Please paste the barcoded label here	TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

MATHEMATICS: PAPER I

EXAMINATION NUMBER													

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 30 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
6. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers.

   Answers only will NOT necessarily be awarded full marks.
7. Diagrams are not necessarily drawn to scale.
8. If necessary, round off answers to **ONE** decimal place, unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. Five blank pages (pages 26 to 30) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
13	14	14	13	11	11	9	14	8	14	13	8	8	/150

SECTION A

QUESTION 1

(a) Given: $3 - 2x = px^2; p \neq 0$

(1) Solve for x in terms of p, leaving your answer in simplest form.

(3)

(2) Hence, or otherwise, determine the values of p for which the roots will be non-real.

(2)

(b) Solve for x, showing all algebraic working.

$$\sqrt{x-2}+4=x$$

(5)

(c) Solve for x if $(x+3)(x-1)\geq 0$.

(3)
[13]

QUESTION 2

(a) Solve for x if: $x^{\frac{2}{3}} = 4$.

(3)

(b) Solve for x and y simultaneously in the following set of equations:

$y + 3x = 2$ and $3^{x^2+1} = 3^{x-y}$

(7)

(c) Lulu invests R20 000 at an interest rate of 4% per annum compounded annually. After how many completed years will it take for her investment to be worth at least R25 000?

(4)
[14]

QUESTION 3

Given the function: $f(x) = 3 - \frac{4}{x-2}$

(a) Determine $f(0)$

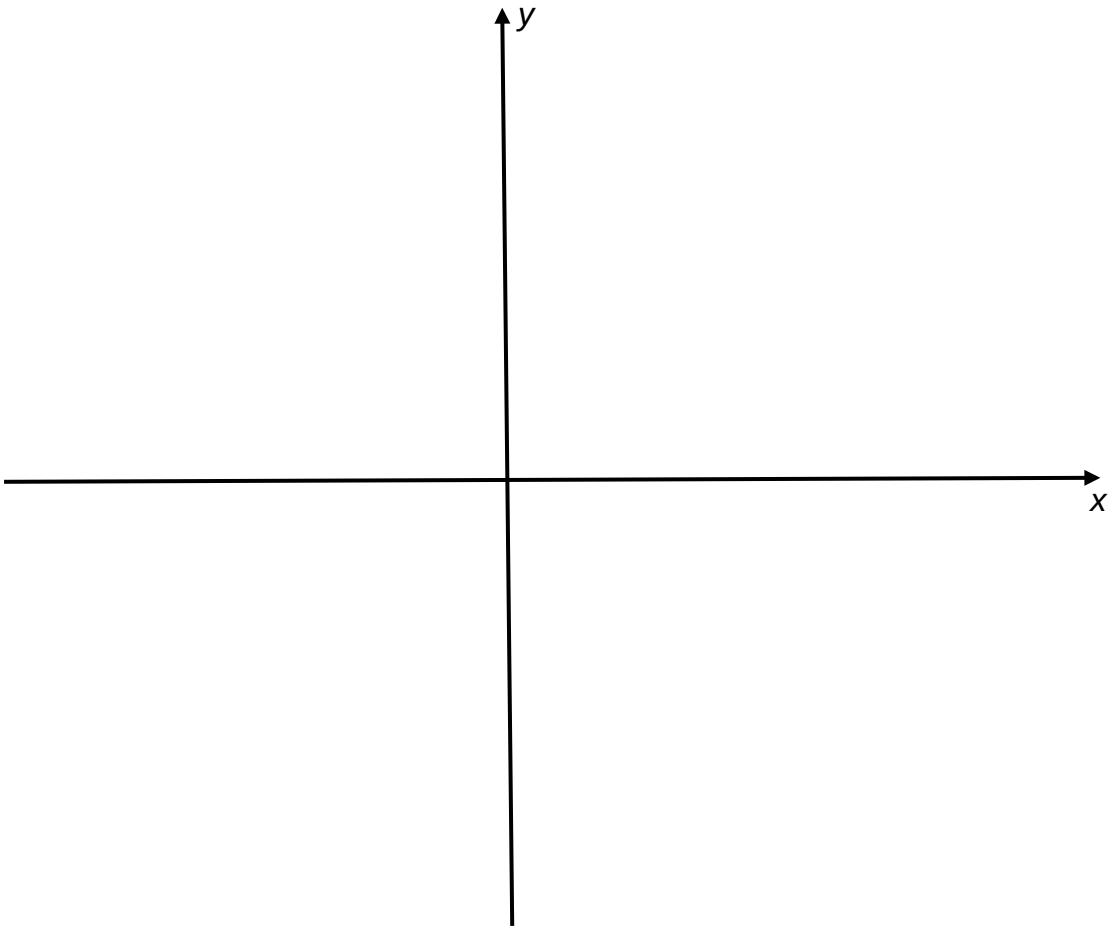
(1)

