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Cutting out the Middleman:  
Crowdfunding, Efficiency, and  
Inequality

**Hans Peter Grüner**

University of Mannheim

**Christoph Siemroth**

University of Mannheim



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# Cutting out the Middleman: Crowdinvesting, Efficiency, and Inequality<sup>a</sup>

Hans Peter Grüner<sup>b</sup>      Christoph Siemroth<sup>c</sup>

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## Abstract

We show how decentralized individual investments can efficiently allocate capital to innovating firms. We consider consumers with privately known but correlated consumption preferences who also act as investors. Consumers identify worthwhile investment opportunities based on their own preferences and invest in firms whose product they like. An efficient capital allocation is achieved if all groups of consumers have enough wealth to invest. If some groups of consumers cannot invest, capital flows reflect preferences of the wealthy but not future demand. Information gathering by financial intermediaries can improve the allocation of capital when wealth inequality prevents an efficient allocation by consumers.

**Keywords:** Capital Markets, Crowdfunding, Crowdinvesting, Financial Markets, Financial Intermediation, Information Aggregation, Wealth Inequality, Welfare

**JEL Classification:** D24, D31, D53, D63, D82, G20

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<sup>b</sup>University of Mannheim, Department of Economics, L7, 3-5. 68131 Mannheim, Germany and CEPR, London. E-mail: gruener@uni-mannheim.de.

<sup>c</sup>University of Mannheim, Department of Economics, L7, 3-5. 68131 Mannheim, Germany. E-mail: christoph.siemroth@gess.uni-mannheim.de.

# 1 Introduction

In most industrialized economies, financial wealth is distributed far more unequally than income. Late last century, 60 percent of American households possessed almost no financial wealth (one percent of total financial wealth), while the top five percent of households held more than two thirds of financial wealth (Wolff, 2002). Income inequality, in comparison, was much lower. The poorest 60 percent of US households received about 22 percent of total income. More recently, Saez and Zucman (2014) found that the bottom 90 percent of American households owned about 23 percent of wealth, but received 60 percent of income in 2012. A similar disparity of income and wealth distributions can be observed in many other countries (Davies et al., 2011). Moreover, according to Piketty (2014), the inequality of the wealth distribution increased over time in several industrialized nations.

In this paper, we show that decentralized investment processes which rely exclusively on the “wisdom of the crowd” can efficiently aggregate information about the potential success of new consumption goods, and channel funds to projects that need them the most. Efficient information aggregation requires that potential consumers of these new products have enough wealth to invest on the capital market. A major mismatch between the income and wealth distribution of consumers instead leads to inefficient investment choices that cannot be fully corrected by financial intermediaries.

Our results are derived from a Bayesian investment game with dispersed and correlated information about the future demand of new products. In our model, a firm invents a novel consumption good and tries to raise capital for production. The more money the firm attracts, the more it can produce for later sale. Consumers look for investment opportunities on the capital market to increase their income for consumption, and may as one option invest in the new firm. We assume there is aggregate demand uncertainty, but tastes among consumers within the same class are correlated. Consequently, consumers can use their own preference for the new product as a signal about future aggregate demand and therefore profitability of the firm. We show that consumers who like the new product are more optimistic about its demand and invest, while consumers who dislike the new product invest elsewhere. Thus, consumption driven investment directs capital towards firms that are likely to find many customers.

We model the investment process as a form of crowdfunding, where shares of the firm are directly sold to many small consumers (“the crowd”), and the proceeds are used to increase production capacity.<sup>1</sup> Our main result is that crowdfundings can efficiently allocate capital to firms if all consumers are wealthy enough to invest. If, however, some groups of

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<sup>1</sup>This kind of crowdfunding differs from traditional forms of financing such as IPOs, where typically a predetermined share of the firm is sold to larger institutional investors, with assistance of an underwriter. Unlike IPOs, our crowdfunding process does not determine a share price; rather, crowdfunders invest an amount of capital and are entitled to a share of firm earnings in proportion to their investment. Otherwise both forms are similar in that equity shares are sold, and indeed a specific form of IPOs, *direct public offerings*, are very close to the crowdfunding process we consider.

consumers cannot invest but will receive income for consumption, then crowdinvestments reflect the preferences of the wealthy, but not necessarily those of all consumers. Thus, capital is misallocated and production does not always scale up with future demand, because consumers who are unable to invest still consume. In a way, the capital market works as an information aggregation device similar to a vote, except consumers without wealth to invest do not get a vote. The same efficiency results obtain if we consider debt instead of equity crowdfunding.

In our baseline model only consumers hold information about preferences for the innovative product in the population. In an extension to our baseline model, we introduce financial intermediaries such as investment funds, who can acquire information about consumer preferences and compete with crowdinvestors on the capital market. We show that these professional investors cannot completely rectify the capital misallocation that arises when some groups of consumers are unable to invest. Moreover, if all consumers can invest a sufficient amount, then financial intermediaries are driven out of the market. This is because the decentralized information about future aggregate demand among consumers is costlessly aggregated on the capital market. Financial intermediaries, on the other hand, have to acquire information at a cost. Thus, the crowd in the aggregate acts like an insider whose superior information drives intermediaries out. Professional investors may be active in the market only if the crowd is not perfectly informed, i.e., cannot aggregate all consumer preferences because not all consumers are able to invest.

Crowdfunding, i.e., financing forms that draw on the masses (e.g., consumers, the general public) rather than a few professional financial intermediaries, is becoming very popular. It encompasses different funding models with equity contracts (as we consider here), debt contracts, material rewards in case of success, or donations, and has grown considerably in recent years and may soon rival traditional funding forms. “In all, crowdfunding platforms have raised some \$2.7 billion and successfully funded more than a million campaigns in 2012, [...] with an 81% increase to \$5.1 billion expected for 2013. By 2025, the global crowdfunding market could reach between \$90 billion and \$96 billion—roughly 1.8 times the size of the global venture capital industry today” (Fortune, 2014). Unlike other sorts of crowdsourcing (e.g., decision making by crowds, software development by crowds), crowdfunding is not primarily designed to rely on the “wisdom of crowds” (Surowiecki, 2005). It was meant to replace conventional financial intermediation for cases when banks, venture capital firms and others were unwilling to provide funding. Yet our main result shows that crowdfunding can achieve both: raise capital, while at the same time aggregating information.

The structure of the paper is as follows. After reviewing the related literature, section 2 presents the basic model and the main result with crowdinvestors. Section 3 extends the model by adding a sector of what we call investment funds, who compete with crowdinvestors on the capital market. In section 4, we discuss further extensions of our basic analytical framework, including a model with sequential investments that permits investors to learn

from other players' investment decisions during a crowdfunding campaign and a model with pre-order crowdfunding. The last section concludes. In the appendix, we analyze the extension to nonlinear production technologies.

## 1.1 Related literature

Our paper is related to several distinct strands of literature, which we shall discuss only briefly. First, our contribution is related to a large literature that studies the effects of wealth inequality on allocative efficiency and in particular on the functioning of capital markets. A non-exclusive list is [Aghion and Bolton \(1997\)](#), [Banerjee and Newman \(1993\)](#), [de Mesa and Webb \(1992\)](#), [Galor and Zeira \(1993\)](#), [Grüner \(2003\)](#), [Grüner and Schils \(2007\)](#), and [Piketty \(1997\)](#). All these papers emphasize the link between credit market imperfections and agents' investments into private production possibilities. Investors with little wealth either do not get credit for their individual investment projects, or they only get credit at a higher interest rate. This is why the distribution of wealth has macroeconomic implications. In the present paper, we instead consider the link between inequality of the wealth distribution and the investment in firms/technologies seeking funding. Moreover, a major difference of our model compared to existing incomplete markets models is that agents hold private information about consumption needs, which is also a signal about the realization of relevant aggregate uncertainty.

Second, our paper is part of an emerging literature on crowdfunding that already spans multiple disciplines including economics, business, and law. Several empirical studies investigate the determinants of fundraising success (e.g., [Agrawal et al., 2011](#); [Ahlers et al., 2012](#); [Mollick, 2014](#); [Li and Martin, 2014](#)). [Agrawal et al. \(2013\)](#) and [Belleflamme et al. \(2015\)](#) review the first empirical findings and discuss economic concepts relating to crowdfunding. According to [Agrawal et al. \(2013\)](#), early results suggest that crowdfunding can replace traditional sources of financing, just as we find in our model. They also remark that, when investments are linked to and motivated by an earlier access to the product, financing by the crowd may be able to provide information about demand to the entrepreneur that would not be available from venture capitalists. Our argument, instead, applies beyond campaigns with presale and is based on the information that the individual consumption preference reveals about others. In one of the first theoretical treatments, [Belleflamme et al. \(2014\)](#) investigate whether an entrepreneur should rather use pre-order or equity-based crowdfunding. In their model, crowdfunders are motivated by “community benefits” (utility from contributing) rather than investment return considerations as in our case.

Third, our paper contributes to the financial intermediation literature (for reviews, see [Bhattacharya and Thakor, 1993](#) or [Gorton and Winton, 2003](#)). Several reasons for the emergence of financial intermediation have been put forth. One of the most prominent reasons given is that financial intermediaries can reduce aggregate transaction costs (e.g., monitoring and screening costs) or benefit from economies of scale (e.g., [Diamond, 1984](#);

Williamson, 1986; Boot and Thakor, 1997). Relatedly, it has been argued that financial intermediaries can solve information asymmetry problems, like the problem of information credibility in markets for information (e.g., Leland and Pyle, 1977; Allen, 1990): A buyer of information (e.g., about asset values, entrepreneur project quality) typically cannot verify its quality at the time of purchase. An intermediary who uses the information to invest and sell shares of his portfolio can credibly signal that the information is good, unlike the original seller. Moreover, in the presence of heterogeneous priors about project success among agents, it has been argued that intermediaries can emerge to channel funds from pessimists, who are unwilling to invest without screening for good projects, to optimistic entrepreneurs, who want to go forward with projects without screening (Coval and Thakor, 2005). Intermediaries are neither optimistic nor pessimistic, and can therefore credibly commit to screening, unlike the optimistic entrepreneurs.

Our model shows that direct financing of firms can be superior in terms of costs and capital allocation, because it utilizes the decentralized information of the crowd. We show that in certain situations crowdfunders have a cost advantage over financial intermediaries, because they can use information they already have and intermediaries first have to purchase. Suppose crowdfunders *could not* use their own preference as free information on the profitability of the firm as they do in our model. Then financial intermediaries would emerge due to a standard cost advantage argument and would be able to drive individual traders out of the market, because they could purchase advantageous information which individual investors cannot afford. Therefore, existing decentralized information among consumers is crucial in our theory.

Our results contribute to the literature comparing market-based and bank-based financial systems (for a review, see Allen and Gale, 2001). Like us, Allen and Gale (1999) consider the problem of financing new technologies, but provide an alternative explanation why market finance (the analogue to our crowdfunding) might emerge instead of financial intermediation. They show that sufficiently strong diversity of opinion (heterogeneous prior beliefs) among traders will favor market finance over intermediation, because an intermediary is more likely to make a suboptimal decision from the perspective of investors. In contrast to their model, our analysis focuses on the effect of the investor information and wealth distribution rather than diversity of opinion on the efficiency of capital allocation and the extent of financial intermediation, and investors in our model have a common prior.

It has also been noted that initial public offerings can aggregate useful information about the future success of projects (e.g., Benveniste and Spindt, 1989). The present paper studies the case where this information concerns the attractiveness of a firm's products for a population of consumers who also act as investors.

Our paper is closely related to Subrahmanyam and Titman (1999), who investigate the firm choice between public (market) financing and private financing (intermediation). In their model, two random variables influence the growth opportunities of an enterprise.

Investors in the financial market can either decide to acquire costly information on one of these variables, or some subset of investors receives free (“serendipitous”) information during their day-to-day activities on the other variable by chance. For example, a retail store employee might come across information about the future demand for a certain product while at work, which he can use to evaluate investment alternatives. The entrepreneur in their model decides whether to go public or use private finance, and he anticipates whether he receives more information about business growth opportunities from the market via stock prices or from the private financier. The entrepreneur uses the information he receives to decide how to invest the proceeds from selling shares of his company in growth opportunities.

While the analysis of [Subrahmanyam and Titman \(1999\)](#) explains firm choices between public and private finance based on informational benefits to the firm, we instead focus on the impact of wealth distribution and information among investors on financial market structure. To examine the effect of the wealth distribution, our analysis requires a more specific microfoundation of the “free information” which investors hold: Consumer preferences are correlated, hence by virtue of having preferences, consumers also possess some information about the aggregate demand for products. The consumer information in our model can therefore be viewed as systematic rather than serendipitous.

The second major difference is that the entrepreneur in [Subrahmanyam and Titman \(1999\)](#) chooses the financing form himself, whereas we assume that the entrepreneur offers equity in a crowdfund campaign, and competition between many small investors and professional investors determines who holds equity in this financial market. One of Subrahmanyam and Titman’s main findings is that market financing is favored if costless information is more widespread, and we show in our context exactly when more of the costless information is incorporated in the capital market as a consequence of the wealth distribution of investors.

