies*

$$\mathbb{E}_0 S_t = (S_0^* + B)(1 + r)^t$$

for $B \geq 0$ and any $t \in \mathbb{N}$ constitutes an equilibrium path.

[10] An example where this condition does not hold true is that where $X_t^{\$}/M_t$ grows indefinitely at a constant rate $g > r$. This is analogous to the familiar result regarding the nonexistence of the fundamental equilibrium price of a security with a cash dividend that grows indefinitely at a constant rate $g > r$.

The initial exchange rate in a bubble equilibrium characterized by the proposition consists of two components: The exchange rate under the baseline equilibrium in Proposition 1 plus a non-negative rational bubble component, $B \geq 0$.

An illustration of an equilibrium with a bubble component in the initial exchange rate is provided by the red dotted line in the upper panel of Figure 2. The exchange rate and the share of cryptocurrency held by investors under this equilibrium are larger than in the baseline equilibrium, and the divergence tends to increase over time. Investors have incentives to hold the cryptocurrency despite of the bubble component because the cryptocurrency is expected to continue to appreciate at a rate that equals the required return on capital. The share of coins held by investors in a bubble equilibrium slowly converges to all coins in circulation as shown by the red dotted line in the lower panel of Figure 2, even though temporary growth in user demand can lead to a short-term reduction in the share held by investors.

The expected exchange rate of a cryptocurrency on a bubble equilibrium path in Proposition 2 depends on the required return on capital and the initial level of the exchange rate. Proposition 2 permits bubble exchange rate paths that contain random elements—it describes the exchange rate path in terms of expectations. Blanchard and Watson (1982) suggest an intuitive example of a bubble equilibrium where the exchange rate grows at a faster speed than the required return as long as the bubble persists, but where there is a positive probability that the bubble will burst. The expected return on such a stochastic bubble path still equals the required return because the higher rate of appreciation is counterbalanced by the downside risk. Alternative stochastic bubble paths with more complex dynamics are possible too.

The previous proposition covers the case in which there is a rational bubble component from the outset. If there was no rational bubble component from the outset, then a rational bubble component to the price could still commence in the future if the expected return in the baseline equilibrium is sufficiently low. The following proposition characterizes the equilibria that involve the inception of a bubble in cryptocurrency prices.

Proposition 3 (Rational bubble inception) *Suppose* $S_t = S_t^*$ *at* $t = \tau(n)$ *for some given value* $n \in \mathbb{N}$. *Then any path for the expected exchange rate that satisfies*

$$\mathbb{E}_{\tau(n)} S_t = (S^*_{\tau(n)+1} + B)(1+r)^{t-\tau(n)-1}$$

for B *s.t.* $0 \leq B \leq S^*_{\tau(n)}(1+r) - S^*_{\tau(n)+1}$ *and all* $t > \tau(n)$ *constitutes an equilibrium path.*

An illustration of an bubble equilibrium path that involves the inception of a bubble in the exchange rate is given by the blue dash-dotted line in Figure 2. The exchange rate under the baseline equilibrium is on a downward trajectory when the inception of the bubble occurs, and, hence, expected returns in the baseline equilibrium are less than the required return. This is important because the size of a bubble at inception as measured by B cannot be so large such that the expected return on the cryptocurrency's exchange rate would exceed the required return in equilibrium. As a consequence, the expected return in the baseline equilibrium must be less than the required return to allow for the possible inception of a bubble in equilibrium. Otherwise, the possible inception of the bubble with non-zero probability would raise the expected return above the required return, which cannot be an equilibrium outcome.[11]

[11] Proposition 3 extends the seminal result of Diba and Grossman (1987) to cryptocurrencies. Diba and Grossman show that the inception of a rational bubble in security prices cannot be an equilibrium outcome

VI. Sustaining a Bubble Equilibrium

The previous sections explored possible equilibrium price paths. This section explores the net investment flows that are implied by the equilibrium price paths and their relationships to Ponzi-schemes.

We first establish the net investment flows relate to the nominal return r_{t+1} on a cryptocurrency's exchange rate. By using the market-clearing condition at time t and $t+1$, we obtain the following result.

Theorem 1 *The nominal return on a cryptocurrency's exchange rate from time t to $t+1$ equals rate r_{t+1} if and only if*

$$\underbrace{\Delta Z_{t+1} S_{t+1}}_{\substack{\text{Value of}\\ \text{additional units}\\ \text{held by investors}}} = \underbrace{\Delta M_{t+1} S_{t+1}}_{\substack{\text{Value of}\\ \text{newly issued}\\ \text{units}}} + \underbrace{r_{t+1} X_t^{\$}}_{\substack{\text{Appreciation}\\ \text{of units held}\\ \text{by users}}} - \underbrace{\Delta X_{t+1}^{\$}}_{\substack{\text{Change in}\\ \text{user}\\ \text{demand}}}. \tag{6}$$

Proof. The proof relies exclusively on the market-clearing condition in Assumption 2. Appendix B provides details. ■

