August, 2020

Miner Collusion and the Bitcoin Protocol

Alfred Lehar
University of Calgary

Christine A. Parlour
UC Berkeley

Miner Collusion and the Bitcoin Protocol

Alfred Lehar[*]
Haskayne School of Business
University of Calgary

Christine A. Parlour[†]
Haas School of Business
UC Berkeley

August 21, 2020
Comments Welcome

Abstract

Bitcoin users can offer fees to miners who record their transactions in the Blockchain. We document high variation of Bitcoin fees, not only over time, but also within blocks. Further, the blockchain rarely runs at capacity, even though fees tend to be higher when blocks are fuller, so miners appear to be leaving "money on the table." We present a simple model of price discrimination to explain our results. We note that mining pools facilitate collusive equilibria, and estimate that they have extracted least 200 million USD a year in excess fees by making processing capacity scarce.

Keywords: Bitcoin, Transaction Costs, Blockchain, Decentralized Finance

[*]Corresponding author, email: `alfred.lehar@haskayne.ucalgary.ca`, Tel: (403) 220 4567.

[†]email:`parlour@berkeley.edu`. We thank Christina Atanasova, Lin William Cong, Jaisun Li, Ye Li, Fahad Saleh, Asani Sarkar, Peter Zimmerman, and seminar participants at CBER, SF BlockChain Week, IWFSAS, Philadelphia Fed FinTech Conference, Toulouse School of Economics, WFA 2020, EFA 2020. Lehar is greateful to the Canadian Securities Institute Research Foundation for financial support.

1 Introduction

On September 10, 2019 Nasdaq announced a new index, Defix, to track firms organized along the principle of decentralized finance. The index is consistent with the growing importance of financial entities that are designed to operate absent a trusted intermediary. While the production function of such entities is clearly novel, it is less obvious if their market conduct is also novel. In this paper, we examine whether the first successful decentralized financial product, Bitcoin, has evolved into a system that provides value transfer to consumers as it was promised to. Understanding how well the Bitcoin system works is important because it is the first widely-adopted fully decentralized finance system; it is also unregulated, and so provides us with insights into the sorts of economic structures that can arise endogenously.

The original white paper by Nakamoto presented a system that obviates the need for trusted intermediaries because "competitive miners" record and settle all Bitcoin transactions. The code assigns Bitcoin as an incentive for miners to do this. The first departure from the zero direct cost settlement, was that spontaneously, over time, consumers have added fees to their transactions to encourage speedy settlement. In this paper, we shed light on the determinants of Bitcoin transaction fees and provide suggestive evidence that miners act in a way consistent with fee maximization, as a monopolist intermediary might. We conclude that even a decentralized finance system can operate non-competitively.

We have three main results. First, we document that the Bitcoin blockchain rarely operates at full capacity even in the presence of fees. Our sample comprises all Bitcoin transactions from the genesis block to November 4, 2018. There is no day over this period in which the BlockChain has run at full capacity. Fees began appearing in 2016, and since then close to 850 million USD have been paid by users of the system.

The existence of excess capacity, in the presence of fees, suggest miners appear to leave "money on the table." Our second result is that we illustrate that this behavior is observationally equivalent to collusive price-discrimination. Mining pools, in as far as they restrict the number of independent players in the system, facilitate such price discrimination. Third, consistent with a conservative interpretation of our model, we define excess fees as those paid above the 10% quantile of the empirical daily fee distribution, which sum close to 560 million USD or approximately 200 million USD a year since the advent of mining pools.

Although the fees submitted by agents to the BlockChain appear akin to bids in a multi-unit first price private value auction, they arise from a different protocol. In particular, there is no commitment by miners to an allocation rule. In a multiunit first price auction, higher bids are more likely to get the good (in this case, immediate settlement). Therefore, whether a particular bid results in the bidder winning the good depends on all the other bids because the seller commits to award the good to higher bids first. Effectively, the rationing rule is one of strict price priority. By contrast, under the Bitcoin protocol, individual miners are not bound by any particular rationing rule, and may choose any set of transactions to work on. To complicate matters, given the decentralized system, miners may even observe different pools of potential transactions.

A fully dynamic model in which sellers have different information sets and do not commit while buyers bid against an unknown number of future other buyers is beyond the scope of this paper. However, to understand how miners' ability to choose any set of buyers to mine affects the consumer cost of using the system, we present a simple two type framework in which users value being processed and bear costs of delay. We then compare bidding behavior in two specific cases. First, we characterize bids if there are capacity constraints, but miners always execute the highest bids first. In this case, the bidders who are most desperate for immediate settlement only have to bid slightly higher than the low type. In other words, the high types retain some surplus. In this case, all processed blocks will be full and fee dispersion will be small. By contrast, if miners can pick and choose when and which bids to process, miners can delay bids that are less than the high types' maximal valuation. This induces high types to bid up to their full valuation, which is effectively price discrimination. Under this rationing rule, the miners extract the high types' full valuation in fees while the low type pay low fees but suffer delays. The empirical content of such rationing is that sometimes processed blocks will either be empty or at less than full capacity with high fee dispersion.

If there are a large number of miners, the probability of "winning the nonce" and processing a block is small, and so individual miners have an incentive to fill up a block with all positive fee transactions, which would not allow price discrimination. It is well known that in repeated games it is more difficult to sustain collusive equilibria as the number of players increases. This suggests that mining pools provide an economic role besides diversifying risk for individual participants. By acting collectively, each mining pool effectively reduces the set of strategic players and so makes it easier to sustain price-discrimination. This observation is consistent with mining pools' habit of "signing" the blocks that they mine. If they do this, other, unsuccessful, mining pools have a credible way of checking whether the block was mined at full capacity (the pool deviated) or under-capacity.

In our sample of over 350 million observations, we document that users who are more likely to have higher valuations for consummated transactions are more likely to pay a higher fee, which is consistent with rent extraction. Specifically, transactions that are more likely to originate from institutional sources (proxied through day of the week), transactions that are more likely to be arbitrage trades (proxied by the Kimchi premium), transactions that involve gambling sites and exchanges and transactions associated with rapid redeployment pay higher fees. To investigate the effect of mining pools, we find that fee dispersion is higher when mining capacity is more concentrated and more mining is done by pools. Consistent with strategic capacity management, blocks tend to be fuller after periods of low mining output and more empty after periods of high mining output. This effect is more pronounced the higher mining concentration. Our empirical analysis does not admit causal identification, but the preponderance of evidence is observationally equivalent to price discrimination.

Our paper fits into a small and growing literature on transaction fees in blockchain systems. David Easley, Maureen O'Hara & Soumya Basu (2019) explain the observed shift from no-fee to fee paying transactions and model the interactions of fee payments and waiting times. While the focus of their empirical analysis is on the time series of average transaction fees our paper documents a huge variation of Bitcoin transaction within blocks analyses the cross section of transaction fees. Gur Huberman, Jacob Leshno & Ciamac C Moallemi (2017) compare Bitcoin to a traditional payment system and derive closed form solutions for equilibrium fees. In their framework, the chain runs at full capacity.

This research on fees fits into a larger body of literature that focuses on the economics and incentives in blockchain ecosystems (among others Joseph Abadi & Markus Brunnermeier (2018), Lin William Cong & Zhiguo He (2019), Eric Budish (2018)) and the impact on financial markets (e.g. Katya Malinova & Andreas Park (2017) or Alexander Brauneis, Roland Mestel, Ryan Riordan & Erik Theissen (2018)). Lin William Cong, Zhiguo He & Jiasun Li (2019) analyze the incentives for miners to form pools to tradeoff risky mining against the amount pools charge to miners. Other research focuses on the pricing of crypto-currencies in the market including frictions causing pricing differences (e.g. Albert S. Hu, Christine A. Parlour & Uday Rajan (2018), Igor Makarov & Antoinette Schoar (2018), Kyoung Jin Choi, Alfred Lehar & Ryan Stauffer (2018)).

Strategic manipulation of capacity to extract rents has been analyzed in the industrial organization literature. For example, Richard J. Gilbert & Paul Klemperer (2000) show that in markets for which consumers have to make a fixed investment to enter a market, rationing may induce more entry and thus be profitable, while Vincenzo Denicolò & Paolo G. Garella (1999) show that rationing may allow a durable good monopolist to maintain high prices. Similarly, in the operations research literature, rationing has been shown to be optimal to induce consumers to accelerate purchases (Qian Liu & Garrett J. van Ryzin (2008)) and to convince consumers to ascribe a higher value to the good (see for example Laurens G. Debo, Christine A. Parlour & Uday Rajan (2011) and Laurens G. Debo, Uday Rajan & Senthil Veeraraghavan (2020)).

An interesting contemporaneous theory paper, Nikhil Malik, Manmohan Aseri, Param Vir Singh & Kannan Srinivasan (2019) considers consumers who decide between processing payments with Bitcoin or going to the banking system. They provide conditions under which increasing Bitcoin capacity leads large miners to collude tacitly to undo such increases by only partially filling blocks; this crowds out low value payments who prefer to use the banking system. They further show that providing incentives for miners to operate at full capacity increases system security risk which could reduce value. We too consider how miners strategically manipulate capacity but our focus is on whether the blockchain system is competitive. Our predictions and findings on the within block dispersion of fees are inconsistent with their framework.

It is important to our discrimination framework that servicers can choose which transactions to include. The lack of commitment in the protocol is the starting point for Soumya Basu, David Easley, Maureen O'Hara & Emin Sirer (2019). The authors observe that over time the prices for the same service have fluctuated, and argue that there is no dominant strategy equilibrium. They propose a mechanism that is manipulation proof as the number of users and miners increases.

2 The Bitcoin Protocol: Block Capacity, Usage and Fees

A Bitcoin transaction comprises inputs and outputs. Once a transaction is submitted, it is broadcast to the network. Then, each node determines if the transaction is valid (i.e., the Bitcoin have not already been spent elsewhere) and stores it, awaiting execution (also known as mining). Each node keeps an inventory of valid transactions that have not yet been mined called the "mempool." It is important to note that the mempool is not a centralized entity but rather is specific to each node (which typically have different capacity RAM). Each time a miner competes to mine a new block, it selects valid transactions from its own mempool to work on. Once a block is mined, it is then broadcast to the network and all nodes remove the relevant transactions from their own mempools. [1]

Miners create blocks by being the first to solve a computational puzzle and are compensated in two ways. First, they receive a mining or block reward, which was set at
btc 50 initially and is cut in half every 210,000 blocks (roughly every four years). The block reward appears as the first transaction in each block and is also called the coinbase. Second, and more germane to our analysis, participants who want their transaction to be included in a block can offer fees to miners. Fees are offered implicitly as the difference between inputs and outputs. For example, a submitted transaction might call ₿2.2 as input but only assign ₿2.18 as output. Miners retain the difference and pay it to themselves as part of the coinbase if they successfully mine a block.

Figure 1 plots the time series of the median as well as the 25 and 75 percentiles of transaction fees in ₿ starting in March 2015. We observe a higher variation of fees over time with the peak in December 2017 when Bitcoin prices peaked. The graph for fees in USD is similar.

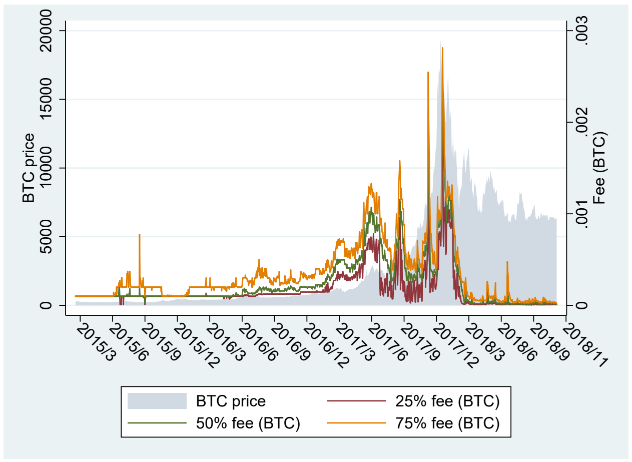


Figure 1. Bitcoin prices (gray area, left axis) and median, 25 and 75 percentile of daily fees (right axis) in Bitcoin. Days are defined over UTC. Transactions exclude coinbase transactions

2.1 The Data and Stylized Facts

Our sample comprises all blocks from the Genesis block (January 3, 2009) to block number 548,684 (Nov 4, 2018) and includes 903,976,764 inputs and 961,308,921 outputs. In total we have 353,306,421 transactions. For each block we observe the coinbase, the inputs and outputs

[1]A theoretical literature considers the possibility of "selfish" mining, in which miners delay publishing of new blocks.

