NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

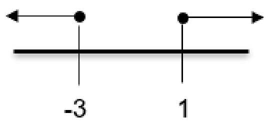
The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

NOTE:

- If a candidate answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

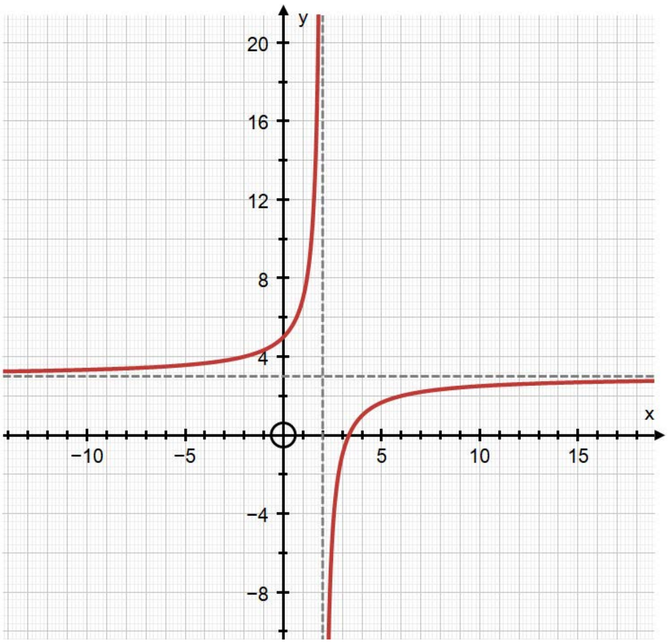
QUESTION 1

(a)(1)	$px^2+2x-3=0$ $x=\dfrac{(-2)\pm\sqrt{(2)^2-4(p)(-3)}}{2(p)}$ $x=\dfrac{-2\pm\sqrt{4+12p}}{2p}$ $x=\dfrac{-2\pm 2\sqrt{1+3p}}{2p}$ $x=\dfrac{-1\pm\sqrt{1+3p}}{p}$	use of quadratic formula $x=\dfrac{-2\pm\sqrt{4+12p}}{2p}$ simplified solution
(a)(2)	Non-real roots for: $1+3p<0$ $p<-\dfrac{1}{3}$	$\Delta<0$ $p<-\dfrac{1}{3}$ No marks for $\Delta>0$
(b)	$\sqrt{x-2}+4=x$ $x^2-9x+18=0$ $(x-2)=(x-4)^2$ $x-2=x^2-8x+16$ $x=6$ or $x=3$ n/v for $x=3$	Isolate surd x^2-4x+4 $x^2-8x+16$ factors answer with selection
(c)	$(x+3)(x-1)\geq 0$ Crit. values: -3 ; 1  $x\leq -3$ or $x\geq 1$	Number line/graph $x\leq -3$ or $x\geq 1$

QUESTION 2

(a)	$x^{\frac{2}{3}}=4$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}}=\left(2^2\right)^{\frac{3}{2}}$ $x=\pm 8$ **Alternate:** $\sqrt[3]{x^2}=4$ $\left(\sqrt[3]{x^2}\right)^3=(4)^3$ $x^2=64$ $x=8$ or $x=-8$	$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}}=\left(2^2\right)^{\frac{3}{2}}$ $x=8$ $x=-8$ $\left(\sqrt[3]{x^2}\right)^3=(4)^3$ $x=8$ $x=-8$
(b)	$x^2+1=x-y$ Sub: $y=2-3x$ $x^2+1=4x-2$ $x^2-4x+3=0$ $x=1$ or $x=3$ When $x=1$; $y=-1$ When $x=3$; $y=-7$ **Alternate:** $y=2-3x$ ... eq 1 $3^{x^2+1}=3^{x-y}$ ... sub. eq 1 $3^{x^2+1}=3^{x-(2-3x)}$ $3^{x^2+1}=3^{4x-2}$ $x^2+1=4x-2$ $x^2-4x+3=0$ $x=1$ or $x=3$ When $x=1$; $y=-1$ When $x=3$; $y=-7$	$x^2+1=x-y$ Sub: $y=2-3x$ $x^2+1=4x-2$ $x=1$ $y=-1$ $x=3$ $y=-7$ $3^{x^2+1}=\frac{3^x}{3^y}$ ... sub. eq 1 $3^{x^2+1}=3^{4x-2}$ $x^2+1=4x-2$ $x=1$ $y=-1$ $x=3$ $y=-7$
(c)	$A=P(1+i)^n$ $25\,000=20\,000\left(1+\frac{4}{100}\right)^n$ $\frac{5}{4}=(1{,}04)^n$ $n=\log_{1{,}04}\left(\frac{5}{4}\right)$ $n\approx 5{,}7$ years After 6 years	$25\,000=20\,000\left(1+\frac{4}{100}\right)^n$ $n=\log_{1{,}04}\left(\frac{5}{4}\right)$ $n\approx 5{,}7$ years 6 years