The ΔZ_{t+1} is the change in the number of cryptocurrency units held by investors in the aggregate. The value of the additional units held by investors, $\Delta Z_{t+1} S_{t+1}$, is interpreted

because, in the absence of a bubble, the expected rate of return must already equal the required return. A nonzero probability for the start of a new bubble would raise the expected return of a security above the required return, which cannot be an equilibrium outcome. Similarly, the inception of a rational bubble in cryptocurrency prices cannot be an equilibrium outcome whenever the expected rate of return in the baseline equilibrium equals to the required return (i.e., whenever investors are willing to hold the cryptocurrency). In the baseline equilibrium, there can be periods where the expected return is less than the required return so that exclusively users are willing to hold the cryptocurrency. By the definition of $\tau(n)$, this can be the case for the expected return over the holding period from $t = \tau(n)$ to $t = \tau(n) + 1$. The possible inception of a rational bubble at $t = \tau(n) + 1$ does not raise the expected return above the required return provided that the initial expected size of the bubble is limited in size, i.e., B s.t. $0 \leq B \leq S^*_{\tau(n)}(1 + r) - S^*_{\tau(n)+1}$.

as the net investment flow.[12] The net investment flow associated with a cryptocurrency return r_{t+1} depends on three components. The first component is the value of newly issued cryptocurrency units. The larger the value of newly issued units, the larger the net inflow of investors' funds for a given return r_{t+1}. The second component stems from the existing user demand. The cryptocurrency return r_{t+1} affects the value of coins held to satisfy user demand. If the return was positive and the user demand in terms of dollars is still the same, then users must have sold coins. Similarly, if the return is negative and the user demand in terms of dollars is still the same, then users have bought coins. The amount of coins sold by users to investors because of this channel depends on the product between the return r_{t+1} and the existing user demand $X_t^{\$}$. The third component arises from the change in user demand. An increase in user demand reduces the number of coins investors have to acquire cryptocurrency for a given cryptocurrency return r_{t+1}. The required net flow could even turn negative if the increase in user demand is sufficiently large.

Based on the result in Theorem 1, we can calculate the expected net investment inflow that is necessary in order to sustain the rational bubble equilibria characterized in Propositions 2–3.

Proposition 4 *A rational bubble equilibrium for a cryptocurrency can persist if and only if*

$$\mathbb{E}_t \Delta Z_{t+1} S_{t+1} = \mathbb{E}_t \Delta M_{t+1} S_{t+1} - \Delta X_{t+1}^{\$} + r X_t^{\$}. \tag{7}$$

Proof. See Appendix C. ■

[12]The notation uses the convention where the difference operator Δ takes precedence over multiplication in the order of operations. For example, $\Delta Z_{t+1} S_{t+1}$ denotes $(Z_{t+1} - Z_t) S_{t+1}$.

A nonzero bubble component requires the expected return to equal r. Consistency requires that the expected investment inflows correspond to the levels associated with achieving such a return in expectation. A bubble equilibrium path implies an expectation of a continued appreciation of the coins held by users, affecting the third component in Theorem 1. Investors need to acquire the coins that users continue to sell in order to sustain the equilibrium. The required net investment inflow in (7) will be lower if investors require a lower return on investments in the cryptocurrency because they perceive it as a digital gold that provides insurance for bad states of the world.

The result in Proposition 4 hints that there may be conditions under which a bubble exchange rate paths may require a continuous inflow of investors funds, like a Ponzi-scheme. A Ponzi-scheme is distinct from a cryptocurrency in the literal sense in that investors can choose to buy or sell them at the prevailing market price, but it could display similarities in that the prolongation of a Ponzi-scheme requires a sustained inflow of aggregate investors' funds like certain equilibria for cryptocurrency prices. We explore the relationship between bubble exchange rate paths and Ponzi-schemes by postulating the following Ponzi-scheme equivalence condition regarding the cash flows for investors.

Condition 1 (Ponzi-scheme equivalance) *The remaining cash flows for investors in the aggregate have, or will have, a negative present value at some point in time in the future, that is,*

$$-\sum_{i=1}^{\infty} \frac{\mathbb{E}_T \Delta Z_{T+i} S_{T+i}}{(1+r)^i} < 0 \quad \textit{for some} \quad T \geq 0.$$

The condition sums all the expected aggregate cash flows to investors at some point in the future and requires the present value to be negative. The minus-sign in front of the sum accounts for the fact that net sales by investors—cash inflows from the perspective of the investors—correspond to negative values of ΔZ_{T+i}. Net purchases—cash outflows for investors—correspond to positive values of ΔZ_{T+i}. A cryptocurrency satisfies the Ponzi-scheme equivalence condition for example if its equilibrium exchange rate path requires a continuous inflow of investors funds: This would imply $\Delta Z_{T+i} > 0$ for all $i \in (1, 2, ...)$, and, hence, violate Condition 1. The condition requires that there will be some point in time where the equilibrium persists even though the remaining discounted cash flows for investors from coin sales will be less than the discounted cost of their remaining coin purchases.