(and hence fees paid to the miners), and the "size" of each transaction. While bytes is the usual metric to determine size, some nuances of the Bitcoin protocol mean that it is more useful to describe the "weight" of transaction. We provide a detailed discussion of this below.

There is a technological limit on the number of transactions that can be processed in a block. In the original design, Satoshi Nakamoto introduced a 1MB limit to Bitcoin blocks in 2010. However, for technical reasons, effective block size was smaller until May 15, 2013.[2] Another upgrade, Segregated Witness (Segwit) was implemented on August 24, 2017. We emphasize that adoption of this technology was voluntary and Bitcoin users slowly converted to the new system. Briefly, Segwit is a way to store signatures and scripts associated with compliant transactions in a special area of a block (the witness section).[3]

It is important to note that post Segwit size in bytes is neither an accurate measure of block capacity nor of transaction size. A fully compliant Segwit transaction takes about 25% the space (in bytes) of a traditional transaction because some components such as scripts are outsourced to the witness area and thus not part of the official block. However if coins are spent from an address locked up before the roll-out of Segwit, the full features of Segwit cannot be used and hence such transactions cannot take full advantage of the capacity increase. In short, they take up more space. The Segwit did not quadruple capacity: As pre-Segwit transactions are replaced with Segwit compliant transactions, block capacity gradually increased. To deal with this heterogeneity in transaction types, the concept of transaction weight was introduced with Segwit. Unless otherwise stated, in the rest of the paper, when we refer to size or block capacity, we work with these weights which reflect true capacity utilization. In Appendix A we provide further institutional details on the measurement of block capacity post Segwit.

The blockchain allows us to observe precisely how much capacity was used. We observe information about capacity demand imprecisely. However, we have obtained two sets of mempool data to measure transaction demand. First, we have partially aggregated mempool data that provides information on the number and size of transactions grouped by fee buckets. Second, we set up two nodes and collected a shorter sample of detailed mempool data that tracks each transaction. This second data set allows us to trace each transaction from the time it entered the mempool to the time that it was mined. Details of both data sets appear in Appendix B.

Figure 2 illustrates the fraction of total blocks per day that are mined either completely full or empty as well as the used capacity per block. In our complete sample the median transaction size has a weight of 904. We count a block as full if there is less than free capacity of 2,000 weight units. Thus, our measure of a full block is conservative. Mining a full and an empty block are equally difficult (empty spaces are also data). From Figure 2, in the early days of Bitcoin, most blocks were mined empty. This is most likely due to a lack of transaction demand.

[2]Block size was limited by the number of database locks required to process a block (at most 10,000). This limit translated to around 500-750 thousand bytes, and was forgotten until March 11, 2013, when an upgrade to V0.8.0 with a switch of databases caused an unplanned fork in the blockchain. After resolving the crisis, the community reached a consensus to remove this unknown limit and a hardfork was scheduled and cleanly activated on May 15, 2013. Subsequently, for the first time, 1MB became the effective maximum block size. Details of this system change are available at `https://en.bitcoin.it/wiki/Block_size_limit_controversy,https://blog.bitmex.com/bitcoins-consensus-forks/`

[3]The first block exceeding 1MB limit was block 481,947 mined on Aug 25, 2017 with a size of 1032 KB.

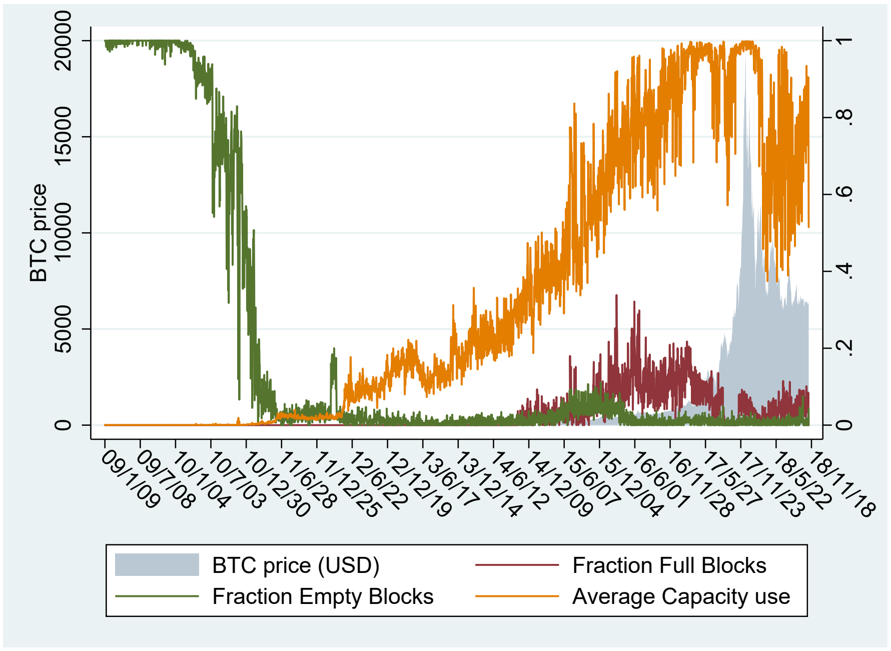


Figure 2. Number of blocks and fraction of empty and full blocks per day. Days are defined over UTC.

Our sample also includes the runup in Bitcoin prices. Even in this period, the blockchain did not run at full capacity. On Dec 17, 2017, for example, when Bitcoin was trading at a record price over USD 19,000, block 499704 and 499763 were mined empty by BTTC pool. On the same day, the same pool also mined 5 non-empty blocks making a technical problem unlikely. We can also rule out lack of transaction demand. Using one of our mempool data sets we find that in the one hour interval around the time that block 499704 was mined more than 130,000 transactions were waiting to be mined. Over December 2017, 40 empty blocks were mined, out of which 39 were mined by 11 different identifiable mining pools. The largest pool in terms of share of mined blocks, AntPool, also mined the largest number of empty blocks.[4] We find that for all blocks mined in or after 2016 about 1.1% of all blocks mined by a given pool are empty. In unreported results we find a similar magnitude for all of the top five mining pools in our sample.

Our results are consistent over time and across pools and so it is unlikely that they are due to random technical problems or due to network latency.[5] This is because most miners participate in Fibre (Fast Internet Bitcoin Relay Engine), which is a special network protocol started in 2016 to deliver Bitcoin blocks around the world with delays as close to the physical limits as

[4]AntPool and BTTCPool mined 9 each, 6 are from BTC.top, 4 from 58coin, and BTC.Com, 2 from Slush pool, and one by 5 smaller pools. Ant pool and BTC top where also the largest mining pools in Dec 2017 with a share of 18.53% and 13.75% of all mined blocks, respectively.

[5]If mining pools experienced a lag, they might let their miners mine empty blocks immediately after a new block was found. In this time pools verify the block that was newly added to the blockchain, delete the transactions from the newly added block in their copy of the mempool, and compose a new block from transactions in the mempool.

possible. At the time of writing the paper we believe the median transmission time of blocks to be 5ms.[6] Anecdotal evidence suggests block validation times for fast hardware to be 45ms.[7] Overall we estimate the process from block discovery to compiling a candidacy block for mining to take at most a few seconds. In our data we observe filled blocks mined within a very short period and empty blocks mined with delays. Starting in 2013 we find that the average time after which an empty block was mined to be 1.96 minutes compared to 9.29 minutes for a full block. However we find 25,552 non-empty blocks mined within less than minute, and 1,566 non-empty blocks mined within less than 5 seconds. Similarly we find 192 empty blocks mined more than 10 minutes after their predecessor.

We note that empty blocks could be consistent with an empty mempool. This was certainly the case in the very early days of bitcoin (the first non-coinbase transaction occurs in block 170). However, using our mempool data which starts in 2016 we find that most empty blocks are mined when there are orders waiting in the mempool. Empty blocks are evenly distributed over time in our sample and do not cluster around times when transaction demand might be low. We therefore conclude that we do not observe empty blocks because of zero transaction demand. Finally, adding transactions to a candidate block is not irreversible. A miner can take all transactions in a mempool and start mining. When a higher fee transaction arrives before the correct nonce is found it can replace a low fee transaction without impacting the probability of finding a valid nonce. Miners do not have to keep block space set aside should higher fee transactions arrive in the future. When confronted over mining empty blocks Jihan Wu from Antpool tweeted in 2016 'sorry, we will continue mining empty blocks. This is the freedom given by the Bitcoin protocol.'

In addition to empty blocks, the second source of unused capacity are blocks that contain some transactions, but are also not completely filled. Excluding empty blocks documented above on the most congested day of our sample, August 22, 2017 (before Segwit), there was room for an additional 625 transactions. On December 17, when Bitcoin peaked, excluding the empty blocks, there was room for an additional 4,957 Segwit compliant transactions or 1,175 non-Segwit transactions. (We base our calculation on the median transaction size of a weight of 250 for Segwit and 1,000 for non-Segwit compliant transactions.) Together, the empty blocks and the empty capacity in filled blocks leave space for several thousand transactions each month. In the blockchain's busiest month so far, December 2017, on average room for 25,838 Segwit compliant transactions was left empty each day (including pace in empty blocks). For the broader sample since Jan 1 2014 on average there was empty space for 909,137 transactions per day.

Throughout this period, unconsummated orders were waiting to be added to the blockchain. To quantify how much money miners would have made if they had filled up all blocks to capacity, we use minute by minute mempool data, which comprises partially aggregated transactions that have been verified and are waiting to be included in a block. (A detailed description of the data set appears in Appendix B). In these data, transactions are grouped by fee buckets based on sat/byte. Two things are worth noting. First, the mempool we observe is not empty: For 78% of the blocks in our sample all the excess capacity could have been completely filled with

[6]See data from `http://bitcoinfibre.org/stats.html`

[7]See for example, `https://bitcoin.stackexchange.com/questions/50349/how-long-does-block-validation-take`.

transactions from the mempool and further 20% could be partially filled. Only for 0.005% of blocks in our sample the mempool was exhausted.[8] Second, specialized mining pools have access to have better hardware and more peer connections and therefore have better and more up-to-date information on potential transactions. We therefore conclude that we have conservative snapshots of their mempools.

Combing the excess capacity per block with information in the mempool about the latent demand, allows us to calculate the direct cost of leaving transactions unmined in the mempool. We define "money left on the table" as the additional fee revenue that could be obtained from filling up blocks with the transactions that offer the highest fees/byte and were in the mempool at the time that the block was mined. Miners also forgo profits by not picking the transactions that pay the highest fee per blockspace which is discussed in Section 3.7. For each block, we match the excess capacity in the block with the excess demand for transactions in the mempool at the time the block was mined. Figure 3 plots the total money left on the table by miners per day. Foregone revenue or money left on the table is observed consistently throughout our sample.

