NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2019

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

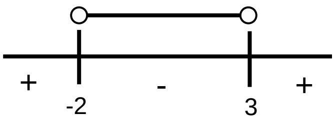
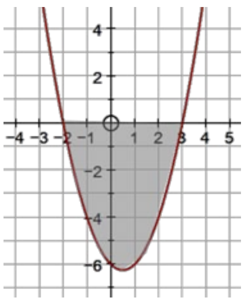
The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a student answers a question more than once, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

(a)(1)	$2(-2)^2+(-2)+k=0$ $8-2+k=0$ $k=-6$	correct subst. of –2 $k=-6$
(a)(2)	$\therefore 2x^2+x-6=0$ $(2x-3)(x+2)=0$ $\therefore$ Other root is $\frac{3}{2}$	Factors/correct subst. in formula $\frac{3}{2}$
(b)(1)	$x-2=3\sqrt{x+2}$ $(x-2)^2=\left(3\sqrt{x+2}\right)^2$ $x^2-4x+4=9(x+2)$ $x^2-13x-14=0$ $(x-14)(x+1)=0$ $x=14$ or $x=-1$ Check: $x=-1$ is not valid	Isolate surd x^2-4x+4 $9(x+2)$ $x^2-13x-14$ factors answer with selection
(b)(2)	$x^2-x-6\leq 0$ $(x-3)(x+2)\leq 0$ Crit. values: 3 ; –2  Solution: $-2\leq x\leq 3$	Factors/critical values Number line/graph $x\geq -2$ $x\leq 3$

QUESTION 2

(a)	$A = P(1+i)^n$ $A = 12\ 349\left(1+\frac{0,123}{52}\right)^1$ $A = \text{R}12\ 378,21$	Sub. P into correct formula $\frac{0,123}{52}$ $n = 1$
(b)	$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$ $12\ 349 = 94,75\left[\frac{1-\left(1+\frac{0,123}{52}\right)^{-52n}}{\frac{0,123}{52}}\right]$ $0,6917... = (1,00236...)^{-52n}$ $\log_{1,00236...} 0,6917... = -52n$ $n \approx 3$ years	$\frac{0,123}{52}$ Correct P & x in correct formula conversion to logs answer
(c)	$A = P(1-in)$ $A = 12\ 349(1-0,2\times 2)$ $A = 7\ 409,40$	$P = 12\ 349$ $0,2\times 2$ answer
(d)	Balance outstanding = A – F $= 12349\left(1+\frac{0,123}{52}\right)^{2\times 52} - \frac{94,75\left[\left(1+\frac{0,123}{52}\right)^{2\times 52}-1\right]}{\frac{0,123}{52}}$ $= 15\ 788,54384 - 11\ 156,97628$ $= \text{R}4\ 631,57$ Depreciated amount $= \text{R}7\ 409,40$ Therefore it would be sufficient. **OR** $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $P = \frac{94,75\left[1-\left(1+\frac{0,123}{52}\right)^{-52}\right]}{\frac{0,123}{52}}$ $P = \text{R}4\ 630,90$ Depreciated amount $= \text{R}7\ 409,40$ Therefore it would be sufficient.	Use of correct formula $n = 104$ in A-F formula $94,75$ rate $\frac{123}{5200}$ Answer Conclusion Correct Pv formula 94,75 in P formula $n \approx 52$ into formula rate $\frac{123}{5200}$ Answer Conclusion

