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Profit Sharing: A Contracting Solution to Harness the Wisdom of the Crowd

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Abstract

When a group of investors with dispersed private information jointly invest in a risky project, how should they divide up the project payoff? A typical common stock contract rewards investors in proportion to their initial investment, but is it really optimal for harnessing all investors' "wisdom of the crowd"? By showing that a *simple* profit-sharing contract with decentralized decision making could *first best* coordinate individuals' investment choices, this paper studies as a general contracting problem the role of profit sharing in harnessing the crowd wisdom, and discusses implications for the security design of investment crowdfunding. Our result connects the traditional diversification insight underpinning portfolio theory with contracting and investment under private information, and has potential implications for the organization of some new business structures such as decentralized autonomous organizations (DAO).

Keywords: contract theory, crowdfunding, disintermediation, financial innovation, FinTech, information economics, investment, security design

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Many business activities feature a "wisdom of the crowd" effect, meaning that a group's collective opinion often dominates the assessment of any single individual.[1] An explanation for this phenomenon is that by aggregating a large number of responses, the idiosyncratic noises associated with each individual judgment tend to cancel out by the law of large numbers – an argument somewhat similar to diversification in traditional portfolio theory. Although the specific term "wisdom of the crowd" was not pushed into the mainstream until the rise of web 2.0 (e.g. Wikipedia or Quora), and only recently gains further popularity with the emergence of the new financing practice of crowdfunding, its underlying idea is rooted in the tradition of economic thoughts, ranging from how the market economy coordinates economic activities under decentralized possession of information (Hayek (1944, 1945)) to theories of rational expectation in the financial market (e.g. Hellwig (1980), Diamond and Verrecchia (1981)). In our information age, how well we can take advantage of wisdom of the crowd affects resource allocation efficiency as well as economic productivity.

Cast in a specific setting of funding a scalable risky investment project from a group of investors, this paper studies the optimal rule to divide up the project payoff among all participating investors. The result touches on a less explored area of how contract designs (rather than market prices as in many rational expectation models) could harness the wisdom of the crowd. It provides general insight for organizing business activities under decentralized possession of information (e.g. how to use blockchain-based smart contracts to structure a decentralized autonomous organization (DAO)), as well as specific guidance for contract design in the emerging financing practice of investment crowdfunding.

The main takeaway from the paper is that the optimal pie-splitting rule among investors to harness their collective wisdom features profit sharing, in which each investor agrees *ex ante* to a share of the project payoff not necessarily proportional to their actual investment

[1]See Surowiecki (2005) for an introduction, Galton (1907) for original empirical evidence from an English weight-judging competition, Da and Huang (2015) for recent empirical evidence from an online earnings forecast platform, and Dindo and Massari (2017) for a theory of behavioral foundation.

amounts. In a broad class of standard settings, the splitting rule may take a particularly simple structure and be completely independent of individual investment amount. In this sense, the optimal contract differs from the often-observed common stock.

To best illustrate, consider the simplest example in which only two investors, Alice and Bob, participate in funding a risky project. Assume that both Alice and Bob are deep pocketed and identically risk averse, and they independently decide how much money to commit to the project, based on their optimal return–risk trade-off. When making their investment decisions, both investors rely on their own private information, which contains idiosyncratic noises, of the return-per-dollar-invested from the project. Neither investor has access to the other's private information.[2] Given these conditions, how should Alice and Bob split the payoff from their investment?

Section 1 analyzes this example in detail and proves a somewhat counterintuitive result: Regardless of how different Alice's and Bob's information quality or their actual investment amounts may be, as a Nash equilibrium outcome they both prefer to split *net* investment profit equally (while each investor still gets back the exact amount of her/his initial investment). For example, if Alice invests \$200, Bob invests \$100, and the project value appreciates 10 percent (i.e. the (net) profit from their investment is (\$200 + \$100) $\times$ 10% = \$30), then the optimal profit sharing rule stipulates that Alice gets back \$200 + \$30/2 =\$215, and Bob gets back \$100 + \$30/2 =\$115. In comparison, if Alice and Bob hold common stocks, which deliver payment in proportion to their initial investment, then assuming *unchanged* investment decisions and the same project performance, Alice gets back \$200 $\times$ (1 + 10%) = \$220, and Bob \$100 $\times$ (1 + 10%) = \$110.

At first sight profit sharing might look like a bad deal for Alice. She could get \$220 un-

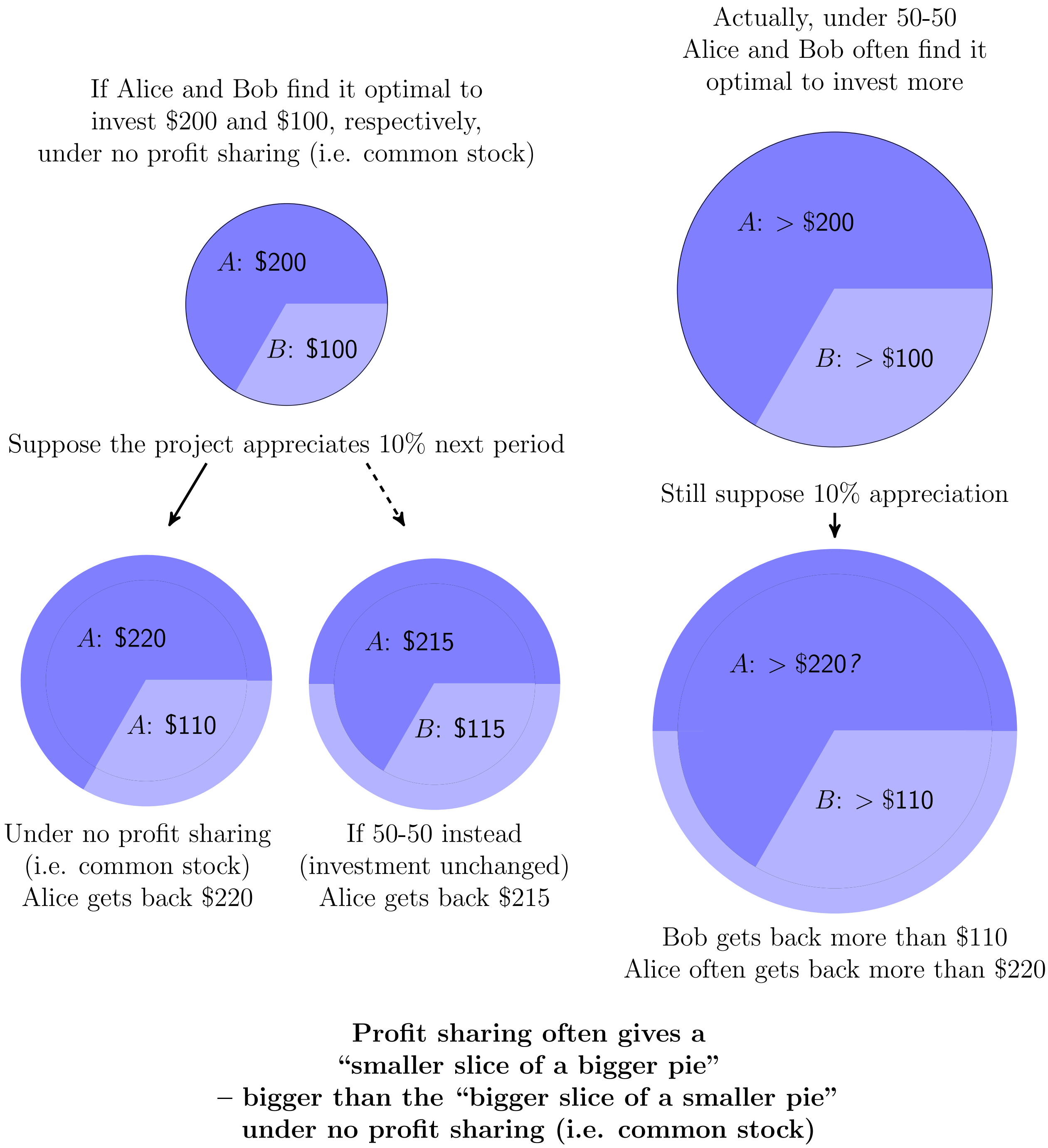
[2]While Alice and Bob could communicate their private information in this simple two-agent example, communication is shut down here to represent general cases with large crowds where aggregating private information is costly. Such cost could be either explicit or incentive-induced. For example, Section 3.3 studies a case of ineffective communication because investors may lie to each other for strategic reasons.

der a common stock arrangement *all else being equal*, so why would she prefer to go 50-50? The answer lies in the fact that profit sharing changes both investors' risk-taking incentives because it provides insurance for the idiosyncratic noises in their private information. Effectively, the improved risk sharing among investors under a profit-sharing agreement enhances their ability to bear risks, allowing them to give more weight to their own private information when deciding on the optimal investment amount. On average, compared with common stocks, profit sharing increases the aggregate amount committed from all investors. Thus even if Alice equally divides net profits with Bob, under profit sharing she will actually be entitled to a smaller slice of a larger pie. Figure 1 illustrates this intuition.

In this particular example profit sharing also brings about an even more surprising result: Under a 50-50 contract, Alice and Bob's total investment in equilibrium pays each of them exactly what she/he could have received had she/he known the other investor's private information, even though she/he actually does not. It is in this sense that profit sharing harnesses Alice's and Bob's collective wisdom. Although the 50-50 arrangement is a special result due to identical risk aversion, it hints at a general insight: In a world featuring decentralized possession of information among many individuals, some *simple* profit-sharing contracts could coordinate individual actions to achieve the first-best full-information outcome.

The general insight is confirmed in settings with a large number of investors with heterogeneous risk aversions. Section 2 derives the general structure of the optimal sharing rule. Overall, a profit-sharing contract has three attractive properties. First, it often achieves the *first-best* outcome in terms of harnessing the collective wisdom of all investors. Section 2 (with further generalizations in Section 3) proves that an optimal profit-sharing contract perfectly harnesses the collective wisdom of all investors and gives them the first-best outcome as long as wealth effects in preferences are negligible and indiosyncratic noises in private information fall into exponential family distributions (which include standard nor-

Figure 1: **Profit Sharing Entitles Alice to a Smaller Piece of a Bigger Pie**



mal idiosyncratic noises).[3] Other than these standard assumptions, results hold for *any* distributions of project return and accommodate potential (dis)economies of scale.

Second, the optimal profit-sharing contract is *simple*. It only requires information about an investor's risk tolerance, and does not depend on how well-informed each individual is, which is private information and often hard for the contract designers to solicit. Such simplicity makes practical implementation of the contract particularly easy. For example, in an application to crowdfunding (to be introduced in Section 2.1), a platform can use answers to standard questionnaire items on income, wealth, investment experience, investment objectives, etc. that investors provide when opening an account to determine the optimal sharing rule among investors participating in any given project.[4] We prove that with an optimal profit-sharing contract, investors will also truthfully report on those questionnaire items.

Third, the contract is *cost-effective*. Because profit sharing does not involve the direct exchange of private information, there is no requirement of sophisticated communication technology, no need to offer incentives to encourage disclosing private information, and no fear of individuals lying. A simple contract gives all.

One specific application of the general theory studied here comes from a new financing practice known as investment crowdfunding. In May 2016, against the backdrop of Title III and Title IV of the JOBS Act to help early-stage business ventures form capital, the SEC further expanded access to investment crowdfunding in which entrepreneurs can directly solicit contributions from a large number of investors in return for monetary payoffs specified by contracts agreed to at the time of investment.[5] Contracts currently used in practice offer returns in the form of common stock, debt, or a mixture of both. It remains an open

[3]The first assumption applies well to investment crowdfunding, in which each investor's investment amount often counts toward a tiny proportion of his/her total wealth. The second assumption is in line with the tradition and recent advances in information economics, see for example Breon-Drish (2015).

[4]Graham, Harvey and Puri (2013) demonstrates the use of psychometric tests to obtain risk-aversion.

[5]Expanding a restricted version of investment crowdfunding under Title II, Title IV allows participation by accredited investors, while Title III further lowers the bar of entry to include non-accredited investors. See https://www.sec.gov/rules/final/2015/33-9974.pdf for the final SEC ruling for Title III crowdfunding.

question, however, as to what the optimal contract should look like. On a separate note, one of the many claimed benefits of crowdfunding is that it can harness the wisdom of the crowd. This argument has been extensively made from the entrepreneur's perspective: By aggregating the investment decisions of a large number of investors, the idiosyncratic noises associated with each individual's judgment tend to be diversified. Because of this diversification, the aggregate investment amount provides crucially useful information to the entrepreneur. There are, however, few studies on how the wisdom of the crowd could similarly benefit investors.[6] An application of our theory to crowdfunding fills the two gaps.