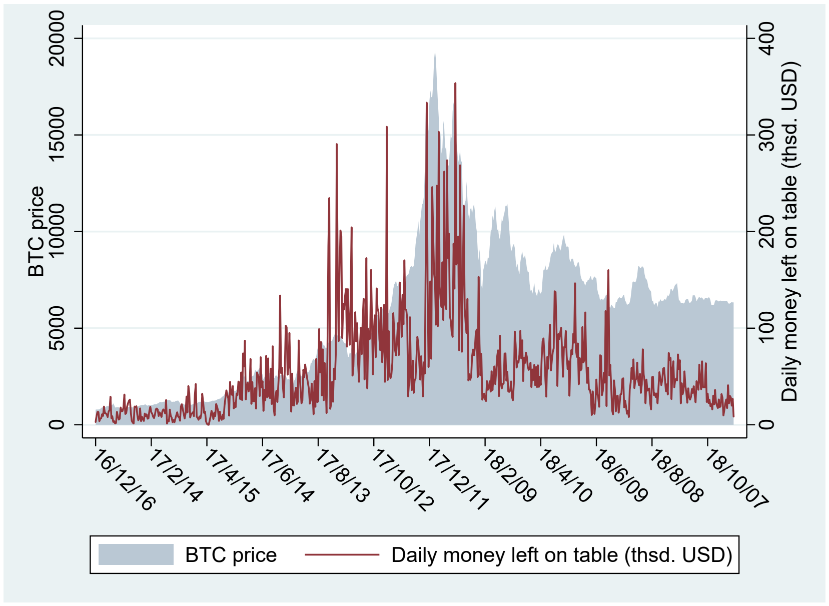


Figure 3. Money left on the table by miners and Bitcoin price. We fill up empty capacity on the blockchain with unmined mempool transactions offering the highest fee/byte.

[8]We are able to match mempool snapshots to 101,045 mined blocks. Out of these 78,820 could be completely filled, and 20,398 could be partially filled. 1,822 blocks were completely full, and for 5 blocks the mempool snapshot was empty.

3 Optimally "leaving money on the table"

To motivate our framework, we present a simple model of on-chain settlement. The population comprises two types of bidders, those with high valuations for settlement v_h, and those with low, v_ℓ, where $v_h > v_\ell > 0$. These agents also differ in the cost incurred for every block that the transaction is delayed, where $c_h > c_\ell = 0$. We refer to the pair $\{c_i, v_i\}$ as the type of the agent i, where $i \in \{\ell, h\}$. For compactness we sometime denote that $c_h - c_\ell = \Delta_c$ and $v_h - v_\ell = \Delta_v$.

Each agent arriving at the market has the same outside option, $\bar{v} > 0$, which determines his willingness to participate. This is the expected utility any agent participating in the Bitcoin system obtains if he chooses his own optimal alternative.

We consider an infinite horizon, discrete time model. Here, one period corresponds to one mining block, so the culmination of each period is the addition of another block to the chain. The sequence of events within a period is as follows: first, λ_i agents of type $i = h, \ell$ arrive, and then submit their transaction (normalized to 1) and bid b_i to the network. Transactions and bids are then submitted to a common mempool and the transaction servicer then picks the bids to process.

Each block has a fixed capacity of κ. We assume that $\lambda_h < \kappa < \lambda_h + \lambda_\ell$, so that there is congestion in the blockchain and some types will have to be rationed. (We note that if there is no congestion, in the case in which the servicer exhausts capacity, the optimal bid is always 0.)

In reality, each miner keeps track of their own mempool. We abstract from this and consider a common mempool. Unprocessed bids remain in the mempool, as agents cannot revise or cancel their bids. However, as a practical matter, a mempool does not grow without bound because each mining pool flushes its mempool depending on its local conditions and capacity. For modeling purposes, we assume that the common mempool is flushed after ever period, but that agents whose transaction was not settled automatically resubmit their transaction at the same price. Thus, the arrival rate λ_i should be understood to include new transactions and those that were submitted earlier but have not been executed. The timeline for one period is presented in Figure 4 below.

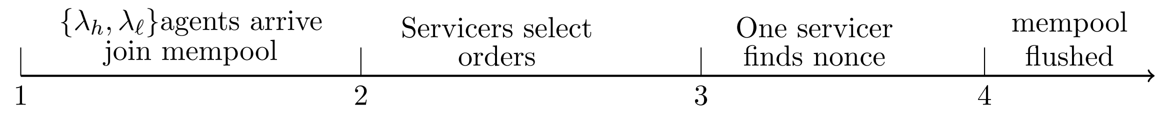


Figure 4. Sequence of Events within a block mining period

We characterize time invariant outcomes both because agents (and econometricians) do not observe the contents of the mempool, and the time between blocks – on average 10 minutes – probably does not admit time trends. We note, however, that in our empirical section, we use the random time between blocks as a proxy for congestion. In our base model, the arrival rates of different types are exogenous.

Let p_i be the time invariant probability that a transaction with bid b_i is executed in the current block. Then, an agent with cost c_i and valuation v_i, who submits a bid of b_i that executes after k periods, garners expected utility of $v_i - b_i - c_i(k-1)$. The costs of waiting are illustrated in Figure 5 below.

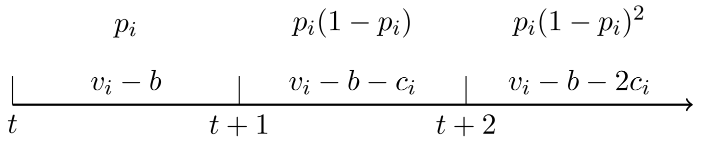


Figure 5. Probability of execution and payoff for an agent with type $\{c_i, v_i\}$. Here t refers to the block height.

Lemma 1 *The expected utility of an agent who submits a bid b_i and anticipates an execution probability p_i is*

$$V(b_i \mid c_i) \;=\; (v_i - b_i) - c_i \frac{1-p_i}{p_i}. \tag{1}$$

Agents take the execution probabilities as given. However, the execution probabilities depend on the actions of the miners, i.e., those who select transactions out of the mempool to process. We characterize two types of liquidity provision, first in which miners execute all orders by price priority and operate at full capacity, and second in which miners execute orders by price priority and ration strategically.

3.1 Different Rationing Rules

We know from the utility function of the consumer, and the participation constraint that the bid is

$$b_i^* \;\leq\; v_i - \bar{v} - c_i \frac{1-p_i}{p_i}.$$

Further, because $v_h > v_\ell$ and $c_h > c_\ell = 0$, we know that the type h has an incentive to bid higher than the type ℓ, and so we consider the minimum bid at which the type h agents execute with probability 1 and the type ℓ are rationed. Let the superscript c denote bids, and execution probabilities if the servicer exhausts all available capacity.

Proposition 2 (Competitive Servicing) *If the servicers execute by price priority and exhaust all capacity then*

i. Type h bids no more than $b_h^c = \lim_{\epsilon \to 0} v_\ell - \bar{v} + \epsilon$, and executes with probability $p_h^c = 1$.

ii. *Type ℓ bids at most $b_\ell^c = v_\ell - \bar{v}$.*

If the servicer commits to exhaust all capacity, then the execution probability of the lower bidder type is determined by the arrival of traders and block capacity. The high types bids just enough to ensure that their orders will always execute. We note in passing that although we have continuous prices, in a model with arbitrarily small prices, the high types would bid the smallest price increment above the lowest types. In short, the high type retains rents and enjoys a surplus by participating in the system.

By contrast, if the servicer strategically chooses the execution probabilities, he can induce higher bids from agents who face higher costs. Specifically, the servicer only executes b_h^d with probability 1, and all other bids with probability $p_\ell^d < 1$.

Proposition 3 (Discriminatory Servicing) *If the servicers execute by price priority and ration, then*

i. *Type h bids $b_h^d = v_h - \bar{v}$, and executes with probability $p_h^d = 1$.*

ii. *Type ℓ bids $b_\ell^d = v_\ell - \bar{v}$ and executes with probability $p_\ell^d = \min\left[\frac{\Delta_c}{\Delta_v + \Delta_c}, \kappa - \lambda_h\right]$*

iii. *The profits of the servicer are $\pi^d = (v_h - \bar{v})\lambda_h + \frac{(v_\ell - \bar{v})\Delta_c}{\Delta_v + \Delta_c}\lambda_\ell$*
.

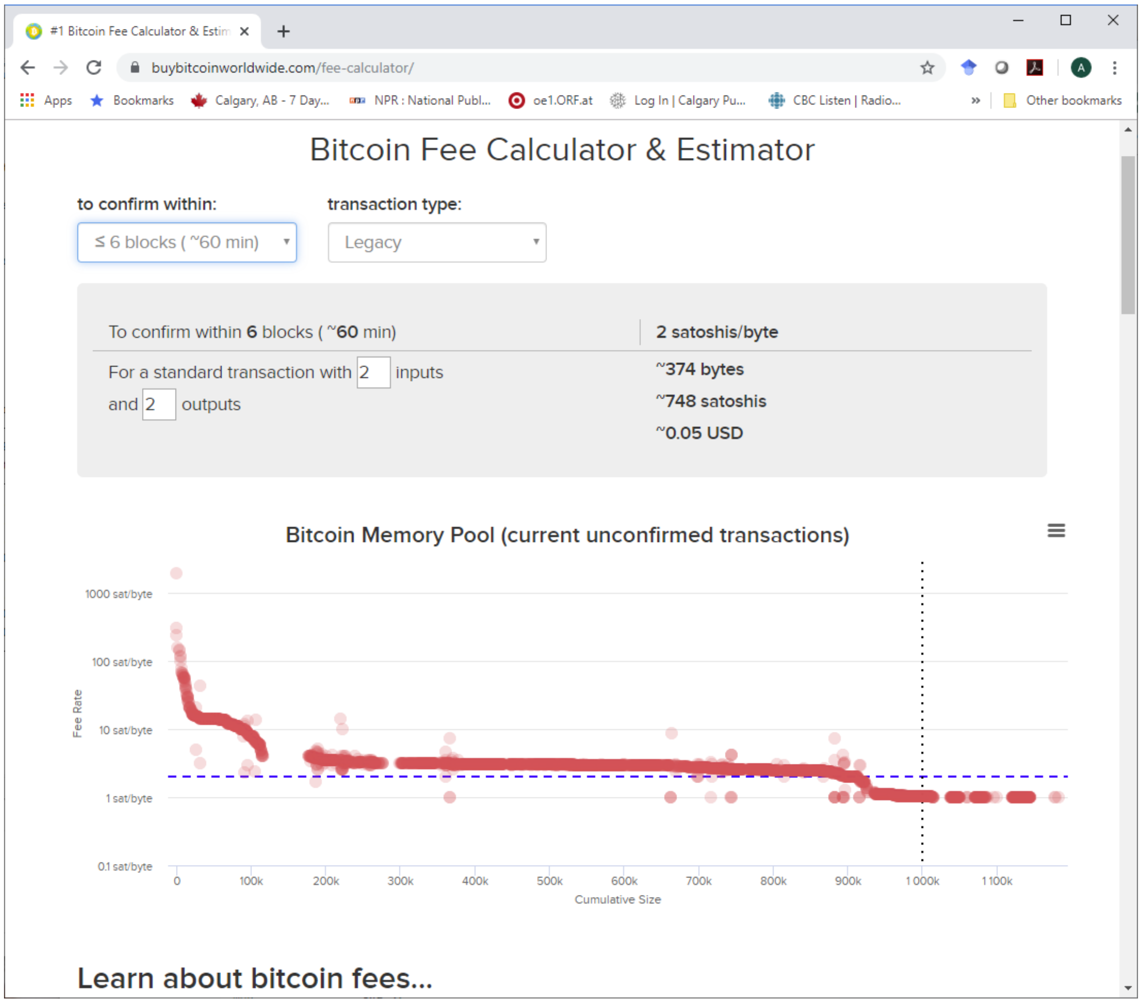
These two different types of servicing strategies will generate different observables. First, it follows immediately from inspection of the two propositions that the maximum bid observed under discriminatory servicing is higher than that under full capacity servicing.

Corollary 1 *The maximum bid observed under discriminatory servicing is higher than the maximum bid under capacity servicing.*

We can therefore expect a wider dispersion in fees under discriminatory pricing than under competitive mining. Furthermore we can expect this price difference to increase when transactions become more valuable for the high type. Under discriminatory pricing we therefore expect large contemporaneous variations in fees and we expect that users who derive greater utility from getting their transaction executed to pay higher fees.

Second, under competitive servicing we should either see full blocks with positive fees when capacity is constrained, or we should see partially empty blocks and zero fees in case of excess capacity. Neither is consistent with the stylized facts presented above. What we see in the data, partially filled blocks with positive fees, is consistent with discriminatory pricing.

