Champagne! (10 points)

Warning: Excessive alcohol consumption is harmful to health and drinking alcohol below legal age is prohibited.

Champagne is a French sparkling wine. Fermentation of sugars produces carbon dioxide ($\mathrm{CO_2}$) in the bottle. The molar concentration of $\mathrm{CO_2}$ in the liquid phase c_ℓ and the partial pressure $P_{\mathrm{CO_2}}$ in the gas phase are related by $c_\ell = k_\mathrm{H} P_{\mathrm{CO_2}}$, known as Henry's law and where k_H is called Henry's constant.

Fig. 1. A glass filled with champagne.

Data

- Surface tension of champagne $\sigma = 47 \times 10^{-3}\,\mathrm{J \cdot m^{-2}}$
- Density of the liquid $\rho_\ell = 1.0 \times 10^3\,\mathrm{kg \cdot m^{-3}}$
- Henry's constant at $T_0 = 20\,°\mathrm{C}$, $k_\mathrm{H}(20\,°\mathrm{C}) = 3.3 \times 10^{-4}\,\mathrm{mol \cdot m^{-3} \cdot Pa^{-1}}$
- Henry's constant at $T_0 = 6\,°\mathrm{C}$, $k_\mathrm{H}(6\,°\mathrm{C}) = 5.4 \times 10^{-4}\,\mathrm{mol \cdot m^{-3} \cdot Pa^{-1}}$
- Atmospheric pressure $P_0 = 1\,\mathrm{bar} = 1.0 \times 10^5\,\mathrm{Pa}$
- Gases are ideal with an adiabatic coefficient $\gamma = 1.3$

Part A. Nucleation, growth and rise of bubbles

Immediately after opening a bottle of champagne at temperature $T_0 = 20\,°\mathrm{C}$, we fill a glass. The pressure in the liquid is P_0 and its temperature stays constant at T_0. The concentration c_ℓ of dissolved $\mathrm{CO_2}$ exceeds the equilibrium concentration and we study the nucleation of a $\mathrm{CO_2}$ bubble. We note a its radius and P_b its inner pressure.

A.1	Express the pressure P_b in terms of P_0, a and σ.	0.2pt

In the liquid, the concentration of dissolved $\mathrm{CO_2}$ depends on the distance to the bubble. At long distance we recover the value c_ℓ and we note c_b the concentration close to the bubble surface. According to Henry's law, $c_\mathrm{b} = k_\mathrm{H} P_\mathrm{b}$. We furthermore assume in all the problem that bubbles contain only $\mathrm{CO_2}$.

Since $c_\ell \neq c_\mathrm{b}$, $\mathrm{CO_2}$ molecules diffuse from areas of high to low concentration. We assume also that any molecule from the liquid phase reaching the bubble surface is transferred to the vapour.

A.2	Express the critical radius a_c above which a bubble is expected to grow in terms of P_0, σ, c_ℓ and c_0 where $c_0 = k_\mathrm{H} P_0$. Calculate numerically a_c for $c_\ell = 4c_0$.	0.5pt

In practice, bubbles mainly grow from pre-existing gas cavities. Consider then a bubble with initial radius $a_0 \approx 40\,\mu\mathrm{m}$. The number of moles of $\mathrm{CO_2}$ transferred at the bubble's surface per unit area and time is noted j. Two models are possible for j.

- model (1) $j = \dfrac{D}{a}(c_\ell - c_\mathrm{b})$ where D is the diffusion coefficient of $\mathrm{CO_2}$ in the liquid.
- model (2) $j = K(c_\ell - c_\mathrm{b})$ where K is a constant here.

Experimentally, the bubble radius $a(t)$ is found to depend on time as shown in **Fig. 2**. Here $c_\ell \approx 4c_0$, and since bubbles are large enough to be visible, the excess pressure due to surface tension can be neglected and $P_\mathrm{b} \approx P_0$.