QUESTION 3

(a)(1)	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-5(x+h)^2+(x+h)-(-5x^2+x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-5(x^2+2xh+h^2)+x+h+5x^2-x}{h}$ $f'(x)=\lim_{h\to 0}\frac{-5x^2-10xh-5h^2+x+h+5x^2-x}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-10x-5h+1)}{h}$ $f'(x)=\lim_{h\to 0}(-10x-5h+1)$ $=-10x+1$ **OR** $f(x+h)=-5(x+h)^2+(x+h)$ $f(x+h)=-5(x^2+2xh+h^2)+x+h$ $f(x+h)=-5x^2-10xh-5h^2+x+h$ $f(x+h)-f(x)=-5x^2-10xh-5h^2+x+h-(-5x^2+x)$ $f(x+h)-f(x)=-10xh-5h^2+h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-10x-5h+1)}{h}$ $f'(x)=\lim_{h\to 0}(-10x-5h+1)$ $=-10x+1$	$-5(x+h)^2+(x+h)$ Squaring & distributing Factorisation notation sub. in 0 to get $-10x+1$ $-5(x+h)^2+(x+h)$ Squaring & distributing Factorisation notation sub. in 0 to get $-10x+1$
(a)(2)	At: $x=1$, $f'(1)=-10(1)+1$ $f'(1)=-9$ $\therefore$ Eq. of tangent: $y=-9x+c$ Substitute: $(1;-4)$ $-4=-9(1)+c$ $c=5$ $\therefore y=-9x+5$ **OR** At: $x=1$, $f'(1)=-10(1)+1$ $f'(1)=-9$ Substitute: $(1;-4)$ $y-(-4)=-9(x-1)$ $\therefore y=-9x+5$	$f'(1)=-9$ Calculating y-coord of -4 Answer $f'(1)=-9$ Calculating y-coord of -4 Answer

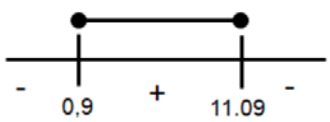
(b)(1)	$y=\dfrac{x^3+x^{\frac{3}{2}}}{x}$ $y=\dfrac{x^3}{x}+\dfrac{x^{\frac{3}{2}}}{x}$ $y=x^2+x^{\frac{1}{2}}$ $\dfrac{dy}{dx}=2x+\dfrac{1}{2}x^{-\frac{1}{2}}$ $\dfrac{dy}{dx}=2x+\dfrac{1}{2\sqrt{x}}$	x^2 $x^{\frac{1}{2}}$ $2x$ $\dfrac{1}{2}x^{-\frac{1}{2}}$
(b)(2)	$D_x\left[\dfrac{(2x-3)(4x^2+6x+9)}{(4x^2+6x+9)}\right]$ $D_x(2x-3)$ $=2$	$(2x-3)(4x^2+6x+9)$ $D_x(2x-3)$ $=2$

QUESTION 4

(4)(a)	$T_n = a + (n-1)d$ $T_n = 5 + (n-1)(4)$ $T_n = 4n + 1$ $100 = 4n + 1$ $4n = 99$ $n = 24\frac{3}{4}$ 24 pentagons	$d = 4$ $T_n = 4n + 1$ $T_n = 100$ Answer
(b)(1)	$T_1 = 3$ and $T_n = 47$ $S_n = 300$ $S_n = \frac{n}{2}(a + l)$ $300 = \frac{n}{2}(3 + 47)$ $n = 12$	Correct formula $300 = \frac{n}{2}(3 + 47)$ Answer
(b)(2)	$T_n = a + (n-1)d$ $47 = 3 + 11d$ $d = 4$	Correct formula $47 = 3 + 11d$ Answer
(c)	Series: $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ $r = \frac{1}{2}$, converging series $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$ $S_\infty = \frac{\frac{1}{4}}{1 - \frac{1}{2}}$ $S_\infty = \frac{1}{2}$	Expansion $r = \frac{1}{2}$ Correct formula Answer
(d)	$\frac{T_5}{T_3} = \frac{T_7}{T_5}$ $\frac{4}{5p+1} = \frac{1}{4}$ $p = 3$	Equating ratios Correct substitution Answer