## 2 Crowdfund: The baseline model

### 2.1 Consumers and endowments

Consider an economy which is populated by a continuum of consumer-investors indexed by  $i \in [0, 1]$ , who we will also call ‘crowdfunders.’ Each consumer has an initial endowment of wealth  $w_i$  in period 1 and receives an exogenous income  $y_i$  in period 2. Income and wealth are measured in monetary units. Individuals consume in period 2 and use the capital market to increase their income in period 2. They can invest any positive amount of money at the riskless rate  $R$ , i.e., one unit invested in period 1 turns into  $R$  units in period 2. The riskless rate  $R$  is exogenously given. In period 2, two consumption goods are available: consumption  $c$  (at a normalized price of 1) and the novel consumption good  $x$ . Consumers have private information about their preference for the novel good. Preferences are represented by the



following utility function:

$$u(c_i, x_i, \theta_i) = c_i + \theta_i x_i^\alpha, \quad (1)$$

with  $0 < \alpha < 1$ . The parameter  $\theta_i$  is private information of consumer  $i$ , with  $\theta_i \in \{0, 1\}$ , i.e., consumers either derive utility from consuming good  $x$ , or they do not.

There is a spot market for goods  $c$  and  $x$  in period 2. But there is neither a credit market on which consumers may borrow against future income  $y_i$  nor a forward market for the innovative good  $x$ . A credit market friction is key to our results because, on a perfect credit market, all consumers could borrow against their future income in order to finance the efficient investment in their preferred technology. Still, the assumption of no credit markets is stricter than necessary and only made for simplicity here.<sup>2</sup> Nonexistence of a forward market is an appropriate assumption if the innovative good  $x$  has important features that are not contractible at the funding stage, which is the case for many of the investment projects financed by crowdfunders. Without a forward market, companies cannot finance their investments drawing on the current sales revenues and must rely on external funding. In section 4.2 we show that the pre-order crowdfunding and forward markets are similarly affected by wealth constraints as a equity crowdfunding market.

## 2.2 The Bayesian investment game

There is aggregate risk regarding the share of consumers who would like to consume good  $x$  in period 2. The share of consumers  $s$  who would like to consume this good is distributed according to

$$s := \int_0^1 \theta_i di = \begin{cases} \beta > 1/2 & \text{with probability } 1/2, \\ 1 - \beta & \text{with probability } 1/2. \end{cases}$$

Observing his private signal  $\theta_i = 1$ , a consumer updates his beliefs that state  $s = \beta$  has realized. The corresponding posterior probability is

$$\Pr(s = \beta | \theta_i = 1) = \frac{\frac{1}{2}\beta}{\frac{1}{2}\beta + \frac{1}{2}(1 - \beta)} = \beta.$$

There are  $m > 1$  firms which have access to a technology for the production of good  $x$ . Each firm produces according to the linear technology:

$$x_{sup}(X) = X,$$

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<sup>2</sup>As will become clear later, it is sufficient to assume a wedge between borrowing and saving rates due to credit market frictions (e.g., [Galor and Zeira, 1993](#)), because borrowing requires an excess return from investing in equilibrium, which is incompatible with efficient capital allocation. Thus, allowing borrowing in imperfect credit markets does not change our efficiency results.



where  $x_{sup}$  denotes the produced amount (supply) of the novel good and  $X$  the aggregate investment made in period 1.<sup>3</sup> Consumers may invest any amount  $\hat{x}_i$  in these firms, and the total size of all firms is determined by the investments of all consumers

$$X = \int_0^1 \hat{x}_i di.$$

All firms act as price takers in period 2 and distribute profits to all shareholders according to their relative investment shares.

In period 2, consumers receive their exogenous income  $y_i$  and the return on their riskless or risky investments. Let  $\tilde{y}_i$  be the total budget available to consumer  $i$  in period 2. An equilibrium is defined as follows.

**Definition 1.** *An equilibrium of the model consists of*

- i. a consumption plan  $x_i(p)$  for each consumer,*
- ii. an investment plan  $\hat{x}_i(\theta_i)$  for each consumer, and*
- iii. a relative price function  $p(X, s)$  for good  $x$ ,*

*such that*

- i. the consumption plan maximizes utility (1) subject to the consumer's period 2 budget constraint,*
- ii. the investment plans constitute a Bayesian Nash equilibrium of the investment game subject to the wealth constraints, taking into account the consumption plans and the relative price  $p(X, s)$ , and*
- iii. at price  $p(X, s)$  the aggregate demand for good  $x$  equals supply  $x_{sup}$ .*

Note that we save on notation by not including wealth  $w_i$  in the investment plan  $\hat{x}_i(\theta_i)$ , since consumer  $i$ 's wealth is already associated with the index  $i$ .

## 2.3 Equilibrium on the goods market

In period 2, at a given price of the novel good  $p$ , a consumer maximizes (1) subject to the budget constraint

$$\tilde{y}_i \geq c_i + px_i.$$

Solving the maximization problem yields the individual demand for good  $x$ ,

$$x_i(p) = \left( \frac{\alpha \theta_i}{p} \right)^{\frac{1}{1-\alpha}} = \theta_i \left( \frac{\alpha}{p} \right)^{\frac{1}{1-\alpha}}, \quad \theta_i \in \{0, 1\}. \quad (2)$$

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<sup>3</sup>The robustness of the main result for nonlinear technologies is discussed in the appendix.

Aggregate demand is therefore

$$x(p) = \int_0^1 \theta_i di \left( \frac{\alpha}{p} \right)^{\frac{1}{1-\alpha}} = s \left( \frac{\alpha}{p} \right)^{\frac{1}{1-\alpha}},$$

leading to inverted aggregate demand

$$p = \alpha \left( \frac{s}{x(p)} \right)^{1-\alpha}.$$

Since producing firms act as price takers on the product market in period 2, aggregate investment  $X = x_{sup}$  determines the good's price in equilibrium according to

$$p = p(X, s) = \alpha \left( \frac{s}{X} \right)^{1-\alpha}.$$

The equilibrium return on investment in the production of good  $x$  simply equals the good's price,

$$r = \frac{p(X, s) X}{X} = p(X, s).$$

## 2.4 Equilibrium investment

Consider now a possible symmetric Bayesian Nash equilibrium of the investment game, where each consumer with preference/signal  $\theta_i = 1$  invests the same amount  $\hat{x}$ , whereas  $\theta_i = 0$  types do not invest. Consumers who invest a positive amount  $\hat{x}$  less than  $w_i$  must be indifferent between an investment in the innovation and an investment at the risk-free rate  $R$ . This follows from condition *ii.* of definition 1. Therefore, the equilibrium investment  $\hat{x}$  of consumers who care about the good can be determined as follows.

$$\begin{aligned} R &= \mathbb{E}_s[p(s \cdot \hat{x}, s) | \theta_i = 1] \\ &= \beta \alpha \left( \frac{\beta}{\beta \hat{x}} \right)^{1-\alpha} + (1 - \beta) \alpha \left( \frac{1 - \beta}{(1 - \beta) \hat{x}} \right)^{1-\alpha} = \alpha \hat{x}^{\alpha-1} \\ \iff \hat{x} &= \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

Hence, the equilibrium aggregate investment in state  $s = \beta$  is

$$X = \beta \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$$

and it is

$$X = (1 - \beta) \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$$

in state  $s = 1 - \beta$ . The model leads to a result contradicting a standard economic intuition. The return on investment in the novel good is not higher in the “good” state of the world

with high demand compared to the state with low demand for the novel good:

$$p = \alpha \left( \frac{s}{s\hat{x}} \right)^{1-\alpha} = R.$$

The reason is that equilibrium investment is proportional to the share of consumers demanding the good in the future, which makes the equilibrium good's price and therefore investment return state-independent.

We now compare this market equilibrium outcome to a planner's solution, assuming the planner knows the realization of  $s$ . An investment in  $x$  has an opportunity cost of  $R$  units of the consumption good  $c$  in period 2. Hence, social welfare is maximized when all individuals consume

$$x_i = \left( \frac{\alpha \theta_i}{R} \right)^{\frac{1}{1-\alpha}}.$$

This is the quantity demanded at a relative price of  $R$ . Any deviation of equilibrium prices from this level reduces social welfare. We state this result formally for later use.

**Lemma 1.** *With a linear production technology, the capital allocation is Pareto-efficient if and only if aggregate investment is  $X = s \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$ . This outcome is realized in a market equilibrium if and only if the good's market clearing price is  $p = R$  independent of the state  $s$ .*

**Proof.** See Appendix. □

Therefore, the symmetric equilibrium derived above maximizes social welfare.

**Proposition 1.** *When all consumers hold wealth  $w_i \geq \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$ , there is a symmetric investment equilibrium in which all consumers invest an amount  $\hat{x} = \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}}$  in the capacity for the production of good  $x$ , if and only if they would like to consume good  $x$  themselves ( $\theta_i = 1$ ). This symmetric equilibrium is Pareto-optimal and maximizes utilitarian welfare.*

According to Proposition 1, crowdinvestments efficiently replace a missing forward market for good  $x$ . The more consumers are interested in the good, the more are willing to invest and finance production. Thus, firms do not have to convince third parties that their business idea is worth investing in; instead, the source of funding is consumers who already find the product attractive.

Clearly, this efficiency result rests on the assumed linearity of the production function. In the presence of nonlinearities, the resulting equilibrium would generally not be efficient anymore. However, equilibrium investment still increases with the share of interested consumers (see the appendix for the model with nonlinear production technology).

## 2.5 Two wealth classes and equality

In the remainder of this paper, we consider an economy which is composed of two groups of consumers of equal size. The fraction of consumers who care about good  $x$  may differ across groups. The shares  $s_1$  and  $s_2$  of consumers who care about good  $x$  are independently distributed according to

$$s_1 = 2 \cdot \int_0^{0.5} \theta_i di = \begin{cases} \beta > 1/2 & \text{with probability } \frac{1}{2} \\ 1 - \beta & \text{with probability } \frac{1}{2} \end{cases}$$

$$s_2 = 2 \cdot \int_{0.5}^1 \theta_i di = \begin{cases} \beta > 1/2 & \text{with probability } \frac{1}{2} \\ 1 - \beta & \text{with probability } \frac{1}{2}. \end{cases}$$

When a consumer from the first group ( $g = 1$ ) observes signal  $\theta_i = 1$ , he receives information regarding the aggregate preference distribution in his own group, but still relies on his prior to estimate demand in the other group. Hence, he attaches the following posterior probabilities to the vector of states  $(s_1, s_2)$ :

$(s_1, s_2)$	$(\beta, \beta)$	$(1 - \beta, 1 - \beta)$	$(1 - \beta, \beta)$	$(\beta, 1 - \beta)$
$\Pr((s_1, s_2)   \theta_i = 1, g = 1)$	$\frac{\beta}{2}$	$\frac{1 - \beta}{2}$	$\frac{1 - \beta}{2}$	$\frac{\beta}{2}$

In case a consumer from group 1 receives signal  $\theta_i = 0$ , the posterior probabilities are

$(s_1, s_2)$	$(\beta, \beta)$	$(1 - \beta, 1 - \beta)$	$(1 - \beta, \beta)$	$(\beta, 1 - \beta)$
$\Pr((s_1, s_2)   \theta_i = 0, g = 1)$	$\frac{1 - \beta}{2}$	$\frac{\beta}{2}$	$\frac{\beta}{2}$	$\frac{1 - \beta}{2}$

Again, there is an equilibrium in which all consumers invest the same positive amount if they care about good  $x$ . The equilibrium investment  $\hat{x}$  of these investors fulfills

$$\begin{aligned} R &= \mathbb{E}_s[p(s \cdot \hat{x}) | \theta_i = 1] \\ &= \frac{\beta}{2} \alpha \left( \frac{\beta}{\beta \hat{x}} \right)^{1 - \alpha} + \frac{1 - \beta}{2} \alpha \left( \frac{1 - \beta}{(1 - \beta) \hat{x}} \right)^{1 - \alpha} + \frac{1}{2} \alpha \left( \frac{\frac{1}{2}}{\frac{1}{2} \hat{x}} \right)^{1 - \alpha} \\ &= \alpha \hat{x}^{\alpha - 1} \\ \iff \hat{x} &= \left( \frac{\alpha}{R} \right)^{\frac{1}{1 - \alpha}}. \end{aligned}$$

Assuming all consumers can afford this investment, equilibrium aggregate investment is

$$X = \begin{cases} \beta \left( \frac{\alpha}{R} \right)^{\frac{1}{1 - \alpha}} & \text{if } s_1 = s_2 = \beta \\ (1 - \beta) \left( \frac{\alpha}{R} \right)^{\frac{1}{1 - \alpha}} & \text{if } s_1 = s_2 = 1 - \beta \\ \frac{1}{2} \left( \frac{\alpha}{R} \right)^{\frac{1}{1 - \alpha}} & \text{if } s_1 \neq s_2. \end{cases}$$

Again, aggregate investment scales up one-to-one with demand, the good's price is independent of the state, and the equilibrium maximizes social welfare.

A model with standard debt contracts promising a certain return of  $R$  instead of equity contracts would yield the same equilibrium outcome if all consumers hold enough wealth. Just as in the efficient equity equilibrium, every unit invested would yield a return of  $R$  to the investor and the firm would make zero profits.