A.3	Express the number of CO_2 moles in the bubble n_c in terms of a, P_0, T_0 and ideal gas constant R. Find $a(t)$ for both models. Indicate which model explains the experimental results in **Fig. 2**. Depending on your answer, calculate numerically K or D.	1.2pt

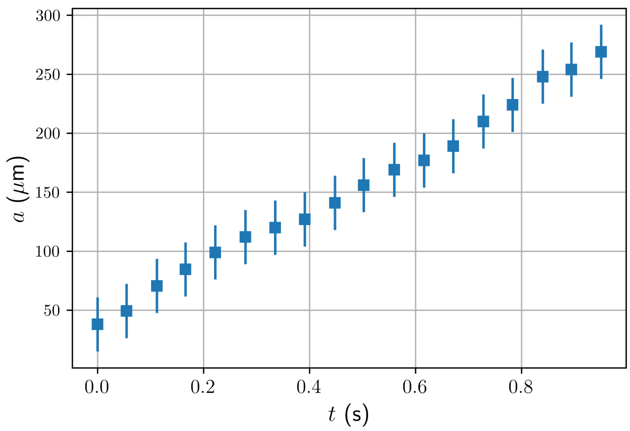

Fig. 2. Time evolution of CO_2 bubble radius in a glass of champagne (*adapted from [1]*).

Eventually bubbles detach from the bottom of the glass and continue to grow while rising. **Fig. 3.** shows a train of bubbles. The bubbles of the train have the same initial radius and are emitted at a constant frequency $f_b = 20\,\mathrm{Hz}$.

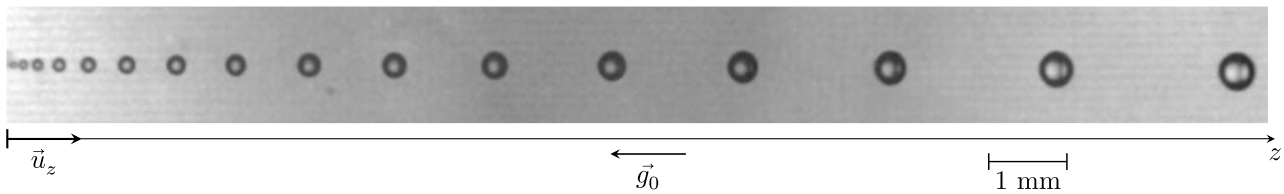

Fig. 3. A train of bubbles. The photo is rotated horizontally for the page layout *(adapted from [1]).*

For the range of velocities studied here, the drag force F on a bubble of radius a moving at velocity v in a liquid of dynamic viscosity η is given by Stokes' law $F = 6\pi\eta a v$. Measurements show that at any moment in time, the bubble can be assumed to be travelling at its terminal velocity.

A.4	Give the expression of the main forces exerted on a vertically rising bubble. Obtain the expression of $v(a)$. Give a numerical estimate of η using ρ_ℓ, g_0 and quantities measured on **Fig. 3.**	0.8pt

The quasi-stationary growth of bubbles with rate $q_a = \frac{\mathrm{d}a}{\mathrm{d}t}$ still applies during bubble rise.

A.5	Express the radius a_{H_ℓ} of a bubble reaching the free surface in terms of height travelled H_ℓ, growth rate $q_a = \frac{\mathrm{d}a}{\mathrm{d}t}$, and any constants you may need. Assume $a_{H_\ell} \gg a_0$ and q_a constant, and give the numerical value of a_{H_ℓ} with $H_\ell = 10\,\mathrm{cm}$ and q_a corresponding to **Fig. 2**.	0.5pt

There are N_b nucleation sites of bubbles. Assume that the bubbles are nucleated at a constant frequency f_b at the bottom of a glass of champagne (height H_ℓ for a volume V_ℓ), with a_0 still negligible. Neglect diffusion of $\mathrm{CO_2}$ at the free surface.

A.6	Write the differential equation for $c_\ell(t)$. Obtain from this equation the characteristic time τ for the decay of the concentration of dissolved $\mathrm{CO_2}$ in the liquid.	1.1pt

Part B. Acoustic emission of a bursting bubble

Small bubbles are nearly spherical as they reach the free surface. Once the liquid film separating the bubble from the air thins out sufficiently, a circular hole of radius r forms in the film and, driven by surface tension, opens very quickly (**Fig. 4.** left). The hole opens at constant speed v_f (**Fig. 4.** right). The film outside the rim remains still, with constant thickness h.