QUESTION 5

(a)	50	50
(b)	$2a = -4$ $\quad 3a + b = 18$ $\quad a + b + c = 2$ $a = -2$ $\quad 3(-2) + b = 18$ $\quad (-2) + 24 + c = 2$ $b = 24$ $\quad c = -20$ $T_n = -2n^2 + 24n - 20$ **OR** $2a = -4$ $a = -2$ $T_n = -2n^2 + bn + c$ $T_1:\quad -2 + b + c = 2 \quad \therefore b + c = 4$ … eq 1 $T_2:\quad -8 + 2b + c = 20 \quad \therefore 2b + c = 28$ … eq 2 $T_2 - T_1:\quad b = 24$ Subst. in eq. 1: $24 + c = 4 \quad \therefore \quad c = -20$ $T_n = -2n^2 + 24n - 20$ **OR** $T_n = T_2(n-1) - T_1(n-2) + \frac{(n-1)(n-2)}{2} \times (2^{nd}$ diff.) $T_n = 20(n-1) - 2(n-2) + \frac{(n-1)(n-2)}{2} \times (-4)$ $T_n = 20n - 20 - 2n + 4 - 2(n^2 - 3n + 2)$ $T_n = 20n - 20 - 2n + 4 - 2n^2 + 6n - 4$ $T_n = -2n^2 + 24n - 20$	Determining differences $a = -2$ $b = 24$ $c = -20$ Determining differences $a = -2$ $b = 24$ $c = -20$ Determining differences $a = -2$ $b = 24$ $c = -20$

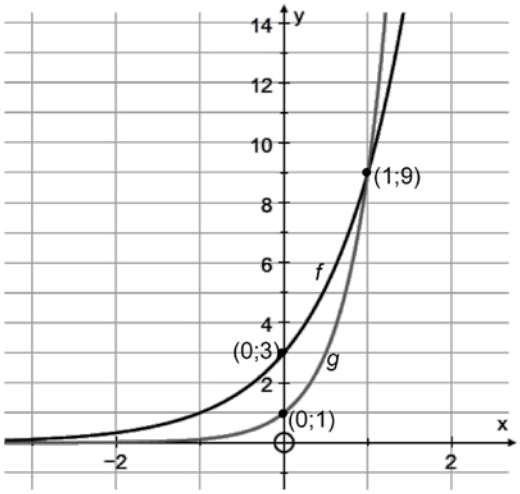
(c)	$T_n = -2n^2 + 24n - 20$ $T_n = -2(n^2 - 12n + 10)$ $T_n = -2\left[(n-6)^2 - 26\right]$ $T_n = -2(n-6)^2 + 52$ Max. passengers is 52 **OR** $T_n' = -4n + 24$ $-4n + 24 = 0$ $n = 6$ Subst. $n = 6$ $T_n = -2(6)^2 + 24(6) - 20$ $T_6 = 52$ Max. passengers is 52	Determining n n = 6 Answer Determining n $n = 6$ Answer
(d)	Let n be 12 stops $T_{12} = -2(12)^2 + 24(12) - 20$ $\therefore T_{12} = -20$ Invalid due to negative answer **OR** $-2n^2 + 24n - 20 \geq 0$ Passengers must be ≥ 0 Crit. Values: $6 \pm \sqrt{26}$  Hence $0{,}9 \leq n \leq 11{,}09$	Substitution Explanation Substitution Explanation