QUESTION 3

(a)	$f(0)=3-\dfrac{4}{0-2}$ $f(0)=5$	$f(0)=5$
(b)	$3-\dfrac{4}{x-2}=0$ $3(x-2)-4=0$ restr. $x\neq 2$ $3x-6-4=0$ $x=\dfrac{10}{3}$ $x=3\dfrac{1}{3}$	$3(x-2)-4=0$ $x=3\dfrac{1}{3}$
(c)		Shape Vertical Asymptote Horizontal Asymptote Intercepts
(d)(1)	$f(x+p)=3-\dfrac{4}{x+p-2}$ $f(x+p)=-\dfrac{4}{[x+(p-2)]}+3$	$f(x+p)=3-\dfrac{4}{x+p-2}$
(d)(2)	Graph of *f* will shift *p* units to the right	Explanation

(e)(1)	For $f^{-1}(x)$: $x = 3 - \frac{4}{y-2}$ $x = 3 - \frac{4}{y-2}$ $\frac{4}{y-2} = 3 - x$ $4 = (3-x)(y-2)$ $4 = 3y - 6 - xy + 2x$ $3y - xy = 4 + 6 - 2x$ $y(3-x) = 10 - 2x$ $y = \frac{10-2x}{3-x}$ $\therefore f^{-1}(x) = \frac{10-2x}{3-x}$ **Alternate final answer:** $f^{-1}(x) = \frac{2x-10}{x-3}$	$x = 3 - \frac{4}{y-2}$ $4 = (3-x)(y-2)$ $\therefore f^{-1}(x) = \frac{10-2x}{3-x}$ **Alternate final answer:** $f^{-1}(x) = \frac{2x-10}{x-3}$ **Alternate final answer:** $y = -\frac{4}{x-3} + 2$
(e)(2)	Domain of $f^{-1}(x)$: $x \in R$; $x \neq 3$	$x \in R$; $x \neq 3$

QUESTION 4

(4)(a)	$ar^2 = 7$ $ar^5 = -2\,401$ $\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$ $\therefore r^3 = -343$ $\therefore r = -7$ $T_n = a\left(-7\right)^{n-1}$ $T_3 = a\left(-7\right)^{3-1} = 7$ $a = \frac{7}{49}$ $\therefore a = \frac{1}{7}$	$ar^2 = 7$ $ar^5 = -2\,401$ $\therefore \frac{ar^5}{ar^2} = -\frac{2\,401}{7}$ $r = -7$ $a = \frac{1}{7}$
(4)(b)(1)	3 7 15 27 Sequence 4 8 12 First Difference 4 4 Constant second differ	Sequence First Difference Constant second differ
(b)(2)	$2a = 4 \quad \therefore a = 2$ $3a + b = 4 \quad \therefore \quad b = -2$ $a + b + c = 3 \quad \therefore \quad c = 3$ $T_n = 2n^2 - 2n + 3$ **Alternate:** $T_n = 7(n-1) - 3(n-2) + \frac{(n-1)(n-2)}{2} \times (4)$ $T_n = 7n - 7 - 3n + 6 + \left(n^2 - 3n + 2\right)\left(2\right)$ $T_n = 2n^2 - 2n + 3$	Method $a = 2$ $b = -2$ $c = 3$ Method $a = 2$ $b = -2$ $c = 3$