Section 3 further validates the robustness of the main result of the paper by relaxing assumptions on return distribution, including costly information acquisition, and looking beyond projects with constant return to scale. We show that the benefit of profit sharing remains intact under non-normal project return distributions, costly information acquisition, and the presence of (dis)economies of scale. For all relaxations, the equilibrium outcome under profit sharing (plus cash transfers such as admission fees or signing bonuses if necessary) sustains the first-best outcome that would have been chosen by a benevolent and omniscient social planner. In other words, a version of the Second Welfare Theorem is obtained even at the presence of asymmetric information and strategic interdependence.

The rest of the paper is organized as follows. Section 1 analyzes the Alice–Bob problem in detail. To illustrate applications of the general theory for multiple investors, Section 2 casts derivations in the context of a investment crowdfunding platform and verifies the incentive compatibility of the optimal sharing contract in practical implementation. Section 3 proves the robustness of all results when standard assumptions are relaxed. Section 4 discusses related literature not yet adequately covered in the context. Section 5 concludes.

[6]Apparently a contract design that most benefits investors of their collective wisdom adds to the attractiveness of the crowdfunding market as well as improves capital allocation efficiency. Hence such a contract indirectly most benefits entrepreneurs with promising projects – the original motivation for Congress to promote investment crowdfunding.

1 Analysis of the Alice–Bob Example

To illustrate intuitions in the simplest form, this section proves that when Alice and Bob have identical risk aversion, they always find it optimal to equally divide net investment profit, regardless of the relative accuracy of their private information or their investment amount.

Let's first formalize the example in precise mathematical terms. Since both investors are deep-pocketed, their preferences feature little wealth effect, and could be summarized by a constant absolute risk aversion (CARA) utility function $u(W) = -e^{-\rho \cdot W}$ for some $\rho > 0$. They individually decide on how much money to invest in a risky, scalable project with *net* return per dollar invested denoted as $\tilde{r}$. In other words, if the gross return to the business is $\tilde{R}$, then $\tilde{r} = \tilde{R} - 1$ when the intertemporal discount rate is normalized to zero. The focus on net rather than gross return distinguishes a profit-sharing contract (to be introduced momentarily) from traditional common stocks. Investor i's independent yet unbiased private information translates into a private signal $s_i = r + \epsilon_i$, where r denotes the realization of the net return $\tilde{r}$, and $\epsilon_i \sim \mathcal{N}(0, \tau_i^{-1})$ is independent of $\tilde{r}$ and ϵ_{-i}, $i \in \{A, B\}$. Without loss of generality, assume that Alice's information is (weakly) more accurate than Bob's, i.e. $\tau_A \geq \tau_B$. Although it is not necessary, for exposition ease the net return $\tilde{r}$ is assumed to follow a normal distribution, i.e. $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$. This simplification helps compare this example with the CARA-normal tradition in standard investment models (e.g. Lintner (1965)). Section 3.1 relaxes normality to arbitrary distributions, Section 3.2 further considers costly acquisition of s_i, and Section 3.3 allows r to be dependent on Alice's and Bob's investment. All results from this section will remain under these generalizations.

1.1 Investment Strategies under Common Stock

Section 1.2 will derive Alice's and Bob's optimal investment strategies under a 50-50 profit-sharing agreement (to be summarized in (6)). Before delving into those derivations, for comparison we first consider a familiar case of common stocks. If both investors agree to hold common stocks instead, then investor i's problem would be to choose the optimal investment amount x_i' given private signal s_i so that

$$x_i'(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho\tilde{r}x}|s_i]. \tag{1}$$

Assume a normal distribution of project return $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$ for ease of exposition (which will be relaxed in Section 3.1), the right-hand side involves maximizing the moment-generating function of a normal variable, which gives

$$\begin{aligned} x_i'(s_i) &= \text{argmax}_x - e^{-\rho\mathbb{E}(\tilde{r}|s_i)x + \frac{1}{2}\text{Var}(\tilde{r}|s_i)\rho^2 x^2} \\ &= \frac{1}{\rho}(\tau_r\bar{r} + \tau_i s_i). \end{aligned}$$

In summary, Alice and Bob's investment strategies under common stocks are

$$\begin{cases} x_A = \frac{1}{\rho}(\tau_r\bar{r} + \tau_A s_A) \\ x_B = \frac{1}{\rho}(\tau_r\bar{r} + \tau_B s_B) \end{cases}. \tag{2}$$

Notice that each investor's investment amount is a weighted sum of the prior mean of investment return $\bar{r}$ and the private signal s_i, with weights being their respective precision normalized by investor risk aversion.

1.2 Equilibrium Investment Strategies under a 50-50 Agreement

Given an equal division of investment profits, investor i's problem is to choose an investment amount x_i based on private signal s_i such that

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2}\tilde{r}[x+\tilde{x}_{-i}(s_{-i})]}|s_i], \tag{3}$$

where $i \in \{A, B\}$ and $-i = \{A, B\}\backslash\{i\}$. Because the optimum to the right hand side depends on i's belief of $x_{-i}(s_{-i})$, the solution constitutes a Nash equilibrium.

Definition *A Nash equilibrium under equal profit sharing in the Alice–Bob example consists of two investment strategy functions $x_A(\cdot)$ and $x_B(\cdot)$ such that each investor's investment strategy is the optimal response to his/her (correct) belief of the other's investment strategy,*

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2}\tilde{r}[x+\tilde{x}_{-i}(s_{-i})]}|s_i], \tag{4}$$

where $i \in \{A, B\}$ and $-i = \{A, B\}\backslash\{i\}$.

Section 3.1 will prove the existence and uniqueness (up to a constant) of the Nash equilibrium under a more general setting. However, in a special case in which all random variables are normally distributed, a linear Nash equilibrium (which happens to be the unique Nash equilibrium) is easily obtained via the guess-and-verify method. If we assume

$$x_i(s_i) = \alpha + \beta_i s_i,$$

then equation (4) leads to

$$\alpha + \beta_i s_i \quad = \quad \text{argmax}_x - \mathbb{E}[e^{\left(-\frac{1}{2}\rho\tilde{r}\right)(x+\alpha+\beta_{-i}\tilde{s}_{-i})}|s_i]. \tag{5}$$

The conditional expectation on the right hand side of (5) could be interpreted as the moment-generating function of a non-central χ^2-distributed random variable (because both $-\frac{1}{2}\rho\tilde{r}$ and $x + \alpha + \beta_{-i}\tilde{s}_{-i}$, that are affine transformations of the normal variable $\tilde{s}_{-i}$, follow normal distributions), which has a closed-form expression given by the following lemma.

Lemma 1.1. *If* $\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}\right)$, *where* $(\sigma_{12}-1)^2 > \sigma_{11}\sigma_{22}$ *then*

$$\mathbb{E}\left[e^{\tilde{y}_1\tilde{y}_2}\right] = \frac{\exp\left\{(\theta_2^2\sigma_{11} - 2\theta_1\theta_2\sigma_{12} + \theta_1^2\sigma_{22} + 2\theta_1\theta_2)/(2[(\sigma_{12}-1)^2 - \sigma_{11}\sigma_{22}])\right\}}{\sqrt{(\sigma_{12}-1)^2 - \sigma_{11}\sigma_{22}}}.$$

Proof. Standard integration. □

Plug in $-\frac{1}{2}\rho\tilde{r}$ and $x + \alpha + \beta_{-i}\tilde{s}_{-i}$ into Lemma 1.1, and notice that conditional on s_i,

$$\begin{bmatrix} -\frac{1}{2}\rho\tilde{r} \\ x + \alpha + \beta_{-i}\tilde{s}_{-i} \end{bmatrix}_{|s_i} \sim \mathcal{N}\left(\begin{bmatrix} -\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \\ x + \alpha + \beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \end{bmatrix}, \begin{bmatrix} \frac{\rho^2}{4(\tau_r+\tau_i)} & -\frac{\rho\beta_{-i}}{2(\tau_r+\tau_i)} \\ -\frac{\rho\beta_{-i}}{2(\tau_r+\tau_i)} & \beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i} + \frac{1}{\tau_{-i}}\right) \end{bmatrix}\right),$$

thus the expectation on the right hand side of (5) is equal to

$$-\frac{\exp\left\{\frac{\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\frac{\rho^2}{4(\tau_r+\tau_i)} - \frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\frac{\rho\beta_{-i}}{\tau_r+\tau_i} + \left(\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right) - \rho\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)}{2\left[\left(-\frac{1}{2}\frac{\rho\beta_{-i}}{\tau_r+\tau_i}-1\right)^2 - \frac{\rho^2}{4(\tau_r+\tau_i)}\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)\right]}\right\}}{\sqrt{\left(-\frac{1}{2}\frac{\rho\beta_{-i}}{\tau_r+\tau_i} - 1\right)^2 - \frac{\rho^2}{4(\tau_r+\tau_i)}\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i} + \frac{1}{\tau_{-i}}\right)}}.$$

Notice that x, the variable we maximize over, only enters the numerator of the exponent in the above expression in a linear-quadratic function, thus (5) leads to

$$\begin{aligned}
\alpha+\beta_i s_i &= \text{argmin}_x \Bigg[\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\frac{\rho^2}{4(\tau_r+\tau_i)} - \frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\frac{\rho\beta_{-i}}{\tau_r+\tau_i} \\
&+ \left(\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right) - \rho\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\Bigg] \\
&= \frac{2}{\rho}(\tau_r\bar{r}+\tau_i s_i)-\alpha.
\end{aligned}$$

Matching coefficients gives $\alpha=\frac{1}{\rho}\tau_r\bar{r}$ and $\beta_i=\frac{2}{\rho}\tau_i$, leading to

$$\begin{cases} x_A = \frac{1}{\rho}(\tau_r\bar{r}+2\tau_A s_A) \\ x_B = \frac{1}{\rho}(\tau_r\bar{r}+2\tau_B s_B) \end{cases}. \tag{6}$$

1.3 How Does Profit Sharing Alter Investment Strategies?

Comparing the investment strategies under profit sharing (6) to those under common stocks (2), the only difference is the additional coefficient 2 in the second term. The first term remains unchanged. This result comes from two competing forces induced by profit sharing, with one encouraging more aggressive investment and the other partially counteracting it.

As mentioned in the introduction, profit sharing diversifies the risk induced by the idiosyncratic noise in each investor's private signal, allowing investors to take on more risks given their risk-bearing capacities. To see this mathematically, the exponent on the right hand side of (4) is the sum of two parts: $-\frac{1}{2}\rho\tilde{r}x$ and $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$. The first part $-\frac{1}{2}\rho\tilde{r}x$, compared to $-\rho\tilde{r}x$ under no profit sharing, divides the sensitivity of i's payoff to her investment by two. Hence it appears *as if* profit sharing makes investor i half less risk averse, allowing her to invest more *aggressively.* This aggressiveness contributes to the higher weights on private information observed in investor i's investment under profit sharing.

Were the more aggressive risk taking the only force in play, investor i would have also (inefficiently) put more weight on her prior of $\tilde{r}$ (the first term in her investment strategy). This is only prevented by the presence of the second part of the exponent on the right hand side of (4) $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$, which involves an interaction between $\tilde{r}$ and investor $-i$'s investment.[7] As private signals are correlated due to the common component r, when Alice receives a high signal of the project return and invests a lot accordingly, she may worry that Bob likely has also received a high signal and invested a lot, exposing her to excessive project risk. As a rational response, this concern would encourage Alice to act more *conservatively* on her prior, balancing her otherwise aggressiveness. An optimal profit-sharing contract is meant to achieve the best balance between such aggressiveness and conservativeness.

The joint force of aggressive and conservative in shaping optimal investment reflects the strategic interdependence that profit sharing introduces. Note that unlike other studies on the efficient use of information (e.g. Angeletos and Pavan (2007), Amador and Weill (2010)) where strategic complementarity/substitutability are embedded in technology, in the Alice–Bob example with a perfectly scalable project the technology itself does not assume any strategic interaction. All strategic interdependence stems from the profit-sharing contract.