SECTION B

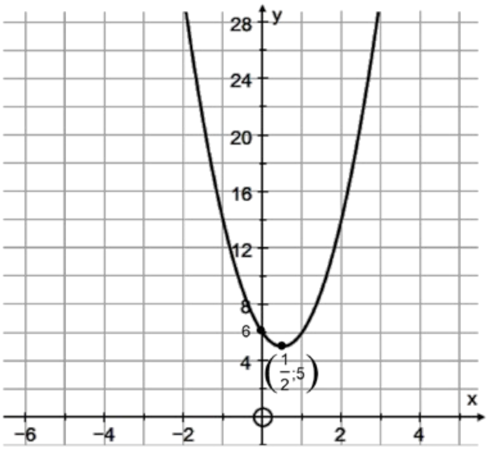
QUESTION 6

(a)	$f(x) > g(x)$ for $0 < x < 3$	$x > 0$ $x < 3$
(b)	$g(x) = \log_a x$ subst. $(3;1)$ $1 = \log_a 3$ $a = 3$ $f(x) = \sqrt{kx}$ subst. $(3;1)$ $1 = \sqrt{3k}$ $(1)^2 = \left(\sqrt{3k}\right)^2$ $k = \frac{1}{3}$	Substitution Answer Substitution Answer
(c)	$f : y = \sqrt{\frac{1}{3}x}$ $f^{-1} : x = \sqrt{\frac{1}{3}y}$ $x^2 = \frac{1}{3}y$ $y = 3x^2$ for $x \geq 0$	Changing x and y $y = 3x^2$ Domain: $x \geq 0$

QUESTION 7

(a)		Shape of f and g Intersection: (1;9) **Graph g:** Horizontal asymptote (0;1) **Graph f:** Horizontal asymptote (0;3)
(b)	$a^{2x} = 3^{x+1}$ $\frac{a^{2x}}{3^x} = 3$ $\left(\frac{a^2}{3}\right)^x = 3$ $\log 3_{\left(\frac{1}{3}a^2\right)} = x$ **OR** $x = \log_{\frac{a^2}{3}} 3$ **OR** $x = \frac{\log 3}{\log a^2 - \log 3}$	Isolating x : $\left(\frac{a^2}{3}\right)^x$ Conversion to logs Answer

QUESTION 8

(a)(1)	$(2x-1)^2 \geq 0$	Answer
(a)(2)		Shape T.P. $\left(\frac{1}{2};5\right)$ y-int.: $(0;6)$
(a)(3)	Shift 5 units down	Answer
(a)(4)	$(2x-1)^2 = k$ $4x^2 - 4x + (1-k) = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1-k)}}{2(4)}$ $x = \frac{4 \pm \sqrt{16k}}{8}$ $x = \frac{4 \pm 4\sqrt{k}}{8}$ $x = \frac{1 \pm \sqrt{k}}{2}$ Roots are real for $k \geq 0$ **OR** $(2x-1)^2 = k$ $4x^2 - 4x + (1-k) = 0$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1-k)}}{2(4)}$ $x = \frac{4 \pm \sqrt{16k}}{8}$ Roots are real for $16k \geq 0 \therefore k \geq 0$ **OR** $(2x-1)^2 = k$ $2x - 1 = \pm\sqrt{k}$ $x = \frac{1 \pm \sqrt{k}}{2}$ Roots are real for $k \geq 0$	Sub in formula $x = \frac{1 \pm \sqrt{k}}{2}$ $k \geq 0$ Sub in formula $x = \frac{4 \pm \sqrt{16k}}{8}$ $16k \geq 0 \therefore k \geq 0$ $2x - 1 = \pm\sqrt{k}$ $x = \frac{1 \pm \sqrt{k}}{2}$ $k \geq 0$

(a)(5)	$y = 4x^2 - 4x + (1+k)$ For real, unequal and rational, $\Delta > 0$ and perfect square $\Delta = (-4)^2 - 4(4)(1+k)$ $\Delta = -16k$ $\therefore k = -1$, $k = -\frac{1}{4}$, $k = -\frac{1}{16}$ etc. **OR** $y = 4x^2 - 4x + (1+k)$ Solve for $4x^2 - 4x + (1+k) = 0$ through trial & improvement: When $k = -1$ roots are real, rational & unequal When $k = -4$ roots are real, rational & unequal, etc.	$\Delta = -16k$ accurate value of k accurate value of k accurate value of k $4x^2 - 4x + (1+k) = 0$ accurate value of k accurate value of k accurate value of k
(b)	$px^2 + qx + r = 0$ $x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$ $\therefore P = \frac{-q + \sqrt{q^2 - 4pr}}{2p}$ $x = \frac{-q \pm \sqrt{q^2 - 4pr}}{2}$ $\therefore Q = \frac{-q + \sqrt{q^2 - 4pr}}{2}$ For: $P : Q$, $\frac{1}{p}\left[\frac{-q + \sqrt{q^2 - 4pr}}{2}\right] : 1\left[\frac{-q + \sqrt{q^2 - 4pr}}{2}\right]$ Ratio: $\frac{1}{p} : 1$ **OR** Ratio: $1 : p$	$P = \frac{-q + \sqrt{q^2 - 4pr}}{2p}$ $Q = \frac{-q + \sqrt{q^2 - 4pr}}{2}$ Answer