QUESTION 5

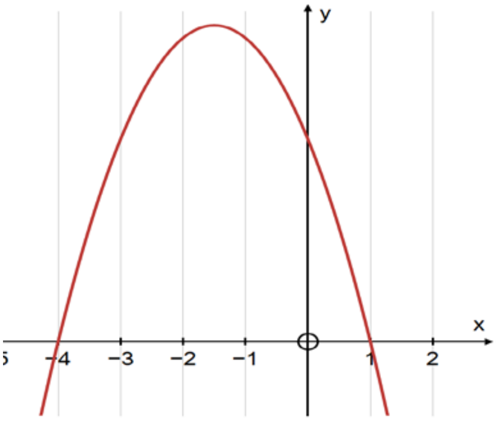
(a)	$g(x)=\log_t x$ sub. $(2;-1)$ $-1=\log_t 2$ $t^{-1}=2$ $t=\frac{1}{2}$	$-1=\log_t 2$ $t=\frac{1}{2}$
(b)	X-int. of normal/standard log graph is always: $(1;0)$ since $\log_t 1=0$ $\therefore \text{C}(1;0)$ **Alternate:** For Co-ord. of C: X-int, let $y=0$ $y=\log_{\frac{1}{2}} x$ $0=\log_{\frac{1}{2}} x$ $x=\left(\frac{1}{2}\right)^0$ $x=1$ $\therefore \text{C}(1;0)$	$\therefore \text{C}(1;0)$ $\therefore \text{C}(1;0)$
(c)	$f(x)=2p^x+q$ $q=-1$ since asymptote passes through $\text{A}(2;-1)$ $f(x)=2p^x-1$ ... sub. $(1;0)$ $0=2p^1-1$ $\therefore p=\frac{1}{2}$	$q=-1$ $0=2p^1-1$ $p=\frac{1}{2}$
(d)	D is the y-int of f: let $x=0$ $f(x)=2\times\left(\frac{1}{2}\right)^x-1$ ... sub. $x=0$ $y=2\times\left(\frac{1}{2}\right)^0-1$ $y=1$ $\therefore \text{D}(0;1)$	$y=2\times\left(\frac{1}{2}\right)^0-1$ $\therefore \text{D}(0;1)$
(e)	$f(x)=2\left(\frac{1}{2}\right)^x-1$ ... sub. $\text{B}(2;y)$ $f(x)=2\left(\frac{1}{2}\right)^2-1$ $f(x)=y=-\frac{1}{2}$ Length of AB $=\frac{1}{2}$	$f(x)=2\left(\frac{1}{2}\right)^2-1$ Length of AB $=\frac{1}{2}$
(f)	Range of f: $y>-1$	$y>-1$