(b) Determine the value of x for which $f(x) = 0$.

(2)

(c) Sketch the graph of f on the axes provided below. Label clearly all asymptotes and intercepts with axes.

(5)



(d) (1) Determine $f(x+p)$ in terms of p.

(1)

(2) What effect would a negative value of p have on the graph sketched in (c)?

(1)

(e) (1) Determine the equation of the inverse of f in the form $f^{-1}(x) = \ldots$

(3)

(2) Write down the domain of $f^{-1}(x) = \ldots$

(1)
[14]

QUESTION 4

(a) The third term of a geometric series is 7 and the sixth term is $-2\,401$.
Determine the first term and the common ratio.

(5)

(b) The diagram below shows a pattern that is being formed as it progresses from Figure 1 to Figure 4.

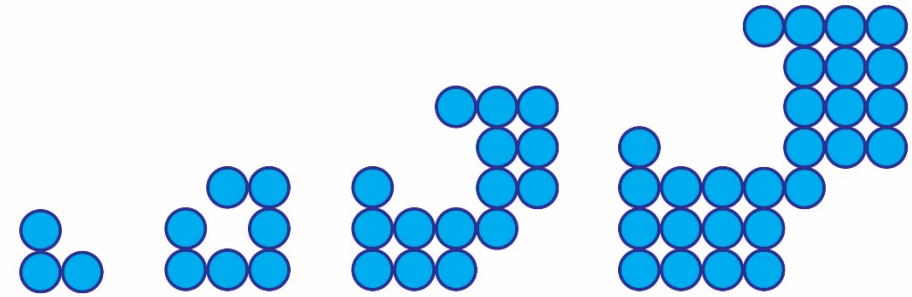


Figure 1 Figure 2 Figure 3 Figure 4

(1) Show that the number of circles in the pattern forms a quadratic sequence.

(4)

(2) Determine a formula for the n^{th} term in the form $T_n = an^2 + bn + c$.

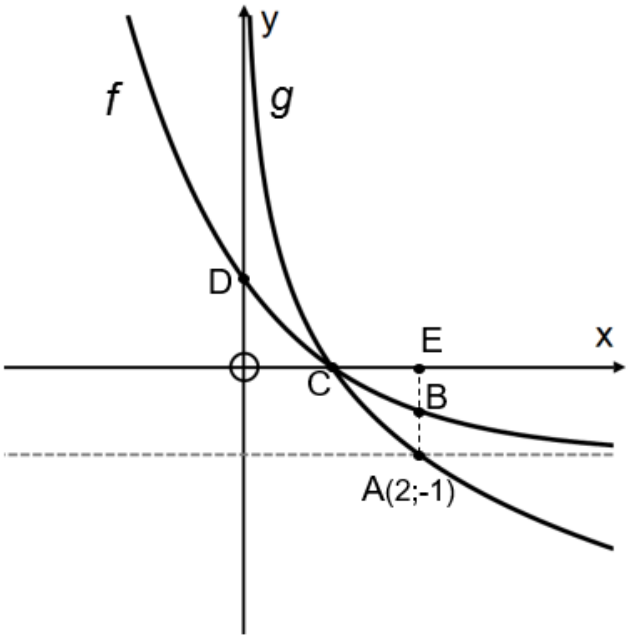
(4)
[13]

QUESTION 5

In the diagram below, the graphs of $f(x) = 2p^x + q$ and $g(x) = \log_t x$ are given.