## 2.6 The role of wealth inequality

Suppose now that consumers of group 2 (the poor) do not hold any wealth ( $w_i = 0$ ) to invest, but they have some income  $y_i > 0$  in period 2 for consumption. The poor cannot borrow against their own future income in order to finance an investment. Therefore, they do not invest on the capital market. Consider an equilibrium in which only wealthy consumers (group 1) with  $\theta_i = 1$ , invest an identical amount  $\hat{x} > 0$ . The individual symmetric investment  $\hat{x}$  of wealthy consumers is determined by equating the expected investment return with its opportunity cost  $R$ :

$$\begin{aligned} R &= \mathbb{E}_s[p(s \cdot \hat{x}) | \theta_i = 1, g = 1] = \sum_{s_1, s_2} \Pr((s_1, s_2) | \theta_i = 1, g = 1) \cdot \alpha \left( \frac{\frac{s_1 + s_2}{2}}{\frac{s_1}{2} \cdot \hat{x}} \right)^{1-\alpha} \\ &= \frac{\beta}{2} \alpha \left( \frac{\beta}{\frac{\beta}{2} \hat{x}} \right)^{1-\alpha} + \frac{1-\beta}{2} \alpha \left( \frac{1-\beta}{\frac{1-\beta}{2} \hat{x}} \right)^{1-\alpha} + \frac{\beta}{2} \alpha \left( \frac{\frac{1}{2}}{\frac{\beta}{2} \hat{x}} \right)^{1-\alpha} + \frac{1-\beta}{2} \alpha \left( \frac{\frac{1}{2}}{\frac{1-\beta}{2} \hat{x}} \right)^{1-\alpha} \\ &= \frac{1}{2} \alpha \left( \frac{2}{\hat{x}} \right)^{1-\alpha} + \frac{\beta}{2} \alpha \left( \frac{1}{\beta \hat{x}} \right)^{1-\alpha} + \frac{1-\beta}{2} \alpha \left( \frac{1}{(1-\beta) \hat{x}} \right)^{1-\alpha}. \end{aligned}$$

Consequently, depending on the state of the world, there are only two aggregate investment levels:

$$X = \begin{cases} \beta \hat{x} & \text{if } s_1 = \beta \\ (1-\beta) \hat{x} & \text{if } s_1 = 1-\beta. \end{cases}$$

Equilibrium investment and thus supply only depends on the preferences of the wealthy, but aggregate demand depends on the preferences of all consumers. The new equilibrium does not maximize social welfare, because it does not take into account the marginal social benefit from an investment in production capacity to satisfy demand of the poor consumer group. We now show that efficient investment is possible if and only if all groups of consumers have sufficient aggregate wealth to invest, so that aggregate investment depends on the preferences of consumers from all groups who later consume the new product.

**Proposition 2.** *Consider the case of two distinct groups of consumers, and suppose that the preference and wealth distribution within each group is independent. Then there exists an efficient equilibrium if and only if aggregate wealth in each group is sufficient for interested consumers to finance production of the group's efficient consumption, i.e.,*

$$2 \cdot \int_0^{0.5} w_i di \geq (\alpha/R)^{\frac{1}{1-\alpha}} \quad \text{and} \quad 2 \cdot \int_{0.5}^1 w_i di \geq (\alpha/R)^{\frac{1}{1-\alpha}}. \quad (3)$$

**Proof.** Sufficiency: If all consumers have  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ , then section 2.5 demonstrates that an efficient equilibrium exists. The derivation is virtually identical with heterogeneous individual investment yielding the same aggregate investment.

Necessity: To be shown: If an efficient equilibrium exists, then aggregate wealth fulfills (3). In an efficient equilibrium, aggregate investment scales up linearly with the share of interested consumers in each group (Lemma 1):

$$X = \frac{s_1 + s_2}{2} (\alpha/R)^{1/(1-\alpha)}. \quad (4)$$

Denote the investment amount of investor  $i$  if  $\theta_i = 1$  by  $\hat{x}_i(\theta_i = 1)$  and if  $\theta_i = 0$  by  $\hat{x}_i(\theta_i = 0)$ . Recall that group 1 are all consumers  $i \in [0, 0.5]$  and group 2 are all consumers  $i \in (0.5, 1]$ . Now we can write aggregate investment  $X$  in terms of investment strategies of all consumers,

$$\begin{aligned} X &= \int_0^{0.5} [s_1 \hat{x}_i(\theta_i = 1) + (1 - s_1) \hat{x}_i(\theta_i = 0)] di + \int_{0.5}^1 [s_2 \hat{x}_i(\theta_i = 1) + (1 - s_2) \hat{x}_i(\theta_i = 0)] di \\ &= \int_0^{0.5} [s_1 (\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)) + \hat{x}_i(\theta_i = 0)] di \\ &\quad + \int_{0.5}^1 [s_2 (\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)) + \hat{x}_i(\theta_i = 0)] di. \end{aligned} \quad (5)$$

Since by assumption an efficient equilibrium exists, both (4) and (5) have to hold for all realizations of  $(s_1, s_2)$ . This is only possible if  $\int_0^1 \hat{x}_i(\theta_i = 0) di = 0$ , i.e., consumers of type  $\theta_i = 0$  do not invest. Hence, simplifying (5) and equating aggregate investment with efficient aggregate investment (4), the following conditions hold in any efficient equilibrium:

$$\begin{aligned} 2 \cdot \int_0^{0.5} \hat{x}_i(\theta_i = 1) di &= (\alpha/R)^{1/(1-\alpha)} \text{ and} \\ 2 \cdot \int_{0.5}^1 \hat{x}_i(\theta_i = 1) di &= (\alpha/R)^{1/(1-\alpha)}. \end{aligned} \quad (6)$$

The investment budget constraint requires  $\hat{x}_i(\theta_i = 1) \leq w_i$  for all  $i$ . Thus, (6) implies

$$\begin{aligned} 2 \cdot \int_0^{0.5} w_i di &\geq (\alpha/R)^{1/(1-\alpha)} \text{ and} \\ 2 \cdot \int_{0.5}^1 w_i di &\geq (\alpha/R)^{1/(1-\alpha)}. \end{aligned} \quad \square$$

Independence of wealth and preference distribution within the group ensures that wealthy consumers can invest on behalf of their less wealthy fellow group members, because they have the same preference distribution. A consequence of Proposition 2 is that a mismatch of the income and wealth distribution on group level may lead to an inefficient allocation of financial capital. The reason is that the mismatch between wealth and income distribution

leads to a mismatch between the ability to invest and the future propensity to consume. The result is related to the limits of arbitrage literature<sup>4</sup>, because consumers from at least one group have information that would allow them to arbitrage away excess returns, but they cannot completely act on it due to wealth constraints.

In Proposition 2, consumer groups are characterized by their correlated preferences. The following corollary considers the special case in which groups are characterized by their wealth endowment and where preferences within the wealth classes are correlated.

**Corollary 1.** *Consider the case of two distinct groups of consumers, and suppose  $w_i$  is constant within each group. Then there exists an efficient equilibrium if and only if consumers in each group hold enough wealth to finance production of their own efficient consumption in case of  $\theta_i = 1$ ,  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ .*

In general, one could imagine additional equilibria to the efficient ones we illustrated if all consumers have sufficient wealth, but the next proposition shows that there are no equilibria where the capital allocation is inefficient.

**Proposition 3.** *If all consumers have wealth  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ , then an inefficient equilibrium does not exist.*

**Proof.** See Appendix. □

The intuition behind this result is as follows. If there were an inefficient equilibrium, then there would be at least one state where the return on investment exceeds  $R$ . Because the preferences of crowdinvestors contain information about the likelihood of such a state, they would best respond by increasing investment. For example, if state  $(\beta, 1 - \beta)$  has a return exceeding  $R$ , then crowdinvestors with  $\theta_i = 1$  in group 1 assign a high probability to this state, and (collectively) increase investment until they expect a return of  $R$ . If, on the other hand, state  $(1 - \beta, \beta)$  has an excess return, then  $\theta_i = 1$  crowdinvestors from group 2 assign a high probability to this state, and increase investment. Together, the two groups of interested consumers can remove any excess return, because they have enough wealth to arbitrage away mispricing.

## 3 Financial intermediaries and market research

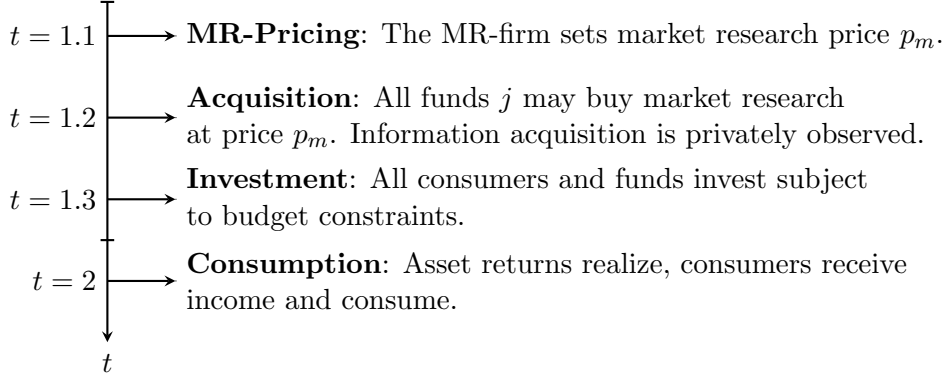
### 3.1 The extended model

In this section, we add a financial sector consisting of  $N \in \mathbb{N}$  investment funds<sup>5</sup>, indexed by  $j$ , with exogenous large endowment  $W_j > 0$ , who may acquire information about consumer

<sup>4</sup>See, for example, Shleifer and Vishny (1997), and Gromb and Vayanos (2010) for a review.

<sup>5</sup>We call the financial market intermediaries “investment funds,” but these may be replaced by any other large investing institutional entity, such as banks, venture capital firms, hedge funds, pension funds, or investment banking divisions.





**Figure 1:** The timing of decisions.

preferences and maximize expected investment returns. They can either make conventional investments with return  $R$ , or they can invest in the novel consumption good with variable return. These funds may be viewed as arbitrageurs, who arbitrage away excess returns in the investment of the firm producing the novel good.

We assume that investment funds have no information<sup>6</sup> on the realization of consumer preferences (unlike consumers, whose preference  $\theta_i$  is informative). Funds may acquire information about the realization of preferences in the consumer population to identify worthwhile investment opportunities. This can be thought of as buying market studies which evaluate the revenue potential of the new product or commissioning consumer surveys.

Formally, we represent the “market research” information by two binary and independent signals about the preference realization in the wealthy (1) and poor (2) consumer group,  $m \in \{0, 1\} \times \{0, 1\}$ . The signal quality is exogenously given by

$$\begin{aligned}
 \gamma &:= \Pr(m_1 = 1 | s_1 = \beta) = \Pr(m_1 = 0 | s_1 = 1 - \beta) \\
 &= \Pr(m_2 = 1 | s_2 = \beta) = \Pr(m_2 = 0 | s_2 = 1 - \beta) > 1/2.
 \end{aligned}$$

Market research is offered by a monopolist market research (MR) firm, which sells the same signal  $m$  to all interested buyers, i.e., signals are perfectly correlated. Neither the assumption that the MR sector is monopolistic nor that signals are perfectly correlated drives our results, as will become clear shortly. For non-triviality, we assume the MR firm can produce market research (i.e., conduct surveys, gather and analyze data) at sufficiently low cost  $c > 0$ , so that it can always offer market research at positive market research price  $p_m$ . If the MR firm sells market research to  $0 \leq n \leq N$  funds, then its profit is given by  $\pi_{MR} = np_m - \mathbf{1}\{n > 0\}c$ .

In contrast to the model of section 2, aggregate investment is now the sum of the infinitesimally small crowdfund investments  $\hat{x}_i$  and the investments of the “large” financial sector

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<sup>6</sup>This assumption is made to simplify the exposition, and any *imperfect* information about the realization of  $s$  for investment funds yields the same results concerning efficient investment for any  $N \in \mathbb{N}$ .

entities  $f_j$ . Thus aggregate investment in good  $x$  is

$$X = \int_0^1 \hat{x}_i di + \sum_{j=1}^N f_j.$$

The timing of decisions is displayed in Table 1. And now that we added new players to the game, we extend the equilibrium definition as follows.

**Definition 2.** *An equilibrium of the extended model consists of*

- i. a market research price  $p_m$  set by the MR-firm at  $t = 1.1$ ,*
- ii. an acquisition plan  $a_j(p_m) \in \{0, 1\}$  to purchase market research  $m \in \{0, 1\} \times \{0, 1\}$  for each investment fund at  $t = 1.2$ ,*
- iii. an investment plan  $\hat{x}_i(\theta_i)$  for each consumer at  $t = 1.3$ ,*
- iv. an investment plan  $f_j(p_m, m_j)$  for each investment fund at  $t = 1.3$ , where  $m_j = m$  iff  $a_j = 1$  and  $m_j = \emptyset$  iff  $a_j = 0$ ,*
- v. a consumption plan  $x_i(p)$  for each consumer,*
- vi. a relative price function  $p(X, s)$  for good  $x$ ,*

*so that*

- i. the market price  $p_m$  maximizes expected profits of the market research firm at  $t = 1.1$ , taking into account  $a_j(p_m)$  of all  $j$ ,*
- ii. the information acquisition plans  $a_j$  and investment plans  $\hat{x}_i$  and  $f_j$  constitute a Bayesian Nash equilibrium of the investment game subject to the wealth constraints, taking into account the consumption plans and the relative price  $p(X, s)$ ,*
- iii. the consumption plan  $x_i$  maximizes utility subject to the consumer's future budget constraint, and*
- iv. at the price  $p(X, s)$ , the demand for good  $x$  equals production capacity  $X$ .*

In our extended model, there are two possible sources of inefficiency, (i) that the creation of market research wastes cost  $c > 0$  (new), and (ii) that state-contingent investment in the novel product is inefficient in the sense of Lemma 1 (as before). Since we assume that there is sufficient aggregate wealth in the economy to fund production of the efficient consumption in every state and also that utility is transferable, Pareto-efficiency from an ex-ante perspective requires that neither of the two kinds of inefficiencies occur, i.e., requires that no market research is carried out and that the capital allocation is efficient.