Finally our model is straightforward to extend to many types. No matter how many types we have under competitive pricing we should only have two outcomes: types above a certain reservation utility should be executed with probability one and all pay the same fee while all


#1 Bitcoin Fee Calculator & Estim

buybitcoinworldwide.com/fee-calculator/

Bitcoin Fee Calculator & Estimator

to confirm within: ≤ 6 blocks (~60 min)

transaction type: Legacy

To confirm within **6** blocks (~**60** min) | **2 satoshis/byte**

For a standard transaction with 2 inputs and 2 outputs | **~374 bytes** **~748 satoshis** **~0.05 USD**

Figure 6. Screenshot from a Bitcoin fee calculator that provides real time estimates of fees that users have to offer to get confirmed with 90% probability within 1,2,3,4,5, and 6 blocks, respectively.

other types should be rationed if capacity constraints exist. Under discriminatory pricing a miner would offer a menu of fees and execution probabilities to price discriminate between users. Anecdotal evidence is consistent with the latter prediction. Online fee calculators like the one shown in Figure 6 provide users with a real time estimate on the fee they have to post to be confirmed with a 90% probability within 1,2,3,4,5, and 6 blocks, respectively.[9]

The stylized facts we have presented are consistent with discriminatory pricing. Further evidence is also consistent with this. We now examine two characteristics that are consistent with discriminatory pricing: fee dispersion per block and the fact that types with a higher willingness to pay bid more and the fees are far from uniform.

[9]See for example `https://www.buybitcoinworldwide.com/fee-calculator/` or `https://bitcoinfees.github.io/#30m`.

3.2 Fee Dispersion

Discriminatory service leads to high fee dispersion. As an example, Figure 7 illustrates Bitcoin fee heterogeneity in December 2017, which was the peak of Bitcoin prices in our sample. The left panel illustrates that fees up to the median were less than USD 35. By contrast, from the right panel the maximum fee was USD 14,174.64. In this quartile, there were many transactions paying several thousand dollars in fees. There are 80 transactions in our sample with fees greater than USD 10,000, of which 51 occur between Dec 20, 2017 and Dec 24, 2017. However over these same five days 1,674,141 transactions were processed out of which 752 had no fee and 16,191 transactions are mined with fees less than USD 5.

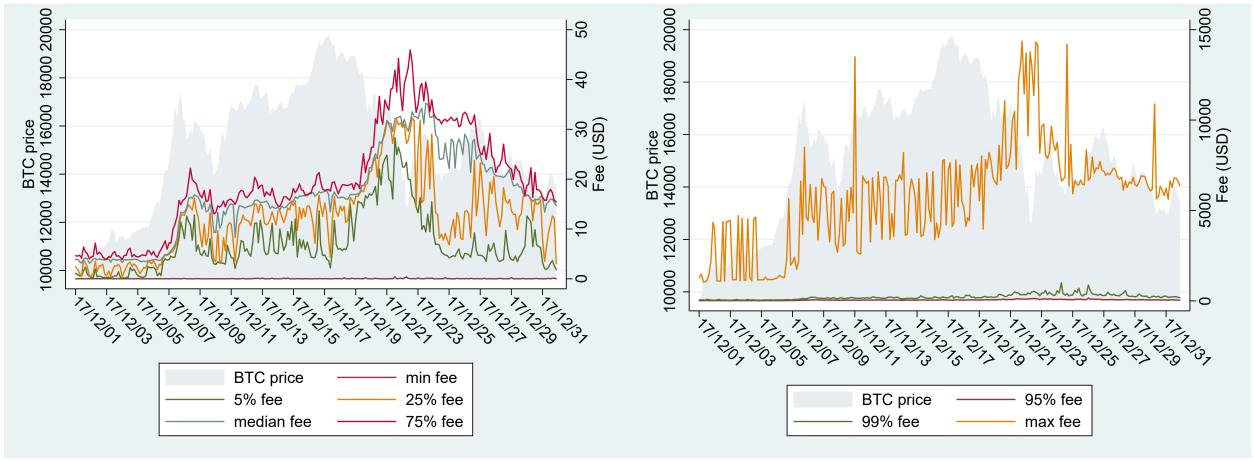


Figure 7. Fees in USD (right axis) and Bitcoin price in USD (left axis) in December 2017.

This pattern is consistent with discriminatory pricing in which miners, by strategic capacity usage, induce induce different fees from different types. If mining was competitive it is difficult to envisage that a rational agent would pay an excessive fee given that transactions with lower fee were recently included in blocks.

3.3 Who pays more?

3.4 Evidence from Blockchain characteristics

We now examine which observable characteristics determine fees. We analyze how transaction and blockchain specific variables drive fees and then provide evidence that is consistent with the idea that high value types pay higher fees. We identity the following variables that differ across users' types and that are plausibly related to their demand for immediate execution. First, the speed with which the output from a transaction is used again. We define rest time as the minimum time (in blocks) until the first output of a transaction is spent again. Recipient wallet owners that spend their funds very quickly have demonstrated an immediate need for funds. Similarly, we use a dummy variable for transactions for which an output is spent in the next block. Second, sometimes a transaction is used to insert data into the blockchain, such transactions have no obvious need for speed. The Data dummy is set to one for all data insertion transactions.[10]

We also include variables that are related to both the capacity of the block and the size of the transactions. Transaction size is the physical size of all inputs and outputs in bytes. The transaction size represents an opportunity cost for miners as block space is limited. Blocksize is the absolute size of the block in weight units. Larger blocks have less space for additional transactions and are an indication for transaction demand. We also include the size of the transaction in both weight units and bytes.

Finally, we include variables as controls. The Sum Inputs variable adds up all input values to a transaction. We choose inputs rather than outputs because some transactions, most of them data-insertion transactions, have almost all their inputs dedicated to fees and only have negligible output.

	Nobs	Mean	Std.Dev.	Median	Min	Max
Fee (Satoshi)	353,306,421	52,406	2,826,555	18,802	0	29,124,090,000
Fee (USD)	291,702,649	2.90	36.86	0.20	0	137,186
Tx-Inputs (thsd. Sat.)	353,306,421	1,367,292.33	40,523,154.35	19,800	0	55,000,000,000
Tx-Inputs (USD)	291,702,649	25,69	556,02	256.64	0	946,541,056
Blocksize (bytes)	353,306,421	834,072	325,063	998,126	267	2,324,736
Blocksize (weight)	353,306,421	3,209,784	1,212,422	3,991,416	624	3,999,120
Tx-Size (bytes)	353,281,736	538	2,211	250	62	999,657
Tx-Size (weight)	353,306,421	2,084	8,544	904	248	3,998,628
Data insertion	353,306,421	.022	.147	0	0	1
Resttime (blocks)	350,031,634	547.54	4,950.34	4	0	474,195
Spent next block	353,306,421	.140	.347	0	0	1
₿ Price (USD)	291,702,649	3,604.66	4,162.71	1,188.28	204.84	19,829.76

Table 1. Summary statistics

Table 1 contains summary statistics. Many variables show some extreme observations. Some of the transactions with unusually high fees are payments miners make to themselves as they consolidate Bitcoin balances with the coinbase under a new address. For example on April 26, 2016 someone paid ₿291.241 to an output with ₿0.0001, leaving fees of ₿291.2409 (or USD 137,186.07 at the time) for the miner.[11] We identified 80 transactions with fees over USD 10,000. These transactions have an average input of 140.89 BTC, and an average fee of ₿7.55 or (USD 16,974.42 at the time). Transactions from the earliest days of Bitcoin may also show unusually high fees denominated in Bitcoin. In December 2011 a transaction can be found with inputs of over ₿207 and outputs of ₿36, leaving a huge fee for the miner. At that time Bitcoin were worth very little, yet since there was no shortage of transaction space in the blockchain, there is no obvious rationale for paying such a high fee.

The average Bitcoin transaction was for ₿13.7 (one Bitcoin equals 100 million Satoshi) while the

[10]These so-called "Op-Ret" transactions are used to store data from 2nd layer applications on the Bitcoin blockchain.

[11]this is the highest fee transaction in BTC and USD terms in our sample, see transaction cc455ae816e6cdafdb58d54e35d4f46d860047458eacf1c7405dc634631c570d.

largest transaction was for ₿550,000 on Nov 16, 2011 at a zero fee.[12] The largest transaction in dollar terms occurred on Dec 17th, 2017 at the peak of the Bitcoin price when ₿48,500 valued at over USD 946 million changed wallet for a fee of 80,908 Satoshi or USD 15.8.[13] In our sample there are 54,907 transactions with a value of more than USD 10 million. Of those, 20,956 were processed with a fee of less than USD 5, while the average fee was USD 90.54.

Most transactions are small as the mean is 538 bytes or 2084 weight units while the median is 250 bytes or 904 weight units. However, the largest transaction in our sample consumes the entire block 364,292 and has a size of 999,657 bytes. This transaction was mined on July 7th, 2015 and consolidates 5,569 inputs of 1,000 Satoshi each into a single output.[14]

For our regressions, we winsorize our fee data, the transaction size, the inputs, and the restime at the 99.9% level (The coinbase of the Genesis block has never been spent.) We find our results to be robust to different levels of winsorizing. Bitcoin fees also vary tremendously over time. To control for this time variation we include day-fixed effects in our regression analysis. To control for variation of fees across miners we cluster standard errors per block.

Our fee regressions appear in Table 2, in terms of both Satoshis and Table 3 in USD. Fees are higher when transaction space is scarce (larger blocksize weight) and when the transaction is larger in bytes (Transaction Size), weight (Transaction Weight) and value (Sum Inputs). The coefficient on transaction weight is roughly one fourth that of transaction size in bytes because all before SegWit transactions have, by definition, a weight equal to four times their size in bytes. Transaction size has an economically significant impact on fees. Specifically, adding one input (with an average size of 180 bytes) to a transaction (the average transaction has 2.5 inputs in our sample) increases the fee by 8,893 Satoshi or USD 0.41. Given that the median fee over the whole sample is USD 0.20 this effect is economically large. Fees are also higher when the funds are spent sooner (lower rest time) and especially if the output was spent in the next block. The dollar value of the transaction(SumInputs), for example, only has a small and economically insignificant effect on fees, which is consistent with the fact that the miner's opportunity cost for including a transaction in a block of limited size is determined by the transaction size in weight and independent of the value. Results are mixed for the data-insertion (OP-Ret) dummy. In terms of BTC we see that data-insertion transactions post lower fees, yet when controlling for other variables we see that they pay a higher fee in USD. We attribute that to the increasing volume of data insertion transactions towards the end of our sample, where BTC prices were generally higher than at the beginning. Outtime is measured in blocks, which are mined every 10 minutes on average. People spending their funds in the next block pay on average USD 0.577 more, and users spending a week ($\sim$1008 blocks) earlier pay on average 1.8 cent more in fees. Receivers that are keen to spend their coins sooner put a higher value on the execution.

[12]see transaction 29a3efd3ef04f9153d47a990bd7b048a4b2d213daaa5fb8ed670fb85f13bdbcf

[13]see transaction 261d69b25896034325d8ad3e0668f963346fd79baefb6a73b4eabd68c58c81ff

[14]According to some forum posts an unknown spammer created all these wallets to test the limits of the UTXO database. Somebody created many small outputs locked up under different addresses. To speed up the time it takes to verify the validity of a transaction each node holds in RAM memory a complete lust of all unspent BTC amounts per address. This list is called the Unspent Transaction Output (UTXO) database. The attack of the spammer forced each of the nodes to expand the UTXO list thereby increasing the memory requirement for each node. The Intentions of the spammer are unclear as all the transactions were protected by very weak private keys such as *'password'* or *'cat'*. Somebody eventually guessed the private keys and consolidated all the small inputs in block 364,292, freeing up space in the UTXO list.