Note: O represents the origin.

The graphs have a common *x*-intercept at C.
The graph of *f* has a *y*-intercept at D.
The asymptote of *f* and the graph of *g* both pass through $A(2;-1)$
EBA is a line perpendicular to the *x*-axis.



(a) Determine the value of *t* in $g(x) = \log_t x$.

(2)

(b) Write down the coordinates of C.

(1)

(c) Determine the value of p showing all working.

(3)

(d) Determine the coordinates of D.

(2)

(e) Calculate the length of AB.

(2)

(f) Write down the range of f.

(1)
[11]

QUESTION 6

(a) If $f(x) = 1 - 2x + x^2$, determine $f'(x)$ from first principles.

(5)

(b) Determine $\frac{dy}{dx}$ if:

(1) $y = x^{10} + 10x$

(2)

(2) $y = \dfrac{5+\sqrt{x}}{x^3}$

(4)
[11]

76 marks

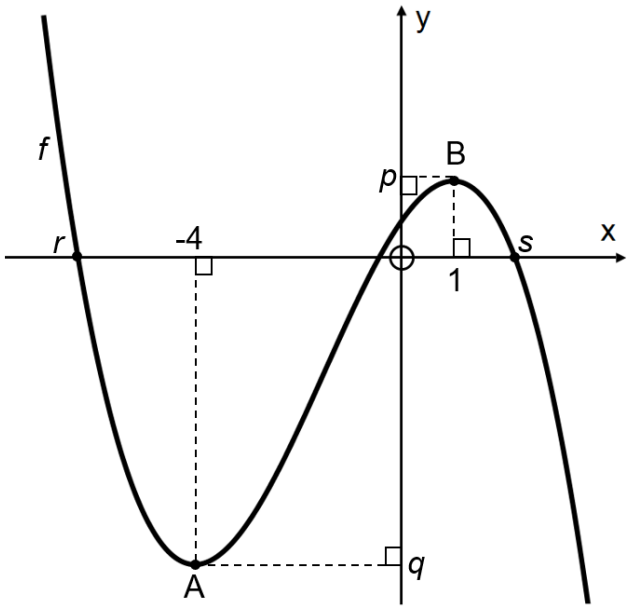
SECTION B

QUESTION 7

In the diagram below, the graph of a cubic function is shown.
Note: O represents the origin.

r and *s* represent the *x*-coordinates of two of the *x*-intercepts of the graph.
A and B represent the stationary points of the graph.
p and *q* are the *y*-values of the respective stationary points B and A of the graph.