A. Non-negative Money Growth

We first consider aggregate payoffs of investors in bubble equilibria for cryptocurrencies that exhibit non-negative money growth ($\Delta M_t \geq 0$ for any $t > T$). Denote the growth rate of user demand by $g_t = (X_t^{\$} - X_{t-1}^{\$})/X_{t-1}^{\$}$. Consider a cryptocurrency for which the growth stabilizes at some point in the—potentially distant—future such that the user growth rate will be lower than the required return on capital, i.e., $g_t < r$ for any $t > T$ given some T. The following proposition summarizes our result for such cryptocurrencies.

Proposition 5 *Consider a cryptocurrency with non-negative issuance $\Delta M_t \geq 0$ and for which the nonzero user demand stabilizes at some distant point in the future such that the growth rate $g_t < r$ for any $t > T$ given some $T \geq 0$.*

The cryptocurrency satisfies the Ponzi-scheme condition if its exchange rate follows a bubble equilibrium path.

Proof. See Appendix D. ■

The intuition for this result is that an ongoing appreciation of the exchange rate is only sustainable if investors purchase cryptocurrency from users, who need fewer and fewer coins as the exchange rate continues to increase. Sustained user growth alleviates the need for investors to purchase units from users, but the user growth will be insufficient to completely avoid the need for investors to purchase coins if the growth rate is less than the required return of capital ($g_t < r$). The continued purchases of cryptocurrency by investors from users ad infinitum imply a negative present value of the remaining aggregate cash flows from the perspective of investors. Paradoxically, every individual investor who acquires cryptocurrency and sells it in the future is expected to earn the required return in the bubble equilibrium, despite of the negative cash flows for investors in the aggregate.

B. Endogenous Negative Money Growth

Some cryptocurrencies exhibit negative money growth. Negative money growth can be a design feature. For example, Ethereum has shown a period of negative money growth after its switch to a mechanism where part of the tokens paid as transaction fees are burned. Coins that are lost because users permanently lost access can be considered as another source of negative money growth.

To illustrate the relationship between the cash flows for investors in Ponzi-schemes and bubble equilibria for cryptocurrencies with negative money growth, we extend the model to allow for a scenario where every period a proportion $f > 0$ of the user demand $X_t^{\$}$ is burnt as transaction fees (or, alternatively, lost) such that the change in the number of coins becomes an endogenous function of the exchange rate as $\Delta M_{t+1} S_{t+1} = -f X_t^{\$}$.[13]

The following proposition states the Ponzi-scheme equivalence result for such a cryptocurrency.

Proposition 6 *Consider a cryptocurrency where every period a fraction of the nonzero user demand $f > 0$ is burnt and for which the growth rate of user demand g is such that $g < r$ for any $t > 0$.*

1. *If $f < r - g$ and the exchange rate follows a bubble equilibrium path, then the cryptocurrency satisfies the Ponzi-scheme equivalence condition.*

2. *If $f > r - g$ and the exchange rate follows a bubble equilibrium path, then the cryptocurrency does not satisfy the Ponzi-scheme equivalence condition.*

Proof. See Appendix E. ■

All bubble equilibria satisfy the Ponzi-scheme equivalence condition if the proportion of the user demand that is burnt every period is less than the difference between the required return and the growth in user demand ($f < r - g$). We find a different result for the situation where the proportion of user demand paid as fees exceeds the difference between the required return and the growth in user demand ($f > r - g$). For such cryptocurrencies, bubble equilibria do not satisfy the Ponzi-scheme equivalence condition. These equilibria are the reason why the set with equilibria with payoffs that are not equivalent to Ponzi-schemes in Figure 1 is non-empty.

[13] The expression assumes $\Delta M_{t+1} = -f X_t^{\$} / S_{t+1}$. Alternatively, one could assume $\Delta M_{t+1} = -f X_t^{\$} / S_t$. This has no impact on Propositions 6 and 7, except that the f in any expression should be replaced by $f/(1+r)$.

A cryptocurrency for which $f > r - g$ is an interesting scenario from an economic point of view in that the user demand per coin and, hence, the reference level would grow forever at a rate $f + g$ which is greater than the required return r. At first sight, this may seem to suggest that there would not be an equilibrium. However, the endogenous money growth in combination with the presence of investors ensures that the exchange rate does not appreciate at a rate higher than the required return in equilibrium. The mechanism functions as follows. Investors who acquire the cryptocurrency drive up the initial exchange rate. The higher initial exchange rate implies that users hold fewer coins, and, hence, burn a smaller number of coins as transaction fees. The number of coins declines at a slower speed resulting in a slower increase in the user demand *per coin* than in the absence of investors. In equilibrium, the user demand per coin will grow at precisely the same rate as the required return. The remaining cash flows to investors will be positive in such an equilibrium. Users have to purchase tokens from investors to replenish their cryptocurrency balances after burning the transaction fees which generates a positive aggregate cash flow from users to investors.

The following proposition illustrates the equilibrium exchange rate path and the evolution in the share of coins held by investors.