	Whole Sample							w/o Peak
Block Size (Weight)	0.000765***						0.000591***	0.000611***
	(0.0000197)						(0.0000244)	(0.0000227)
Transaction Size		49.39***						
		(0.138)						
Transaction Weight			12.90***				12.92***	11.10***
			(0.0355)				(0.0356)	(0.0330)
Sum Inputs (Sat)				0.00000104***			0.000000561***	0.000000384***
				(8.65e-09)			(5.58e-09)	(4.42e-09)
OPRET					-10371.5***		-2107.3***	-2196.3***
					(232.5)		(231.6)	(223.9)
Spent next block						10049.4***	10417.5***	8580.7***
						(68.54)	(62.50)	(53.69)
Time until spent						-0.211***	-0.366***	-0.278***
						(0.00362)	(0.00420)	(0.00377)
R^2	0.0951	0.369	0.374	0.0973	0.0952	0.0957	0.376	0.345
Observations	353,306,421	353,281,736	353,306,421	353,306,421	353,306,421	350,031,634	350,031,634	320,966,799

Table 2: **Regression results of fees in Satoshis (1 BTC is 100 million Satoshis). Regressions include day fixed effects. Standard errors are clustered by block. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.**

Consistent with discriminatory service those users pay higher fees as miners are able to price discriminate by delaying users that put a low priority on execution.

We stress that our findings are not driven by the peak in Bitcoin prices around December 2017. The last column of each table shows the results for the subsample excluding all transactions between November 1, 2017 and January 31, 2018.

3.5 Evidence from trader specific characteristics

Different trader types plausibly have different values for transactions and different costs of waiting. More sophisticated investors such as institutional investors or hedge funds are more likely to be active during the week and when the Bitcoin futures at CME are trading.[15] To investigate this, we augment the regression presented in Tables 2 and 3 to include time dummies. The results are presented in Table 4. Average transaction fees on weekends are 20 cents lower than week days which, while small, is economically meaningful as it is the median transaction fee for the whole sample. Also, fees gradually increase during the work week so that the highest observed fees are on Fridays (which is also the settlement day for futures). Fees are also higher by about half a dollar, or twice the median fee, whenever the futures market is open (column (3)).

It is possible that CME trading hours are picking up higher transaction demand that is unrelated to futures and hence institutional trading. To ensure that our findings are not driven by specific characteristics of CME futures trading hours we perform a robustness check using a 15 day window around December 18, 2017 when futures were first traded. These results appear in column (4). The dummy *CME trading hours* is one during the regular trading hours of Bitcoin futures at CME, *Post Dec 18th* is a dummy equal to one after December 18, 2017 and *CME Futures trading* is the product of *CME trading hours* and *Post Dec 18th.* We find that fees during CME trading hours are significantly higher once futures trading starts.

Bitcoin is a pseudo-anonymous system, while wallet addresses are public and payments can be observed moving from one wallet to another, the identity of the wallet owner is usually unknown. In some cases, e.g. voluntary disclosure, or court proceedings, the owner of some addresses becomes public. In addition, gambling sites often use vanity addresses, such as '1dice....' or '1Lucky....' which are easily identified and more importantly are reused.[16] Once an address is known, other addresses controlled by the same wallet can be inferred in a process commonly known as address clustering see e.g. Fergal Reid & Martin Harrigan (2013) or Sean Foley, Jonathan R Karlsen & Tālis J Putniņš (2018). The idea is that if multiple addresses are used

[15]BTC futures at CME trade Sunday to Friday from 6pm to 5pm EDT with a daily one hour break between 5pm and 6pm EDT. Futures were first traded on December 18, 2017. Settlement is on the last Friday of the contract month. For this analysis we convert all block timestamps from UTC to Eastern Time with the appropriate adjustment for summer daylight savings time.

[16]Addresses are encoded in a Base58 alphabet (i.e. there are 58 possible 'letters' consisting of upper case, lower case letters and numbers with some combinations dropped that are often mixed up when printed on paper, e.g. capital i and lower case L) and start with 1. To get an address starting with '1Lucky' one has to try $58^5 \approx 656 million$ combinations. Vanity address companies offer computing resources to find custom Bitcoin addresses.

	Whole Sample							w/o Peak
Block Size (Weight)	2.48e-08***						2.24e-08***	2.18e-08***
	(1.07e-09)						(1.30e-09)	(1.08e-09)
Transaction Size		0.00233***						
		(0.00000989)						
Transaction Weight			0.000603***				0.000595***	0.000391***
			(0.00000256)				(0.00000252)	(0.00000184)
Sum Inputs (USD)				0.00000769***			0.00000615***	0.00000283***
				(6.65e-08)			(5.86e-08)	(2.14e-08)
Data Insertion					-0.368***		0.0463***	-0.0363***
					(0.0168)		(0.0152)	(0.00864)
Spent next block						0.636***	0.576***	0.298***
						(0.00646)	(0.00600)	(0.00247)
Time until spent						-0.0000146***	-0.0000178***	-0.00000728***
						(0.000000281)	(0.000000312)	(0.000000194)
R^2	0.274	0.400	0.400	0.286	0.274	0.275	0.408	0.265
Observations	291,702,649	291,678,054	291,702,649	291,702,649	291,702,649	288,555,299	288,555,299	259,490,464

Table 3: **Regression results of fees in USD. Regressions include day fixed effects. Standard errors are clustered by block. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.**

	Whole Sample			Event
	(1)	(2)	(3)	(4)
Block Size (Weight)	1.27e-08***	1.76e-08***	2.02e-08***	0.00000786***
	(1.68e-09)	(1.59e-09)	(1.61e-09)	(0.00000266)
Transaction Weight	0.000595***	0.000595***	0.000595***	0.00264***
	(0.00000252)	(0.00000252)	(0.00000252)	(0.0000150)
Sum Inputs (USD)	0.00000613***	0.00000613***	0.00000613***	0.00000689***
	(5.91e-08)	(5.92e-08)	(5.92e-08)	(0.000000178)
OPRET	0.00511	0.00669	0.00242	1.142***
	(0.0181)	(0.0180)	(0.0180)	(0.276)
Spent next block	0.580***	0.582***	0.581***	2.564***
	(0.00679)	(0.00682)	(0.00681)	(0.118)
Time until spent	-0.0000182***	-0.0000182***	-0.0000183***	-0.000158***
	(0.000000323)	(0.000000323)	(0.000000323)	(0.00000601)
Monday	0.272***			
	(0.0205)			
Tuesday	0.380***			
	(0.0196)			
Wednesday	0.441***			
	(0.0188)			
Thursday	0.527***			
	(0.0196)			
Friday	0.643***			
	(0.0221)			
Saturday	0.485***			
	(0.0212)			
Weekend		-0.200***		
		(0.0120)		
CME Futures trading			0.492***	2.845***
			(0.0368)	(0.509)
CME trading hours				-1.356***
				(0.282)
Post Dec 18th				10.85***
				(0.430)
R^2	0.395	0.395	0.395	0.393
Observations	288,555,299	288,555,299	288,555,299	10,898,250

Table 4. Regression results of fees in USD - day of the week and opening hours of the futures market. Regressions include week fixed effects. Days are defined in UTC. Standard errors are clustered by block. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

as inputs in the same transaction these addresses most likely belong to the same person because the private key has to be used to sign the transaction.[17]

We use data from lists of known addresses and are able to identify the sender for 18,123,498 transactions, out of which 7,250,374 (2.05% of all transactions) were initiated by an exchange

[17]One notable exception are anonymizing services that for a fee combine transactions of several users into one large transaction so that it is not that clear who paid whom. See e.g. Malte Möser & Rainer Böhme (2017).

and 10,873,124 (3.07% of all transactions) that were initiated by a gambling site.[18] Similarly, we are able to identify 32,771,960 payments to an exchange and 23,099,021 payments to a gambling site. Table 5 presents our findings for fees in USD. Notably, flows to and from exchanges transact at substantially higher than average fees. Since we control for day fixed effects our results cannot be driven by more exchange flows occurring on days when fees are generally higher. Our findings are also not driven by outliers as the data is winsorized. Transactions flowing into exchanges pay on average USD 3.97 more than the average fee paid on the same day. This is economically large, given that the median fee for the whole sample is USD 0.20. Flows from exchanges pay USD 2.20 more than same-day average. Gamblers also pay significantly higher fees. Traders moving funds in and out of exchanges and gamblers put a high value on immediate execution. Consistent with discriminatory pricing, such transactions pay higher fees.

To Exchange	6.928***	3.973***		
	(125.78)	(104.91)		
To Gambling	0.392***	0.235***		
	(99.88)	(32.19)		
Block Size (Weight)		2.82e-08***		2.45e-08***
		(19.25)		(16.75)
Transaction Weight		0.000604***		0.000608***
		(237.63)		(236.52)
Sum Inputs (USD)		0.00000763***		0.00000774***
		(96.14)		(97.68)
OPRET		-0.224***		0.00812
		(-14.63)		(0.53)
Spent next block		0.616***		0.621***
		(89.28)		(91.25)
Time until spent		-0.0000171***		-0.0000164***
		(-53.47)		(-52.15)
From Exchange			2.198***	1.239***
			(156.89)	(124.17)
From Gambling			0.772***	0.524***
			(129.46)	(103.30)
R^2	0.268	0.408	0.264	0.406
Observations	294,586,994	291,439,644	294,586,994	291,439,644

Table 5. Regression results of fees in USD - day of the week. Regressions include week fixed effects. Days are defined in UTC. Standard errors are clustered by block. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

Arbitrageurs who take advantage of cross exchange pricing differences also value quick execution. Arbitrageurs' preference for immediacy is higher the higher the price differential between exchanges as they need to process transactions on the blockchain to move Bitcoin from one exchange to the next. Under competitive pricing we should find that fees are independent of the value that the high types put on immediacy. Under discriminatory pricing we should find that miners can extract at least part of that surplus and therefore charge arbitrageurs a higher fee when the price differential is high. To test this prediction we match minute time-stamped prices from Korea and the US with block creation times. The Kimchi premium is the relative price difference of Bitcoin in the US and Korea.[19]We interact a dummy for payments being made

[18]The data are primarily from `walletexplorer.com` and are available from the authors upon request.

[19]The Kimchi premium is calculated as the Bitcoin price in Korea converted to USD minus the Bitcoin price

to and from wallets that can be identified as exchanges with the absolute value of the Kimchi premium and find that fees on transactions to and from exchanges increase in the Kimchi premium. Since arbitrage profits are increasing in the absolute value of the price difference between BTC markets our evidence is consistent with discriminatory pricing and the idea that miners can extract more from high value types.

Our findings are in Table 6. Exchange is a dummy that identifies 35,163,580 payments to and from known exchange wallets, the Kimchi Premium is measured as relative price difference or the BTC price between Korbit in Korea and coinbase in the US, respectively. The interaction term of Exchange and Kimchi premium is statistically and economically significant. For an increase in the Kimchi premium of 10 percentage points, users are willing to pay USD 3.22 more in fees. We note that our analysis is most likely an underestimate of how the demand for immediacy affects fees for two reasons. First, we cannot identify all payments to exchanges. Observed high fees to exchanges might therefore coincide with similar high fee payments to unidentified exchanges making it harder to identify any effect in the data. Second, arbitrage between KRW and USD is only one of many potential trading strategies to exploit price differences, within one country, or between countries. We might therefore also observe high fees for payments to exchanges at times when two different markets have a large price difference which would not be captured in our regression. The results are robust to the introduction of Segwit and other control variables. Our finding cannot be driven by general variation in fees over time as we include day fixed effects.

3.6 Transaction fees over time and mining concentration

Our model has implications for within block fee dispersion: we predict that dispersion of fees is higher under discriminatory service. In this section, we define feespread as the difference between the 90% and the 10% quantile of fees in a given block, standardized by the average fee. We choose this measure of fee dispersion over, say, a standard deviation, to reduce the influence of outliers. We then investigate how this variable is affected by mining concentration.

Suppose that discriminatory pricing is optimal. It is straight forward to show that even a decentralized financial system such as the Bitcoin protocol can lead to discriminatory pricing equilibrium. Suppose that there are $i = 1, \ldots N$ servicers of varying size. Let χ_i denote a servicer's hash rate as the proportion of the total Bitcoin system hash capacity. We assume that this is also the probability that a servicer wins the nonce.