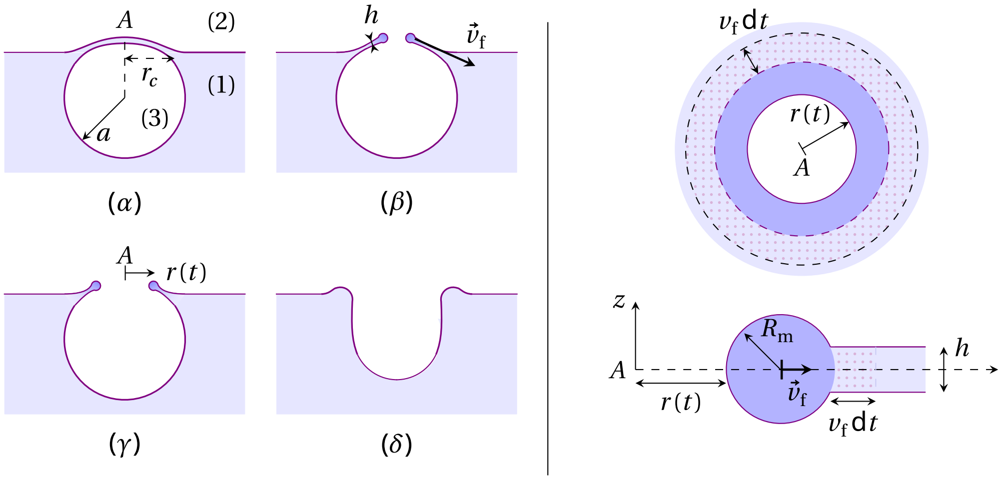

Fig. 4. *(Left)* (α) Bubble at the surface: (1) liquid, (2) air at pressure P_0 and (3), $\mathrm{CO_2}$ at pressure P_b, (β) and (γ) retraction of the liquid film, where the rim is in dark blue, (δ) bubble collapse. *(Right)* Retraction of the liquid film at time t. Top: sketch of the pierced film seen from above. Bottom: cross-section of the rim and the retracting film. During $\mathrm{d}t$ the rim accumulates nearby liquid (dotted).

Due to dissipative processes, only half of the difference of the surface energy between t and $t+\mathrm{d}t$ of the rim and the accumulated liquid is transformed into kinetic energy. We further assume that the variation of the surface of the rim is negligible compared to that of the film.

B.1	Express v_f in terms of ρ_ℓ, σ and h.	1.1pt

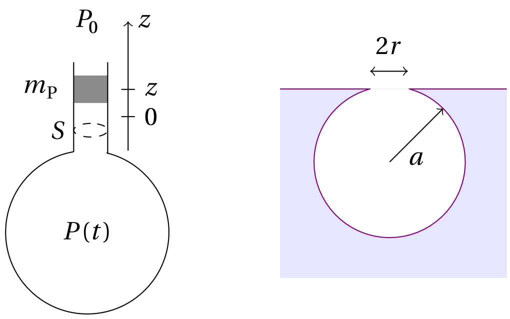

Fig. 5. *(Left)* a Helmholtz resonator. *(Right)* a bubble as an oscillator.

When the film bursts, it releases internal pressure and emits a sound. We model this acoustic emission by a Helmholtz resonator: a cavity open to the atmosphere at P_0 through a bottleneck aperture of area S (**Fig. 5**. left). In the neck, a mass m_p makes small amplitude position oscillations due to the pressure forces it experiences as the gas in the cavity expands or compresses adiabatically. The gravity force on m_p is negligible compared to pressure forces. Let V_0 be the volume of gas under the mass m_p for $P = P_0$ as $z = 0$.

B.2	Express the frequency of oscillation f_0 of m_p. Hint: for $\varepsilon \ll 1$, $(1+\varepsilon)^\alpha \approx 1 + \alpha\varepsilon$.	1.1pt

The Helmholtz model may be used for a bubble of radius a. V_0 is the volume of the closed bubble. From litterature, the mass of the equivalent of the piston is $m_p = 8\rho_g r^3/3$ where r is the radius of the circular aperture and $\rho_g = 1.8\,\mathrm{kg\cdot m^{-3}}$ is the density of the gas (**Fig. 5**. right). During the bursting process, r goes from 0 to r_c, given by $r_\mathrm{c} = \dfrac{2}{\sqrt{3}}a^2\sqrt{\dfrac{\rho_\ell g_0}{\sigma}}$. At the same time, the frequency of emitted sound increases until a maximum value of $40\,\mathrm{kHz}$ and the bursting time is $t_b = 3\times10^{-2}\,\mathrm{ms}$.

B.3	Find the radius a and the thickness h of the champagne film separating the bubble from the atmosphere.	1.1pt

Part C. Popping champagne

In a bottle, the total quantity of CO_2 is $n_\mathrm{T} = 0.2\,\mathrm{mol}$, either dissolved in the volume $V_\mathrm{L} = 750\,\mathrm{mL}$ of liquid champagne, or as a gas in the volume $V_\mathrm{G} = 25\,\mathrm{mL}$ under the cork (**Fig. 6**. left). V_G contains only CO_2. The equilibrium between both CO_2 phases follows Henry's Law. We suppose that the fast gaseous CO_2 expansion when the bottle is opened, is adiabatic and reversible. Ambient temperature T_0 and pressure $P_0 = 1\,\mathrm{bar}$ are constant.