**Definition 3.** *Pareto-efficiency from an ex-ante perspective involves all agents in the economy (consumers, funds, market research firm), and requires that*

- i. the market research cost  $c > 0$  is not wasted, and*
- ii. the state-contingent capital allocation is efficient (Lemma 1).*

The following analysis focuses on the possibility of efficient state-contingent investment, i.e., efficiency of the capital allocation (ii), which is necessary but not sufficient for Pareto-optimality. Our results show that Pareto-efficiency with an unequal wealth distribution fails not only because the market research cost is wasted, but because the capital allocation cannot be efficient even if market research is acquired in equilibrium.

## 3.2 Equilibrium existence

We first establish the existence of an equilibrium in the extended version of our model.

**Proposition 4.** *An equilibrium in which all crowdinvestors play pure strategies exists.*

**Proof.** See Appendix. □

The following sections analyze the equilibrium properties, especially with respect to the wealth distribution of crowdinvestors, in more detail.

## 3.3 The impossibility of efficient investment with active funds

To characterize the set of possible equilibria in more detail, we next show that efficient state dependent investment and active funds are inconsistent. The main obstacle to achieving efficient investment with active investment funds is an informational friction: Funds first have to buy the information that allows them to adjust their investment, but there are no excess returns in an efficient equilibrium that would incentivize them to buy market research. We discuss these obstacles in more detail in section 3.5.

**Proposition 5.** *There exists no equilibrium with an efficient state-dependent capital allocation in which investment funds invest.*

**Proof.** See Appendix. □

This result is independent of the wealth distribution of consumers. The proof proceeds in two main steps. First, suppose there is an efficient equilibrium where funds invest. Efficiency implies the investment return is  $R$  in every state (Lemma 1). But then it does not pay to buy market research for price  $p_m > 0$ , since return  $R$  can be realized elsewhere without this additional cost. Second, given that funds must be uninformed in an efficient equilibrium, their investment is constant over states  $s$ . Aggregate investment may still react to changes in

$s$ , since consumers may invest depending on their preferences. However, they do not invest as much as they would if investment funds were inactive, i.e., not as much as in the efficient equilibrium, since this would imply an expected return of less than  $R$ . But if consumers invest less, then the slope of aggregate investment  $X(s)$  in  $s$  cannot be equal to  $(\alpha/R)^{1/(1-\alpha)}$  as in the efficient equilibrium. That is, investment cannot scale up one-to-one with future aggregate demand. Consequently, there exists at least one state where aggregate investment is inefficient, which contradicts the earlier assumption that an efficient equilibrium in which funds invest exists.

### 3.4 Equilibrium if all consumers can invest

As benchmark, we again consider the case where all consumers have wealth  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ . In this case, the equilibrium of section 2 persists after adding investment funds: All consumers with type  $\theta_i = 1$  invest, which is efficient and gives an investment return of  $R$  in each state (Proposition 1). Given this investment strategy by crowdinvestors, it does not pay for funds to participate; they do not buy market research and do not invest.

**Proposition 6.** *If all consumers have wealth  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ , then there exists an equilibrium where the consumer investment strategies are the same investment strategies as in Proposition 1 ( $\hat{x}_i = \theta_i(\alpha/R)^{1/(1-\alpha)}$ ), and investment funds neither acquire information nor invest. This equilibrium is efficient.*

**Proof.** Suppose all consumers with  $\theta_i = 1$  invest  $\hat{x}_i = (\alpha/R)^{1/(1-\alpha)}$ .

Investment stage: The profit of one of the  $N$  corporate investors when using investment strategy  $f_j$  with opportunity cost  $R$  and information set  $\mathcal{I}_j$ , given the investment strategies  $\hat{x}_i$  of all consumers, is

$$\mathbb{E}_s[\pi_j(f_j, f_{-j}, \hat{x})|\mathcal{I}_j] = f_j(\mathbb{E}_s[p(f, \hat{x})|\mathcal{I}_j] - R).$$

The first order condition of Cournot competition with respect to  $f_j$ , taking investment strategies of all other players as given, is

$$0 = \mathbb{E}_s[p'(f, \hat{x})|\mathcal{I}_j]f_j + \mathbb{E}_s[p(f, \hat{x})|\mathcal{I}_j] - R \iff \mathbb{E}_s[p(f, \hat{x})|\mathcal{I}_j] = R - \mathbb{E}_s[p'(f, \hat{x})|\mathcal{I}_j]f_j, \quad (7)$$

hence funds aim to realize a price  $p > R$ , since  $p' < 0$ . However, the investments of the consumers are enough to realize a price  $p = R$  in all states. Hence, first order condition (7) cannot be fulfilled with equality for any positive  $f_j$ , and the optimal choice is a corner solution  $f_j = 0$  for all  $j$ .

Acquisition stage: Since investment funds do not invest, buying market research is strictly dominated for  $p_m > 0$ .  $\square$

### 3.5 Equilibrium if one group of consumers cannot invest

If a group of consumers is poor and cannot invest, then there may be investment opportunities for the financial sector. If the poor are interested in the novel good and the wealthy are not, then future demand for the novel good will be large but investment by the wealthy and consequently supply will be small. Hence, the price of the novel good  $p$ —which is also the per unit return of an investment in the novel good—is larger than  $R$ . In this state it would pay for the financial sector to swoop in and arbitrage away (part of) the excess return on investment, because wealthy investors underestimate future demand for the novel good.

However, as a consequence of Proposition 2 and Proposition 5, there will be some inefficiency in capital allocation whenever there is a group of consumers that does not have enough wealth to invest. Throughout this section, we assume that all consumers of group 2 (the poor) have no wealth, i.e.,  $\int_{0.5}^1 w_i di = 0$ .

**Corollary 2.** *There exists no equilibrium with an efficient state-dependent capital allocation if aggregate wealth in any group is less than  $1/2 \cdot (\alpha/R)^{1/(1-\alpha)}$ .*

**Proof.** *If investment funds do not invest, then the equilibrium cannot be efficient.* This follows from Proposition 2.

*If investment funds invest, then the equilibrium cannot be efficient.* This follows from Proposition 5.  $\square$

In order to see why an efficient outcome is impossible if some consumer groups cannot invest, we describe the frictions involved in more detail. One obstacle to efficiency is the market power of investment funds if  $N < \infty$ . Efficient investment implies that all investors make zero profits compared to the outside option at rate  $R$ , but if the fund sector is not perfectly competitive, then funds will withhold some investment to drive up prices (and therefore investment returns). This can be directly seen in the first order condition (7) of the fund investment problem. Thus, even if funds were perfectly informed about the state of consumer preferences  $s$ , they would not want to remove all inefficiency, as this would imply zero profits (or in fact a loss, since becoming informed is costly).

If the fund sector is competitive ( $N \rightarrow \infty$ ), then an efficient equilibrium is still not possible. To understand why, consider the following proposition, which establishes that, if the investment fund sector is competitive, then aggregate investment will not be affected by market research in equilibrium.

**Proposition 7.** *Suppose the investment fund sector is competitive ( $N \rightarrow \infty$ ), so that  $X = \int \hat{x}_i di + \int f_j dj$ . Then there exists no equilibrium where a positive mass of funds buys market research for  $p_m > 0$ .*

**Proof.** Suppose there is an equilibrium with a positive mass of funds buying market research and investing in the novel good using the superior information. Because a single investment

fund  $j$  is small and its investment does not influence  $p$ ,  $j$  can deviate by not buying research, keep investing, and making the same investment return as before, yet saving cost  $p_m > 0$ .  $\square$

Proposition 7 shows that information acquisition is subject to a free-rider problem in a continuum of investment funds. As soon as aggregate investment reacts to market research information—which can only be the case if a positive probability mass of funds acquire it—then it pays to deviate for informed funds to not buying market research, and free-ride on the information incorporated in the aggregate investment by others.<sup>7</sup> Consequently, even if there is a continuum of investment funds, no or only finitely many funds will become informed in equilibrium, but their impact on aggregate investment is negligible.<sup>8</sup>

Thus, with a competitive fund sector, the market for information breaks down. This result has a similar flavor as the one in [Grossman and Stiglitz \(1980\)](#) for financial markets, who show that there is no fully revealing equilibrium with costly information acquisition, because uninformed traders can free-ride on the information of informed traders.

Finally, even if a competitive fund sector somehow got hold of the market research signal for free, this would still not lead to efficient investment, unless market research was noiseless ( $\gamma = 1$ ). That is, a noisy signal ( $\gamma < 1$ ) prevents efficiency, because a wrong market research signal—which occurs with positive probability—leads to an inefficiently high or low investment.

Thus, an efficient equilibrium if not all groups of consumers can invest the efficient amount exists only if  $\gamma = 1$ ,  $N \rightarrow \infty$ , and market research is costlessly available. But this is equivalent to a situation where the consumer preference realization is common knowledge, which is not realistic.

Our results show that financial intermediaries cannot fully correct the inefficiency that arises when wealth and income distribution do not match. However, they may still play a useful role in increasing social welfare in such situations. To see why this is so, consider as a simple example the case where no consumer holds any wealth. Then the addition of intermediaries is unambiguously welfare improving—even without the possibility to purchase market research.

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<sup>7</sup>The same argument would apply to crowdinvestors if they were allowed to buy market research. Hence, assuming that consumers may also buy market research would not change our results.

<sup>8</sup>Moreover, independent market research signals cannot yield efficient investment either. Although a law of large numbers guarantees that many independent market research draws  $m_j$ ,  $j = 1, \dots, N$  reveal the state as  $N \rightarrow \infty$  perfectly even for  $\gamma < 1$ , the market for information would break down, because it does not pay for funds to become informed (Proposition 7).

## 4 Further extensions of the baseline model

### 4.1 Crowdinvestors and intermediaries

As we established in sections 2 and 3, efficient investment in our setting is possible if and only if aggregate wealth in both groups is large enough. This result is robust to a number of modifications and extensions of our model. It holds for any finite number of investment funds. When the number of funds approaches infinity, then there is a free-rider problem and the market for information breaks down (Proposition 7). Consequently, even a competitive fund sector is not sufficient for an efficient outcome.

The inefficiency results are also unaffected if several market research firms engage in Bertrand competition. In this case funds instead of MR firms extract profits, but the price of market research must stay positive in equilibrium, since it is costly to produce. As we showed above, acquisition of market research at positive prices is incompatible with an efficient equilibrium.

Our results are robust to changes of our assumption concerning market research signal correlation. If market research signals are independent rather than perfectly correlated, then for  $N < \infty$  the Cournot objective (7) of the funds still precludes efficient investment; for  $N \rightarrow \infty$  the free-rider problem still prevents efficient aggregate investment. For the same reasons, our results hold for any market research signal quality  $\gamma \in [0, 1]$ .

It is also easy to see that extensions such as a larger number of states, a larger number of consumer groups, or the possibility for crowdinvestors to acquire market research do not change our conclusions. On the latter, as in the case of a competitive fund sector, an incentive to free-ride on the (costly) market research information of others prevents efficiency.

Consequently, the only situation where financial intermediaries can bring about efficient investment is if (i) perfect information about the preference realization in the population is (ii) costlessly available, and (iii) the investment fund sector is competitive.

### 4.2 Forward markets and pre-order crowdfunding

An important segment of the crowdfunding industry permits consumers to pre-order products. This segment essentially provides firms with an organized forward market that also serves as a financing device for innovations. It requires that the properties of the innovation are contractible at the date of pre-ordering. We now briefly discuss the robustness of the link between wealth/income distribution and efficiency when a pre-order crowdfunding instead of an equity crowdfunding market exists.