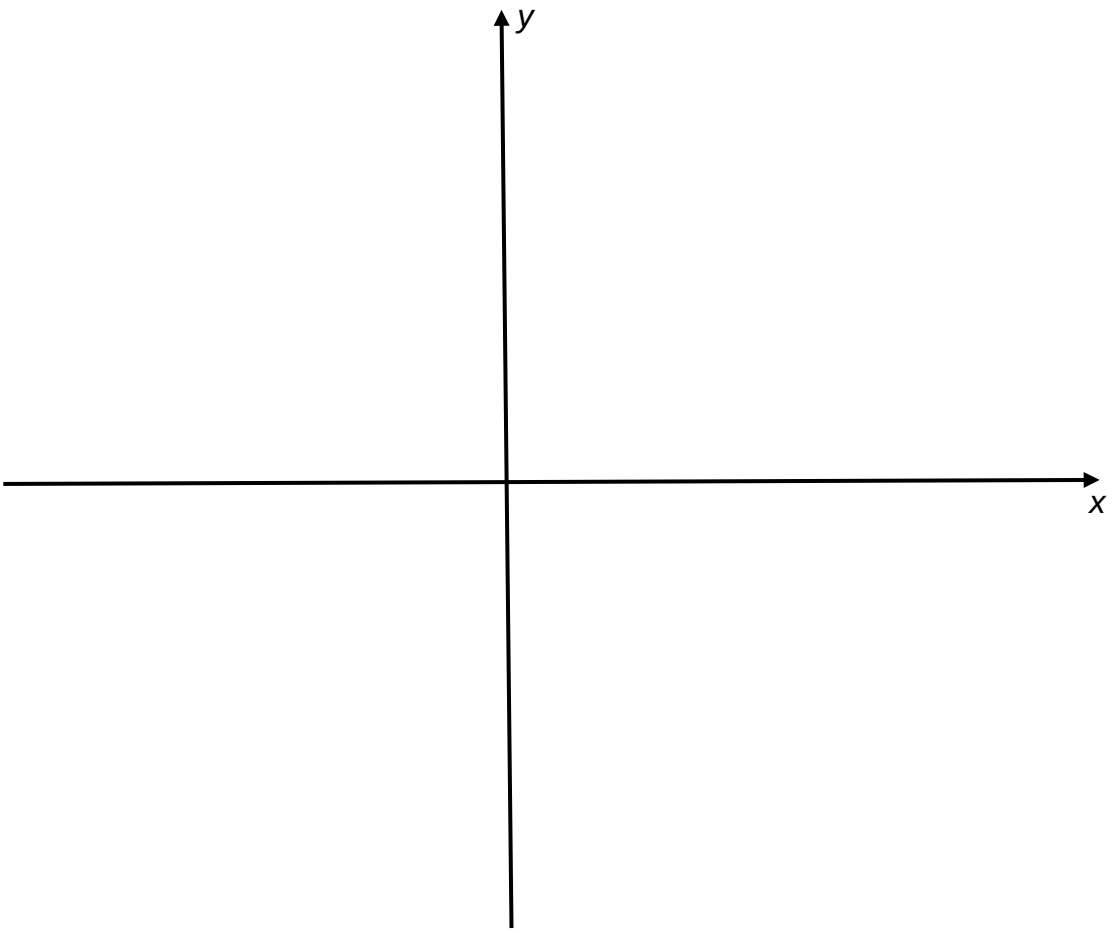
All answers may be given in terms of *p, q, r, s* and the numerical values given on the diagram.



(a) For which value(s) of x is $f'(x) < 0$?

(2)

(b) Draw a sketch graph on the grid provided, of the derivative $f'(x)$, indicating only the x-intercepts of the graph.



(3)

(c) The equation $f(x) - k = 0$ has only one real root. What are all the possible value(s) of k?

(2)

(d) Determine the value(s) of x for which the given graph of f is concave down.

(2)
[9]

QUESTION 8

(a) Consider the word: OLYMPIAD

(1) How many six-letter word arrangements can be made if the letters may be repeated?

(2)

(2) How many six-letter word arrangements can be made if the letters may not be repeated?

(2)

(b) A box contains **three** *40 watt* bulbs, **five** *60 watt* bulbs and **seven** *100 watt* bulbs. Two bulbs are taken out at random and at the same time from the box.

(1) Represent the above information on a tree diagram.

(3)

(2) Calculate the probability that the sum of the *watts* of the bulbs taken out is 160 watts.

(3)

(c) Maria decides to enter two different Mathematics Olympiads.

The probability that she wins the one is 0,02 and the probability that she wins the other is 0,08.

Assuming that winning the different Olympiads are independent events, determine the probability that she will win at least one of them.
Give your answer correct to 4 decimal places.