When Alice and Bob have the same preference, the optimal profit-sharing contract that achieves the best balance between aggressiveness and conservativeness is indeed a 50-50 one. To understand why, notice that because investor $-i$'s investment is a function of $-i$'s private information, the second part of the exponent on the right hand side of (4) $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$ *effectively* exposes investor i to $-i$'s private information. Given identical risk tolerance, only under a 50-50 contract would Bob (Alice) act exactly the same as how Alice (Bob) would like to had she (he) gotten access to his (her) private information. In other words, a 50-50 profit-sharing contract perfectly aligns both investors' incentives and makes each investor a perfect "agent" for the other.

[7] To see more concretely, consider a hypothetical case in which Alice is forced to get only half of her investment profit, and does not enjoy the half contributed by Bob. Alice would then invest $\frac{1}{\rho}(2\tau_r\bar{r} + 2\tau_A s_A)$ instead of $\frac{1}{\rho}(\tau_r\bar{r} + 2\tau_A s_A)$ as Alice's investment under profit sharing x_A.

The 2s in the second terms of the two right-hand side expressions in equation set (6) reflect the aggressiveness under profit sharing, while the absence of 2s in the first terms reflect the counteraction from the conservativeness.

1.4 Profit Sharing Harnesses the Wisdom of the Crowd

As immediate from (6), for any realization of project return and private signals, investor i's payoff under equal profit sharing is

$$r\frac{x_A(s_A)+x_B(s_B)}{2}=\frac{r}{\rho}(\tau_r\bar{r}+\tau_A s_A+\tau_B s_B). \tag{7}$$

Let's compare this outcome with a full-information benchmark, in which both investors (hypothetically) have each other's private information and no information asymmetry exists. In this case, without profit sharing investor i's optimal investment amount is given by

$$\begin{aligned} x_i'(s_A,s_B) &= \text{argmax}_x \mathbb{E}[-e^{-\rho r x}|s_A,s_B] \\ &= \text{argmax}_x - e^{-\rho\mathbb{E}(r|s_A,s_B)x+\frac{1}{2}\text{Var}(r|s_A,s_B)\rho^2x^2} \\ \Rightarrow x_i'(s_A,s_B) &= \frac{\mathbb{E}(r|s_A,s_B)}{\rho\text{Var}(r|s_A,s_B)} \\ &= \frac{1}{\rho}(\tau_r\bar{r}+\tau_A s_A+\tau_B s_B). \end{aligned}$$

Hence investor i's payoff under a full-information benchmark is given by

$$x_i'(s_A,s_B)=\frac{r}{\rho}(\tau_r\bar{r}+\tau_A s_A+\tau_B s_B). \tag{8}$$

Comparing (7) with (8) shows that the payoff under profit sharing exactly equals that under a full-information benchmark. This observation is summarized below.

Theorem 1.2. *For all realizations of the state of nature $\{r, s_A, s_B\}$, each investor's payoff* *under equal division of profits is always equal to that under a full-information benchmark. A 50-50 profit-sharing contract perfectly harnesses the collective wisdom of both investors.*

1.5 Further Digestions of the Alice–Bob Example

It is admittedly counterintuitive at first sight to imagine that when Alice and Bob have the same level of risk aversion, Alice would be willing to go 50-50 with Bob even when she is strictly better informed. To help better appreciate this result, this section further explains the intuition based on several comments received from earlier readers. Readers without doubt so far may simply skip this section and head directly to Section 2.

We first study an extreme case in which Bob's signal precision is zero (while Alice still has positive signal precision). According to the results above, even in this extreme case, Alice should still prefer, or at least be indifferent to, dividing profits 50-50 with Bob. Section 1.5.1 confirms this conjecture by drawing an analogy to the literature of delegated wealth management. Section 1.5.2 dissipates a false impression that Alice may dilute profit and lower investment return by sharing with Bob. Section 1.5.3 reconciles our new profit sharing contract with often-observed common stocks by showing that common stocks are just special cases of optimal profit-sharing contracts when investors have no private information (i.e. when the wisdom of the crowd effect is irrelevant). Section 1.5.4 further dissipates concerns over an account-dividing attack.

1.5.1 Analogy to Delegated Wealth Management

When Bob's private signal has no information content, i.e. $\tau_B = 0$, by Equation (6), we have

$$\begin{cases} x_A &= \frac{1}{\rho}(\tau_r \bar{r} + 2\tau_A s_A) \\ x_B &= \frac{1}{\rho}\tau_r \bar{r} \end{cases} . \tag{9}$$

One interpretation of Equation (9) is that Bob invests according to the public prior $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$ only (as he has no private signal), while at the same he hires the better-informed Alice as a "fund manager" to use her superior information and take care of his money. Following the terminology of Admati and Pfleiderer (1990), Alice *indirectly* sells her private information to Bob via delegated wealth management. Because Alice and Bob have the same utility function, Bob would like Alice to treat his delegated funds exactly the same way as she would treat her own money. Hence, Bob would strictly prefer to get exactly one half of the investment profit that Alice makes. On the other hand, the scalability of the project ensures that Alice is indifferent between alternative sharing rules, because she can always adjust the mapping from her private information to her investment amount so that she exposes herself to the optimal amount based on her own return–risk trade-off. Under a 50-50 rule, however, whatever Alice chooses to invest that best suits her own interests happens to be what Bob would choose had he had access to Alice's private signal. Following the principal-agent language, a 50-50 deal perfectly aligns Alice's (agent's) and Bob's (principal's) incentives.

The above special example relates to several papers on delegated wealth management. As mentioned earlier, Admati and Pfleiderer (1990) use the insight of indirect sale of information to explain the rise of institutional investors. Ross (2005) suggests how delegated wealth management could disrupt a classic noisy rational expectation equilibrium.[8] García and Vanden (2009) considers wealth delegation with endogenous information acquisition.

In the general case case in which Bob is not completely uninformed (i.e. $\tau_B > 0$), Alice and Bob are both better informed than the other in different dimensions. Following the logic in the extreme case, Alice and Bob would also like to delegate their wealth to each other, for which the optimal delegation contract would be to equally divide the profit made by

[8] In a different setting, but similar in spirit, Indjejikian, Lu and Yang (2014) suggest that the strategic Kyle (1985) equilibrium is not stable, because the most informed investor has incentive to leak information to an uninformed one so that the other informed investors will trade less aggressively.

the more informed investor. Such mutual delegation would then appear like (equal) profit sharing.

1.5.2 Do Investors Care about Terminal Wealth or Investment Return?

As is standard in economics, the investors' problems in all derivations above are to maximize expected utility as a function of wealth (see (3) and (1)). A few readers, however, find it hard to reconcile with the common practice of (risk-adjusted) return maximization as in investment theory. They argue that when Alice and Bob go 50-50 and Alice invests more than Bob, she effectively "subsidizes" Bob and "dilutes" her own investment return, defined as the dollar amount of net profit received over the amount of initial investment in the project. They wonder why would Alice invest more in exchange for a "diluted" return?

The apparent conflict is resolved by an appropriate choice of denominator for return calculation. As Alice actively chooses how much to invest in the risky project (while the rest of her wealth earns a zero risk-free rate), her investment decision in the project is indeed part of an asset allocation decision between investing and saving. Hence the rate of return that Alice actually looks at should be a weighted average of project return and risk-free rate, with the weights determined by her investment amount in the project. In other words, Alice's rate of return is the ratio between her overall final wealth and her total initial wealth. With the correct choice of the denominator in return calculation, there is no longer a conflict between utility maximizing and the pursuit of the highest (risk-adjusted) return.

1.5.3 How to Reconcile Profit Sharing with Common Stocks?

An editor of an earlier draft has raised the question of why we do not see such profit-sharing contracts in the crowdfunding environment, or in any other capital market context. For the first part of the question, as we have discussed in the introduction, investment crowdfunding has really only been in practice for several months, with various types of contracts currently used in practice and few theoretical guidance in the literature yet. For the second part of the question, we shall point out that widely used common stocks are indeed in no conflict with profit sharing. When investors have homogeneous information about project payoffs (an assumption implicit in most other capital market contexts), the optimal profit-sharing contract coincides with a common stocks, which divides payoffs in proportional to initial investment. In this sense, a common stock is just a special case of profit sharing when the wisdom of the crowd effect is weak.

1.5.4 Is Profit Sharing Robust under Sybil Attacks?

Named after the case study of a woman with multiple personality disorder, a Sybil attack is a term in computer science that refers to a type of security threat when a node in a network claims multiple identities. We borrow the term here to characterize another comment raised by the editor: if Alice spreads her investment across two accounts, she seems to be able to secure two-thirds instead of one-half of net profits. Does the central result survive such account divisibility considerations?

The short answer is yes. As will be clear once we get to Theorem 2.3, forging duplicate accounts will endogenously lead to an (incentive compatible) different sharing ratio that exactly cancels off its effect. Furthermore, even if the sharing ratio has to be fixed, Alice's launching a Sybil attack would only hurt herself as well as Bob.

2 Profit Sharing with Many Heterogeneous Investors

With intuitions from the Alice–Bob example, a general harnessing-the-wisdom-of-the-crowd contract is easily derived. For exposition concreteness, Section 2.2 derives the optimal contract within the specific context of a crowdfunding platform, although the theory can also be applied to other business organizations in which harnessing the wisdom of the crowd is an important concern. Section 2.3 further discusses potential incentive problems in implementing the optimal contract on a crowdfunding platform. Before moving on, Section 2.1 first provides an overview of the crowdfunding market and highlight our contributions.

2.1 The Crowdfunding Market

Crowdfunding has emerged in recent years as an alternative financing method in which entrepreneurs directly solicit contributions from a large number of investors. In return for their contributions, investors receive non-pecuniary rewards (reward-based crowdfunding), private benefits (denotation-based crowdfunding),[9] or since May 16, 2016, monetary payoffs according to a contract agreed upon at the time of investment (investment crowdfunding).[10]

Currently all funding activities take place on intermediaries known as crowdfunding platforms, which list projects, manage investor accounts, and supervise the entire funding process. Platforms often differentiate themselves from peers by setting unique rules that apply to their own projects. For example, within the category of reward-based crowdfunding, most platforms (e.g. Kickstarter) implement the all-or-nothing rule (i.e. an entrepreneur will post her intended fundraising goal and deadline, and she receives financing only if her fundraising goal is met before the deadline), with a few exceptions (e.g. Indiegogo) which allow the entrepreneur to keep whatever she has raised even if her funding goal is not met.[11] In the realm of investment crowdfunding, different platforms often offer different types of securities to their investors. For example, Texas-based NextSeed mainly offers debt contracts, while

[9] See e.g. Boudreau et al. (2015).

[10] Agrawal, Catalini and Goldfarb (2013) provide an overview of the investment crowdfunding industry. Vulkan, Åstebro and Sierra (2016) provide European evidence.

[11] While the all-or-nothing feature has proven to be effective for some reward-based crowdfunding projects (see Cimon (2017) for a real option argument), several recent studies have also questioned its efficiency from alternative perspectives. For example, in the context of reward-based crowdfunding, Kumar, Langberg and Zvilichovsky (2015) find that due to price discrimination against pivotal investors, existing crowdfunding structures may lead to a distorted phenomenon in which reducing the cost of capital to entrepreneurs may unintentionally reduce production and welfare. In the context of investment crowdfunding, Brown and Davies (2015) find that with all-or-nothing as well as fixed funding target and pro-rata payoff in place, a well-informed crowd could collectively behave as if uninformed due to coordination failure.

many peers (e.g. WeFunder) offer common stocks. Some platforms also offer more flexible and complicated hybrid contracts.[12] Despite the wide range of contract forms adopted in practice, few theories exist yet to guide the optimal security design.

Specific to harnessing the wisdom of the crowd, regulators are well aware of crowdfunding's potential. In the first paragraph of its final rule of Regulation Crowdfunding, the SEC highlights that "individuals interested in the crowdfunding campaign – members of the 'crowd' ... fund the campaign based on the collective 'wisdom of the crowd'".[13] On the other hand, there seems to be little consensus on the best way to harness the wisdom of the crowd. In the same document, the SEC notes that "(investors) share information about the project, cause, idea or business with each other" as ways to implement the wisdom of the crowd. This interpretation, however, is challenged by recent academic research. Brown and Davies (2015) point out that 1) "the small size and dispersed nature of investments likely make individual communication impractical", and 2) unlike in IPO, "a setting in which underwriters aggregate information from investors during the bookbuilding process, neither platforms nor entrepreneurs have allocation discretion in crowdfunding. As a result, truthfully reporting information cannot be rewarded with underpriced allocations".[14] Given these obstacles, it is natural to investigate whether alternative mechanism designs could help.