QUESTION 6

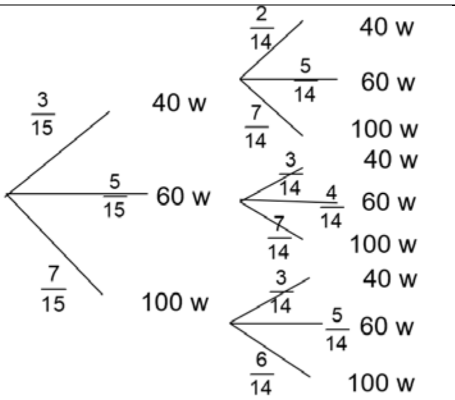
(a)	$f(x)=1-2x+x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{1-2(x+h)+(x+h)^2-(1-2x+x^2)}{h}$ $f'(x)=\lim_{h\to 0}\frac{1-2x-2h+x^2+2xh+h^2-1+2x-x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2h+2xh+h^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-2+2x+h)}{h}$ $f'(x)=\lim_{h\to 0}(-2+2x+h)$ $2x-2$	$1-2(x+h)+(x+h)^2$ $-(1-2x+x^2)$ Squaring and distributing Factorisation Notation Sub. to get: $2x-2$
(b)	$y=x^{10}+10x$ $\frac{dy}{dx}=10x^9+10$	$10x^9$ 10
(c)	$y=\frac{5}{x^3}+\frac{x^{\frac{1}{2}}}{x^3}$ $y=5x^{-3}+x^{-\frac{5}{2}}$ $\frac{dy}{dx}=-15x^{-4}-\frac{5}{2}x^{-\frac{7}{2}}$	$y=5x^{-3}+x^{-\frac{5}{2}}$ $\frac{dy}{dx}=-15x^{-4}-\frac{5}{2}x^{-\frac{7}{2}}$ Penalise 1 for notation

SECTION B

QUESTION 7

(a)	For: $x < -4$ and $x > 1$	$x < -4$ $x > 1$
(b)		Shape X-Intercepts
(c)	$k > p$ or $k < q$	$k > p$ $k < q$
(d)	$x > -1\frac{1}{2}$	$x > -1\frac{1}{2}$

QUESTION 8

(a)(1)	8^6	8^6
(a)(2)	$= 20\,160$	$8 \times 7 \times 6 \times 5 \times 4 \times 3$ 20 160
(b)(1)		$\frac{3}{15}$; $\frac{5}{15}$ and $\frac{7}{15}$ $\frac{\square}{14}$ Branches with correct values
(b)(2)	$\left(\frac{5}{15} \times \frac{7}{14}\right) + \left(\frac{7}{15} \times \frac{5}{14}\right)$ $= \frac{1}{3}$	$\left(\frac{5}{15} \times \frac{7}{14}\right)$ $\left(\frac{7}{15} \times \frac{5}{14}\right)$ $= \frac{1}{3}$
(c)	$P(A \cap B) = P(A) \times P(B)$ $\therefore P(A \cap B) = 0{,}08 \times 0{,}02$ $= 0{,}0016$ but $P(AUB) = P(A) + P(B) - P(A \cap B)$ $\therefore P(AUB) = 0{,}08 + 0{,}02 - 0{,}0016$ $= 0{,}0984$ **Alternate:** P(at least one win) $=$ P(one or more wins) $= 1 -$ P(no wins) $= 1 - P(L) \times P(L)$ $= 1 - 0{,}98 \times 0{,}92$ $= 0{,}0984$	$\therefore P(A \cap B) = 0{,}0016$ $P(AUB) = P(A) + P(B) -$ $P(A \cap B)$ $\therefore P(AUB) = 0{,}08 + 0{,}02 - ...$ $= 0{,}0984$ $= 1 -$ P(no wins) 0,98 0,92 $= 0{,}0984$

QUESTION 9

(a)	$a = 725$ $b = 190$	$a = 725$ $b = 190$
(b)	$h = k(x-a)^2 + b$ $h = k(x-725)^2 + 190$ sub. $(0;315)$ $315 = k(0-725)^2 + 190$ $k = \frac{1}{4\ 205}$ $h = \frac{1}{4\ 205}(x-725)^2 + 190$ ... sub. $(x;210)$ $210 = \frac{1}{4\ 205}(x-725)^2 + 190$ $x = 1015$ or $x = 435$ Therefore the horizontal distance of hygrometer from the left tower is 435 m.	$h = k(x-725)^2 + 190$ $k = \frac{1}{4\ 205}$ $210 = \frac{1}{4\ 205}(x-725)^2$ $+190$ $x = 1015$ or $x = 435$ Therefore the horizontal distance of hygrometer from the left tower is 435 m.