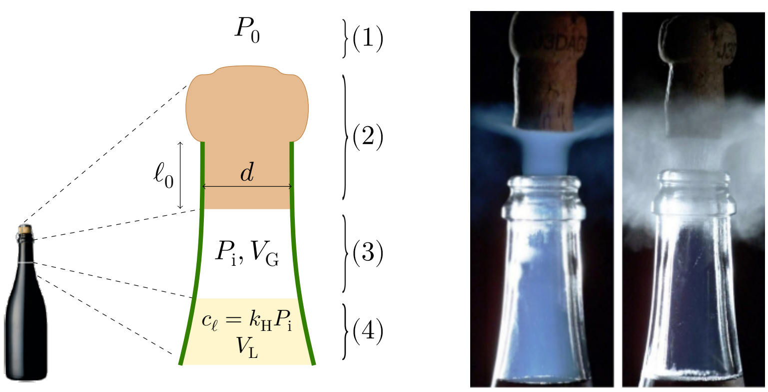

Fig. 6. *Left:* traditional bottleneck: (1) surrounding air, (2) cork stopper, (3) headspace, (4) liquid champagne. *Right*: Two phenomena observed while opening the bottle at two different temperatures (adapted from *[2]*).

C.1	Give the numerical value of the pressure P_i of gaseous $\mathrm{CO_2}$ in the bottle for $T_0 = 6\,°\mathrm{C}$ and $T_0 = 20\,°\mathrm{C}$.	0.4pt

Another step of champagne production (not described here) leads to the following values of P_i that we will use for the next questions: $P_\mathrm{i} = 4.69\,\mathrm{bar}$ at $T_0 = 6\,°\mathrm{C}$ and $P_\mathrm{i} = 7.45\,\mathrm{bar}$ at $T_0 = 20\,°\mathrm{C}$.

During bottle opening, two different phenomena can be observed, depending on T_0 (Fig. 6. right).

- either a blue fog appears, due to the formation of solid $\mathrm{CO_2}$ crystals (but water condensation is inhibited);
- or a grey-white fog appears, due to water vapor condensation in the air surrounding the bottleneck. In this latter case, there is no formation of $\mathrm{CO_2}$ solid crystals.

The saturated vapor pressure $P_\mathrm{sat}^{\mathrm{CO_2}}$ for the $\mathrm{CO_2}$ solid/gas transition follows : $\log_{10}\left(\dfrac{P_\mathrm{sat}^{\mathrm{CO_2}}}{P_0}\right) = A - \dfrac{B}{T+C}$ with T in K, $A = 6.81$, $B = 1.30 \times 10^3\,\mathrm{K}$ and $C = -3.49\,\mathrm{K}$.

C.2	Give the numerical value T_f of the $\mathrm{CO_2}$ gas at the end of the expansion, after opening a bottle, if $T_0 = 6\,°\mathrm{C}$ and if $T_0 = 20\,°\mathrm{C}$, if no phase transition occured. Choose which statements are true (several statements possible): 1. At $T_0 = 6\,°\mathrm{C}$ a grey-white fog appears while opening the bottle. 2. At $T_0 = 6\,°\mathrm{C}$ a blue fog appears while opening the bottle. 3. At $T_0 = 20\,°\mathrm{C}$ a grey-white fog appears while opening the bottle. 4. At $T_0 = 20\,°\mathrm{C}$ a blue fog appears while opening the bottle.	0.7pt

During bottle opening, the cork stopper pops out. We now determine the maximum height H_c it reaches. Assume that the friction force F due to the bottleneck on the cork stopper is $F = \alpha A$ where A is the area of contact and α is a constant to determine. Initially, the pressure force slightly overcomes the friction force. The cork's mass is $m = 10\,\mathrm{g}$, its diameter $d = 1.8\,\mathrm{cm}$ and the length of the cylindrical part initially stuck in the bottleneck is $\ell_0 = 2.5\,\mathrm{cm}$. Once the cork has left the bottleneck, you can neglect the net pressure force.

C.3	Give the numerical value of H_c if the external temperature is $T_0 = 6\,°\mathrm{C}$.	1.3pt

[1] Liger-Belair *et al,* Am. J. Enol. Vitic., Vol. 50, No. 3 (1999).

[2] Liger-Belair *et al.*, *Sc. Reports* **7**, 10938 (2017).