In addition, despite extensive discussions on how crowdfunding could harness the wisdom of the crowd from investors and provide early feedback to entrepreneurs that facilitates their learning (e.g. Chemla and Tinn (2016) and Xu (2016)) or guide follow-up investors (Li (2015*a*)), the literature is silent on how investors themselves could ever benefit from their own wisdom of the crowd and hence make better investment decisions.[15] This paper takes this new perspective and offers a solution.

[12] See Belleflamme, Omrani and Peitz (2015) for a detailed discussion on various crowdfunding platforms.

[13] See 17 CFR Parts 200, 227, 232, 239, 240, 249, 269, and 274.

[14] See Ritter and Welch (2002) for a review of the IPO literature.

[15] Kovbasyuk (2011) investigates a related but different question of how uninformed investors learn the crowd wisdom of experts.

As a new financing practice in its infancy, investment crowdfunding is currently subject to several protective regulations. For example, under the current version of Regulation Crowdfunding, issuers may only raise $1M in a rolling 12-month period, and investors are limited to investing a certain dollar amount based on their income or net worth.[16] Despite these restrictions, investment crowdfunding has been growing rapidly. According to incomplete statistics collected from five crowdfunding platforms (Flashfunders, NextSeed, SeedInvest, StartEngine, and WeFunder) by third-party service NextGen, since the the first deal on May 19, 2016, the total successful capital commitment to Title III crowdfunding has surpassed $25 million by the beginning of March, 2017.[17]

The vast heterogeneity in specific rules across platforms and the likely future evolution in regulations as the market grows make it almost impossible for a theory to match all existing institutional details. As a result, derivations below abstract from platform- or regulation-specific details (such as whether issuers impose fixed funding target or receive all or nothing), but instead focus on three general features on investment crowdfunding. First, projects are entrepreneurial in nature and therefore investors only have imprecise ideas about their returns, although across all investors the wisdom of the crowd exists. Second, each individual's contribution to a particular project is small compared to his or her total wealth, and thus there is little wealth effect that would confound an investor's funding decision. Third, as projects are all in their early stages, they are generally flexible in size. Of course, these assumptions are not meant to downplay other features of investment crowdfunding, but rather to focus the current discussion.

[16] The constraint on investment amount, however, is likely not binding. In reward-based crowdfunding, where there is no investment cap, Mollick (2014) documents an average investment size of $64.

[17] To put the rapid growth in perspective, Kickstarter, the pioneer in reward-based crowdfunding, has received $2.6 billion in pledges over the seven years since its launch on April 28, 2009.

2.2 The General Optimal Contract Features Profit Sharing

A crowdfunding platform launches a new security that stipulates a compensation scheme among its n investors who participate in funding a risky project. In period $t = 0$, the security stipulates that investor i, who has a constant absolute risk aversion parameter ρ_i, receives a_i of the project's residual earnings, which will be realized by the end of period $t = 1$. $\sum_{i=1}^{n} a_i = 1$. The project renders net payoff $\tilde{Y} = \tilde{r}X$, where $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$ denotes the return-per-dollar-invested, and X is total amount of initial investment contributed by all investors. Denote x_i as investor i's investment contribution, then $\sum_{j=1}^{n} x_i = X$. Assume that investor i has private information about the project return $s_i = r + e_i$, where $r \perp\!\!\!\perp e_i$ and $e_i \sim \mathcal{N}(0, \tau_j^{-1})$, and each investor independently decides on how much to invest.

The investment provided by the n investors is given in a Nash equilibrium. In particular, investor i chooses x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_i r(x_i + \sum_{k \neq i} x_k)\right]\right)\middle| s_i\right], \tag{10}$$

given her perception of other players' equilibrium investment x_k, $k \neq i$. The following theorem provides a linear Nash equilibrium solution for a given player.

Theorem 2.1. *A Nash equilibrium exists only when the pre-agreed profit ratio is proportional to each investor's risk tolerance, i.e.*

$$a_i = \frac{1/\rho_i}{\sum_{i=1}^{n} 1/\rho_i},$$

and in equilibrium investor i's investment is given by

$$x_i = \frac{\tau_r \bar{r}}{\rho_i} + \left(\sum_{k=1}^{n} \frac{1}{\rho_k}\right)\tau_i s_i. \tag{11}$$

Proof. To be subsumed in the proof for Theorem 2.3. Notice that when $n = 2$ and $\rho_1 = \rho_2 = \rho$, we go back to the special case with Alice and Bob. □

Notice that if investor i has full information, her investment would be

$$x_i = \frac{\tau_r \bar{r}}{\rho_i} + \frac{\sum_{i=1}^{n} \tau_i}{\rho_i} i^*, \text{ where } i^* = \frac{\sum_{k=1}^{n} \tau_k s_k}{\sum_{k=1}^{n} \tau_k}.$$

Thus investor i's payoff is

$$r\left(\frac{\tau_r \bar{r}}{\rho_i} + \frac{\sum_{i=1}^{n} \tau_i}{\rho_i} i^*\right)$$

under both full information and profit sharing. This observation is summarized below.

Theorem 2.2. *Conditional on her own private information, an investor's expected utility from agreeing to a well-designed n-investor profit-sharing contract is identical to the expected utility she could obtain had she known the other $n - 1$ investors' private information while taking common stocks.*

Similar to in the Alice–Bob example, the benefit from profit sharing has a diversification explanation. Profit sharing effectively takes the average over a large number of conditionally independent signals, and by the law of large numbers it goes toward canceling the idiosyncratic noises associated with each individual's private information. This insight is reminiscent of what has become conventional wisdom since the seminal work of Markowitz (1952) that proper diversification achieves optimal return-risk trade-off. There are, however, several differences between Theorem 2.2 and traditional diversification arguments. First, diversification in portfolio theory relies on pooling multiple assets, while diversification in Theorem 2.2 comes from pooling multiple signals, and applies to cases of only one risky project. Second, portfolio theory usually does not involve asymmetric information, while Theorem 2.2 requires dispersed private information, without which (i.e., $\tau_e = 0$) profit sharing would make no difference. Third, as Appendix 3.1 will show, Theorem 2.2 extends beyond cases with normal distributions, while traditional portfolio theory depends on the absence of higher (than second) moments.[18]

2.3 Incentive to Truthfully Report Risk-Tolerances

As Theorem 2.1 clarifies, an optimal contract that harness the wisdom of the crowd only requires information about investors' risk tolerances. From the crowdfunding platform's perspective, how could they extract this information when designing the contract? This section shows that at the contract signing stage, each investor has a strict incentive to truthfully report his/her risk tolerance. This is because if they misreport their risk tolerances so that the crowdfunding platform stipulates an alternative sharing contract based on distorted information, all investors' expected utility will be lowered (even though this negative effect could be partially but never completely diminished by investors making a "private" investment in the project outside the funding platform). To see this, consider how investor i, who is entitled to an arbitrary share a_i of the firm's residual earnings, chooses x_i and X_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_j r(x_i+\sum_{k\neq i} x_k)+rX_i\right]\right)\middle| s_i\right], \tag{12}$$

given her belief of other investors' equilibrium investment strategies x_k, $k \neq i$. X_i denotes investor i's "private" investment in the project outside the profit-sharing agreement, if any. Investors may have incentives to engage in "private" investment to (partially) offset their undesired risk exposure to the project whenever the profit-sharing rule is not optimal. Theorem 2.3 summarizes results in this setting.

Theorem 2.3. *Each investor's expected utility is maximized when profits from the project are divided according to investors' risk tolerance, i.e.* $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^{n}\frac{1}{\rho_i}}$*. In the resulting linear*

[18] For example, Conine and Tamarkin (1981) show that given the existence of positive skewness a rational investor may hold an optimal limited number of homogeneous risk assets.

Nash equilibrium, investors do not make "private" investments, that is $X_i = 0$.

Proof. A linear symmetric equilibrium is given by $x_k + \frac{X_k}{a_k} = \pi_k + \gamma_k s_k + \frac{\Pi_k + \Gamma_k s_k}{a_k}$ for some π_k and γ_k. Because

$$\begin{bmatrix} -a_i\rho_i r \\ x_i + \frac{X_i}{a_i} + \sum_{k\neq i} x_k \end{bmatrix}_{|s_i} \sim \mathcal{N}\left(\begin{bmatrix} -\rho_i a_i \mathbb{E}(v|s_i) \\ x_i + \frac{X_i}{a_i} + \sum_{k\neq i}\pi_k + \sum_{k\neq i}\gamma_k \mathbb{E}(v|s_i) \end{bmatrix},\right.$$
$$\left.\begin{bmatrix} \rho_i^2 a_i^2 \mathrm{Var}(v|s_i) & -\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) \\ -\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) & (\sum_{k\neq i}\gamma_k)^2 \mathrm{Var}(v|s_i) + \sum_{k\neq i}\gamma_k^2\tau_k^{-1} \end{bmatrix}\right)$$

by Lemma 1.1, investor i equivalently minimizes

$$\theta_2^2\rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1\theta_2\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) + \theta_1^2[(\sum_{k\neq i}\gamma_k)^2 \mathrm{Var}(v|s_i) + \sum_{k\neq i}\gamma_k^2\tau_k^{-1}] + 2\theta_1\theta_2$$
$$\xrightarrow{\text{FOC}} \quad 2\theta_2\rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) + 2\theta_1 = 0,$$
$$\text{where } \theta_1 = -\rho_i a_i \mathbb{E}(v|s_i) \text{ and } \theta_2 = x_i + \frac{X_i}{a_i} + \sum_{k\neq i}\pi_k + \sum_{k\neq i}\gamma_k \mathbb{E}(v|s_i).$$

Plugging in $x_i + \frac{X_i}{a_i} = \pi_i + \gamma_i s_i + \frac{\Pi_i + \Gamma_i s_i}{a_i}$ leads to

$$[\sum_{k\neq i}\pi_k + \pi_i + \gamma_i s_i + \frac{\Pi_i + \Gamma_i s_i}{a_i} + \sum_{k\neq i}\gamma_k \mathbb{E}(v|s_i)]\rho_i^2 a_i^2 \mathrm{Var}(v|s_i)$$
$$-\rho_i a_i \mathbb{E}(v|s_i)\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) - \rho_i a_i \mathbb{E}(v|s_i) = 0.$$

Matching coefficients renders $(\gamma_i + \frac{\Gamma_i}{a_i})\rho_i a_i = \tau_i$, $(\Pi + \frac{\Pi_i}{a_i})\rho_i a_i = \tau_r \bar{r}$ (where $\Pi \doteq \sum_{i=1}^n \pi_i$). □

Plug in each investor's equilibrium investment amount into (12), and with Lemma 1.1

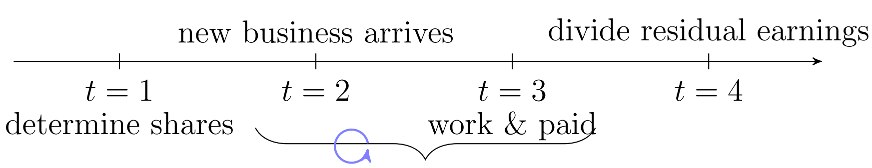
we obtain investor i's expected utility

$$-\frac{\sqrt{\tau_r+\tau_i}\exp\{-\frac{(\tau_r+\tau_i)[\mathbb{E}(v|s_i)]^2}{2}\}}{\sqrt{\tau_r+\tau_i+2\rho_i a_i\sum_{k\neq i}\gamma_k-\rho_i^2 a_i^2\sum_{k\neq i}\gamma_k^2\tau_k^{-1}}}, \tag{13}$$

which is maximized at $\gamma_k = \frac{\tau_k}{\rho_i a_i}$. Plugging in $\gamma_i + \frac{\Gamma_i}{a_i} = \frac{\tau_i}{\tau_r \bar{r}}(\Pi + \frac{\pi_i}{a_i})$ and $\frac{1}{\tau_r \bar{r}}(\Pi + \frac{\pi_i}{a_i})\rho_i a_i = 1$ shows that at the optimal γ_i, $\Gamma_i = 0$. Thus for any given sharing rule a_k ($k = 1, \cdots, n$), there exists a linear equilibrium in which each investor optimally chooses her amount of investment both within and outside of a firm. In particular, when a_i is chosen to be $\frac{\frac{1}{\rho_i}}{\sum_{i=1}^n \frac{1}{\rho_i}}$, investment in the firm can be stipulated so that no investor has incentive to work outside of the firm, and the resulting equilibrium gives the highest expected utilities to all investors.