Consider the payoff to servicer i. Mining garners a block reward in addition to fees. For simplicity, we normalize the block reward to zero as this is awarded mechanically. Under pricing regime $j = c, d$, a servicer obtains a per block profit at time t of π_t^j. Thus, under either regime $j = d, c$, the servicer anticipates from the next block onwards of

$$\Pi_i^j = \sum_{t=1}^{\infty} \delta^{t-1} \chi_i \pi_t^d \qquad j = d, c. \tag{2}$$

in the US as a percentage of the US price.

Kimchi Premium × exchange	40.20***	35.05***	32.46***
	(120.07)	(91.89)	(95.94)
Exchange		0.928***	-0.0962***
		(60.02)	(-7.10)
Kimchi Premium		-1.521**	-0.850
		(-2.23)	(-1.31)
Post Segwit			1.859***
			(4.66)
Block Size (Weight)			2.27e-08***
			(15.61)
Transaction Weight			0.000607***
			(237.15)
Sum Inputs (USD)			0.00000759***
			(97.90)
OPRET			0.0280*
			(1.84)
Spent next block			0.625***
			(91.43)
Time until spent			-0.0000159***
			(-51.11)
R^2	0.270	0.271	0.412
Observations	293,822,933	293,822,933	290,678,793

Table 6. Regression results of fees in USD including payments to exchanges and the size of the Kimchi premium. Regressions include day fixed effects. Days are defined in UTC. Standard errors are clustered by block. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

Thus, a servicer who is adhering to discriminatory pricing and wins the nonce receives a payoff of:

$$\Pi_i^{coop} = (v_h - \bar{v})\lambda_h + (v_\ell - \bar{v})\lambda_\ell p_\ell^d + \delta \Pi_i^d$$

Discriminatory pricing requires each miner to refrain from executing low bids to ensure that their expected execution probability is sufficiently low that the agents who value a rapid settlement are willing to bid aggressively up to their valuation. A miner who deviates would include too many transactions from the low bid types. If he finds the nonce, this would become public. Full capacity mining in perpetuity is a natural punishment strategy.[20] The maximum profit to deviating is thus:

$$\Pi_i^{dev} = (v_h - \bar{v})\lambda_h + (\kappa - \lambda_h)(v_\ell - \bar{v}) + \delta \Pi_i^c \qquad (3)$$

It is immediate in this framework that if the payoff to discriminating is higher than the payoff to

[20]Another punishment strategy would be to purposefully orphan blocks of deviating miners. Orphan blocks are rare in the data. An examination of the approximately 100 orphan blocks in our data exhibit no relationship between capacity usage and orphaning.

full capacity mining, then full capacity as a punishment strategy will sustain the discriminatory pricing outcome. The condition to sustain collusive outcome is

$$\Pi_i^{coop} \geq \Pi_i^{dev} \tag{4}$$

As with most collusive equilibria, the fewer the participants, the easier it is to sustain collusive equilibria. In this framework, fewer participants is equivalent to a higher probability that a particular miner or mining pool finds the nonce and is successful. This suggests that an effect of mining pools is to make collusive equilibria much easier to sustain. To see this note, the continuation payoffs in equation (4) are scaled by the probability that a miner wins the nonce. Thus, to sustain collusion, a minimum mining capacity as a function of the total capacity is required. Finally, we note that the miners' habit of signing the blocks they mine ensures that other pools can easily verify that a pool did not exhaust capacity.

We measure mining concentration by the market share of mining pools. To do this, we collect miners' signatures from each block's coinbase transaction. Unlike any other transaction on the Bitcoin blockchain the Bitcoin in the coinbase are newly created and therefore do not originate from another wallet. Miners use the space reserved for the input script to insert data into the blockchain. This space is used to send messages to the community like the miners' opinion on Bitcoin improvement proposals, it can contain other philosophical statements,[21] but most important for our purpose it usually contains a signature identifying the mining pool. We automatically search for commonly used signatures and then manually examine unidentified blocks for reoccurring signature patterns.[22]

We compute the daily Hirschman Herfindahl index (HHI) of mining concentration as the sum of the squared shares of each mining pool computed over the day where the block is mined. We also compute the mining pools' daily share of all mining activity. Table 7 presents our findings. We find that the feespread (i.e., dispersion) increases in both the HHI and mining pool's aggregate share of mining activity. This finding is consistent with the idea that more mining by pools and more concentrated mining makes it easier to maintain the collusive discriminatory equilibrium in which the dispersion of fees increase within blocks.

Another way of identifying strategic behavior on the part of mining pools is to consider capacity management. The Bitcoin protocol calibrates the difficulty, i.e. the number of leading zeros that a block hash has to have to qualify as valid, in such a way that on average a new block is added every ten minutes. Yet the times between blocks as they are recorded on the blockchain vary widely because mining a successful block is purely random and so sometimes blocks are found very quickly and sometimes it takes a long time.[23] Figure 8 illustrates the time between blocks. In the graph we focus on blocks after block 100,000 because dispersion in times between blocks

[21]e.g. 'Welcome to the real world.' in block 328465, or 'smile to life and life will smile back at you' in block 328444, or 'the Lord of the harvest, that he send forth labourers into his harvest' in Block 143822.

[22]It is unclear why pools sign blocks. There is no clear gain in expected revenue from joining a larger pool. While larger pools are expected to mine a block more often, the reward has to be shared among a larger group. In the context of our model mining pools find it optimal to disclose their identity to show that they do not deviate from the collusive equilibrium.

[23]Another source of variation is that the time a block was mined is self reported by the miner and miners can

HHI mining activity	3.274***	3.084***		
	(0.0380)	(0.0368)		
Fraction mined by pools			0.959***	0.996***
			(0.0130)	(0.0135)
Mined by pool	0.0329***	0.0298***	-0.0150***	-0.0124***
	(0.00456)	(0.00445)	(0.00482)	(0.00466)
Post Segwit		0.177***		0.171***
		(0.00302)		(0.00303)
Block weight		-3.85e-08***		-5.92e-08***
		(9.12e-10)		(9.54e-10)
Average tx weight		0.00000443***		0.00000475***
		(0.000000120)		(0.000000121)
Sum Inputs (USD)		-2.12e-08		-0.000000292***
		(4.81e-08)		(4.84e-08)
OPRET		1.074***		0.965***
		(0.0253)		(0.0254)
Time until spent		0.0000907***		0.0000758***
		(0.00000224)		(0.00000227)
R^2	0.0395	0.0996	0.0303	0.0927
Observations	198,111	198,105	198,111	198,105

Table 7. Regression results of fee spread per block defined as the difference of the 90% and 10% quantile over the average fee. HHI is the Hirschman Herfindahl index of mining pool activity and Agg Pool Share is the share of mining done by mining pools. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

was higher in the early days of Bitcoin. Very few blocks have more than 50 minutes between them and these observations are omitted from the graph.

Price discrimination in our model works because miners can credibly delay low types. To keep delay consistent miners can compensate for the variation in the arrival-time of blocks as documented in Figure 8 by adjusting block usage. Specifically, we would expect that if a few blocks arrive close together, subsequent blocks would be more empty as miners delay patient types. Similarly after a long interval between blocks, subsequent blocks would be fuller to accommodate the patient types. Under collusion this effect should be more pronounced when mining is more concentrated. We test this intuition by regressing block-weight on dummies for terciles of the distribution of the number of blocks mined in the last hour. In column 2 we find that block weight increases when few blocks were mined in the last hour and that block weight decreases when many blocks were mined. When we interact the tercile dummies with the HHI of mining

have their local clocks not aligned with the world time. These clock mis-alignments can be substantial. Out of the 548,648 blocks in our sample we find 13,848 cases in which a block has an earlier time-stamp than its predecessor. This is technically impossible. Each block contains information from the previous block, which links the blocks together in a blockchain. The only rational explanations for the inconsistency in time-stamps is improper alignment of miners' clocks. To accommodate potential synchronization problems in miners' clocks the Bitcoin protocol allows a block to have time-stamp up to two hours earlier than the previously mined block. We adjust for these mis-measurements heuristically by assuming that a block with an impossible timestamp has been mined half way between the two neighbouring blocks. Despite these problem cases for the vast majority of the sample the time-stamps seem to be properly recorded.

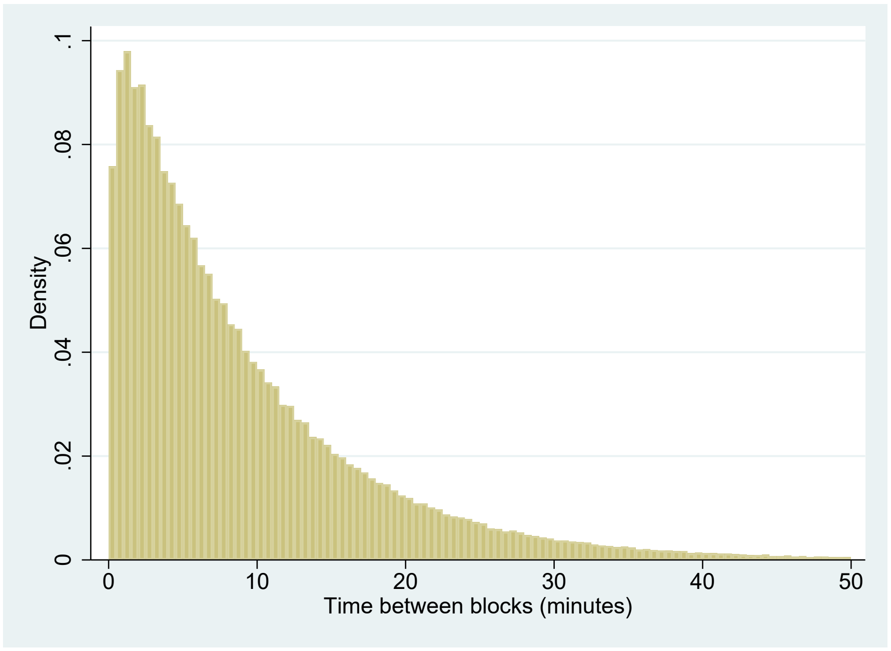


Figure 8. Histogram of time between blocks with blockheight larger than 100,000, capped at 50 minutes.

concentration we find a much larger effect. Our findings are consistent with the idea that when miners collude, capacity management by mining pools is more prevalent.

Low tercile recent blocks	83799.4	440940.1***	35386.9
	(61137.8)	(11970.6)	(68535.8)
HHI x low tercile recent blocks	3016703.1***		3429945.0***
	(525303.4)		(586159.6)
High tercile recent blocks	75236.4	-317545.1***	39521.3
	(106066.9)	(14592.9)	(114589.2)
HHI x high tercile recent blocks	-3295341.1***		-2894436.0***
	(875125.6)		(952862.4)
Sum Inputs (USD)	7.235***	7.224***	
	(0.834)	(0.838)	
OPRET	-1095141.8***	-1082040.9***	
	(251794.3)	(252135.6)	
Time until spent	293.9***	297.1***	
	(27.50)	(27.76)	
R^2	0.101	0.0974	0.0616
Observations	198,551	198,551	198,577

Table 8. Regression results of blockweight on the frequency of recent blocks. Standard errors are clustered by day. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

3.7 Detailed Mempool data

To investigate how individual transaction get delayed we collect detailed mempool data on a transaction level for a total of 671,025 transactions out of which 660,870 eventually get mined. A detailed description of our data can be found in Appendix B. Figure 9 illustrates how long transactions had to wait in the mempool for processing conditional on their place in the fee distribution. Specifically we look at the subset of transactions that eventually got mined and for the purpose of the graph drop all transactions that waited more than 10 blocks. For each transaction we compute the fee distribution of all unmined transactions at the time that this transaction entered the mempool. The left panel shows waittimes in blocks for transactions that were in the top 5% of the fee distribution at the time they entered the mempool. Similarly the middle panel and right panel show waittimes for transaction in the 5% around the median and at the bottom of the fee distribution, respectively. Consistent with discriminatory service we find that transactions with lower fees have to wait longer to be processed than high fee transactions.