Proposition 7 *Consider a cryptocurrency with no new issuance, with a constant growth rate of user demand $g < r$, and with users who burn a proportion $f > r - g$ of their coins as transaction fees at any t.*

The following exchange rate path constitutes an equilibrium

$$S_t = (1+r)^t \frac{f}{r-g} \frac{X_0^{\$}}{M_0 - U} \quad \textit{for any} \quad 0 \leq U < M_0. \tag{8}$$

The equilibrium where $U = 0$ corresponds to the baseline equilibrium; any equilibrium where U such that $0 < U < M_0$ corresponds to a bubble equilibrium. The equilibrium share of coins held by investors evolves as

$$\frac{Z_t}{M_t} = \frac{\frac{f-(r-g)}{f}(M_0 - U)\left(\frac{1+g}{1+r}\right)^t + U}{(M_0 - U)\left(\frac{1+g}{1+r}\right)^t + U}.$$

Proof. See Appendix F. ■

The share of coins held by investors slowly converges to all coins in a bubble equilibrium ($0 < U < M_0$) as was the case for the earlier bubble equilibria where the money issuance was exogenous. The evolution of the share of coins held by investors with the endogenous burning of coins is different for the baseline equilibrium ($U = 0$). Rather than converging to zero as was the case with the exogenous money issuance, the share of coins held by investors is constant: Investors in the baseline equilibrium sell precisely the amount to "restore" the balance between the shares held by users and investors.

For cryptocurrencies with negative money growth,t here are many rational bubble equilibria that do not satisfy the Ponzi-scheme equivalence condition (Propositions 6-7). However, this does not mean that the *net present value* of the cash flows for investors of investing in a cryptocurrency following such an equilibrium will be non-negative in the aggregate. The Ponzi-scheme equivalence condition is satisfied only if the future cash flows for investors have a negative present value in the aggregate, which excludes the initial cost for investors to acquire the coins.

To analyze which equilibria in Proposition 7 result in a non-negative net present value, we need to deduct the cost of the initial position at $t = 0$ from the position, which equals $S_0 Z_0$. Then, we can derive the following corollary.

Corollary 1 *Consider the equilibria characterized in Proposition 7. The baseline equilibrium (i.e., $U = 0$) is the only equilibrium with a non-negative net present value for investors in the aggregate, i.e.,*

$$-S_0 Z_0 - \sum_{t=1}^{\infty} \frac{\mathbb{E}_0 \Delta Z_t S_t}{(1+r)^t} \geq 0.$$

Investors experience positive cash flows in the aggregate in bubble equilibria where $f > r - g$, so the Ponzi-scheme equivalence condition is not satisfied, but the present value of the aggregate cash flows is less than the initial cost of the cryptocurrency at $t = 0$. Note that, despite the negative net present value in the aggregate for equilibria where $U > 0$, every individual investor who acquires cryptocurrency to sell it in the future still expects to earn the required return, and investors receive a positive cash flows in the aggregate.

VII. Discussion

A. Endogenous user demand

The user demand in the model is assumed to be inelastic with respect to the expected return on the cryptocurrency. Although this assumption simplifies the model considerably, it ignores how the demand for a means of payment depends on the opportunity cost of holding it. Alternatively, one could consider a reduced-form generalization of the model where the user demand at time t equals $X_t^{\$}(\mathbb{E}_t R_{t+1}) = A(\mathbb{E}_t R_{t+1})\widehat{X}_t^{\$}$. The $\widehat{X}_t^{\$}$ is assumed to be exogenous and the $A(\cdot)$ is a non-negative finite scaling factor that increases in the expected return $\mathbb{E}_t R_{t+1} = \mathbb{E}_t(S_{t+1}/S_t)$ for $\mathbb{E}_t R_{t+1} \geq 0$. The original model can be considered as a special case where $A(\mathbb{E}_t R_{t+1}) = 1$ for any $\mathbb{E}_t R_{t+1}$. The more general specification allows user demand to depend on time-specific fundamental factors as measured by $\widehat{X}_t^{\$}$ as well as the expected rate of appreciation.

The main results regarding the investment flows in bubble equilibria and their relationship to investors' payoffs in Ponzi-schemes extend immediately to the more general specification, provided that $A(1+r)$ is finite. If this is true, then Theorem 1, Propositions 4, 5, 6 and 7, and Corollary 1 all hold true in the more general model.[14] The reason why the bubble results also hold true in the more general model is straightforward. Investors hold the cryptocurrency in any equilibrium where the exchange rate contains a bubble component, so the expected return must equal the required return in any bubble equilibrium (Assumption 1). Hence, we must have $A(\mathbb{E}_t R_{t+1}) = A(1 + r)$ for all t whenever the cryptocurrency price contains a bubble component in equilibrium, so that the user demand $X_t^{\$}$ becomes a re-scaled version of the underlying fundamental factor $\widehat{X}_t^{\$}$. The user demand takes its maximum possible value because the expected return attains its maximum possible equilibrium value—that is, the required return—in a bubble equilibrium.