QUESTION 9

(a)(1)	$y = a(x - x_1)(x - x_2)(x - x_3)$ $y = a(x+3)(x+3)\left(x - \frac{1}{2}\right)$ Subst.: (0;9) $a = -2$ $y = -2(x+3)^2\left(x - \frac{1}{2}\right)$ $y = -2\left(x - \frac{1}{2}\right)\left(x^2 + 6x + 9\right)$ $y = -2\left(x^3 + 6x^2 + 9x - \frac{1}{2}x^2 - 3x - 4\frac{1}{2}\right)$ $y = -2\left(x^3 + 5\frac{1}{2}x^2 + 6x - 4\frac{1}{2}\right)$ $y = -2x^3 - 11x^2 - 12x + 9$ **OR** $y = a(x+3)(x+3)(2x-1)$ Subst.: (0;9) $a = -1$ $y = -1(x+3)(x+3)(2x-1)$ $y = -1\left(x^2 + 6x + 9\right)(2x-1)$ $y = -1(2x^3 + 12x^2 + 18x - x^2 - 6x - 9)$ $y = -1(2x^3 + 11x^2 + 12x - 9)$ $y = -2x^3 - 11x^2 - 12x + 9$	formula Substitution of intercepts Subst. of (0;9) $a = -2$ Answer showing a, b, c and d formula Substitution of intercepts Subst. of (0;9) $a = -1$ Answer showing a, b, c and d
(a)(2)	$f(x) = -2x^3 - 11x^2 - 12x + 9$ $f'(x) = -6x^2 - 22x - 12$ $f''(x) = -12x - 22$ $-12x - 22 = 0$ $x = -\frac{11}{6}$	$f'(x) = -6x^2 - 22x - 12$ $f''(x) = -12x - 22$ $x = -\frac{11}{6}$
(b)	$f'(x) = 8$ $-6x^2 - 22x - 12 = 8$ $-6x^2 - 22x - 20 = 0$ $x = -\frac{5}{3}$ or $x = -2$ $E(-2;5)$	$f'(x) = 8$ $x = -2$ $y = 5$

(c)	$y = \frac{2}{x+p} + q$ $p = 3$ $y = \frac{2}{x+3} + q$ subst. (–2;5) $5 = \frac{2}{-2+3} + q$ $q = 3$ $\therefore y = \frac{2}{x+3} + 3$	$x + 3$ Substitution $q = 3$
(d)	$y = x + 6$ **OR** Line goes through (–3;3) $y = x + c$ ... substitute (–3;3) $\therefore y = x + 6$	$y = x$ $y = x + 6$ (–3;3) $y = x + 6$
(e)	$(-\infty;-3) \cup [-2;0]$	$(-\infty;-3)$ ( ) due to asymptote $[-2;0]$
(f)	$h(x) - k = -2x^3 - 11x^2 - 12x + 9$ For h: y-int. (0;9) For g: y-int. $\left(0;\frac{11}{3}\right)$ $k < -\frac{16}{3}$	y-int. (0;9) y-int. $\left(0;\frac{11}{3}\right)$ $k < -\frac{16}{3}$