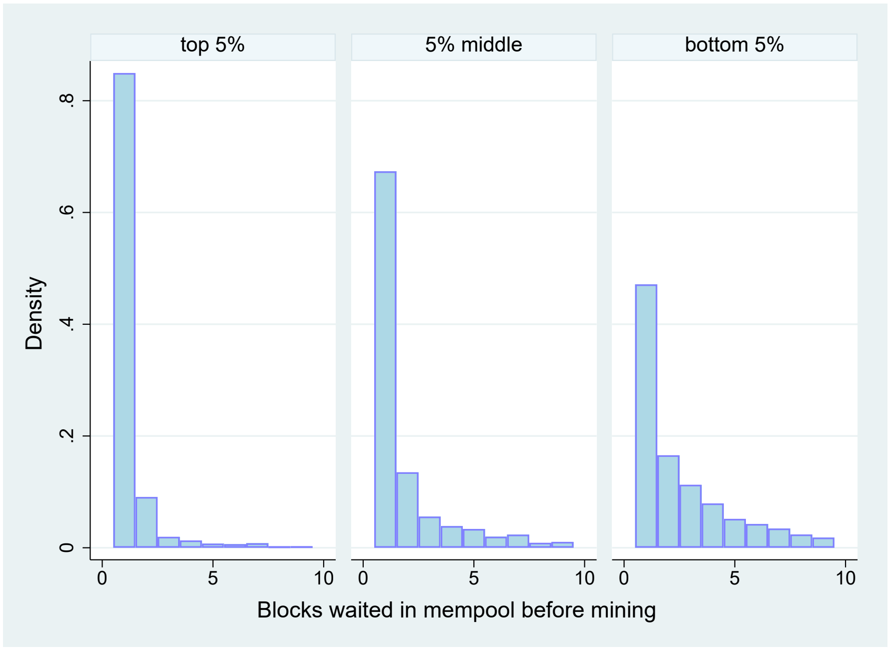


Figure 9. Histogram how long transactions that eventually got mined had to wait in the mempool for transactions that were in the top 5% of the fee distribution at they time they entered the mempool (left panel), in the 5% around the median (middle panel), and in the bottom 5% (right panel), respectively.

To quantify which transactions get delayed we define a proiority violation as an instance where a transaction that was paying a higher fee per weight unit was not mined at the expense of a lower fee transaction. Table 9 shows the results of a probit regression explaining the probability of a transaction experiencing a priority violation. We find that higher fee transactions are significantly less likely to be delayed. The blockweight and the number of transactions in the pool do not determine the probability of priority violations. We also find no significant difference across pools with the exception of BTC.TOP, which has a smaller probablity of priority violations in our sample.

Fee/Wght	−0.0132***	−0.0142***	−0.0150***
	(−3.59)	(−3.92)	(−4.00)
Blockweight		0.0000974	−0.0000197
		(0.31)	(−0.80)
Number TX in pool		0.00000354	0.00000344
		(0.83)	(0.81)
TX weight		−0.00000785***	−0.00000793***
		(−4.87)	(−4.90)
AntPool		0	
		(.)	
BTC.COM		−0.122	
		(−0.42)	
BTC.TOP		−0.719**	
		(−2.57)	
Bitcoin.com		−0.0929	
		(−0.49)	
F2Pool		−0.596	
		(−0.35)	
Huobi		0.299	
		(0.65)	
SlushPool		−0.139	
		(−0.49)	
Unknown		0.236	
		(1.26)	
ViaBTC		−0.106	
		(−0.46)	
poolin		0.0190	
		(0.09)	
R^2			
Observations	1,226,983	1,226,983	1,226,983

Table 9. Probit regression explaining the probability of a priority valuation. Standard errors are clustered per block. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

3.8 Economic impact of collusion

The total amount of fees paid in all transactions of our sample is USD 844,537,503.30. How much was extracted from miners via collusion is hard to estimate. According to our model the equilibrium fee under competitive mining should be approximately, the low type's valuation. An estimate of maximum fees under competitive mining we approximate the fees that the lowest type would be willing to bid with the 10%-quantile of the realized fee distribution per block. We note that we do not observe the transactions that were not incorporated in a block. We then define excessive fees as the sum of all fees above the 10%-quantile. For the whole sample these excessive fees sum to USD 557,568,542.21.

To make the result robust to outliers we re-compute excessive fees and winsorize fees per block at the 99% quantile. With winsorizing, excessive fees for the whole sample add to USD 416,006,674.88. Overall we argue that excessive fees paid because of collusion are between half and two thirds of total fees paid. Figure 10 shows the daily ratio of excessive fees over total fees, Bitcoin prices, and the mining concentration as measured by the HHI. Excess fees seem to be positively correlated with mining concentration.

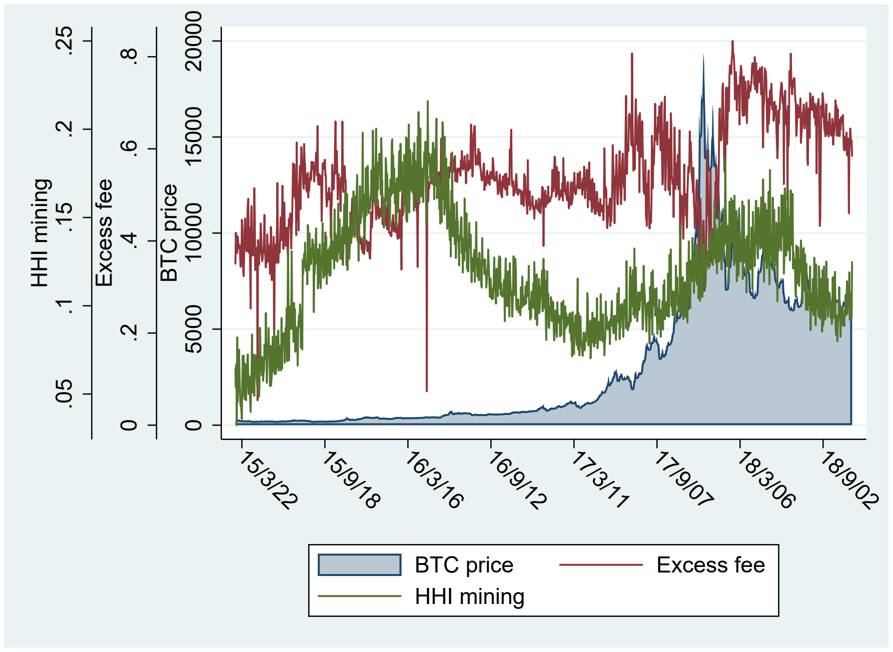


Figure 10. Daily excessive fees, mining concentration as measured by the HHI, and Bitcoin price. Days are defined over UTC.

4 Conclusion

We have documented stylized facts about the Bitcoin protocol. In particular, we observe that there appears to be excess capacity. A significant portion of blocks are empty or not at capacity. We note that this is consistent with collusive price discrimination. Indeed, the rise of fees coincided with the rise of mining pools. Given that the idea behind the Bitcoin system was to provide a completely decentralized way of transferring value based on competitive mining, the possibility that there could be collusive equilibria raises questions about the viability of decentralized finance.

Our preliminary evidence suggests that one implementation of decentralized finance may operate in a way that is observational equivalent to traditional finance. Indeed, our analysis has highlighted the cost to the consumer of the higher fees they pay because of strategic unused capacity. However, we note that there is a positive side to these rents. Higher profits are one way to ensure that miners will view participating as a valuable exercise which ensures the continuity and stability of the Bitcoin protocol. Similar to financial intermediaries, market power and the ability to extract rents provide an incentive to continue.

References

Abadi, Joseph, and Markus Brunnermeier. 2018. "Blockchain economics." National Bureau of Economic Research.

Basu, Soumya, David Easley, Maureen O'Hara, and Emin Sirer. 2019. "Towards a Functional Fee Market for Cryptocurrencies." *Cornell Working Paper.*

Brauneis, Alexander, Roland Mestel, Ryan Riordan, and Erik Theissen. 2018. "A high-frequency analysis of bitcoin liquidity."

Budish, Eric. 2018. "The economic limits of bitcoin and the blockchain." National Bureau of Economic Research.

Choi, Kyoung Jin, Alfred Lehar, and Ryan Stauffer. 2018. "Bitcoin Microstructure and the Kimchi premium."

Cong, Lin William, and Zhiguo He. 2019. "Blockchain disruption and smart contracts." *The Review of Financial Studies*, 32(5): 1754–1797.

Cong, Lin William, Zhiguo He, and Jiasun Li. 2019. "Decentralized mining in centralized pools." National Bureau of Economic Research.

Debo, Laurens G., Christine A. Parlour, and Uday Rajan. 2011. "Signallng Quality via Queues." *Management Science*, 58(5): 44–55.

Debo, Laurens G., Uday Rajan, and Senthil Veeraraghavan. 2020. "Signaling Quality via Long Lines and Uninformative Prices." *Manufacturing and Service Operations Management*, 22(3): 513–537.

Denicolò, Vincenzo, and Paolo G. Garella. 1999. "Rationing in a Durable Goods Monopoly." *Rand Journal of Economics*, 30(1): 44–55.

Easley, David, Maureen O'Hara, and Soumya Basu. 2019. "From mining to markets: The evolution of bitcoin transaction fees." *Journal of Financial Economics.*

Foley, Sean, Jonathan R Karlsen, and Tālis J Putniņš. 2018. "Sex, drugs, and bitcoin: how much illegal activity is financed through cryptocurrencies?" *Review of Financial Studies, Forthcoming.*

Gilbert, Richard J., and Paul Klemperer. 2000. "An Equilibrium Theory of Rationing." *Rand Journal of Economics*, 31(1): 1–21.

Hu, Albert S., Christine A. Parlour, and Uday Rajan. 2018. "Cryptocurrencies: stylized facts on a new investible instrument." working paper.

Huberman, Gur, Jacob Leshno, and Ciamac C Moallemi. 2017. "Monopoly without a monopolist: An economic analysis of the bitcoin payment system."

Liu, Qian, and Garrett J. van Ryzin. 2008. "Strategic Capacity Rationing to Induce Early Purchases." *Management Science*, 54(6): 1115–1131.

Makarov, Igor, and Antoinette Schoar. 2018. "Trading and arbitrage in cryptocurrency markets." working paper.

Malik, Nikhil, Manmohan Aseri, Param Vir Singh, and Kannan Srinivasan. 2019. "Why Bitcoin will fail to scale?" *Tepper Working Paper.*

Malinova, Katya, and Andreas Park. 2017. "Market design with blockchain technology." *Available at SSRN 2785626.*

Möser, Malte, and Rainer Böhme. 2017. "The price of anonymity: empirical evidence from a market for Bitcoin anonymization." *Journal of Cybersecurity*, 3(2): 127–135.

Reid, Fergal, and Martin Harrigan. 2013. "An analysis of anonymity in the bitcoin system." In *Security and privacy in social networks.* 197–223. Springer.

A Block capacity post Segwit

A big part of the physical space that transactions take up in a block are the locking and unlocking scripts. Segregated Witness (Segwit) compliant transactions outsource these scripts into a separate data structure, the witness. The witness structure is organized as Merkel tree, a data structure where leaves hold data and each node is a hash of the underlying nodes. The root of the tree is linked to the Bitcoin block by including the root-hash in the coinbase transaction.

Segwit transactions are designed to be backward compatible. There are two basic types of Segwit transactions, Pay-to-Witness-Public-Key-Hash (P2WPKH) and Pay-to-Witness-Script-Hash (P2WPSH). In the former the locking script is marked as Segwit by including the Segwit version number (currently 0) followed by a 20 byte hash of the public key. The signature and the full public key required for unlocking the Bitcoin are outsourced to the witness block. For P2WPSH transactions the locking script consists of the version number followed by a 32 byte hash of the unlocking script. These transactions are also often referred to as Bech32 transactions. A non-native and inefficient way of implementing Segwit transactions is to embed them in a classic Pay-to-Script-Hash (P2SH) transaction.