The aforementioned generalization of the bubble results is conditional upon $A(1 + r)$ being finite. This requires the user demand for cryptocurrency in dollar terms to be finite even if the opportunity cost of holding cryptocurrency were zero. Although many micro-founded models of money do imply finite real transactional balances if the opportunity

[14] The growth rates referred to in those propositions should in the more general model be understood as growth rates of the underlying fundamental $\widehat{X}_t$, i.e., $g_t = (\widehat{X}_t - \widehat{X}_{t-1})/\widehat{X}_{t-1}$. The solution for the baseline equilibrium is more challenging in the general model. The path of the user demand per coin need not be unique in the baseline equilibrium for a given sequence of $\widehat{X}_t^{\$}$.

cost of holding money converges to zero, this does not hold true for all models. Famously, Tirole (1985, Proposition 7) considers an environment with money-in-the-utility where the marginal benefit of real money balances is strictly positive. In this environment, agents are incentivized to increase their real money balances to infinitely large levels as the opportunity cost approaches zero. Intuitively, a rational bubble in the value of a currency—implying a zero opportunity cost of holding that currency—cannot coexist with that currency being used for transactional purposes in such an environment. The present model would yield a similar impossibility result if we were to assume $A(\mathbb{E}_t R_{t+1}) \to \infty$ as $\mathbb{E}_t R_{t+1} \to 1 + r$. If $A(1 + r)$ is finite, then the bubble results generalize as aforementioned to the setting with the reduced-form endogenous money demand.

The theoretical result that a price bubble may stimulate user demand for a cryptocurrency by reducing the opportunity cost of holding it makes cryptocurrency quite different from a commodity that serves as an input at fixed quantities for other goods or services. Brunnermeier and Oehmke (2013) note that the emergence of a price bubble would make a commodity relatively expensive, so that users would be incentivized to seek for substitutes. In contrast, a higher exchange rate of a cryptocurrency does not directly change the cost of using that cryptocurrency for making payments. Users simply need a smaller quantity of the cryptocurrency to make the same dollar-amount of payments. What may matter to users is the expected rate of appreciation of the cryptocurrency in the bubble equilibrium, which is either higher than or equal to the rate of appreciation in the baseline equilibrium.

VIII. Concluding Remarks

This paper focused on the question how cryptocurrency price paths relate to concepts such as new asset classes, bubbles, Ponzi schemes and digital gold. The analysis reveals how those terms relate to differences in underlying beliefs regarding future peak values of the user demand per coin and discount rates.

High crypto prices can be justified by a high expected peak value in terms of user demand. Describing a cryptocurrency as equivalent to a Ponzi-scheme is not justified if the current price reflects the discounted value of the expected peak value in user demand. This holds true even if the current price seems to be driven mostly by investors' actions rather than user dynamics. Even though investors experience outflows when they acquire cryptocurrency, they expect to profit from selling crypto to users in the future. An observer could easily mistake a high price observed in a baseline equilibrium for a bubble when underestimating the expected peak value in user demand. Moreover, higher prices could be justified by a lower discount rate resulting from investors perceiving cryptocurrency as a digital gold that provides insurance against bad states of the world.

The picture is bleaker if high prices are the consequence of investors expecting price increases solely due to higher future investment demand, and not due to the expected peak level of future user demand. Such bubble price paths are possible in equilibrium and are associated with a gradually increasing share of coins that is held exclusively for investment purposes. The analysis reveals that, for cryptocurrencies with nonnegative money growth, such price paths are associated with Ponzi-scheme equivalent payoffs to investors in the aggregate. Even though individual investors are expected to earn their required return if the bubble persists, they do experience negative cash flows in the aggregate. Finally, investors do not need to experience Ponzi-scheme equivalent payoffs in the aggregate if they invest in a bubble equilibrium for a cryptocurrency with negative money growth.

From an empirical or experimental perspective, our results provide theoretical foundations for several testable implications regarding factors that could make cryptocurrencies more or less susceptible to explosive price paths. Proposition 4 shows that a bubble equilibrium requires higher net investment inflows to sustain a bubble equilibrium with (1) *higher new issuance*, (2) *lower growth in user demand*, (3) *higher existing user demand*, and (4) a *higher required return.* If an equilibrium is considered less plausible if its persistence requires continuous high net investment inflows, then these factors must be associated with a smaller probability of bubbles in empirical or experimental settings.

From a theoretical point of view, the condition in Corollary 1 may prove useful to rule out rational bubble equilibria in different settings if the baseline equilibrium is the object of study. One may be tempted to ruling out bubble equilibria by imposing an condition that limits the asymptotic growth of the exchange rate so that the present value of the future exchange rate converges to zero, akin to a transversality condition. The disadvangage of such a transversality-like condition is that it may rule out both the bubble equilibria *and* the baseline equilibrium as we have seen in the case studied in Proposition 7.[15]

[15]From Eq. (8), it is immediate that the baseline equilibrium condition in Proposition 7 violates the condition $\lim_{t\to\infty} S_t/(1+r)^t = 0$ since $\lim_{t\to\infty} S_t/(1+r)^t > 0$ for $U = 0$ provided that $X_0^{\$} > 0$.

Appendix: Proofs

A. *Proof of Proposition 1*

We first consider t such that $0 < t \leq \tau(1)$.