The analysis of a forward market requires a modified equilibrium concept. In addition to the objects listed in definition 1, an equilibrium at a given borrowing rate  $B > R$  (due to credit market frictions) consists of a forward market price  $p_1$ , preference contingent



forward market demand  $x_i^F(\theta_i, p_1)$  (replacing all consumer investment plans) and a credit demand schedule  $d_i(\theta_i, p_1)$  for each consumer. As before, we assume the innovative firm invests its entire revenue. Note that if  $p_1 > 1$ , then there is supply left after serving forward contract obligations in  $t = 2$  and a market clearing spot market price  $p_2$  realizes. Once more we consider the case with two wealth classes from Section 2, one wealthy and one poor. Since the forward market is the only way of raising capital for production, an unconstrained efficient outcome requires that the forward market revenue per interested consumer is  $(\alpha\theta_i/R)^{1/(1-\alpha)}$ . Obviously, this can only be achieved if interested consumers of *both* groups pay on average  $(\alpha\theta_i/R)^{1/(1-\alpha)}$ . Moreover, if there is no resale between consumers at  $t = 2$ , then efficiency requires that forward market payments by all interested consumers must equal  $(\alpha\theta_i/R)^{1/(1-\alpha)}$  exactly. Due to  $B > R$ , wealthy and poor consumers face different opportunity costs in terms of  $t = 2$  consumption when buying good  $x$  on the forward market. However, different opportunity costs of consumption and the fact that all interested agents consume the same amount of  $x$  is not compatible with utility maximization of all consumers. Therefore, pre-order crowdfunding can work efficiently only if the poor are sufficiently wealthy, just as we find in the case of equity crowdfunding.

### 4.3 Sequential investments

On most crowdfunding and crowdfund investment platforms, the current aggregate investment into a project is observable at any point in time for potential investors. An important question is whether our previous inefficiency results change if aggregate investment to date is observable. In that case wealthy crowdfunders might learn something about the preferences of the poor, and consequently adjust their investment. In order to study this question, we extend the simultaneous investment game from section 2 to a simple sequential two stage investment game.

Consider the following modification of the baseline setup from section 2. In  $t = 0$ , all crowdfunders may condition their investment plans  $\hat{x}_i^{t=0}(\theta_i)$  only on their own private information, leading to aggregate investment  $X_0 = \int_0^1 \hat{x}_i^0(\theta_i) di$ . In  $t = 1$ , all crowdfunders may condition their investment plans  $\hat{x}_i^1(\theta_i, X_0)$  on their private information and aggregate investment from the previous investment stage. The equilibrium concept from definition 1 can be readily extended to the present setup by replacing the one stage by the two stage investment plans. In equilibrium, investors can adjust their investment to the realization of  $X_0$ , and use the information contained in  $X_0$  about the distribution of  $\theta_i$  when investing at  $t = 1$ . Overall investment by crowdfunder  $i$  in the company is  $\hat{x}_i^0(\theta_i) + \hat{x}_i^1(\theta_i, X_0)$ , i.e., the sum of the investments in  $t = 0$  and  $t = 1$ , with  $\hat{x}_i^t \geq 0$  as before.

It is straightforward to show that all equilibria from the baseline model can be extended to equilibria in this dynamic model. Hence, the set of equilibria is weakly larger in the dynamic model.

**Proposition 8.** *Any equilibrium with investment strategy profile  $\{\hat{x}_i(\theta_i)\}_i$  from the baseline model in section 2 can be extended to an outcome-identical equilibrium in the dynamic model.*

**Proof.** Take any equilibrium investment strategy profile  $\{\hat{x}_i(\theta_i)\}_i$  from the baseline model. Consider the following equilibrium candidate for the dynamic model:

$$\begin{aligned}\hat{x}_i^0(\theta_i) &= 0 \quad \forall i, \\ \hat{x}_i^1(\theta_i, X_0) &= \hat{x}_i(\theta_i) \quad \forall i.\end{aligned}$$

Since nobody invests in  $t = 0$ ,  $X_0 = 0$  in all states, so aggregate investment is uninformative. Consequently, at  $t = 1$ , investors have the same information they have in the baseline model, so if  $\hat{x}_i(\theta_i)$  is an equilibrium strategy in the baseline model, it also must be an equilibrium strategy in the last investment period of the dynamic model.

It remains to be shown that there is no profitable deviation at  $t = 0$ . A unilateral deviation of investing at  $t = 0$  means investor  $i$  has the same information compared to the candidate strategy, and it does not change the investments by other investors, since  $i$  has no mass and does not affect  $X_0$ . Consequently,  $i$  is indifferent between investing earlier or investing according to the equilibrium candidate strategy.  $\square$

The question now is whether efficient equilibria exist in the dynamic model that do not exist in the baseline model (Proposition 2) due to the possibility of reacting to aggregate investment. First, note that an efficient equilibrium does not exist if consumers of one of the groups do not have any wealth. The intuition is quite simple: If investors of a group cannot invest at all, then nothing can be learned about their preferences from observing aggregate investment. However, efficient equilibria exist if the poor consumers cannot invest enough on their own, but enough so that aggregate investment becomes informative about their preferences.<sup>9</sup> More specifically, efficient equilibria exist as long as the poor consumers have some wealth and the wealthy have enough to cover the rest. The efficient equilibria are coordination equilibria in the sense that the poor consumers first invest and reveal their preference distribution to the wealthy consumers, who later invest on behalf of the poor.<sup>10</sup>

Proposition 8 implies that inefficient equilibria exist along with the efficient coordination equilibria. A reasonable equilibrium refinement is to require that no weakly dominated strategies are played in equilibrium. In our dynamic investment game, investing at  $t = 0$  is a weakly dominated strategy: Clearly, any investment at  $t = 0$  can be postponed to  $t = 1$  without any drawbacks. However, if—off equilibrium—a large amount is invested at  $t = 0$ ,

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<sup>9</sup>See the online appendix for a detailed example at [http://gruener.vwl.uni-mannheim.de/fileadmin/user\\_upload/gruener/pdf/crowd-appendix.pdf](http://gruener.vwl.uni-mannheim.de/fileadmin/user_upload/gruener/pdf/crowd-appendix.pdf).

<sup>10</sup>There can be different efficient equilibria that overcome the wealth constraints of the poor, but all rely on the fact that the poor group reveals its preferences via investments at  $t = 0$ , so that others know how much more they have to invest in order to arbitrage away mispricing. This is why these efficient equilibria do not exist in the static model, or whenever aggregate investment is not observable. And clearly these efficient equilibria survive if we added more investment stages in the model or even set up a continuous time investment game.

so that the investment return will be below  $R$  in any state, then a player who postponed his own investment to  $t = 1$  could still react by observing  $X_0$  and not investing. A player who already committed at  $t = 0$  has no such option.<sup>11</sup> Hence, if we restrict attention to equilibria without weakly dominated strategies, then efficient equilibria exist in the dynamic model only if they also exist in the static model of section 2, and our main results carry over to the dynamic model.

## 4.4 Nonlinear production technologies

We analyze nonlinear production technologies in detail in the appendix and only give a brief summary here. So far we only considered a linear production technology. A concave or convex technology implies that the efficient aggregate investment from a planner's perspective is nonlinearly increasing in the share of interested consumers. The aggregate investment made by crowdinvestors, on the other hand, is linearly increasing in the share of interested consumers if all consumers invest, and is not strictly increasing if some groups never invest. Thus, the market cannot achieve efficient investment as in the case of a linear production technology.

The main question is whether our previous results hold in terms of (ex ante) welfare, i.e., whether welfare is higher if all consumers can invest compared to the case where the poor consumer group cannot. In the appendix we compare two scenarios: In the equal wealth case, all consumers have sufficient wealth to make their investments. In the unequal wealth case, consumers of the poor group have no wealth whereas the wealthy have twice the wealth. The income distribution is the same in both scenarios.

We find that ex ante welfare is larger in the equal wealth scenario for concave, linear, and slightly convex production functions. Welfare is larger in the unequal wealth scenario only if there is a sufficiently large convexity in the production technology. Thus, our results generalize except for sufficiently convex production technologies in the sense that crowdinvesting yields higher welfare when all consumers have enough wealth to invest.

Given small nonlinearities, the reason why welfare is higher when wealth and income distributions match is the same as in the linear case: Since all consumers can invest, aggregate investment reacts to changes in the share of interested consumers, which is not the case with a wealth/income distribution mismatch. For a more detailed explanation of these findings we refer the reader to the appendix.

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<sup>11</sup>Indeed, in parimutuel betting—where as in our case the profits and losses are shared among all who invest—it is typically observed that bettors wait to place their bets until the very last moment in order to be able to react to new information (and not reveal their information to others), see, for example, (Ottaviani and Sørensen, 2009) and the references therein.

## 5 Conclusion

In most industrialized countries, wealth is far more concentrated in the population than income. We investigate the implications of this empirical fact when consumers invest in the capital market to increase their income for consumption, as can be observed in crowdinvest-ment campaigns. If tastes in the population are correlated, then consumers can use their own consumption preferences as signal for the profitability of firms. Consequently, consumers invest in companies whose products they like,<sup>12</sup> and firms can attract more capital for production if their product is well received among consumers.

We show that this pattern leads to an efficient capital allocation if all consumers who later consume also invest in the capital market. In this case, firms who need the most funding to build production capacity get the most funding. However, we also show that a wealth and income distribution mismatch may lead to an inefficient capital allocation. This is because firms with products favored by the wealthy will attract the most funding, but these are not necessarily the firms that meet the highest demand and therefore need the most funding.

We also show that financial intermediaries cannot completely fix the capital misallocation that arises with a mismatch of wealth and income distribution. The reason is that the acquisition of information about consumer preferences is costly. And even if the intermediaries were perfectly informed, they would not want to fully fund the new product, since the efficient investment implies zero profits. Unlike most of the financial intermediation literature, our setup is one where financial intermediaries are at an information cost disadvantage compared to consumers, who together hold enough information to perfectly predict future demand and therefore profitability of investments in firms.

By allowing intermediaries to compete with crowdinvestors, we endogenously determine the extent to which markets rely on intermediaries. If all consumers can invest in the capital market, then efficient investment is possible, which leaves no margin for profit and hence no room for intermediaries. The picture changes if some groups of consumers cannot invest, leading to over- or underinvestment in the new product, depending on the realization of consumer preferences. In this case it can pay for intermediaries to acquire information and reduce underinvestment.

Our analysis generates several testable predictions. First, if preferences are correlated among consumers, then consumers should tend to invest in firms whose products they like. This behavior should not (only) be driven by a sympathy for a brand name or the firm, but by the favorable information that the own preference for a product contains. Second,

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<sup>12</sup>In the present model, the risk-neutral crowdinvestors make investment decisions based on the expected investment return. A different mechanism that may lead to similar investment patterns as we describe consists in consumers trying to hedge against price increases of products they like. As one option, consumers could hedge by investing in the company making the product, because price increases also lead to higher returns on equity. An analysis investigating the impact of such a hedging motive would have to assume that investors are risk-averse.

our model predicts that a very unequal wealth distribution relative to the income distribution limits the scope of direct financing mechanisms such as crowdfinancing (compared to intermediated finance). If wealth is concentrated only among few consumers, then the information aggregation function of crowdfunding campaigns—a strong advantage compared to intermediated finance—is impaired. The (mis-)match of income and wealth distribution among the actual consumers of the product, and not the population, is crucial: Information aggregation of preferences for luxury products aimed at wealthy consumers may work even with very unequal wealth distributions in the population, because these consumers have sufficient wealth and income to invest and buy. But information aggregation may not work with products aimed at less fortunate consumers, who consume but cannot invest. Third, and relatedly, funding outcomes should on average be more efficient when the wealth distribution of consumers better matches the income distribution. This could either be tested across countries, or alternatively, within a country by comparing product success after different crowdfunding campaigns that target consumers from different wealth and income groups.

Recent technological advances and the widespread use of the internet made it possible to match a large amount of investors with projects or firms seeking funding at substantially lower cost.<sup>13</sup> Thus, firms and projects that were previously too small to offer equity directly to the public, and therefore had to rely on financial intermediaries, now have access to the money and wisdom of crowds. Our results show that the improved access to financing from crowdfinancers increases the efficiency of capital allocation for those small firms, if the mismatch of wealth and income distribution of consumers is not too large. Hence, our paper shows that crowdfunding may be a valuable financial innovation, which can improve social welfare.

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<sup>13</sup>The UK crowdfunding-platform crowdfunder is one example. On this platform, 178 businesses collected on average about £281,000 from crowdfinancers, who on average invested about £415 (official statistics from 7th of January 2015, [crowdfunder.com](http://crowdfunder.com)).

## Appendix A: Proofs

**Proof of Lemma 1.** Concavity of the utility function (1) in consumption  $x_i$  implies that  $x_i$  must be equal for all  $\theta_i = 1$  types in the social optimum, and equal zero for all  $\theta_i = 0$  types. No waste and feasibility requires that per capita production equals per capita consumption for  $\theta_i = 1$  types, i.e.,  $x_i = \hat{x}_i$ . Thus, the social planner determines a constant per capita investment  $\hat{x}_i = \hat{x}$  for all  $\theta = 1$  types.

Because an investment  $\hat{x}$  has opportunity cost of  $R$  units of  $c_i$  consumption, the budget constraint of the economy is

$$\int w_i - c_i - \theta_i R \hat{x} \, di = 0 \iff \int c_i \, di = \int w_i - \theta_i R \hat{x} \, di.$$

The planner's problem determines  $\hat{x}$  to maximize total welfare,

$$\max_{\hat{x}} \int \theta_i x_i^\alpha + c_i \, di \text{ s.t. } \int c_i \, di = \int w_i - \theta_i R \hat{x} \, di.$$

Substituting from the budget constraint and using  $x_i = \hat{x}$ , this is equivalent to the unconstrained problem

$$\max_{\hat{x}} \int \theta_i \hat{x}^\alpha + w_i - \theta_i R \hat{x} \, di.$$

The first order necessary and sufficient condition of the concave objective is

$$0 = \int \alpha \theta_i \hat{x}^{\alpha-1} - \theta_i R \, di \iff \hat{x} = \left( \frac{\alpha}{R} \right)^{\frac{1}{1-\alpha}},$$

where  $x_i = \hat{x} = (\alpha/R)^{\frac{1}{1-\alpha}}$  is also the socially optimal per capita consumption for  $\theta_i = 1$  types. The corresponding efficient aggregate investment is  $X = s\hat{x}$ .