Side remark: profit-sharing and partnership structure A profit-sharing contract shares similarities with the structure of many partnership firms. As Figure 2 illustrates, at a high level a partnership's business could be summarized by a risky production technology, to which a set of "partners" provide (usually relatively homogeneous) production inputs (capital, time, or raw ingredients, etc.) based on their assessment of the business productivity, and from which the same set of partners share residual earnings according to a pre-specified rule.

Figure 2: **The Structure of a Partnership**



This primitive description of a partnership firm with emphases on profit sharing and risk taking follows the corporate law tradition (Hansmann (2009)), and is linguistically consistent with English convention – a firm is also known as a "company" (involving multiple owners) or a "venture" (a risky business). A companion paper, Li (2015*b*), develops a profit sharing-based theory of the firm, and discusses the relationship between profit sharing within a firm and rational expectations within the market in terms of coordinating economic activities.

3 Generalizations and Robustness

This section relaxes the assumptions in Section 2.2. We will confirm the superiority of profit sharing even if the project return follows arbitrary distributions, investors' private information has to be costly acquired, and the project features (dis)economy of scale. For exposition ease, we will restrict ourselves to a two-agent case, although accommodating more than two investors is straightforward except for more tedious math. All proofs are left to the Appendix.

3.1 Arbitrary Distributions of Project Return $\tilde{r}$

So far in all derivations we have assumed that the project return $\tilde{r}$ follows a normal distribution. Empirically, however, returns from entrepreneurial projects are likely to be highly skewed. Most ventures fail and only a small number of projects eventually take off and win big. This section verifies the main result from Section 2.2 (that profit sharing perfectly harnesses the wisdom of the crowd) does not rely on the normality of $\tilde{r}$. Indeed it could work under any arbitrary distribution. A modest sufficient condition for this to hold follows the spirit of Breon-Drish (2015) that the likelihood function of $\tilde{r}$ given private signals $s_i, i \in \{A, B\}$ lies in an *exponential family*.[19] Because this condition includes normally distributed idiosyncratic noises, empirically it is likely to hold, especially when the number of investors is large as in crowdfunding. The following theorem summarizes.

[19] Exponential family distributions are extensively used in Bayesian statistics (e.g. deriving conjugate priors) and decision theories to preserve closed-form expression.

Theorem 3.1. *For any arbitrary distributions of project return $\tilde{r}$ and an exponential family likelihood function of $\tilde{r}$ given private signals $s_i, i \in \{A, B\}$, an optimally designed profit-sharing contract gives the same payoff for both investors as in a full-information benchmark.*

3.2 Costly Information Acquisition

Our discussion so far has assumed that each investor's private information comes from endowment. Although this is a sensible assumption for many crowdfunding projects, it is natural to think of cases in which investors' private information has been acquired at some cost. If investors cannot externalize their private costs in information acquisition, a moral hazard in team concern (Holmström (1982)) may arise as each investor would like to save his/her private information acquisition cost and free ride on the others. The free-rider problem under endogenous information acquisition may counter the benefits from the wisdom of the crowd, and hence discourage investors from entering into a profit-sharing agreement. This section shows that, under standard assumptions on information acquisition costs, this concern is nonexistent, as summarized in the following theorem.

Theorem 3.2. *With a constant cost to acquire an extra unit of signal precision, investors strictly prefer more participants in profit sharing. In other words, in this case the free-riding concern is not large enough to cancel out the wisdom of the crowd benefit from profit sharing.*

3.3 Projects with (Dis)economy of Scale

Our discussion so far has been focusing on investment projects that have a constant return to scale. Although this is a sensible assumption in the context of investment crowdfunding, in applications beyond crowdfunding we are more likely to see businesses that feature (dis)economies of scale. This section shows that in the presence of (dis)economy of scale, the optimal profit-sharing contract derived in previous sections is still first-best optimal.

To this end consider an Alice–Bob model similar to the one in Section 1 except that the net return per dollar invested from the project is influenced by the total dollar amount invested. Specifically, assume that the net return is $\tilde{r} - \lambda(x_1 + x_2)$. Note $\lambda = 0$ corresponds to a constant-return-to-scale project discussed in Section 1. $\lambda > 0$ corresponds to a decreasing-return-to-scale project, and $\lambda < 0$ an increasing-return-to-scale project. The following lemmas summarize investor i's expected utilities under profit sharing and a full-information benchmark.

Lemma 3.3 (**Profit Sharing**). *Under a profit-sharing contract in which the sharing rule divides the net payoff proportional to the investors' risk tolerances, a Nash equilibrium exists under which strategy functions are given as $x_i(s_i) = \alpha + \beta_i s_i$, where $i \in \{A, B\}$ such that*

$$\beta_i = \frac{\tau_i}{\frac{\rho_i \rho_{-i}}{\rho_i + \rho_{-i}} + 2\lambda(\tau_r + \tau_i + \tau_{-i})} \tag{14}$$

$$\alpha = \frac{1}{2}\frac{\tau_r \bar{r}}{\frac{\rho_i \rho_{-i}}{\rho_i + \rho_{-i}} + 2\lambda(\tau_r + \tau_i + \tau_{-i})} \tag{15}$$

and investor i's expected utility is

$$-\frac{\exp\left(-\frac{(\bar{r}\tau_r + s_i\tau_i)^2}{2(\tau_i+\tau_r)}\frac{\rho_i\rho_{-i}}{2\lambda(\rho_i+\rho_{-i})(\tau_i+\tau_r)+\rho_i\rho_{-i}}\right)}{\sqrt{\frac{\tau_{-i}+\tau_i+\tau_r}{\tau_i+\tau_r}\frac{2\lambda(\rho_i+\rho_{-i})(\tau_i+\tau_r)+\rho_i\rho_{-i}}{2\lambda(\rho_i+\rho_{-i})(\tau_{-i}+\tau_i+\tau_r)+\rho_i\rho_{-i}}}} \tag{16}$$

Lemma 3.4 (**Full-Information Benchmark**). *In a full-information benchmark, investor i chooses $x_i(s_i) = \alpha_i + \beta_{iA}s_A + \beta_{iB}s_B$, where $i \in \{A, B\}$, such that*

$$\alpha_i = \frac{\bar{r}\tau_r\left(\lambda\left(\tau_A + \tau_B + \tau_r\right) + \rho_{-i}\right)}{\rho_{-i}\left(2\lambda\left(\tau_A + \tau_B + \tau_r\right) + \rho_i\right) + \lambda\left(\tau_A + \tau_B + \tau_r\right)\left(3\lambda\left(\tau_A + \tau_B + \tau_r\right) + 2\rho_i\right)}$$

$$\beta_{iA} = \frac{\tau_A\left(\lambda\left(\tau_A + \tau_B + \tau_r\right) + \rho_{-i}\right)}{\rho_{-i}\left(2\lambda\left(\tau_A + \tau_B + \tau_r\right) + \rho_i\right) + \lambda\left(\tau_A + \tau_B + \tau_r\right)\left(3\lambda\left(\tau_A + \tau_B + \tau_r\right) + 2\rho_i\right)}$$

$$\beta_{iB} = \frac{\tau_B\left(\lambda\left(\tau_A + \tau_B + \tau_r\right) + \rho_{-i}\right)}{\rho_{-i}\left(2\lambda\left(\tau_A + \tau_B + \tau_r\right) + \rho_i\right) + \lambda\left(\tau_A + \tau_B + \tau_r\right)\left(3\lambda\left(\tau_A + \tau_B + \tau_r\right) + 2\rho_i\right)},$$

and investor i's expected utility is

$$-\frac{\exp\left(-\frac{(\bar{r}\tau_r+s_i\tau_i)^2}{2(\tau_i+\tau_r)}\frac{\rho_i(\tau_i+\tau_{-i}+\tau_r)(\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_{-i})^2(2\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_i)}{\tau_{-i}\rho_i(2\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_i)(\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_{-i})^2+(\tau_i+\tau_r)(\rho_{-i}(2\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_i)+\lambda(\tau_i+\tau_{-i}+\tau_r)(3\lambda(\tau_i+\tau_{-i}+\tau_r)+2\rho_i))^2}\right)}{\sqrt{\frac{1}{\tau_i+\tau_r}}\sqrt{\tau_i+\tau_r+\frac{\tau_{-i}\rho_i(2\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_i)(\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_{-i})^2}{(\rho_{-i}(2\lambda(\tau_i+\tau_{-i}+\tau_r)+\rho_i)+\lambda(\tau_i+\tau_{-i}+\tau_r)(3\lambda(\tau_i+\tau_{-i}+\tau_r)+2\rho_i))^2}}} \tag{17}$$

As apparent from Theorems 3.3 and 3.4, the expected utilities under profit sharing and that under a full-information benchmark are no longer identical when the project features (dis)economy of scale. However, in this case profit sharing could still dominate a full-information benchmark. Indeed, we have a stronger result. If we take a mechanism design approach and compare the expected utilities achieved under profit sharing with the first-best outcome a benevolent and omniscient social planner can get, we find that with the introduction of cash transfers such as lump-sum admission fees or signing bonuses, profit sharing could always sustain the first-best outcome. Theorem 3.5 summarizes this result.

Theorem 3.5. *In an Alice–Bob example (Section 1) where the project features (dis)economy of scale, the first-best allocation chosen by an omniscient and benevolent social planner could be sustained with a Nash equilibrium under profit sharing plus some cash transfers. The profit-sharing rule sustaining the Pareto optimal outcome divides net investment profit in proportion to investors' risk tolerances.*

To facilitate further discussion we highlight two features of the first-best outcome (derived in the proof of Theorem 3.5). First, an omniscient and benevolent social planner would order A and B to jointly invest

$$x = \frac{\bar{r}\tau_r + \tau_A s_A + \tau_B s_B}{2\lambda(\tau_A + \tau_B + \tau_r) + \frac{\rho_A\rho_B}{\rho_A+\rho_B}}$$

in the risk project. Second, when the project return realizes, the social planner would give investor A (in addition to her initial investment)

$$\frac{\log\left(\frac{\rho_A\gamma_A}{\rho_B\gamma_B}\right)+\rho_B(r-\lambda x)x}{\rho_A+\rho_B},$$

and give investor B (in addition to his initial investment)

$$\frac{-\log\left(\frac{\rho_A\gamma_A}{\rho_B\gamma_B}\right)+\rho_A(r-\lambda x)x}{\rho_A+\rho_B}.$$

Here γ_A and γ_B are Pareto weights assigned to A and B by the social planner.

When $\gamma_i=\frac{1}{\rho_i}$ where $i\in\{A,B\}$, we have the first-best outcome to be investor A getting a share of $\frac{\frac{1}{\rho_A}}{\frac{1}{\rho_A}+\frac{1}{\rho_B}}$ of the net investment profit, while investor B gets a share of $\frac{\frac{1}{\rho_B}}{\frac{1}{\rho_A}+\frac{1}{\rho_B}}$. In other words, the social planner's first-best choice is also to divide (net) investment profit between the two investors in proportion to each investor's risk-tolerance. When Pareto weights (γ_i-s) change, cash transfers of amount $\pm\frac{\log\left(\frac{\rho_A\gamma_A}{\rho_B\gamma_B}\right)}{\rho_A+\rho_B}$ are necessary to sustain the first-best allocation, but the sharing rule does not change at all.

Furthermore, when we compare the sum of each investor's optimal investment amount under profit sharing (Lemma 3.3) with the first-best joint investment amount chosen by the social planner (Theorem 3.5), it is easily verified that they are exactly the same. In other words, even in the presence of decreasing return to scale, a profit-sharing contract perfectly replicates the first-best outcome.

Theorem 3.5 is reminiscent of a generalized Second Welfare Theorem. Despite the presence of strategic interdependence and asymmetric information, any Pareto optimal outcome could be sustained with an equilibrium under profit sharing with lump sum transfers.[20] Theorem 3.5 is significant also because it shows that when investors have strict incentives

[20] I conjecture but have not yet been able to elucidate that this result might serve as a path toward a formal statement and proof of the Coase theorem that with no transaction costs, an efficient set of inputs/outputs to and from a production-optimal distribution will be selected regardless of how property rights are divided.

to lie to each other (at the presence of a decreasing-return-to-scale project), so that the first-best outcome is hard to obtain by conventional approaches, profit sharing provides a simple alternative.