Suppose $X^{\$}_{\tau(1)} > 0$. Lemma 1 and $Z_{\tau(1)} \geq 0$ imply that any $S_{\tau(1)} < X^{\$}_{\tau(1)}/M_{\tau(1)}$ would be inconsistent with market-clearing (Assumption 2). This proves the lower bound for $t = \tau(1)$. Moreover, if $S_{\tau(1)} = X^{\$}_{\tau(1)}/M_{\tau(1)}$, then any level of the exchange rate $S_t(1+r)^{\tau(1)-t} < X^{\$}_{\tau(1)}/M_{\tau(1)}$ for $0 < t < \tau(1)$ would violate Assumption 1. This proves the lower bound for any $0 < t < \tau(1)$. Assumption 3 also holds true on the path since $X^{\$}_{\tau(1)} > 0$ implies $S^*_t > 0$ for $0 < t \leq \tau(1)$, so the path S^*_t in the proposition constitutes an equilibrium for any t such that $0 < t \leq \tau(1)$.

Repeating the argument for $n = 2, 3, ...$ provides the proof for any t such that $\tau(n-1) < t \leq \tau(n)$.

The special case where $X^{\$}_{\tau(1)} = 0$ implies $X^{\$}_t = 0$ for any $t \in \mathbb{N}$, so that $\tau(n) = n$. In this case, the path described in the proposition implies $S_t = X^{\$}_t/M_t = 0$ for any t, which is the lowest possible level of the exchange rate that does not violate Assumption 3.

B. *Proof of Theorem 1*

For the proof of Theorem 1, we consider four different cases, depending on whether $X^{\$}_t$ and $X^{\$}_{t+1}$ equal zero or not.

The case $X^{\$}_t = 0, X^{\$}_{t+1} = 0.$

The market-clearing condition in Assumption 2 implies $M_t = Z_t$ since $X^{\$}_t = 0$ and $M_{t+1} =$

Z_{t+1} since $X^{\$}_{t+1} = 0$. Hence, $(Z_{t+1} - Z_t)S_{t+1} = (M_{t+1} - M_t)S_{t+1}$. Moreover, $-\Delta X^{\$}_{t+1} + r_{t+1}X^{\$}_t = 0$ since $X^{\$}_t = 0$ and $X^{\$}_{t+1} = 0$, so that the equality in Theorem 1 holds true.

The case $X^{\$}_t > 0, X^{\$}_{t+1} > 0.$

Since $X^{\$}_t > 0$ and $X^{\$}_{t+1} > 0$, one can use Lemma 1 for both S_{t+1} and S_t. From Lemma 1, we have

$$\Delta S_{t+1} = \frac{X^{\$}_{t+1}}{M_{t+1} - Z_{t+1}} - \frac{X^{\$}_t}{M_t - Z_t}.$$

The theorem specifies the relationship for nominal return r_{t+1}. Substituting $\Delta S_{t+1} = r_{t+1}S_t$ gives

$$r_{t+1}S_t = \frac{X^{\$}_{t+1}(M_t - Z_t) - X^{\$}_t(M_{t+1} - Z_{t+1})}{(M_{t+1} - Z_{t+1})(M_t - Z_t)}, \tag{9}$$

and, since $S_t = X^{\$}_t/(M_t - Z_t)$,

$$\begin{aligned}
r_{t+1}X^{\$}_t &= \frac{X^{\$}_{t+1}(M_t - Z_t) - X^{\$}_t(M_{t+1} - Z_{t+1})}{(M_{t+1} - Z_{t+1})}, \\
&= \frac{X^{\$}_{t+1}(M_t - Z_t) - X^{\$}_{t+1}(M_{t+1} - Z_{t+1}) + \Delta X^{\$}_{t+1}(M_{t+1} - Z_{t+1})}{M_{t+1} - Z_{t+1}}, \\
&= S_{t+1}(M_t - Z_t) - S_{t+1}(M_{t+1} - Z_{t+1}) + \Delta X^{\$}_{t+1}, \\
&= -\Delta M_{t+1}S_{t+1} + \Delta Z_{t+1}S_{t+1} + \Delta X^{\$}_{t+1}.
\end{aligned}$$

Rearranging the last line gives the expression for $\Delta Z_{t+1}S_{t+1}$ in Theorem 1.

The case $X^{\$}_t = 0, X^{\$}_{t+1} > 0.$

The market-clearing condition in Assumption 2 at time $t+1$ gives

$$Z_{t+1}S_{t+1} = M_{t+1}S_{t+1} - X^{\$}_{t+1}.$$

Subtracting $M_t S_{t+1}$ from both sides gives

$$\begin{aligned}(Z_{t+1} - M_t)S_{t+1} &= \Delta M_{t+1}S_{t+1} - X^{\$}_{t+1},\\ \Delta Z_{t+1}S_{t+1} &= \Delta M_{t+1}S_{t+1} - X^{\$}_{t+1},\end{aligned}$$

where the last equality holds because market-clearing at time t implies $M_t = Z_t$ for $X^{\$}_t = 0$. The last equation equals that in Theorem 1 because $-\Delta X^{\$}_{t+1} + r_{t+1}X^{\$}_t = -X^{\$}_{t+1}$ if $X^{\$}_t = 0$.

The case $X^{\$}_t > 0, X^{\$}_{t+1} = 0$.