In a market equilibrium, consumption choices  $x_i(p) = (\alpha/p)^{\frac{1}{1-\alpha}}$  for  $\theta_i = 1$  types depend on market clearing price  $p$ , and are socially optimal if and only if aggregate investment is such that  $p = R$  in every state.  $\square$

**Proof of Proposition 3.** In a first step, we will show that consumers with type  $\theta_i = 0$  do not invest in equilibrium if  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$ . In a second step, we will show that if consumers from group 1 with  $\theta_i = 1$  invest, then so do consumers from group 2 with  $\theta_i = 1$ , and vice versa. Finally, we show that equilibrium investment must be efficient.

*First step: consumers with  $\theta_i = 0$  do not invest in equilibrium.* Denote the price in state  $s = (\beta, \beta)$  by  $p_{11}$ , the price in state  $s = (\beta, 1 - \beta)$  by  $p_{10}$  and so on. Then we can write the expected returns of crowdinvestors of type  $\theta_i = 1$  in group 1 and 2 as

$$\begin{aligned} \mathbb{E}_s[p|\theta_i = 1, g = 1] &= \frac{\beta}{2}p_{11} + \frac{\beta}{2}p_{10} + \frac{1-\beta}{2}p_{00} + \frac{1-\beta}{2}p_{01}, \\ \mathbb{E}_s[p|\theta_i = 1, g = 2] &= \frac{\beta}{2}p_{11} + \frac{1-\beta}{2}p_{10} + \frac{1-\beta}{2}p_{00} + \frac{\beta}{2}p_{01}. \end{aligned}$$

Similarly, the expected returns of consumers with  $\theta_i = 0$  are

$$\begin{aligned}\mathbb{E}_s[p|\theta_i = 0, g = 1] &= \frac{1-\beta}{2}p_{11} + \frac{1-\beta}{2}p_{10} + \frac{\beta}{2}p_{00} + \frac{\beta}{2}p_{01}, \\ \mathbb{E}_s[p|\theta_i = 0, g = 2] &= \frac{1-\beta}{2}p_{11} + \frac{\beta}{2}p_{10} + \frac{\beta}{2}p_{00} + \frac{1-\beta}{2}p_{01}.\end{aligned}\tag{8}$$

We want to show that consumers with  $\theta_i = 0$  always expect a weakly lower investment return compared to consumers with  $\theta_i = 1$ . Thus, comparing  $\mathbb{E}_s[p|\theta_i = 1, g = 2]$  with  $\mathbb{E}_s[p|\theta_i = 1, g = 1]$ ,

$$\begin{aligned}\mathbb{E}_s[p|\theta_i = 1, g = 1] &\geq \mathbb{E}_s[p|\theta_i = 0, g = 2] \\ \iff \frac{2\beta-1}{2}p_{11} &\geq \frac{2\beta-1}{2}p_{00} \iff p_{11} \geq p_{00}.\end{aligned}\tag{9}$$

Denote the investment amount of investor  $i$  if  $\theta_i = 1$  by  $\hat{x}_i(\theta_i = 1)$  and if  $\theta_i = 0$  by  $\hat{x}_i(\theta_i = 0)$ . Recall that group 1 are all consumers  $i \in [0, 0.5]$  and group 2 are all consumers  $i \in (0.5, 1]$ . Now we can rewrite condition (9) in terms of investment strategies. After simplifying, (9) is equivalent to

$$\int_0^{0.5} \hat{x}_i(\theta_i = 0)di + \int_{0.5}^1 \hat{x}_i(\theta_i = 0)di \geq 0,\tag{10}$$

which always holds true. Moreover, any positive aggregate investment by consumers with  $\theta_i = 0$  from either group leads to  $\mathbb{E}_s[p|\theta_i = 1, g = 1] > \mathbb{E}_s[p|\theta_i = 0, g = 2]$ . Using the same reasoning, we also get  $\mathbb{E}_s[p|\theta_i = 1, g = 2] \geq \mathbb{E}_s[p|\theta_i = 0, g = 1]$ , and if (10) holds with strict inequality, then  $\mathbb{E}_s[p|\theta_i = 1, g = 2] > \mathbb{E}_s[p|\theta_i = 0, g = 1]$ .

Note that consumers only invest if their expected return is equal to or exceeds  $R$ , otherwise investing at the riskless rate  $R$  is a profitable deviation. Therefore, whenever consumers with  $\theta_i = 0$  from either group invest, i.e., (10) holds with strict inequality, then consumers with  $\theta_i = 1$  expect a return exceeding  $R$ . However, this cannot occur in equilibrium if  $w_i \geq (\alpha/R)^{1/(1-\alpha)}$  for all  $i$ . Suppose (10) holds with strict inequality, then it is optimal for all consumers with  $\theta_i = 1$  to increase their investment  $\hat{x}_i(\theta_i = 1)$  until their expected return equals  $R$ . For at least one consumer this deviation must be feasible, since the wealth endowment  $w_i$  is sufficient for all consumers with  $\theta_i = 1$  to invest  $\hat{x}_i(\theta_i = 1) = (\alpha/R)^{1/(1-\alpha)}$ , which guarantees a return of  $R$  or less. But if consumers with  $\theta_i = 1$  expect a return of  $R$ , then by (9), consumers with  $\theta_i = 0$  expect a return below  $R$ , which contradicts that (10) holds with strict inequality. Consequently, no consumer of type  $\theta_i = 0$  invests in equilibrium.

*Second step: If consumers with  $\theta_i = 1$  from one group invest, then so do consumers  $\theta_i = 1$  from the other group in equilibrium.* Suppose that there is an equilibrium where some consumers with type  $\theta_i = 1$  from group 1 (without loss of generality) invest in  $x$ , which implies  $\mathbb{E}_s[p|\theta_i = 1, g = 1] \geq R$ , otherwise not investing would be a profitable deviation. Now suppose to the contrary that investors from group 2 do not invest in this case, which



implies they expect a weakly lower return from investing,

$$\begin{aligned} \mathbb{E}_s[p|\theta_i = 1, g = 1] &\geq \mathbb{E}_s[p|\theta_i = 1, g = 2] \\ \iff \frac{2\beta - 1}{2}p_{10} &\geq \frac{2\beta - 1}{2}p_{01} \iff p_{10} > p_{01}. \end{aligned} \quad (11)$$

Since the aggregate demand is the same in state  $s = (\beta, 1 - \beta)$  and  $s = (1 - \beta, \beta)$ , price differences between these two states must be due to differences in aggregate investment. Rewriting condition (11) in terms of investment strategies gives

$$\begin{aligned} &\int_0^{0.5} [\beta \hat{x}_i(\theta_i = 1) + (1 - \beta) \hat{x}_i(\theta_i = 0)] di + \int_{0.5}^1 [(1 - \beta) \hat{x}_i(\theta_i = 1) + \beta \hat{x}_i(\theta_i = 0)] di \\ &\leq \int_0^{0.5} [(1 - \beta) \hat{x}_i(\theta_i = 1) + \beta \hat{x}_i(\theta_i = 0)] di + \int_{0.5}^1 [\beta \hat{x}_i(\theta_i = 1) + (1 - \beta) \hat{x}_i(\theta_i = 0)] di \\ &\iff \int_0^{0.5} [\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)] di \leq \int_{0.5}^1 [\hat{x}_i(\theta_i = 1) - \hat{x}_i(\theta_i = 0)] di \\ &\iff \int_0^{0.5} \hat{x}_i(\theta_i = 1) di \leq \int_{0.5}^1 \hat{x}_i(\theta_i = 1) di, \end{aligned} \quad (12)$$

where the last line follows from the fact that consumers with  $\theta_i = 0$  do not invest in equilibrium (see first step). Thus, if some consumers from group 1 with  $\theta_i = 1$  invest (i.e.,  $\int_0^{0.5} \hat{x}_i(\theta_i = 1) di > 0$ ), then consumers from group 2 with  $\theta_i = 1$  must also invest ( $\int_{0.5}^1 \hat{x}_i(\theta_i = 1) di > 0$ ). The same reasoning holds in the opposite direction as well: If consumers from group 2 with  $\theta_i = 1$  invest, then so must those from group 1.

Condition (12) implies that whichever group of consumers with  $\theta_i = 1$  invests less in the aggregate has a larger expected return. The argument in the next paragraph uses fact (12) to establish that, in equilibrium, we must have

$$\int_0^{0.5} \hat{x}_i(\theta_i = 1) di = s_1(\alpha/R)^{1/(1-\alpha)}, \quad \int_{0.5}^1 \hat{x}_i(\theta_i = 1) di = s_2(\alpha/R)^{1/(1-\alpha)}, \quad (13)$$

which leads to a price of  $R$  in all states and is efficient (Lemma 1).

To show (13), suppose  $\int_0^{0.5} \hat{x}_i(\theta_i = 1) di > s_1(\alpha/R)^{1/(1-\alpha)}$  and  $\int_{0.5}^1 \hat{x}_i(\theta_i = 1) di > s_2(\alpha/R)^{1/(1-\alpha)}$ , then  $\mathbb{E}_s[p|\theta_i = 1, g = 1] < R$  and  $\mathbb{E}_s[p|\theta_i = 1, g = 2] < R$ , and not investing is a profitable deviation. Suppose  $\int_0^{0.5} \hat{x}_i(\theta_i = 1) di < s_1(\alpha/R)^{1/(1-\alpha)}$  and  $\int_{0.5}^1 \hat{x}_i(\theta_i = 1) di < s_2(\alpha/R)^{1/(1-\alpha)}$ , then  $\mathbb{E}_s[p|\theta_i = 1, g = 1] > R$  and  $\mathbb{E}_s[p|\theta_i = 1, g = 2] > R$ , and investing more is profitable and feasible. Suppose  $\int_0^{0.5} \hat{x}_i(\theta_i = 1) di > s_1(\alpha/R)^{1/(1-\alpha)}$  and  $\int_{0.5}^1 \hat{x}_i(\theta_i = 1) di < s_2(\alpha/R)^{1/(1-\alpha)}$ , then either  $\mathbb{E}_s[p|\theta_i = 1, g = 1] < R$  or  $\mathbb{E}_s[p|\theta_i = 1, g = 2] > R$ . Suppose  $\int_0^{0.5} \hat{x}_i(\theta_i = 1) di < s_1(\alpha/R)^{1/(1-\alpha)}$  and  $\int_{0.5}^1 \hat{x}_i(\theta_i = 1) di > s_2(\alpha/R)^{1/(1-\alpha)}$ , then either  $\mathbb{E}_s[p|\theta_i = 1, g = 1] > R$  or  $\mathbb{E}_s[p|\theta_i = 1, g = 2] < R$ .  $\square$

**Proof of Proposition 4.** We shall confirm that all equilibrium requirements of definition 2 can be fulfilled.

A unique market clearing price  $p$  exists for all aggregate investment levels  $X$  and all realizations of preferences  $(s_1, s_2)$ . In the consumption stage, consumers use the demand function (2), which by construction maximizes utility.

Every price  $p_m$  set by the market research firm induces a Bayesian investment game at the acquisition and investment stage. In this investment game, all crowdinvestors  $i$  choose  $\hat{x}_i \in [0, w_i]$  for each  $\theta_i \in \{0, 1\}$  and all funds choose  $(a_j, f_j) \in \{0, 1\} \times [0, W_j]$  for each  $p_m \in \mathbb{R}_+$  and  $m_j \in \{\{0, 1\}^2, \emptyset\}$ , where  $w_i \in [0, \infty)$  and  $W_i \in [0, \infty)$ .

Consider first a reduced game, where the strategy space for funds is  $f_j \in [0, W_j]$  and information acquisition decisions  $(a_1, a_2, \dots, a_N)$  are exogenous. Then strategy spaces of all investors are compact and convex, and strategy  $f_j$  is concave and continuous in the expected payoff  $\pi_j$  for a given strategy profile  $(f_{-j}, \hat{x})$  of all other investors, where

$$\mathbb{E}[\pi_j(f_j, f_{-j}, \hat{x}) | \mathcal{I}_j(a_j)] = f_j(\mathbb{E}[p(f_j, f_{-j}, \hat{x}) | \mathcal{I}_j(a_j)] - R),$$

and  $\hat{x}_i$  is quasi-concave and continuous for crowdinvestors  $i$ . Thus, the Debreu-Glicksberg-Fan theorem (e.g., Theorem 1 in [Reny, 2008](#)) guarantees the existence of a pure strategy equilibrium for any exogenous profile  $(a_1, a_2, \dots, a_N)$ .