(4)
[14]

QUESTION 9

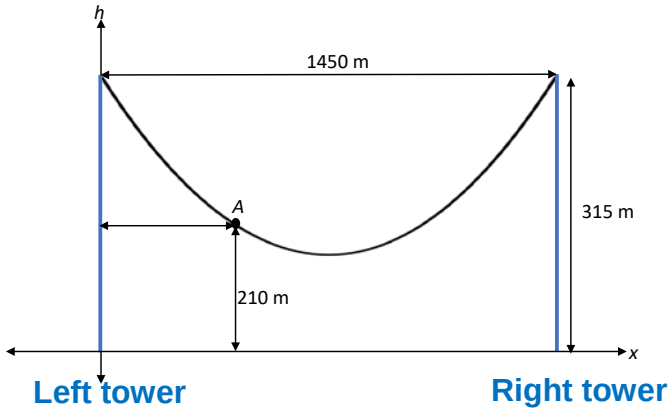
The Maputo-Katembe twin tower bridge extends over the Maputo Bay. The diagram shows a representation of a suspension cable above the mean water level.

The height h in metres of the suspension cable at a horizontal distance x metres from the base of the left tower can be modelled by the function $h = k(x-a)^2 + b$.

[<https://allafrica.com/stories/201811130758.html>]

At the mid-point between the two towers, the suspension cables are 190 metres above the mean water level. The distance between the towers is 1 450 metres and the towers are both 315 metres tall, measured from the mean water level (x-axis).

A hygrometer (shown as A in the diagram below) to measure humidity is placed on the cable at a height of 210 metres above the mean water level.



(a) Write down the values of a and b. (2)

(b) Calculate the horizontal distance of the hygrometer from the left tower. Show all working.

(6)
[8]

QUESTION 10

(a) A company sold shares at a fixed price of R875,50 per share over 5 years.

A minimum guaranteed interest rate earned over the 5-year period is 6,7% per annum compounded quarterly.

Colleen purchased 10 shares every quarter over the 5-year period. Her first purchase was at the end of the first quarter.

Determine the percentage (%) minimum profit on the total cost of shares bought at the end of the 5 year period.

(6)

(b) Colleen took a bank loan of R300 000 at the beginning of the same year as she bought her first 10 shares indicated in (a) and paid back the loan through monthly payments. Her first repayment was made at the end of January of that year.

The bank offered her an interest rate of 9,5% per annum compounded monthly over 15 years. If she sold her shares at the end of the 5-year period based on the minimum interest rate, would she be able to pay the balance of her loan at that stage?

(8)
[14]

QUESTION 11

(a) Determine the smallest value of k for which $\sum_{i=1}^{\infty}\frac{k}{2^i}+\sum_{i=1}^{10}2^{2i} > 1\,000\,000$ for $k \in \mathbb{Z}$.

(6)

(b) Consider the following arithmetic series: $5+\frac{15}{2}+10+\ldots+\frac{505}{2}$

(1) Determine the number of terms in the series.

(3)

(2) Calculate the sum of the middle 30 terms of the series.

(4)
[13]

QUESTION 12

The graphs of $g(x) = x^3 - ax^2 + 6$ and $h(x) = 2x^2 + bx + 3$ touch when $x = 1$. Therefore, the tangent to the curve of g at $x = 1$ is also the tangent to the curve of h at $x = 1$.

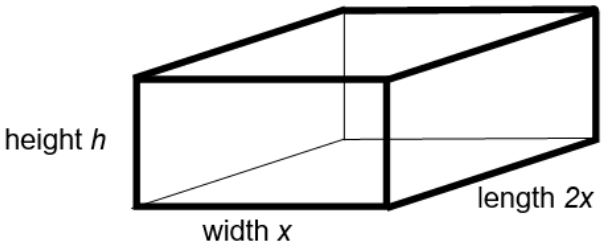
Determine the coordinates of this point of contact of the two graphs.

(8)
[8]

QUESTION 13

The frame of the box indicated in the diagram is made up of 12 plastic rods that have a total length of P cm.

The width of the box is half the length.



If the volume is a maximum, determine the length of the box in terms of P.
Show all working.

$$\text{Volume of Rectangular Prism} = \text{Length} \times \text{Width} \times \text{Height}$$

[8]

74 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