Market-clearing at time $t+1$ implies $M_{t+1} = Z_{t+1}$ since $X^{\$}_{t+1} = 0$. Subtracting $Z_t S_{t+1}$ from both sides implies

$$\begin{aligned}\Delta Z_{t+1}S_{t+1} &= \Delta M_{t+1}S_{t+1} - Z_t S_{t+1},\\ &= \Delta M_{t+1}S_{t+1} + (M_t - Z_t)S_{t+1}.\end{aligned}$$

Since $X^{\$}_t > 0$, we can use $S_t = X^{\$}_t/(M_t - Z_t)$ in Lemma 1 to write the last condition as

$$\begin{aligned}\Delta Z_{t+1}S_{t+1} &= \Delta M_{t+1}S_{t+1} + X^{\$}_t S_{t+1}/S_t.\\ &= \Delta M_{t+1}S_{t+1} + X^{\$}_t + r_{t+1}X^{\$}_t. \qquad (10)\end{aligned}$$

The last expression equals the right-hand-side in Theorem 1 since $-\Delta X^{\$}_{t+1} = X^{\$}_t$ if $X^{\$}_t > 0$ and $X^{\$}_{t+1} = 0$. This concludes the proof of Theorem 1.

C. *Proof of Proposition 4*

A nonzero bubble component requires the expected return to equal r, or, $\mathbb{E}_t \Delta S_{t+1} = rS_t$. The proof follows the proof of Theorem 1 for $r_{t+1} = r$, except that expectations are taken over both sides of the equation before substituting $\mathbb{E}_t \Delta S_{t+1} = rS_t$ in equations (9) and (10).

D. *Proof of Proposition 5*

Aggregating the relationship for the net inflows from investors in Proposition 4 for all $t > T$ with discounting and iterating expectations back to $t = T$ gives

$$\sum_{i=1}^{\infty} \frac{\mathbb{E}_T \Delta Z_{T+i} S_{T+i}}{(1+r)^i} = \sum_{i=1}^{\infty} \frac{\mathbb{E}_T \Delta M_{T+i} S_{T+i}}{(1+r)^i} + \sum_{i=1}^{\infty} \frac{-\Delta X^{\$}_{T+i} + rX^{\$}_{T+i-1}}{(1+r)^i}. \tag{11}$$

Using $M_{T+i} \geq 0$ (non-negative money growth) and $\Delta X_{T+i} = (1 + g_{T+i})X^{\$}_{T+i-1} - X^{\$}_{T+i-1} = g_{T+i}X^{\$}_{T+i-1}$ gives

$$\sum_{i=1}^{\infty} \frac{\mathbb{E}_T \Delta Z_{T+i} S_{T+i}}{(1+r)^i} \geq \sum_{i=1}^{\infty} \frac{rX^{\$}_{T+i-1} - g_{T+i}X^{\$}_{T+i-1}}{(1+r)^i}, \tag{12}$$

$$\geq 0, \tag{13}$$

where the last inequality holds because $g_t < r$ for all $t \geq T$. The present value of the net cash inflows from investors is larger than zero which is the same as saying that that present value of the remaining cash flows is negative from the perspective of investors.

E. *Proof of Proposition 6*

If a fraction f of the coins paid as transaction fees by users are burned, and if there is no further issuance of coins, then $\mathbb{E}_T \Delta M_{T+i} S_{T+i} = -f X^{\$}_{T+i-1}$. Then, from (11), we have

$$\begin{aligned}\sum_{i=1}^{\infty} \frac{\mathbb{E}_T \Delta Z_{T+i} S_{T+i}}{(1+r)^i} &= \sum_{i=1}^{\infty} \frac{-f X^{\$}_{T+i-1} - g X^{\$}_{T+i-1} + r X^{\$}_{T+i-1}}{(1+r)^i}, \\ &= \frac{r-f-g}{1+g} \sum_{i=1}^{\infty} \frac{(1+g)^i}{(1+r)^i} X^{\$}_T, \\ &= \frac{r-f-g}{r-g} X^{\$}_T.\end{aligned}$$

This value can be both positive and negative, depending on the level of the transaction fees. If $f < r-g$, then the present value of the future cash flows to investors is negative, so that the Ponzi-scheme equivalence condition is satisfied for any equilibrium where coins continually appreciate at an expected rate r (i.e., bubble equilibria in this case). If the fees $f > r-g$, then the present value of the remaining cash flows to investors would be positive, so that the Ponzi-scheme equivalence condition is not satisfied in an equilibrium where coins continuously appreciate at an expected rate r (i.e., the equilibria described by Proposition 7 in this case).