QUESTION 10

	$V=\pi r^2h+\frac{4}{3}\pi r^3$ $1000=\pi r^2h+\frac{4}{3}\pi r^3$ $\pi r^2h=1000-\frac{4}{3}\pi r^3$ $h=\frac{1}{\pi r^2}\left(1000-\frac{4}{3}\pi r^3\right)$ …. *Eq. 1* *Total S.A.(S) = S.A. Cylinder + S.A. Sphere* $S=2\pi rh+4\pi r^2$ …. *Sub. Eq. 1* $S=2\pi r\left[\frac{1}{\pi r^2}\left(1000-\frac{4}{3}\pi r^3\right)\right]+4\pi r^2$ $S=\frac{2}{r}\left(1000-\frac{4}{3}\pi r^3\right)+4\pi r^2$ $S=2000r^{-1}-\frac{8}{3}\pi r^2+4\pi r^2$ $S=2000r^{-1}+\frac{4}{3}\pi r^2$ $\frac{dS}{dr}=-2000r^{-2}+\frac{8}{3}\pi r$ $\frac{8\pi r}{3}-\frac{2000}{r^2}=0$ $8\pi r^3-6000=0$ $r^3=\frac{6000}{8\pi}$ $r\approx 6{,}2$ for S.A to be a minimum	$V=\pi r^2h+\frac{4}{3}\pi r^3$ Making *h* subject of formula $h=\frac{1}{\pi r^2}\left(1000-\frac{4}{3}\pi r^3\right)$ $S=2\pi rh+4\pi r^2$ $2000r^{-1}+\frac{4}{3}\pi r^2$ $\frac{dS}{dr}=-2000r^{-2}+\frac{8}{3}\pi r$ Answer

QUESTION 11

(a)	$\frac{10!}{5!\times 2!\times 3!}$ $= 2\,520$	$\frac{10!}{[\ ]}$ $\frac{[\ ]}{5!\times 2!\times 3!}$ Answer
(b)(1)	$P(\text{not picked}) = 0{,}3 \times 0{,}65$ $= 0{,}195$	$0{,}3$ and $0{,}65$ multiplication of above Answer
(b)(2)	$P(\text{made into juice})$ $= (0{,}7 \times 0{,}6) + (0{,}3 \times 0{,}35 \times 0{,}6)$ $= 0{,}483$ $\therefore$ Approx. 48,3% will be made into juice **OR** $P(\text{made into juice})$ $= (1 - 0{,}195) \times 0{,}6$ $= 0{,}483$ $\therefore$ Approx. 48,3% will be made into juice	indicating 0,6 or 60% $(0{,}7 \times 0{,}6)$ $(0{,}3 \times 0{,}35 \times 0{,}6)$ $0{,}483$ indicating 0,6 or 60% $(1 - ...)$ $(1 - 0{,}195) \times 0{,}6$ $0{,}483$

(b)(3)	Total oranges exported $= 120 \times 172$ $= 20\ 640$ $P(e\text{xported})$ $= (0{,}7 \times 0{,}09) + (0{,}3 \times 0{,}35 \times 0{,}09)$ $= 0{,}07245$ $\therefore 7{,}245\%$ exported Let total number of oranges $= x$ $\frac{20\ 640}{x} \times 100 = 7{,}245$ $\therefore x = 284\ 886$ oranges in total **OR** Total oranges exported $= 120 \times 172$ $= 20\ 640$ $P(e\text{xported})$ $= (1 - 0{,}195) \times 0{,}09$ $= 0{,}07245$ $\therefore 7{,}245\%$ exported Let total number of oranges $= x$ $\frac{20\ 640}{x} \times 100 = 7{,}245$ $\therefore x = 284\ 886$ oranges in total	20 640 $(0{,}7 \times 0{,}09)$ $(0{,}3 \times 0{,}35 \times 0{,}09)$ Answer 20 640 $(1 - 0{,}195)$ $(1 - 0{,}195) \times 0{,}09$ Answer

Total: 150 marks