Going back to the actual game with fund strategy space  $(a_j, f_j) \in \{0, 1\} \times [0, W_j]$ , which is not convex, every information acquisition profile  $(a_1, a_2, \dots, a_N)$  induces a reduced game for which we just showed a pure strategy equilibrium exists. By allowing mixed strategies in  $a_j$ , we can convexify the strategy space to  $[0, 1] \times [0, W_j]$ , and the expected payoffs from the mixed strategies are just linear combinations of the payoffs of the reduced game. Since a linear combination is quasi-concave and continuous, the Debreu-Glicksberg-Fan theorem guarantees existence of an equilibrium of the investment game, with possible mixing in  $a_j$  and corresponding  $f_j$  for all  $j$  and pure strategies for crowdinvestors  $\hat{x}_i$ .

We still have to show that, given the outcomes of the investment game for every  $p_m$ , there exists a profit maximizing price  $p_m$  for the MR firm. For a given  $p_m$ , all funds  $j$  determine the information acquisition decision by solving the problem

$$\max_{a_j \in [0, 1]} \mathbb{E}[\pi_j | a_j, a_{-j}, f, \hat{x}] - a_j p_m,$$

where the set of mixed strategies  $[0, 1]$  is compact, and  $\mathbb{E}[\pi_j | a_j, a_{-j}, f, \hat{x}] - a_j p_m$  is continuous in  $p_m$ . Berge's maximum theorem implies that  $a_j(p_m)$ —the expected demand for market research by fund  $j$ —is upper hemi-continuous (uhc) in  $p_m$ . Aggregate expected demand for market research is  $\sum_j a_j(p_m)$ . The profit function for the market research firm is given by

$$\pi_{MR}(p_m) = p_m \sum_j a_j(p_m) - \mathbf{1} \left\{ \sum_j a_j(p_m) > 0 \right\} c.$$

Since summation and integration preserves upper hemi-continuity,  $\sum_j a_j(p_m)$  is uhc. More-

over, the product of two non-negative uhc correspondences  $p_m$  and  $\sum_j a_j(p_m)$  is uhc. The negative of the last term  $\mathbf{1}\left\{\sum_j a_j(p_m) > 0\right\}c$  is lower hemi-continuous, since the indicator function  $\mathbf{1}\{x \in X\}$  is lower hemi-continuous if and only if  $X$  is an open set. Consequently,  $-\mathbf{1}\left\{\sum_j a_j(p_m) > 0\right\}c$  is uhc, and thus  $\pi_{MR}(p_m)$  is uhc.

We can find an upper bound for a profit maximizing  $p_m$ , since no fund will buy market research if  $p_m$  is larger than the maximally possible earnings in the capital market, which are bounded. Denote such a bound by  $0 < P < \infty$ . Then, the market research firm chooses  $p_m \in [0, P]$ , which is a compact set, hence the Weierstrass extreme value theorem implies there exists a  $p_m$  which maximizes  $\pi_{MR}(p_m)$ .  $\square$

**Proof of Proposition 5.** Suppose an equilibrium with efficient investment exists in which some investment funds invest, which implies that the return on investment is  $R$  in every state (Lemma 1). In this case it does not pay for funds to buy market research at any price  $p_m > 0$ , as funds can by assumption obtain an investment return  $R$  by investing elsewhere without paying  $p_m$ . Consequently, investment funds must be uninformed in any efficient equilibrium, and invest a state independent amount  $F := \sum_j f_j > 0$  in every state.

In any equilibrium, each consumer can condition his investment plan  $\hat{x}_i$  on  $\theta_i$ . Consequently, aggregate investment by consumers depending on the preference realization can be written as

$$\int_0^1 \hat{x}_i di = \int [s\hat{x}_i(\theta_i = 1) + (1 - s)\hat{x}_i(\theta_i = 0)] di.$$

Efficiency requires that the price in each state equals  $R$ . In particular,

$$R = \alpha \left( \frac{\beta}{F + \int [\beta\hat{x}_i(\theta_i = 1) + (1 - \beta)\hat{x}_i(\theta_i = 0)] di} \right)^{1-\alpha} \quad \text{if } s = \beta, \quad (14)$$

$$R = \alpha \left( \frac{1 - \beta}{F + \int [(1 - \beta)\hat{x}_i(\theta_i = 1) + \beta\hat{x}_i(\theta_i = 0)] di} \right)^{1-\alpha} \quad \text{if } s = 1 - \beta, \quad (15)$$

and combining (14) and (15) implies

$$(2\beta - 1)F = (1 - 2\beta) \int \hat{x}_i(\theta_i = 0) di.$$

This condition is fulfilled with  $F = \int \hat{x}_i(\theta_i = 0) di = 0$ , which contradicts the assumption that investment funds invest. For  $F > 0$  it implies  $\int \hat{x}_i(\theta_i = 0) di < 0$ , but this is impossible, thus contradicting efficiency.  $\square$

## Appendix B: Nonlinear production technology

In this section, we study the robustness of our main results when the production technology is nonlinear. We assume that aggregate investment  $X$  translates into supply according to production function

$$x_{sup}(X) = X^\lambda = \left[ \int \hat{x}_i di \right]^\lambda, \quad \lambda > 0.$$

Although this looks as if only one firm produces the novel good, the production function is up to a constant factor identical to a situation where  $1 \leq M < \infty$  firms receive an  $1/M$ -share of the investment and produce, so that aggregate supply is given by

$$x_{sup}(X) = M \left[ \int \hat{x}_i / M di \right]^\lambda = M^{1-\lambda} \left[ \int \hat{x}_i di \right]^\lambda.$$

For  $0 < \lambda < 1$  the production function will be concave (decreasing returns to scale), and for  $\lambda > 1$  it will be convex (increasing returns to scale).  $\lambda = 1$  is the linear case considered throughout the main part of the paper. For  $\lambda > 1$ , we require  $1/\lambda > \alpha$ , otherwise the planner's problem may have a corner solution.

Consumer demand for given prices remains unchanged:

$$x_i(p) = \left( \frac{\alpha \theta_i}{p} \right)^{1/(1-\alpha)}.$$

The generalized market clearing condition and spot market price is

$$X^\lambda = s \left( \frac{\alpha}{p} \right)^{1/(1-\alpha)} \iff p = \alpha \left( \frac{s}{X^\lambda} \right)^{1-\alpha}.$$

### The social optimum

We first determine the planner's solution for the optimal aggregate state dependent investment  $X^*$  in the novel good. In the aggregate, market clearing requires that  $x_i = x_{sup}/s$ , where  $s = \int \theta_i di$ ,  $x_i$  is the symmetric consumption level for  $\theta_i = 1$  types in the population, and  $x_{sup}$  is the aggregate supply (or production) of the novel good. The cost function for producing the novel good is  $c(x_{sup}) = RX = Rx_{sup}^{1/\lambda}$ , since every unit of investment  $X$  has an opportunity cost of  $R$ , and the marginal cost is  $MC_2 = x_{sup}^{1/\lambda-1} R/\lambda$ . In the social optimum, the marginal rate of substitution for a  $\theta_i = 1$  consumer has to equal the ratio of marginal costs of production (investment) of the two goods,

$$\begin{aligned} MRS &= \frac{MU_1}{MU_2} = -\frac{1}{\alpha x_i^{\alpha-1}} = -\frac{1}{\alpha (x_{sup}/s)^{\alpha-1}} \stackrel{!}{=} -\frac{MC_1}{MC_2} = -\frac{1}{x_{sup}^{1/\lambda-1} R/\lambda} \\ &\iff x_{sup}^* = \left[ \frac{\lambda \alpha}{R} s^{1-\alpha} \right]^{\frac{\lambda}{1-\lambda\alpha}} \iff X^* = x_{sup}^{*1/\lambda} = \left[ \frac{\lambda \alpha}{R} s^{1-\alpha} \right]^{\frac{1}{1-\lambda\alpha}}. \end{aligned} \tag{16}$$

Consequently, the optimal aggregate investment  $X^*$  depends nonlinearly on state  $s$  whenever  $\lambda \neq 1$ . The Pareto-optimal investment yields a market clearing price  $p^*$ ,

$$x_{sup}^* = \left[ \frac{\lambda\alpha}{R} s^{1-\alpha} \right]^{\frac{\lambda}{1-\lambda\alpha}} \stackrel{!}{=} x_i s = s \left( \frac{\alpha}{p} \right)^{\frac{1}{1-\alpha}} \iff p^* = \alpha s^{1-\alpha} \left( \frac{R}{\lambda\alpha s^{1-\alpha}} \right)^{\frac{\lambda-\lambda\alpha}{1-\lambda\alpha}},$$

which depends on state  $s$  whenever  $\lambda \neq 1$ . Moreover,  $p^* < R \iff \lambda < 1$ . Intuitively, if  $\lambda < 1$ , then firms can produce more for a given aggregate investment  $X < 1$  compared to the linear case, since  $X^\lambda > X \iff \lambda < 1$ . Thus, in the social optimum, the planner produces more the smaller  $\lambda$ , which means  $p^*$  decreases when  $\lambda$  decreases.

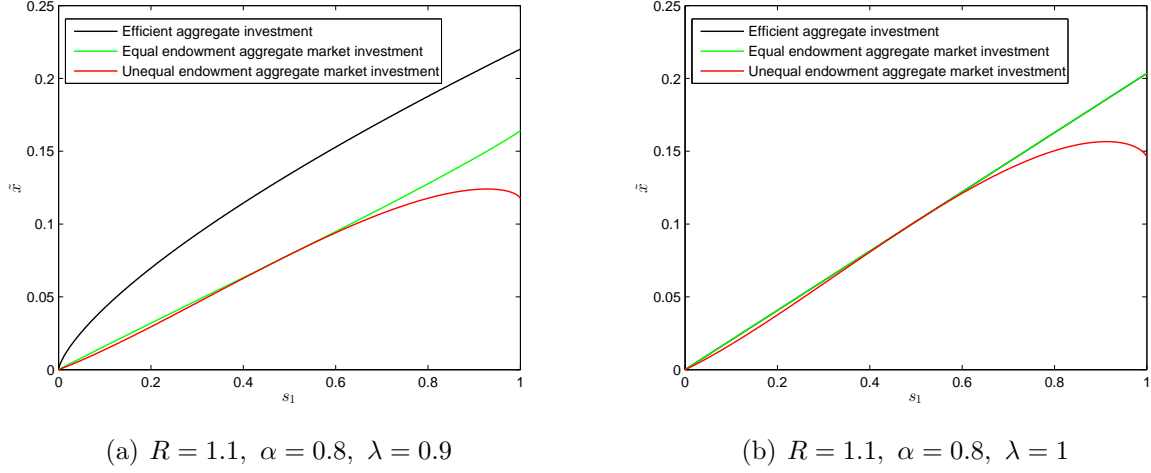
## Market investment

Given sufficient wealth,  $\theta_i = 1$  types will invest such that  $\mathbb{E}_s[p|\theta_i = 1] = R$  in any market equilibrium. For  $\lambda < 1$  this implies underprovision of funds by the market compared to the planner's solution, because more than the investment/production that results in  $p = R$  is efficient (see previous section).

As before, we assume there are two groups of mass  $1/2$  each, where a share  $s_1 \in \{1-\beta, \beta\}$  and  $s_2 \in \{1-\beta, \beta\}$  of consumers is interested in the novel good, respectively. Moreover, we assume  $s_1$  and  $s_2$  are independently distributed. If all consumers have enough wealth to invest, then the investment  $\hat{x}_e$  in a symmetric equilibrium where all  $\theta_i = 1$  types invest is

$$\begin{aligned} R &= \mathbb{E}_s[p|\theta_i = 1] = \alpha \mathbb{E}_s \left[ \left( \frac{\int \theta_i di}{(\int \hat{x}_e di)^\lambda} \right)^{1-\alpha} \middle| \theta_i = 1 \right] \\ &= \alpha\beta/2 \left( \frac{\beta}{(\beta\hat{x}_e)^\lambda} \right)^{1-\alpha} + \alpha(1-\beta)/2 \left( \frac{(1-\beta)}{((1-\beta)\hat{x}_e)^\lambda} \right)^{1-\alpha} + \alpha/2 \left( \frac{1/2}{(\hat{x}_e/2)^\lambda} \right)^{1-\alpha} \\ \iff \hat{x}_e &= (\alpha/R [\beta^{(1-\lambda)(1-\alpha)+1}/2 + (1-\beta)^{(1-\lambda)(1-\alpha)+1}/2 + 2^{-1-(1-\lambda)(1-\alpha)}])^{\frac{1}{\lambda(1-\alpha)}}. \end{aligned} \tag{17}$$

In a symmetric equilibrium where only half of the population has wealth to invest (“un-



**Figure 2:** Plot of aggregate investment  $X$  in states  $s_1 = s_2 = \beta$ , depending on the share of interested consumers  $s_1$ .

equal wealth”), the symmetric investment  $\hat{x}_u$  by wealthy  $\theta_i = 1$  types fulfills

$$\begin{aligned}
R &= \alpha \mathbb{E}_s \left[ \left( \frac{\int \theta_i di}{(\int \hat{x}_u di)^\lambda} \right)^{1-\alpha} \middle| \theta_i = 1 \right] \\
&= \alpha \beta / 2 \left( \frac{\beta}{(\beta \hat{x}_u / 2)^\lambda} \right)^{1-\alpha} + \alpha (1 - \beta) / 2 \left( \frac{(1 - \beta)}{((1 - \beta) \hat{x}_u / 2)^\lambda} \right)^{1-\alpha} \\
&\quad + \alpha \beta / 2 \left( \frac{1/2}{(\beta \hat{x}_u / 2)^\lambda} \right)^{1-\alpha} + \alpha (1 - \beta) / 2 \left( \frac{1/2}{((1 - \beta) \hat{x}_u / 2)^\lambda} \right)^{1-\alpha} \quad (18) \\
\iff \hat{x}_u &= \left( \alpha / R \left[ \beta^{1+(1-\lambda)(1-\alpha)} 2^{\lambda(1-\alpha)-1} + (1 - \beta)^{1+(1-\lambda)(1-\alpha)} 2^{\lambda(1-\alpha)-1} \right. \right. \\
&\quad \left. \left. + \beta^{1-\lambda(1-\alpha)} 2^{-1-(1-\lambda)(1-\alpha)} + (1 - \beta)^{1-\lambda(1-\alpha)} 2^{-1-(1-\lambda)(1-\alpha)} \right] \right)^{\frac{1}{\lambda(1-\alpha)}}.
\end{aligned}$$

Thus, if all consumers invest, aggregate investment is  $X_e = \int_0^1 \theta_i \hat{x}_e di$ , and if group 2 cannot invest, aggregate investment is  $X_u = \int_0^{1/2} \theta_i \hat{x}_u di$ . In Figure 2, we consider the two states  $s_1 = s_2 = s$ , where the realization of both random variables is the same, in order to make aggregate investment with  $\hat{x}_e$  and  $\hat{x}_u$  comparable. The figure plots the aggregate investment depending on the share of interested consumers when  $s = s_1 = s_2$  for a specific parameter profile  $(R, \alpha, \lambda)$  with concave and linear production technology. The plots depict all  $\beta \in [0, 1]$  by setting  $\beta = s$ .