Outsourcing part of the transaction to the witness section reduces the amount of effective space a transaction takes up in the block. The measure of transaction size in bytes includes both the transaction and the witness data to make the measure comparable to pre-segwit transactions. For most purposes, e.g. to measure capacity use, the transaction size in bytes is not a useful measure because segwit-, partial segwit, and non-segwit transactions can be included in a block. To address this problem Bitcoin introduced a measure of transaction "weight." The weight of a transaction that does not take advantage of segwit is 4 times its size in bytes. The weight for a fully segwit compliant transaction is obtained by multiplying components that are part of the block (inputs, outputs, input- and output counts, version, and lock-time) by 4 and multiplying witness components by 1 and then adding up the weighted components. In our sample the weight is between 1.2 and 4 times the size in bytes.

The Segwit update did not have a big impact on variables of economic interest. The blue line in Figure 11 shows that the introduction of SegWit brought no immediate increase in capacity. The average weight per block stays between 3 and 4 MB, the latter being the maximum amount. The reason that Segwit brought no sharp increase unused transaction capacity is because of its slow adoption. The red line shows the fraction of transactions that use some Segwit features. Adoption is slow peaking at 15% after fifty days. With most transactions using the pre-Segwit format not much new capacity on the blockchain is being created. The green line illustrates the total fee revenue per block. While there is a peak around the introduction of Segwit the variation in fee revenue per block seems of similar or smaller magnitude than other variations in total fee revenue. It seems that there is no unusual variation in miners' fee revenue around the Segwit inroduction.

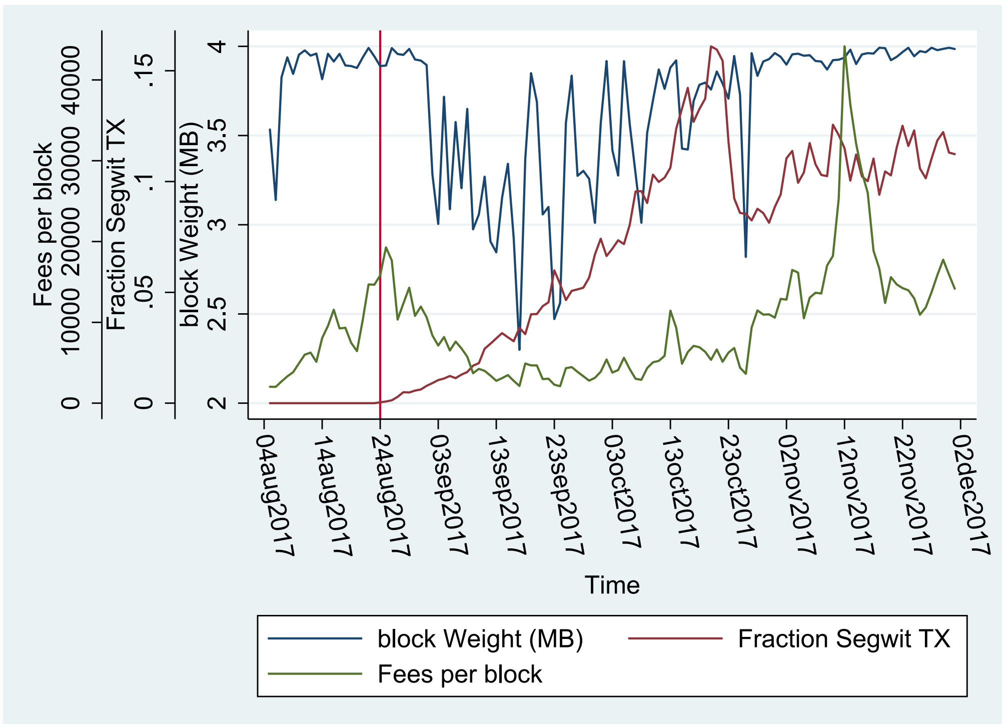


Figure 11. Average Weight per block, fraction of Segwit Transactions, and mining Revenue per block 20 days before to 100 days after the introduction of Segwit. Days are defined over UTC.

B Mempool data

We collect two sets of mempool data to examine transaction demand for Bitcoin. The partially aggregated dataset is used for the monay-left-on-the-table calculation in Section 2.1, the detailed mempool data is used in Section 3.7.

B.1 Partially aggregated data

We collect minute by minute snapshot data of the mempool from Jochen Hoenicke's website, `https://jochen-hoenicke.de/queue/#0,all`. The data ranges from Dec 16, 2016 to the end of our sample period. For each snapshot, transactions are grouped into 45 fee buckets based on sat/byte and contain for each bucket the number of transactions in that bucket at that time, the sum of fees offered by all transaction in that bucket, and the size of all transactions in the bucket. The sample contains over 1.14 million snapshots with a total of 51 million time/bucket observations. There are some gaps in the data, most likely because outages of the server collecting the data. Out of 1,140,218 snapshots we observe 852 snapshots that are more than 70 seconds apart, with the longest gap being 35 hours.

We match mined blocks to mempool data based on the timestamp that the block was mined and by looking for sharp drops in the size and the number of transactions in the pool. We identify these drops as blocks being mined. We cannot reconcile blocks based on the timestamp alone, as timestamps of blocks are sometimes inaccurate. We therefore have a record of the mempool immediately after a block was mined. For our estimate of money left on the table we start filling any empty blockspace with transactions from the highest fee/byte bucket, until we exhaust this bucket and so forth until the block is full.

Because mempool data is specific to each node, any individual miner may face a different mempool. However, we note that transactions which enter the mempool are shared via peer-to-peer communication. We expect that miners have better hardware, faster connections, and are connected to more peers than our data source. Therefore we provide a conservative estimate of the money miners appear to leave on the table.

B.2 Detailed data

We set up our own Bitcoin node and collect the precise composition of the mempool on a transaction level for a small sample of 277 blocks from block 629,408 to 629,684. We observe a total of 671,025 transactions out of which 660,870 eventually get mined. For each transaction we observe precisely when it entered the mempool, its weight, the fee, any dependencies on other unmined transactions, and if and when it was eventually mined. We also collect information on the weight, time, and transaction count of the mined blocks.

C Proofs

Proof of Lemma 1

The utility of an agent with cost c_i who bids b and executes in period k is $v - b(v, c) - c(k - 1)$. If the probability of execution is p_i, they obtain utility

$$\begin{aligned} V_i(b) &= \sum_{k=1}^{\infty} p_i[1-p_i]^{k-1}(v - b - c_i(k-1)) \\ &= (v-b) - c_i p_i \sum_{j=0}^{\infty} [1-p_i]^j j \\ &= (v-b) - c_i \frac{1-p_i}{p_i}. \end{aligned} \tag{5}$$

■

Proof of Proposition 2

The high types incur a cost to delaying, and so they will always bid to ensure that their orders execute. We are looking for an equilibrium in which the high type executes with probability 1, and the low types randomize over a range. The utility of the low type is

$$V_\ell(b) \quad = \quad v_\ell - b - c_\ell \frac{1-p(b)}{p(b)}.$$

The highest bid a low type would make is $\bar{b_\ell^c} = v_\ell - \bar{v}$. Suppose that a fraction of at least $\frac{\kappa-\lambda_h}{\lambda_\ell}$ of the low types bid $\bar{b_\ell^c}$, then if an agent bids lower, her order will never execute. If she bids higher, she obtains negative utility. Thus, there is an equilibrium in which $[\frac{\kappa-\lambda_h}{\lambda_\ell}, 1]$ of the low types bid $\bar{b_\ell^c} = v_\ell - \bar{v}$.

Now, suppose that there is an equilibrium in which less than $\frac{\kappa-\lambda_h}{\lambda_\ell}$ bid $\bar{b_\ell^c}$. Agents have an incentive to reduce their bid to obtain positive surplus. Suppose all agents bid a maximum $b*$, which is strictly less than $\bar{b_\ell^c}$. An agent has a strict incentive to increase his bid and obtain execution for sure.

The high type will bid no more than $b_h^c = lim_{\epsilon \to 0}(v_\ell - \bar{v} + \epsilon)$ as at this price, he will obtain immediate execution.

Proof of Proposition 3

The high type bids $b_h^d = v_h - \bar{v}$ and is executed with probability 1.

The second highest type is executed with probability p_ℓ^d, which satisfies incentive compatibility, that ensures that type ℓ doesn't want to bid b_h^d.

$$\begin{aligned} v_\ell - b_h^d &\leq v_\ell - b_\ell^d - c_\ell \frac{1-p_\ell^d}{p_\ell^d} \\ v_\ell - (v_h - \bar{v}) &\leq v_\ell - b_\ell^d - c_\ell \frac{1-p_\ell^d}{p_\ell^d} \\ b_\ell^d &\leq (v_h - \bar{v}) - c_\ell \frac{1-p_\ell^d}{p_\ell^d} \end{aligned}$$

Further, the highest type does not want to bid lower and experience waiting costs:

$$\bar{v} \geq v_h - b_\ell^d - c_h \frac{1-p_\ell^d}{p_\ell^d} \tag{6}$$

$$b_\ell^d \geq v_h - \bar{v} - c_h \frac{1-p_\ell^d}{p_\ell^d} \tag{7}$$

Subtracting these two equations from each other yields:

Type	Bid	Execution Probability	Quantity
1	$v_h - \bar{v}$	1	λ_h
2	$v_\ell - \bar{v} - c_\ell\Big(\frac{\Delta_v}{\Delta_c}\Big)$	$\frac{\Delta_c}{\Delta_v+\Delta_c}$	$(\lambda_\ell)p_\ell^d$

Table 10. Bids and quantity executed if the servicer discriminates

$$0 \leq (c_h - c_\ell)\Big(\frac{1-p_\ell^d}{p_\ell^d}\Big)$$

As the RHS is strictly positive, these conditions are satisfied. To pin down b_ℓ^d, we note that

$$b_\ell^d \;=\; v_\ell - \bar{v} - c_\ell \frac{1-p_\ell^d}{p_\ell^d},$$

and use this in the incentive compatibility constraint Equation 6:

$$\begin{aligned}
\bar{v} &\geq v_h - b_\ell^d - c_h \frac{1-p_\ell^d}{p_\ell^d} \\
\bar{v} &\geq v_h - \Bigg(v_\ell - \bar{v} - c_\ell \frac{1-p_\ell^d}{p_\ell^d}\Bigg) - c_h \frac{1-p_\ell^d}{p_\ell^d} \\
0 &\geq \Delta_v - \Delta_c \frac{1-p_\ell^d}{p_\ell^d}
\end{aligned}$$

$$\begin{aligned}
\frac{1-p_\ell^d}{p_\ell^d} &\geq \frac{\Delta_v}{\Delta_c}, && (8) \\
\Longrightarrow p_\ell^d &= \frac{\Delta_c}{\Delta_v + \Delta_c} && (9) \\
1 - p_\ell^d &= \frac{\Delta_v}{\Delta_v + \Delta_c} && (10)
\end{aligned}$$

In this case, the flow revenue of the servicer is:

$$\begin{aligned}
\pi^d &= b_h^d \lambda_h + b_\ell^d p_\ell^d (\lambda_\ell) \\
&= (v_h - \bar{v})\lambda_h + \Big(v_\ell - \bar{v} - c_\ell \frac{1-p_\ell}{p_\ell}\Big) p_\ell^d (\lambda_\ell) \\
&= (v_h - \bar{v})\lambda_h + \Big((v_\ell - \bar{v}) - c_\ell \frac{\Delta_v}{\Delta_c}\Big)\Big(\frac{\Delta_c}{\Delta_v + \Delta_c}\Big)(\lambda_\ell) \\
&= (v_h - \bar{v})\lambda_h + \frac{(v_\ell - \bar{v})(\Delta_c) - c_\ell(\Delta_v)}{\Delta_v + \Delta_c}(\lambda_\ell)
\end{aligned}$$

■