4 Literature

There is a growing economic literature on the wisdom of the crowd. Kremer, Mansour and Perry (2014) studies one form of implementation of the wisdom of the crowd by characterizing the optimal disclosure policy of a planner who maximizes social welfare in a setting where agents arrive sequentially and choose one from a set of actions with unknown payoffs. Da and Huang (2015) run experiments on Estimize.com, a crowd-based earnings forecast platform, in which they restrict the information set available to randomly selected users. Their experiments confirm that independent forecasts lead to more accurate consensus and suggest that the wisdom of the crowd can be better harnessed by encouraging independent voices from the participants, preventing information cascade (Bikhchandani, Hirshleifer and Welch (1992) and Welch (2000), etc.).[21] In the setting of investment crowdfunding platforms, Brown and Davies (2015) show that naive investors, possessing weak but on average correct signals, are required for efficient financing in the presence of the all-or-nothing clause. They develop a model showing that sophisticated investors, who are better informed and anticipate other investors' actions, cannot by themselves use their information to improve financing efficiency.

There is also an emerging literature studying various crowdfunding mechanisms, mostly in reward-based crowdfunding. For example, Cumming, Leboeuf and Schwienbacher (2015) compare keep-it-all versus all-or-nothing financing, and show that keep-it-all mechanisms are better for small, scalable projects. On the other hand, Chang (2015) shows that all-or-

[21] Information cascade could also potentially plague investment crowdfunding, as described by Hornuf and Schwienbacher (2016) in a sample of equity crowdfunding platforms from Germany.

nothing funding generates more revenue than keep-it-all funding by helping the entrepreneurs learn market value as all-or-nothing funding complements borrowing. Kumar, Langberg and Zvilichovsky (2015) derive the optimal crowdfunding contract of a financially constrained monopolist and analyze its implications for production, investment and welfare. They emphasize that crowdfunding contracts may serve as a price-discrimination mechanism. Strausz (2016) and Ellman and Hurkens (2015) study the optimal reward-based crowdfunding design with a focus on a trade-off between improved screening/adaption and worsening entrepreneur moral hazard/rent extraction, respectively. Belleflamme, Lambert and Schwienbacher (2014) emphasize the role of private benefits in determining an entrepreneur's choice between crowdfunding via pre-orders and selling equity claims. Hakenes and Schlegel (2014) analyze a model with endogenous information production and debt-based crowdfunding, and highlight the winner's curse and the natural hedge from not financing bad projects. Grüner and Siemroth (2015) consider crowdfunding as a mechanism in which consumers signal future product market demand via investment.

5 Conclusion

In our information age, many business activities could benefit from taking advantage of a wisdom of the crowd effect. By highlighting the role of profit sharing, this paper provides a contracting solution to harness the wisdom of the crowd. As a concrete example, we also examine how the general theoretical results could be applied to guide security design for the emerging JOBS Act Title III crowdfunding market. For small, risky, and scalable investment projects, a profit-sharing contract different from often-observed common stocks, yet similar to some partnership structures, could help investors achieve better (risk-adjusted) investment performance, as well as ease financing for entrepreneurs with positive-return projects.[22]

[22]The requirement of a project being small, as in the application of crowdfunding, helps bypass concerns over bankruptcy as well as wealth effects in investor preference. For applications where project size is large,

The optimal profit-sharing contract described in the paper is easy to implement on crowdfunding platforms, as the sharing rule does not depend on investors' private information about the project, nor does it require potentially costly direct communication among a large group of investors. As a word of caution, however, the effectiveness of profit sharing in harnessing the wisdom of the crowd, as mathematically proved by a Nash equilibrium, does depend on investors being rational. This rationality requirement emphasizes the crucial importance of investor education to accompany the launch of any profit-sharing contracts.

As investors gradually get more familiar with the new practice of investment crowdfunding, right now seems to be the best time to begin educating investors regarding why profit-sharing could benefit them, and what their optimal investment strategies should be when they share profits with each other. It would be much easier to carry out investor education when investors are still learning about a new market than to change their entrenched understanding once the market matures. Although outside of the scope of the current paper, we provide a brief perspective on the timely topic of an effective plan of investor education.

1. An alternative term to "profit-sharing" could be coined to more sharply distinguish from common stock or other existing contract forms.

2. Investor education could start by first educating entrepreneurs/issuers because

   (a) entrepreneurs are often more open-minded and eager to learn;

   (b) entrepreneurs with good projects are direct beneficiaries of a profit-sharing contract, hence they would have strong incentives to educate their investors, too;

   (c) once many entrepreneurs contribute to investor education, some of them may be able to come up with better ways for investor education, and "the wisdom of the crowd" (of entrepreneurs) will be harnessed.

future work could consider including several model refinements such as margin requirements and mappings from changes in risk aversion to changes in project return.

Outside of the application of Title III crowdfunding, many other implications from the general theory are left for future research. For example, does profit sharing speak to the organizational structures within large private equity/venture capital partnerships? What are the implications for optimal compensation within team-managed mutual funds? What is the connection between profit sharing and strategic alliances in R&D activities? How does profit sharing relate to classic theory of syndicates (Wilson (1968)) and financial intermediaries (e.g. Diamond (1984), Ramakrishnan and Thakor (1984))? Does profit sharing have implications for the design of alternative governance models such as the Decentralized Autonomous Organization (DAO) on smart contract?[23] How does it compare to traditional models based on voting (see e.g. Feddersen and Pesendorfer (1997))? What is the relationship between a profit-sharing structure and that of a Bitcoin mining pool (Baldimtsi et al. (2017))? Further developments are down the road.

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[23]See *The Economist* article (http://www.economist.com/news/finance-and-economics/21699159-new-automated-investment-fund-has-attracted-stacks-digital-money-dao) for an introduction.

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Appendix

A Proof of Theorem 3.1

Proof. Denote $u(W) = -e^{-\rho W}$, and consider general distributions of r and $s_i, i \in \{A, B\}$. Under a full-information benchmark $x_i(s_i, s_{-i})$ maximizes

$$
\begin{aligned}
& \mathbb{E}\left[u(x\tilde{r})|s_i, s_{-i}\right] \\
= & \int u(xr) f(r|s_i, s_{-i}) dr \\
= & \int u(xr) f(s_{-i}|r, s_i) f(s_i|r) f(r) \frac{1}{f(s_i, s_{-i})} dr \\
= & \int u(xr) f(s_{-i}|r) f(s_i|r) f(r) \frac{1}{f(s_i, s_{-i})} dr, (\because s_i \perp\!\!\!\perp s_{-i}|r),
\end{aligned}
$$

or equivalently $x(s_i, s_{-i})$ maximizes

$$\int u(xr) f(s_{-i}|r) f(s_i|r) f(r) dr \tag{18}$$

Assume the profit-sharing agreement stipulates that investor i gets α_i of the total profit ($\sum_i \alpha_i = 1$), then under profit sharing $x_i(s_i)$ maximizes (in a Nash equilibrium)

$$
\begin{aligned}
& \mathbb{E}\left[u\left(\alpha_i x\tilde{r} + \alpha_i x_{-i}(s_{-i})\tilde{r}\right)|s_i\right] \\
= & \iint u\left(\alpha_i xr + \alpha_i x_{-i}(s_{-i})r\right) f(r, s_{-i}|s_i) ds_{-i} dr \\
= & \iint u\left(\alpha_i xr + \alpha_i x_{-i}(s_{-i})r\right) f(s_{-i}|r) f(s_i|r) f(r) \frac{1}{f(s_i)} ds_{-i} dr,
\end{aligned}
$$

or equivalently $x(s_i)$ maximizes

$$\iint u\left(\alpha_i xr + \alpha_i x_{-i}(s_{-i})r\right) f(s_{-i}|r) f(s_i|r) f(r) ds_{-i} dr \tag{19}$$

Taking first-order conditions we have that

$$(18) \Rightarrow \int u'(x_i(s_i, s_{-i})r) r f(s_{-i}|r) f(s_i|r) f(r) dr = 0 \tag{20}$$

$$(19) \Rightarrow \iint u'\left(\alpha_i x_i(s_i)r + \alpha_i x_{-i}(s_{-i})r\right) r f(s_{-i}|r) f(s_i|r) f(r) ds_{-i} dr = 0, \tag{21}$$

where (with some abuse of notation) $x(s_i, s_{-i})$ denotes the optimal investment amount given signal s_i and s_{-i} under the full-information benchmark, while $x_i(s_i)$ denotes investor i's investment amount given signal s_i in the Nash equilibrium under profit sharing.

As the likelihood function of r given private signals $s_i, i \in \{A, B\}$ lies in an *exponential family*, there exists constant k_i as well as positive functions $h(\cdot)$ and $g(\cdot)$ such that

$$f(s_i|r) = h_i(s_i) e^{rk_is_i} g_i(r).$$

Hence

$$(20) \Rightarrow \quad -\int e^{-\rho x_i(s_i,s_{-i})r} r h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} g_{-i}(r) h_i(s_i) e^{rk_i s_i} g_i(r) f(r) dr = 0 \tag{22}$$

$$(21) \Rightarrow \quad -\iint e^{-\rho(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r)} r h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} g_{-i}(r) h_i(s_i) e^{rk_i s_i} g_i(r) f(r) ds_{-i} dr = 0 \tag{23}$$

or (factoring out $-h_i(s_i)$)

$$(22) \quad \Rightarrow \quad \int e^{-\rho x_i(s_i,s_{-i})r + rk_{-i}s_{-i} + rk_i s_i} r g_i(r) g_{-i}(r) f(r) dr = 0 \tag{24}$$

$$(23) \quad \Rightarrow \quad \iint e^{-\rho(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r)} r h_{-i}(s_{-i}) e^{rk_{-i}s_{-i} + rk_i s_i} g_i(r) g_{-i}(r) f(r) ds_{-i} dr = 0$$

$$\Rightarrow \quad \int e^{-\rho\alpha_i x_i(s_i) r + rk_i s_i} r g_i(r) g_{-i}(r) f(r) \left(\int e^{-\rho\alpha_i x_{-i}(s_{-i}) r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} \right) dr = 0 \tag{25}$$

Notice that under the full-information benchmark, equation (24) has a unique solution, which is linear in s_i and s_{-i}. To see this consider the equation of x

$$\int e^{xr} r g_i(r) g_{-i}(r) f(r) dr = 0. \tag{26}$$

Taking derivative with respect to x immediately tells that equation (26) has at most one solution, denoted as X. Compared to equation (24) we get $x_i(s_i, s_{-i}) = \frac{1}{\rho}(k_{-i}s_{-i} + k_i s_i - X)$.