The left figure shows the efficient aggregate investment (black line) for  $\lambda = 0.9$ , which is concave in the share of interested consumers  $s$ , since the average cost of production is increasing in  $s$ . It shows that the market invests less than what would be efficient in equilibrium. And the market investment if all consumers invest (green line) is weakly larger than the market investment if only one group invests, i.e., weakly more efficient (proven in Proposition 9 for all  $\lambda < 1$ ).

The reason why aggregate investment tends to be larger when all consumers can invest is most easily seen in the linear case  $\lambda = 1$ . Also in this special case, aggregate investment when all consumers invest is larger compared to when not all consumers can invest (see the right plot in Figure 2). Crowdinvestors aim to equalize the expected return of investment—which is equal to the market clearing price of the novel good  $p$ —with the risk free rate  $R$ . The expected market clearing price is given by

$$\mathbb{E}_s[p|\theta_i = 1] = \alpha \mathbb{E}_s \left[ \left( \frac{s}{X(s)} \right)^{1-\alpha} \middle| \theta_i = 1 \right].$$

In the linear case, if all consumers invest, the price is state independent and equal to  $R$ . Thus, there is no price risk. If not all consumers can invest, however, then prices vary depending on the state: It is higher if more poor consumers have a preference for the good ( $s_2 = \beta$ ), and lower if not ( $s_2 = 1 - \beta$ ), keeping  $s_1$  and therefore aggregate investment constant. Because the market clearing price is a concave function of  $s$  for a given aggregate investment, crowdinvestors in the economy with unequal wealth and price risk expect a lower price than crowdinvestors in the economy with equal wealth for the same aggregate investment  $X$ , i.e.,

$$\mathbb{E}_s[p_e|\theta_i = 1, X] > \mathbb{E}_s[p_u|\theta_i = 1, X],$$

which follows from the strict concavity of the price and Jensen's inequality. Thus, because their investment return expectations are more optimistic, crowdinvestors invest more in the economy where everyone invests. Intuitively, although crowdinvestors are risk neutral, the investment return is concave in the random variable, so they need higher returns with more uncertainty for the same expected return, leading to reduced investment if the investment return is risky. The intuition carries over to the nonlinear case  $\lambda \neq 1$ , because price risk in the economy where all consumers can invest is lower—since aggregate investment scales up monotonically with the share of interested consumers—than in the economy where only wealthy consumers can invest. This is reflected in the price expectations in (17) and (18), where the price can take three different values depending on the state when all consumers invest, but four different values if not all can invest.

**Proposition 9.** *Consider the states in which  $s_1 = s_2$ . If  $\lambda < 1$ , then (ex post) utilitarian welfare is weakly larger if both groups invest compared to the case where only one group can invest, and strictly larger if  $\beta \neq 1/2$ .*

**Proof.** From (16), if  $\lambda < 1$ , the efficient aggregate investment is larger than the investment leading to price  $R$ . Thus, we have to show that the aggregate investment when both groups invest is weakly larger than the aggregate investment when only one group can invest. The



corresponding condition is

$$\begin{aligned}
& (s/2 + s/2) \left( \alpha/R \left[ \beta^{(1-\lambda)(1-\alpha)+1}/2 + (1-\beta)^{(1-\lambda)(1-\alpha)+1}/2 + 2^{-1-(1-\lambda)(1-\alpha)} \right] \right)^{\frac{1}{\lambda(1-\alpha)}} \\
& \geq s/2 \left( \alpha/R \left[ \beta^{1+(1-\lambda)(1-\alpha)} 2^{\lambda(1-\alpha)-1} + (1-\beta)^{1+(1-\lambda)(1-\alpha)} 2^{\lambda(1-\alpha)-1} \right. \right. \\
& \quad \left. \left. + \beta^{1-\lambda(1-\alpha)} 2^{-1-(1-\lambda)(1-\alpha)} + (1-\beta)^{1-\lambda(1-\alpha)} 2^{-1-(1-\lambda)(1-\alpha)} \right] \right)^{\frac{1}{\lambda(1-\alpha)}} \\
& \iff \beta^{(1-\lambda)(1-\alpha)+1} + (1-\beta)^{(1-\lambda)(1-\alpha)+1} + 2^{-(1-\lambda)(1-\alpha)} \\
& \geq 2^{-\lambda(1-\alpha)} \left[ \beta^{1+(1-\lambda)(1-\alpha)} 2^{\lambda(1-\alpha)} + (1-\beta)^{1+(1-\lambda)(1-\alpha)} 2^{\lambda(1-\alpha)} \right. \\
& \quad \left. + \beta^{1-\lambda(1-\alpha)} 2^{-(1-\lambda)(1-\alpha)} + (1-\beta)^{1-\lambda(1-\alpha)} 2^{-(1-\lambda)(1-\alpha)} \right] \\
& = \beta^{1+(1-\lambda)(1-\alpha)} + (1-\beta)^{1+(1-\lambda)(1-\alpha)} + \beta^{1-\lambda(1-\alpha)} 2^{-(1-\alpha)} + (1-\beta)^{1-\lambda(1-\alpha)} 2^{-(1-\alpha)} \\
& \iff 2^{-(1-\lambda)(1-\alpha)} \geq \beta^{1-\lambda(1-\alpha)} 2^{-(1-\alpha)} + (1-\beta)^{1-\lambda(1-\alpha)} 2^{-(1-\alpha)} \\
& \iff 2^{\lambda(1-\alpha)} \geq \beta^{1-\lambda(1-\alpha)} + (1-\beta)^{1-\lambda(1-\alpha)} \tag{19}
\end{aligned}$$

The right hand side of the inequality is a sum of concave functions, and it achieves its unique maximum at  $\beta = 1/2$ . Evaluating the RHS at its maximum, the inequality (19) changes to

$$2^{\lambda(1-\alpha)} \geq (1/2)^{1-\lambda(1-\alpha)} + (1/2)^{1-\lambda(1-\alpha)}.$$

At the maximum  $\beta = 1/2$ , the condition holds with equality, and  $2^{\lambda(1-\alpha)} > \beta^{1-\lambda(1-\alpha)} + (1-\beta)^{1-\lambda(1-\alpha)}$  whenever  $\beta \neq 1/2$  immediately follows.  $\square$

The proposition does not imply, however, that welfare is larger in every state if all consumers invest and  $\lambda < 1$ . If  $s_1 = \beta$  and  $s_2 = 1 - \beta$ , then  $s = 1/2$ . Group 1 is the wealthy one, so for  $\beta$  large enough aggregate investment will be larger in the economy where only one group can invest, because wealthy consumers overestimate aggregate demand in the economy. This is the only state where endowment inequality among consumers may be better in terms of welfare.

Since welfare from an ex post perspective may depend on the state, the main question is whether ex ante welfare (expectation over all four states) is still larger if all consumers invest. We investigate this question numerically in the next section.

## Numerical welfare analysis

To compare welfare for equal and unequal wealth among consumers, we assume aggregate wealth in the economy is constant, but in the unequal distribution case half of the consumers holds investment endowment  $2w$  whereas the other half holds zero. In the equal distribution

case, every consumer has  $w_i = w > 0$  to invest. Wealth  $w$  and income  $y$  are chosen so that budget constraints when investing in  $x$  or consuming  $x$  are never binding.

Ex ante utilitarian welfare in the market with equal wealth distribution (all consumers can invest) is

$$W_e = \mathbb{E}_s [s(x_i(p_e(s))^\alpha + y + R(w - \hat{x}_e) - p_e(s)(x_i(p_e(s)) - \hat{x}_e)) + (1 - s)(y + Rw)],$$

where  $p_e(s)$  denotes the market clearing price in state  $s$ . Similarly, ex ante utilitarian welfare in the market with unequal wealth (only half of consumers can invest) is

$$W_u = \mathbb{E}_s [s_1(x_i(p_u(s))^\alpha + y + R(2w - \hat{x}_u) - p_u(s)(x_i(p_u(s)) - \hat{x}_u)) + (1 - s_1)(y + 2Rw) + (1 - s_2)y + s_2(x_i(p_u(s))^\alpha + y - p_u(s)x_i(p_u(s)))]/2.$$

The following results assume  $\beta > 1/2$ , since there is no demand uncertainty for  $\beta = 1/2$ , and investment and welfare is always the same for equal and unequal wealth distribution. Result 1 is based on numerical calculations with the following parameter values in all possible combinations, where (following the Matlab syntax)  $\{a : z : b\} := [a, b] \cap \{a + kz\}_{k=0,1,2,\dots}$  is the parameter grid.

$$\beta \in \{0.6 : 0.1 : 1\}, R \in \{1 : 0.1 : 2\}, \lambda \in \{0.1 : 0.1 : 1\}, \alpha \in \{0.1 : 0.1 : 0.9\}^{14}$$

**Result 1.** *For  $\lambda \leq 1$ , ex ante utilitarian welfare is always strictly larger if all consumers invest compared to the case where only one group invests.*

Thus, the results of section 2 generalize to concave production technologies in the sense that market outcomes yield higher (ex ante) welfare when all consumers invest. The reason, as explained in the previous subsection, is that crowdinvestors tend to invest more if there is less price uncertainty, and it is efficient to invest more if the production function is strictly concave.

If the production technology is convex, i.e.,  $\lambda > 1$ , then market investment tends to be lower if only one group can invest, and thus (for  $\lambda \gg 1$ ) tends to be closer to the efficient aggregate investment. However, aggregate investment still scales better with consumer preferences if all consumers can invest. This trade-off suggests that welfare is not unambiguously better for one or the other wealth distribution with  $\lambda > 1$ , which is confirmed in our next result. Result 2 is based on numerical calculations with the following parameter values in all possible combinations (which satisfy  $1/\lambda > \alpha$ , see above).

$$\beta \in \{0.6 : 0.1 : 1\}, R \in \{1 : 0.1 : 2\}, \lambda \in \{1.1 : 0.1 : 3\}, \alpha \in \{0.1 : 0.1 : 0.9\}, 1/\lambda > \alpha^{15}$$

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<sup>14</sup>We have not found a parameter profile where the results do not hold. Matlab scripts of the numerical calculations are available upon request.

<sup>15</sup>In this parameter profile, the equal wealth economy has larger ex ante total welfare for  $\lambda < 1.9$ , for

**Result 2.** *For  $\lambda > 1$  and sufficiently close to 1, ex ante utilitarian welfare is larger if all consumers invest compared to the case where only one group invests. For  $\lambda \gg 1$ , ex ante utilitarian welfare is larger if only one group invests.*

Thus, ex ante total welfare is still greater when all consumers invest if  $\lambda$  is close to but above 1. For very strong convexity in the production function, however, an unequal wealth distribution is superior in terms of ex ante welfare.

The intuition is as before: When not all consumers invest, the investment return is more risky, and crowdinvestors tend to invest less. Due to convexity of the production function, production for  $X < 1$  is more expensive compared to a linear or concave production function, hence socially optimal production is less than in the linear/concave case. Thus, less investment/production is socially more desirable with convex production technology, which explains why the unequal wealth economy is superior in terms of ex ante welfare for large convexity. The trade-off is that aggregate investment reacts better to changes in consumer preferences when all consumers invest, which from an ex ante point of view is welfare improving. Thus, for small convexity the equal wealth economy still has larger ex ante welfare. Our numerical simulation suggests that the equal wealth economy fares better even for moderate convexity (up to  $\lambda = 1.9$ , see the footnote above).

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$\lambda = 1.9$  it depends on the remaining parameter values, and for  $\lambda > 1.9$  the unequal wealth economy is always superior in terms of ex ante welfare.

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