F. *Proof of Proposition 7*

Market-clearing (Assumption 2) requires that, for any $t \geq T$,

$$S_{t+1} = \frac{X^{\$}_{t+1}}{M_{t+1} - Z_{t+1}} = \frac{(1+g) X^{\$}_t}{M_t - f X^{\$}_t / S_{t+1} - Z_{t+1}} = \frac{(1+f+g) X_t}{M_t - Z_{t+1}}.$$

The rational expectation market model (Assumption 1) requires

$$\mathbb{E}_t \frac{S_{t+1}}{S_t} = (1+f+g)\mathbb{E}_t \frac{M_t - Z_t}{M_t - Z_{t+1}} = (1+r).$$

For non-stochastic Z_{t+1}, one can rewrite this condition into the following difference equation for the speculative position

$$Z_{t+1} = M_t - \frac{1+f+g}{1+r}(M_t - Z_t) = \frac{1+f+g}{1+r} Z_t - \frac{f+g-r}{1+r} M_t.$$

Similarly, we derive the number of cryptocurrency units as

$$M_{t+1} = M_t - fX_t/S_{t+1} = M_t - \frac{f}{1+f+g}(M_t - Z_{t+1}) = \frac{1+g}{1+f+g} M_t + \frac{f}{1+f+g} Z_{t+1}.$$

Plugging in the difference equation for Z_{t+1} yields the difference equation for the existing number of currency units as

$$M_{t+1} = \frac{f}{1+r} Z_t + \frac{1+r-f}{1+r} M_t.$$

Thus, we have the system of difference equations

$$\begin{pmatrix} Z_{t+1} \\ M_{t+1} \end{pmatrix} = \begin{bmatrix} \frac{1+f+g}{1+r} & -\frac{f+g-r}{1+r} \\ \frac{f}{1+r} & \frac{1+r-f}{1+r} \end{bmatrix} \begin{pmatrix} Z_t \\ M_t \end{pmatrix}, \tag{14}$$

with distinct and real eigenvalues $(\lambda_1, \lambda_2) = \left(\frac{1+g}{1+r}, 1\right)$, and eigenvectors $v_1 = \left(\frac{f+g-r}{f}, 1\right)$ and $v_2 = (1, 1)$. The system has the following solution

$$\begin{pmatrix} Z_{t+i} \\ M_{t+i} \end{pmatrix} = C \begin{bmatrix} \frac{f+g-r}{f} \\ 1 \end{bmatrix} \left(\frac{1+g}{1+r}\right)^i + U \begin{bmatrix} 1 \\ 1 \end{bmatrix} 1^i. \tag{15}$$

From the solution of M_{t+i} for $i = 0$, we solve $C = M_t - U$, so that we find the generic solution

$$\begin{aligned} Z_{t+i} &= \frac{f+g-r}{f}(M_t - U)\left(\frac{1+g}{1+r}\right)^i + U, \\ M_{t+i} &= (M_t - U)\left(\frac{1+g}{1+r}\right)^i + U. \end{aligned}$$

What values of U correspond to valid equilibria? Note that $\lim_{i\to\infty}(Z_{t+i}, M_{t+i}) = (U, U)$. Hence, a valid equilibrium requires $U \geq 0$: Otherwise, Z_{t+i} and M_{t+i} would converge to negative numbers. Similarly, a valid equilibrium also requires $M_t > U$. Any solution with a value of U such that $0 \leq U < M_t$ corresponds to a valid equilibrium.

The floor for the equilibrium exchange rate corresponds to the case where $U = 0$, so that $Z_t = M_t(f + g - r)/f$, and $S_t = (X_t/M_t)(f/(r - g))$. Any values of U such that $0 < U < M_t$ corresponds to equilibrium exchange rate paths with higher levels. The share of coins held by speculators can be calculated for all equilibria as

$$\frac{Z_{t+i}}{M_{t+i}} = \frac{\frac{f+g-r}{f}(M_t - U)\left(\frac{1+g}{1+r}\right)^i + U}{(M_t - U)\left(\frac{1+g}{1+r}\right)^i + U}.$$

This number converges to one for all equilibria corresponding to $U > 0$. Hence, these equilibria exhibit a similar pattern in investment holdings as bubble equilibria. If $U = 0$, then the share held by speculators will be constant at $(f+g-r)/f$.

G. *Proof of Corollary 1*

From the proof of Proposition 6, we have

$$-\sum_{i=1}^{\infty} \frac{\mathbb{E}_0 \Delta Z_{t+i} S_{t+i}}{(1+r)^i} = \frac{f+g-r}{r-g} X_0^{\$}. \tag{16}$$

Moreover, from Proposition 7 and its proof, we have

$$\begin{aligned} -S_0 Z_0 &= -\left(\frac{f}{r-g}\frac{X_0^{\$}}{M_0-U}\right)\left(\frac{f+g-r}{f}(M_0-U)+U\right), \\ &= -\frac{f+g-r}{f}X_0^{\$} - U\left(\frac{f}{r-g}\frac{X_0^{\$}}{M_0-U}\right). \end{aligned} \tag{17}$$

Summing (16) and (17) gives for the net present value

$$-S_0 Z_0 - \sum_{i=1}^{\infty} \frac{\mathbb{E}_0 \Delta Z_{t+i} S_{t+i}}{(1+r)^i} = -U\left(\frac{f}{r-g}\frac{X_0^{\$}}{M_0-U}\right).$$

The only value of U such that $0 \leq U < M_0$ for which the net present value is non-negative is $U = 0$. This value of U corresponds to the baseline equilibrium.

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