Similarly, under any profit-sharing agreement (i.e. $\forall \alpha_i$), equation (25) has a unique Nash equilibrium, in which investor i's strategy is linear in s_i, $\forall i$. To see this, consider the equation of x

$$\int e^{rx} r g_i(r) g_{-i}(r) f(r) H_{-i}(r) dr = 0,$$

where $H_{-i}(r) = \int e^{-\rho\alpha_i x_{-i}(s_{-i}) r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} > 0$. Taking derivative with respect to x immediately tells that the equation features at most one solution (for a given $x_{-i}(s_{-i})$). Compared to equation (25) we get that $x_i(s_i) = \frac{k_i s_i - C}{\rho\alpha_i}$, where C is a constant such that

$$\int e^{rC} r g_i(r) g_{-i}(r) f(r) H_{-i}(r) dr = 0. \tag{27}$$

By the same logic, $x_{-i}(s_{-i}) = \frac{k_{-i}s_{-i} - C'}{\rho\alpha_{-i}}$, where C' is also a constant such that

$$\int e^{rC'} r g_i(r) g_{-i}(r) f(r) H_i(r) dr = 0, \tag{28}$$

where $H_i(r) = \int e^{-\rho\alpha_{-i} x_i(s_i) r} h_i(s_i) e^{rk_i s_i} ds_i > 0$. Plug in x_i and x_{-i} into (27) , we have

$$\int e^{rC} r g_i(r) g_{-i}(r) f(r) \int e^{\frac{\alpha_i}{\alpha_{-i}}(C' - k_{-i}s_{-i}) r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} dr = 0. \tag{29}$$

If $\alpha_i = \alpha_{-i} = \frac{1}{2}$, equation (29) simplifies into (after factoring out $\int h_{-i}(s_{-i})ds_{-i}$)

$$\int e^{r(C'+C)} r g_i(r) g_{-i}(r) f(r) dr = 0. \tag{30}$$

Since equation (26) has at most one solution, we have $C + C' = X$. Thus under profit-sharing the payoff to investor i for a given realization of r and private signals is

$$\begin{aligned}
& \alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r \\
= \; & \alpha_i \frac{k_i s_i - C}{\rho \alpha_i} r + \alpha_i \frac{k_{-i} s_{-i} - C'}{\rho \alpha_{-i}} r \\
= \; & \frac{r}{\rho}(k_i s_i - C + k_{-i} s_{-i} - C'), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2} \\
= \; & \frac{r}{\rho}(k_i s_i + k_{-i} s_{-i} - X), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2},
\end{aligned}$$

which exactly equals to $rx(s_i, s_{-i})$, or the payoff under a full-information benchmark. When the profit-sharing agreement is optimally designed, profit-sharing obtains the same payoff as under the full-information benchmark. □

B Proof of Theorem 3.2

Proof. From (13) it is immediate that investor i's expected utility conditional on her private signal under a properly designed profit-sharing contract is given by

$$\begin{aligned}
& -\frac{\sqrt{\tau_r + \tau_i} \exp\{-\frac{(\tau_r+\tau_i)[\mathbb{E}(v|s_i)]^2}{2}\}}{\sqrt{\tau_r + \tau_i + 2\frac{1}{\sum_{i=1}^n \frac{1}{\rho_i}} \sum_{k \neq i} \frac{\tau_k}{\frac{1}{\sum_{i=1}^n \frac{1}{\rho_i}}} - \left(\frac{1}{\sum_{i=1}^n \frac{1}{\rho_i}}\right)^2 \sum_{k \neq i} \left(\frac{\tau_k}{\frac{1}{\sum_{i=1}^n \frac{1}{\rho_i}}}\right)^2 \tau_k^{-1}}} \\
= \; & -\frac{\exp\left(-\frac{(\tau_i s_i + \tau_v \bar{v})^2}{2(\tau_i + \tau_v)}\right)}{\sqrt{\frac{\sum_{k=1}^n \tau_k + \tau_v}{\tau_i + \tau_v}}}.
\end{aligned} \tag{31}$$

Hence if one unit of precision costs c, investor i's *ex ante* expected utility would be

$$\mathbb{E}\left(-\frac{\exp\left(-\frac{(\tau_i s_i + \tau_v \bar{v})^2}{2(\tau_i + \tau_v)}\right)}{\sqrt{\frac{\sum_{k=1}^n \tau_k + \tau_v}{\tau_i + \tau_v}}}\right) - c\tau_i = -\frac{\sqrt{\tau_v} \exp\left\{\frac{-\tau_v \bar{v}^2}{2}\right\}}{\sqrt{\sum_{k=1}^n \tau_k + \tau_v}} - c\tau_i, \tag{32}$$

for an optimally chosen τ_i. To pin down the optimal τ_i, consider a symmetric equilibrium in which the FOC is given by taking derivative of the right hand side of (32) with respect to τ_i:

$$\frac{1}{2}\sqrt{\tau_v} \exp\left(\frac{-\tau_v \bar{v}^2}{2}\right) (n\tau_i + \tau_v)^{-\frac{3}{2}} = c. \tag{33}$$

From (33), we see that the optimal τ_i is decreasing in n, the number of participants in profit sharing. This observation is in line with the Holmström (1982) free-rider effect. However, such free-rider effect is not enough to offset the benefit from wisdom of the crowd. To see this, plug (33) into the right-hand side of (32), we get the expected utility for investor i can be expressed as

$$-(2c)^{\frac{1}{3}}\left[\sqrt{\tau_v}\exp\left(\frac{-\tau_v\bar{v}^2}{2}\right)\right]^{\frac{2}{3}} - c\tau_i, \tag{34}$$

which decreases with τ_i, and hence increases with n. □

C Proof of Lemma 3.3

Proof. Under a profit-sharing contract, investor i chooses x_i to maximize

$$\mathbb{E}[-e^{-a_i\rho_i[r-\lambda(x_i+x_{-i})](x_i+x_{-i})}|s_i].$$

Still guess and verify a symmetric equilibrium $x_i(s_i) = \alpha + \beta_i s_i$, where $i \in \{A, B\}$ gives

$$x_i \quad = \quad \text{argmax}_x \mathbb{E}[-e^{-a_i\rho_i[r-\lambda(x+\alpha+\beta_{-i}s_{-i})](x+\alpha+\beta_{-i}s_{-i})}|s_i]. \tag{35}$$

Plug in $-a_i\rho_i[r - \lambda(x + \alpha + \beta_{-i}s_{-i})]$ and $x + \alpha + \beta_{-i}s_{-i}$ into Lemma 1.1, and since

$$\begin{bmatrix} -\rho_i a_i[r-\lambda(x+\alpha+\beta_{-i}s_{-i})] \\ x+\alpha+\beta_{-i}s_{-i} \end{bmatrix}_{|s_i} \sim \mathcal{N}\left(\begin{bmatrix} -\rho_i a_i\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} + \rho_i a_i\lambda(x+\alpha) + \rho_i a_i\lambda\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \\ x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \end{bmatrix}, \begin{bmatrix} \frac{\rho_i^2 a_i^2(1-\lambda\beta_{-i})^2}{\tau_r+\tau_i} + \frac{\rho_i^2 a_i^2\lambda^2\beta_{-i}^2}{\tau_{-i}} & -\frac{\rho_i a_i\beta_{-i}(1-\lambda\beta_{-i})}{\tau_r+\tau_i} + \frac{\rho_i a_i\lambda\beta_{-i}^2}{\tau_{-i}} \\ -\frac{\rho_i a_i\beta_{-i}(1-\lambda\beta_{-i})}{\tau_r+\tau_i} + \frac{\rho_i a_i\lambda\beta_{-i}^2}{\tau_{-i}} & \beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right) \end{bmatrix} \right),$$

the expectation on the right hand side of (35) is equal to

$$-\frac{\exp\left\{\frac{\left[\begin{array}{c} a_i^2\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\left(\frac{\rho_i^2(1-\lambda\beta_{-i})^2}{\tau_r+\tau_i}+\frac{\rho_i^2\lambda^2\beta_{-i}^2}{\tau_{-i}}\right) \\ +2a_i^2\left(-\rho_i\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}+\rho_i\lambda(x+\alpha)+\rho_i\lambda\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\left(\frac{\rho_i\beta_{-i}(1-\lambda\beta_{-i})}{\tau_r+\tau_i}-\frac{\rho_i\lambda\beta_{-i}^2}{\tau_{-i}}\right) \\ +a_i^2\left(-\rho_i\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}+\rho_i\lambda(x+\alpha)+\rho_i\lambda\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)^2\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right) \\ +2a_i\left(-\rho_i\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}+\rho_i\lambda(x+\alpha)+\rho_i\lambda\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right)\left(x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i}\right) \end{array}\right]}{2\left[\left(-\frac{\rho_i a_i\beta_{-i}(1-\lambda\beta_{-i})}{\tau_r+\tau_i}+\frac{\rho_i a_i\lambda\beta_{-i}^2}{\tau_{-i}}-1\right)^2-a_i^2\left(\frac{\rho_i^2(1-\lambda\beta_{-i})^2}{\tau_r+\tau_i}+\frac{\rho_i^2\lambda^2\beta_{-i}^2}{\tau_{-i}}\right)\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)\right]}\right\}}{\sqrt{\left(-\frac{\rho_i a_i\beta_{-i}(1-\lambda\beta_{-i})}{\tau_r+\tau_i}+\frac{\rho_i a_i\lambda\beta_{-i}^2}{\tau_{-i}}-1\right)^2-a_i^2\left(\frac{\rho_i^2(1-\lambda\beta_{-i})^2}{\tau_r+\tau_i}+\frac{\rho_i^2\lambda^2\beta_{-i}^2}{\tau_{-i}}\right)\beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right)}}. \tag{36}$$

Notice that x, the variable we maximize over, only enters the numerator of the exponent in the above expression in a linear-quadratic function, thus

$$
\begin{aligned}
\alpha + \beta_i s_i &= \operatorname{argmin}_x \Bigg[a_i^2 \left(x + \alpha + \beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right)^2 \left(\frac{\rho^2 (1 - \lambda \beta_{-i})^2}{\tau_r + \tau_i} + \frac{\rho^2 \lambda^2 \beta_{-i}^2}{\tau_{-i}} \right) \\
&\quad + 2a_i^2 \left(-\rho \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} + \rho\lambda(x + \alpha) + \rho\lambda\beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right) \left(x + \alpha + \beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right) \left(\frac{\rho \beta_{-i} (1 - \lambda \beta_{-i})}{\tau_r + \tau_i} - \frac{\rho \lambda \beta_{-i}^2}{\tau_{-i}} \right) \\
&\quad + a_i^2 \left(-\rho \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} + \rho\lambda(x + \alpha) + \rho\lambda\beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right)^2 \beta_{-i}^2 \left(\frac{1}{\tau_r + \tau_i} + \frac{1}{\tau_{-i}} \right) \\
&\quad + 2a_i \left(-\rho \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} + \rho\lambda(x + \alpha) + \rho\lambda\beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right) \left(x + \alpha + \beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right) \Bigg] \\
&= \frac{(\tau_r \bar{r} + \tau_i s_i)(1 - 2\beta_{-i}\lambda)}{a_i \rho + 2\lambda(\tau_r + \tau_i)} - \alpha
\end{aligned}
$$

Matching coefficients gives

$$
\begin{aligned}
\beta_i &= \frac{\tau_i (1 - 2\beta_{-i}\lambda)}{a_i \rho + 2\lambda(\tau_r + \tau_i)} && (37) \\
2\alpha &= \frac{\tau_r \bar{r} (1 - 2\beta_{-i}\lambda)}{a_i \rho + 2\lambda(\tau_r + \tau_i)} && (38)
\end{aligned}
$$

Interchanging i and $-i$ into (37) it is easy to verify that

$$
\begin{aligned}
\beta_{-i} &= \frac{\tau_{-i} (a_i \rho_i + 2\lambda \tau_r)}{a_i \rho_i a_{-i} \rho_{-i} + 2\lambda(\tau_r + \tau_{-i}) a_i \rho_i + 2\lambda(\tau_r + \tau_i) a_{-i} \rho_{-i} + 4\lambda^2 \tau_r (\tau_r + \tau_i + \tau_{-i})} && (39) \\
\beta_i &= \frac{\tau_i (a_{-i} \rho_{-i} + 2\lambda \tau_r)}{a_i \rho_i a_{-i} \rho_{-i} + 2\lambda(\tau_r + \tau_{-i}) a_i \rho_i + 2\lambda(\tau_r + \tau_i) a_{-i} \rho_{-i} + 4\lambda^2 \tau_r (\tau_r + \tau_i + \tau_{-i})} && (40) \\
2\alpha &= \frac{\tau_r \bar{r} (a_{-i} \rho_{-i} + 2\lambda \tau_r)}{a_i \rho_i a_{-i} \rho_{-i} + 2\lambda(\tau_r + \tau_{-i}) a_i \rho_i + 2\lambda(\tau_r + \tau_i) a_{-i} \rho_{-i} + 4\lambda^2 \tau_r (\tau_r + \tau_i + \tau_{-i})} && (41) \\
&= \frac{\tau_r \bar{r} (a_i \rho_i + 2\lambda \tau_r)}{a_i \rho_i a_{-i} \rho_{-i} + 2\lambda(\tau_r + \tau_{-i}) a_i \rho_i + 2\lambda(\tau_r + \tau_i) a_{-i} \rho_{-i} + 4\lambda^2 \tau_r (\tau_r + \tau_i + \tau_{-i})} && (42)
\end{aligned}
$$