QUESTION 10

(a)	$F = 8\,755\left[\dfrac{\left(1+\dfrac{6,7}{400}\right)^{(5\times 4)}-1}{\dfrac{6,7}{400}}\right]$ $F = 205\,973,485$ Total cost of shares $= 8\,755 \times 4 \times 5$ Total cost of shares $= 175\,100$ Total Profit $= 30\,873,485$ % Profit $= \dfrac{30\,873,485}{175\,100} \times 100$ $= 17,6319\,\%$ $\approx 17,6\%$	Inside square bracket Correct ***X*** in correct formula $F = 205\,973,485$ $175\,100$ $30\,873,485$ $\approx 17,6\%$
	Alternate: $F = 8\,755\left[\dfrac{\left(1+\dfrac{6,7}{400}\right)^{(5\times 4)}-1}{\dfrac{6,7}{400}}\right]$ $F = 205\,973,485$ Total cost of shares $= 8\,755 \times 4 \times 5$ Total cost of shares $= 175\,100$ $\therefore$ %Profit $= \dfrac{205\,973,485}{175100}$ $= 1,176319$ $\therefore 17,6\%$	Inside square bracket Correct ***X*** in correct formula $F = 205\,973,485$ $175\,100$ % Profit $= \dfrac{205\,973,485}{175\,100}$ $\approx 17,6\%$

(b)	$300\,000 = x\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(15\times12)}}{\dfrac{9,5}{1200}}\right]$ $x = 3\,132,674049$ Balance of loan $= A - F$ $A = 300\,000\left(1+\dfrac{9,5}{1200}\right)^{12\times5}$ $A = 481\,502,8408$ $F = 3\,132,674049\left[\dfrac{\left(1+\dfrac{9,5}{1200}\right)^{(12\times5)}-1}{\dfrac{9,5}{1200}}\right]$ $F = 239\,405,9954$ Balance of loan $=(481\,502,8408)-(239\,405,9954)$ $= 242\,096,8454$ $\approx 242\,096,85$ **Alternate:** $P = 3\,132,674049\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(10\times12)}}{\dfrac{9,5}{1200}}\right]$ $P = 242\,096,8454$ $\approx 242\,096,85$ No, there would be a shortfall of R36 123,36	300 000 Inside the square bracket $x = 3\,132,674049$ $A = 481\,502,8408$ $F = 239\,405,9954$ $=(481\,502,8408)-(239\,405,9954)$ Comparison between 10a and 10b with conclusion ………………………………… 300 000 Inside the square bracket $x = 3\,132,674049$ $P = 3\,132,674049\left[\dfrac{1-\left(1+\dfrac{9,5}{1200}\right)^{-(10\times12)}}{\dfrac{9,5}{1200}}\right]$ No. of years: -120 $\approx 242\,096,85$ Comparison between 10a and 10b with conclusion

QUESTION 11

(a)	$\sum_{i=1}^{\infty}\frac{k}{2^i}+\sum_{i=1}^{10}2^{2i}>1000\,000$ Working with: $\sum_{i=1}^{\infty}\frac{k}{2^i}$ $T_1=\frac{k}{2}$; $T_2=\frac{k}{4}$; $T_3=\frac{k}{8}$ Common ratio: $\frac{k}{4}\div\frac{k}{2}$ $r=\frac{1}{2}$ $S_\infty=\frac{a}{1-r}$ for $-1<r<1$ $S_\infty=\frac{\frac{k}{2}}{1-\frac{1}{2}}$ $S_\infty=k$ Working with: $\sum_{i=1}^{10}2^{2i}$ $T_1=2^2$; $T_2=2^4$; $T_3=2^6$ Common ratio: $r=4$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq 1$ $S_{10}=\frac{4\left(4^{10}-1\right)}{4-1}$ $S_{10}=1\,398\,100$ $\therefore\sum_{i=1}^{\infty}\frac{k}{2i}+\sum_{i=1}^{10}2^{2i}>1\,000\,000$ can be rewritten as $k+1\,398\,100>1000\,000$ $k>-398\,100$ $\therefore k=-398\,099\quad\left(k\in\mathbb{Z}\right)$	$r=\frac{1}{2}$ Correct substitution into correct formula to get $S_\infty=k$ $r=4$ Correct substitution into correct formula to get $S_{10}=1\,398\,100$ $k+1\,398\,100>1\,000\,000$ $\therefore k=-398\,099\quad\left(k\in\mathbb{Z}\right)$
(b)(1)	$5+\frac{15}{2}+10+...+\frac{505}{2}$ Common difference of $\frac{5}{2}$; series is arithmetic $T_n=a+(n-1)d$ $\frac{505}{2}=5+(n-1)\left(\frac{5}{2}\right)$ $250=\frac{5}{2}n$ $n=100$	$\text{d}=\frac{5}{2}$ Correct substitution in the correct formula $n=100$