With (41) and (42) we have $a_i \rho_i = a_{-i} \rho_{-i} \Rightarrow a_i = \frac{\frac{1}{\rho_i}}{\frac{1}{\rho_i} + \frac{1}{\rho_{-i}}}$. In another word, with presence of scale economy, as long as the sharing rule is proportional to investors' risk tolerances, a Nash equilibrium exists, under which strategy functions are given as

$$
\begin{aligned}
\beta_i &= \frac{\tau_i}{\frac{\rho_i \rho_{-i}}{\rho_i + \rho_{-i}} + 2\lambda(\tau_r + \tau_i + \tau_{-i})} && (43) \\
\beta_{-i} &= \frac{\tau_{-i}}{\frac{\rho_i \rho_{-i}}{\rho_i + \rho_{-i}} + 2\lambda(\tau_r + \tau_i + \tau_{-i})} && (44) \\
\alpha &= \frac{1}{2} \frac{\tau_r \bar{r}}{\frac{\rho_i \rho_{-i}}{\rho_i + \rho_{-i}} + 2\lambda(\tau_r + \tau_i + \tau_{-i})} && (45)
\end{aligned}
$$

Plug in (36) we get i's expected utility to be

$$-\frac{\exp\left(-\frac{(\bar{r}\tau_r+s_i\tau_i)^2}{2(\tau_i+\tau_r)}\frac{\rho_i\rho_{-i}}{2\lambda(\rho_i+\rho_{-i})(\tau_i+\tau_r)+\rho_i\rho_{-i}}\right)}{\sqrt{\frac{\tau_{-i}+\tau_i+\tau_r}{\tau_i+\tau_r}\frac{2\lambda(\rho_i+\rho_{-i})(\tau_i+\tau_r)+\rho_i\rho_{-i}}{2\lambda(\rho_i+\rho_{-i})(\tau_{-i}+\tau_i+\tau_r)+\rho_i\rho_{-i}}}}$$

□

D Proof of Lemma 3.4

Proof. Under a full-information benchmark, investor i chooses

$$\begin{aligned} & -\mathbb{E}[e^{-\rho_i(r-\lambda x_{-i})x_i}|s_A,s_B]\cdot e^{\rho_i\lambda x_i^2} \\ = & -e^{-\rho_i\mathbb{E}[(r-\lambda x_{-i})|s_A,s_B]x_i}e^{\frac{1}{2}\mathrm{Var}[(r-\lambda x_{-i})|s_A,s_B]\rho_i^2x_i^2}\cdot e^{\rho_i\lambda x_i^2}, \end{aligned}$$

hence we have

$$x_i = \mathrm{argmax}_x - e^{-\rho_i\mathbb{E}[(r-\lambda x_{-i})|s_A,s_B]x}e^{\frac{1}{2}\mathrm{Var}[(r-\lambda x_{-i})|s_A,s_B]\rho_i^2x^2}\cdot e^{\rho_i\lambda x^2} \quad (46)$$

Still by the guess and verify method and assume that $x_i(s_i)=\alpha_i+\beta_{iA}s_A+\beta_{iB}s_B$, where $i\in\{A,B\}$

$$x_i = \mathrm{argmax}_x - e^{-\rho_i\mathbb{E}[(r-\lambda\alpha_{-i}-\lambda\beta_{-iA}s_A-\lambda\beta_{-iB}s_B)|s_A,s_B]x}e^{\frac{1}{2}\mathrm{Var}[r|s_i]\rho_i^2x^2}\cdot e^{\rho_i\lambda x^2} \quad (47)$$

thus by FOC

$$\begin{aligned} x_i &= \mathrm{argmax}_x\rho_i\mathbb{E}[(r-\lambda\alpha_{-i}-\lambda\beta_{-iA}s_A-\lambda\beta_{-iB}s_B)|s_A,s_B]x-\frac{1}{2}\mathrm{Var}[r|s_A,s_B]\rho_i^2x^2-\rho_i\lambda x^2 \\ &= \frac{\mathbb{E}[r|s_A,s_B]-\lambda\alpha_{-i}-\lambda\beta_{-iA}s_A-\lambda\beta_{-iB}s_B}{\mathrm{Var}[r|s_A,s_B]\rho_i+2\lambda} \quad (48) \\ &= \frac{\frac{\tau_r\bar{r}+\tau_As_A+\tau_Bs_B}{\tau_r+\tau_A+\tau_B}-\lambda\alpha_{-i}-\lambda\beta_{-iA}s_A-\lambda\beta_{-iB}s_B}{\frac{\rho_i}{\tau_r+\tau_A+\tau_B}+2\lambda} \quad (49) \end{aligned}$$

Plug in the expressions for x_i and match coefficients we have

$$\alpha_i = \frac{\frac{\tau_r\bar{r}}{\tau_r+\tau_A+\tau_B}-\lambda\alpha_{-i}}{\frac{\rho_i}{\tau_r+\tau_A+\tau_B}+2\lambda} \quad (50)$$

$$\beta_{iA} = \frac{\frac{\tau_A}{\tau_r+\tau_A+\tau_B}-\lambda\beta_{-iA}}{\frac{\rho_i}{\tau_r+\tau_A+\tau_B}+2\lambda} \quad (51)$$

$$\beta_{iB} = \frac{\frac{\tau_B}{\tau_r+\tau_A+\tau_B}-\lambda\beta_{-iB}}{\frac{\rho_i}{\tau_r+\tau_A+\tau_B}+2\lambda} \quad (52)$$

Along with similar equations derived from investor $-i$'s problem gives

$$\begin{aligned}
\alpha_i &= \frac{\bar{r}\tau_r\left(\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_{-i}\right)}{\rho_{-i}\left(2\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_i\right)+\lambda\left(\tau_A+\tau_B+\tau_r\right)\left(3\lambda\left(\tau_A+\tau_B+\tau_r\right)+2\rho_i\right)} \\
\beta_{iA} &= \frac{\tau_A\left(\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_{-i}\right)}{\rho_{-i}\left(2\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_i\right)+\lambda\left(\tau_A+\tau_B+\tau_r\right)\left(3\lambda\left(\tau_A+\tau_B+\tau_r\right)+2\rho_i\right)} \\
\beta_{iB} &= \frac{\tau_B\left(\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_{-i}\right)}{\rho_{-i}\left(2\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_i\right)+\lambda\left(\tau_A+\tau_B+\tau_r\right)\left(3\lambda\left(\tau_A+\tau_B+\tau_r\right)+2\rho_i\right)}
\end{aligned}$$

Hence expected utility is

$$-\exp\left(-\frac{\rho_i\left(\bar{r}\tau_r+s_A\tau_A+s_B\tau_B\right)^2\left(\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_{-i}\right)^2\left(2\lambda\left(\tau_A+\tau_B+\tau_r\right)+\rho_i\right)}{2\left(\tau_A+\tau_B+\tau_r\right)\left(3\lambda^2(\tau_A+\tau_B+\tau_r)^2+2\lambda\rho_{-i}(\tau_A+\tau_B+\tau_r)+2\lambda\rho_i(\tau_A+\tau_B+\tau_r)+\rho_i\rho_{-i}\right)^2}\right)$$

Taking expectation conditional on s_A only, we get that investor A's *ex ante* utility is

$$-\frac{\exp\left(\begin{array}{c}-\frac{(\bar{r}\tau_r+s_A\tau_A)^2}{2(\tau_A+\tau_r)} \\ \frac{\rho_i(\tau_A+\tau_B+\tau_r)(\lambda(\tau_A+\tau_B+\tau_r)+\rho_{-i})^2(2\lambda(\tau_A+\tau_B+\tau_r)+\rho_i)}{\tau_B\rho_i(2\lambda(\tau_A+\tau_B+\tau_r)+\rho_i)(\lambda(\tau_A+\tau_B+\tau_r)+\rho_{-i})^2+(\tau_A+\tau_r)(\rho_{-i}(2\lambda(\tau_A+\tau_B+\tau_r)+\rho_i)+\lambda(\tau_A+\tau_B+\tau_r)(3\lambda(\tau_A+\tau_B+\tau_r)+2\rho_i))^2}\end{array}\right)}{\sqrt{\frac{1}{\tau_A+\tau_r}}\sqrt{\tau_A+\tau_r+\frac{\tau_B\rho_i(2\lambda(\tau_A+\tau_B+\tau_r)+\rho_i)(\lambda(\tau_A+\tau_B+\tau_r)+\rho_{-i})^2}{(\rho_{-i}(2\lambda(\tau_A+\tau_B+\tau_r)+\rho_i)+\lambda(\tau_A+\tau_B+\tau_r)(3\lambda(\tau_A+\tau_B+\tau_r)+2\rho_i))^2}}}$$

□

E Proof of Theorem 3.5

Proof. Assume that the omniscient and benevolent social planner orders A and B to jointly invest x. The planner also rules that A receives $q(r)$ in addition to her initial invest when project return realizes, and B receives (in addition to his original investment) the remaining $(r-\lambda x)x-q(r)$ in net profit. The social planner informs both A and B of the content and quality of each other's private information. Then the social planner chooses x and $q(\cdot)$ to maximize the social welfare function

$$\gamma_A\mathbb{E}[-e^{-\rho_A q(r)}|s_A,s_B]+\gamma_B\mathbb{E}[-e^{-\rho_B(r-\lambda x)x+\rho_B q(r)}|s_A,s_B], \tag{53}$$

where γ_A and γ_B are Pareto weights.

Expression (53) could be equivalently expressed as to choose x and $q(\cdot)$ to maximize

$$-\int\left(\gamma_A e^{-\rho_A q(r)}+\gamma_B e^{-\rho_B(r-\lambda x)x+\rho_B q(r)}\right)e^{r\tau_A s_A+r\tau_B s_B-\frac{(\tau_A+\tau_B)r^2}{2}-\frac{\tau_r(r-\bar{r})^2}{2}}dr \tag{54}$$

Hence for given x and r, $q(r)$ maximizes

$$\begin{aligned}
&-\left(\gamma_A e^{-\rho_A q(r)}+\gamma_B e^{-\rho_B(r-\lambda x)x+\rho_B q(r)}\right)e^{r\tau_A s_A+r\tau_B s_B-\frac{(\tau_A+\tau_B)r^2}{2}-\frac{\tau_r(r-\bar{r})^2}{2}} \\
\Rightarrow\quad & q(r)=\frac{\log\left(\frac{\rho_A\gamma_A}{\rho_B\gamma_B}\right)+\rho_B(r-\lambda x)x}{\rho_A+\rho_B} \qquad (55)
\end{aligned}$$

Plug (55) in (54) we have that x maximizes

$$
\begin{aligned}
& -\int \left(\gamma_A e^{-\rho_A \frac{\log\left(\frac{\rho_A \gamma_A}{\rho_B \gamma_B}\right) + \rho_B (r-\lambda x) x}{\rho_A + \rho_B}} + \gamma_B e^{-\rho_B (r-\lambda x) x + \rho_B \frac{\log\left(\frac{\rho_A \gamma_A}{\rho_B \gamma_B}\right) + \rho_B (r-\lambda x) x}{\rho_A + \rho_B}} \right) \cdot \\
& e^{r\tau_A s_A + r\tau_B s_B - \frac{(\tau_A+\tau_B) r^2}{2} - \frac{\tau_r (r-\bar{r})^2}{2}} dr \\
= \; & -\left(\gamma_A \left(\frac{\rho_A \gamma_A}{\rho_B \gamma_B} \right)^{-\frac{\rho_A}{\rho_A+\rho_B}} + \gamma_B \left(\frac{\rho_A \gamma_A}{\rho_B \gamma_B} \right)^{\frac{\rho_B}{\rho_A+\rho_B}} \right) \int e^{-\frac{\rho_A \rho_B}{\rho_A+\rho_B}(r-\lambda x)x + r\tau_A s_A + r\tau_B s_B - \frac{(\tau_A+\tau_B) r^2}{2} - \frac{\tau_r (r-\bar{r})^2}{2}} dr,
\end{aligned}
$$

or equivalently x maximizes

$$
-\int e^{-\frac{\rho_A \rho_B}{\rho_A+\rho_B}(r-\lambda x)x + r\tau_A s_A + r\tau_B s_B - \frac{(\tau_A+\tau_B) r^2}{2} - \frac{\tau_r (r-\bar{r})^2}{2}} dr.
$$

The first-order condition gives

$$
\frac{\rho_A \rho_B}{\rho_A + \rho_B} \int (r - 2\lambda x) e^{-\frac{\rho_A \rho_B}{\rho_A+\rho_B}(r-\lambda x)x + r\tau_A s_A + r\tau_B s_B} g^2(r) f(r) dr = 0,
$$

and with further simplification and solving the equation we get

$$
x = \frac{\bar{r}\tau_r + \tau_A s_A + \tau_B s_B}{2\lambda(\tau_A + \tau_B + \tau_r) + \frac{\rho_A \rho_B}{\rho_A + \rho_B}}.
$$

□