(b)(2)	Middle 30 terms would be: T_{36} to T_{65} $T_{36}=5+(35)\left(\frac{5}{2}\right)$ $T_{36}=\frac{185}{2}$ Let $a=\frac{185}{2}$; $d=\frac{5}{2}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $S_{30}=\frac{30}{2}\left[2\left(\frac{185}{2}\right)+(29)\left(\frac{5}{2}\right)\right]$ $S_{30}=3\,862{,}5$ **Alternate:** Middle 30 terms would be: T_{36} to T_{65} $T_{36}=5+(35)\left(\frac{5}{2}\right)$ $T_{36}=\frac{185}{2}$ $T_{65}=5+(64)\left(\frac{5}{2}\right)$ $T_{65}=165$ $S_n=\frac{n}{2}(a+l)$ $S_{30}=\frac{30}{2}\left(\frac{185}{2}+165\right)$ $S_{30}=3\,862{,}5$	T_{36} $T_{36}=\frac{185}{2}$ Correct substitution into correct formula $S_{30}=3\,862{,}5$ T_{36} $T_{36}=\frac{185}{2}$ Correct substitution into correct formula $S_{30}=3\,862{,}5$

QUESTION 12

12	Let: $g(1) = h(1)$ $(1)^3 - a(1)^2 + 6 = 2(1)^2 + b(1) + 3$ $1 - a + 6 = 2 + b + 3$ $a = 2 - b$ ... eq1 $g'(x) = 3x^2 - 2ax$ $h'(x) = 4x + b$ $g'(1) = h'(1)$ $3(1)^2 - 2a(1) = 4(1) + b$ $3 - 2a = 4 + b$ ... sub eq1: $a = 2 - b$ $3 - 2(2 - b) = 4 + b$ $b = 5$ $a = -3$ $h(x) = 2x^2 + 5x + 3$ $h(1) = 10$ Point of contact is: $(1;10)$	$g(1) = h(1)$ $a = 2 - b$ ... eq1 $g'(x) = 3x^2 - 2ax$ $h'(x) = 4x + b$ $g'(1) = h'(1)$ $2a + b = -1$ $b = 5$ $(1;10)$

QUESTION 13

13	$8x+4x+4h=P$ $P=12x+4h$ $P-12x=4h$ $\therefore h=\frac{1}{4}P-3x$ $V=l\times b\times h$ $V=(2x)(x)(h)$ ... sub.: $h=\frac{1}{4}P-3x$ $V=(2x)(x)\left(\frac{1}{4}P-3x\right)$ $V=\frac{1}{2}x^2P-6x^3$ $V'=Px-18x^2$ $0=x\left(P-18x\right)$ $x=0$ or $x=\frac{P}{18}$ Hence, length of the box is $2x=\frac{P}{9}$ $\therefore$ length of box is $\frac{1}{9}P$ cm when the volume is a maximum.	$8x+4x+4h=P$ $h=\frac{1}{4}P-3x$ $V=(2x)(x)\left(\frac{1}{4}P-3x\right)$ $V=\frac{1}{2}x^2P-6x^3$ $V'=Px-18x^2$ $0=x\left(P-18x\right)$ $P=18x$ Length of the box is $2x$ and $P=18x$

Total: 150 marks