NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2014

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $x^2 - 5x = -6$

$x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2 \quad \text{or} \quad x = 3$ (3)

(2) $(3x+1)(x-4) < 0$

$-\frac{1}{3} < x < 4$ (3)

(3) $\log_2(x+6) = 1$

$x + 6 = 2$

$x = -4$ (2)

(4) $2x + \sqrt{x+1} = 1$

$\sqrt{x+1} = 1 - 2x$

$x + 1 = 1 - 4x + 4x^2$

$4x^2 - 5x = 0$

$x(4x-5) = 0$

$x = 0 \quad \text{or} \quad x = \frac{5}{4}$

Check $x = 0$:

$\text{LHS} = 2 \times 0 + \sqrt{0+1}$

$= 1 = \text{RHS}$

Check $x = \frac{5}{4}$

$\text{LHS} = 2\left(\frac{5}{4}\right) = \sqrt{\frac{5}{4}+1}$

$= 4 \neq \text{RHS}$ (6)

(5) $12^{5+3x} = 1$

$5 + 3x = 0$

$x = \frac{-5}{3}$ (2)

(b) $2x - y = 8$ …… ①

$x^2 - xy + y^2 = 19$ …… ②

① : $y = 2x - 8$

② : $x^2 - x(2x-8) + (2x-8)^2 = 19$

$x^2 - 2x^2 + 8x + 4x^2 - 32x + 64 = 19$

$3x^2 - 24x + 45 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ or $x = 5$

$y = 2\times 3 - 8$ or $y = 2\times 5 - 8$

$= -2$ $= 2$ (7)

(c) $f(x) = x^{10} - 2x^5 + c$

$f(-1) = (-1)^{10} - 2(-1)^5 + c = 0$

$1 + 2 + c = 0$

$c = -3$ (3)

(d) $y = x^2$

$\frac{dy}{dx} = 2x$

At (–1; 1): $m = 2(-1)$

$= -2$ (2)

[28]

QUESTION 2

(a) A.P. $a = 4$; $d = 3$

$Tn = 4 + (n - 1)\,3$

$= 3n + 1$ (2)

(b) (1) $d : 6;8;10$

$s : 7;10;13$

$d_4 = 12$

$s_4 = 16$ (2)

(2) $d_n = 6 + (n-1) \times 2$

$= 2n + 4$

$s_n = 7 + (n-1) \times 3$

$= 3n + 4$ (4)

(3) $2n + 4 = 100$

$2n = 96$

$n = 48$

$s_{48} = 3 \times 48 + 4$

$= 148$ (4)

(c) (1) $T_6 = 20 \times 1{,}1^5$

$= 32{,}2$ km (2)

(2) $S_6 = \dfrac{20\left[1{,}1^6 - 1\right]}{1{,}1 - 1}$

$= 154{,}3$ km (3)

[17]

QUESTION 3

(a) $f(x) = \dfrac{x}{2} - 1$

$y = -\dfrac{(x-4)}{2} + 1$

$y = -\dfrac{x}{2} + 2 + 1$

$= -\dfrac{x}{2} + 3$ (3)

(b) $g(x) = \frac{-x^2}{4}$

(1) Domain: $x \in \mathbb{R}$
Range: $y \leq 0$ (2)

(2) $x \leq 0$ (1)

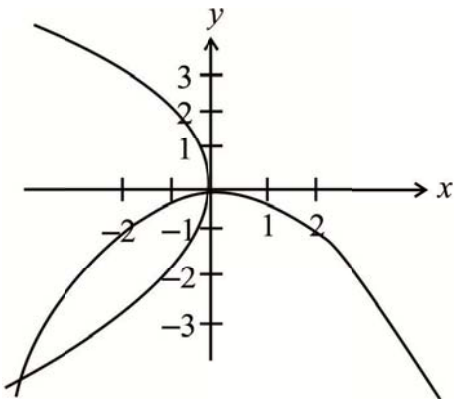
(3) Inverse:

$x = -\frac{y^2}{4}$

$y^2 = -4x$

$y = \pm\sqrt{-4x}\ ,\ x \leq 0$ (3)

(4)



(4)

(c) (1) $j(x) = 2(x-2)^2 + q$

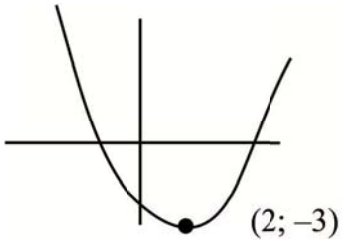
$j(x) = 2x^2 - 8x + 8 + q$

$\therefore \quad q = -3$ (2)

(2) $j(x) = 2(x-2)^2 - 3 = c$

$2(x-2)^2 = 3 + c \geq 0$ (3)

$c \geq -3$



[18]

QUESTION 4

(a) $f(x) = 6x^2 - 5x$

$$f(x+h) = 6(x+h)^2 - 5(x+h)$$
$$= 6(x^2 + 2hx + h^2) - 5x - 5h$$
$$= 6x^2 + 12hx + 6h^2 - 5x - 5h$$
$$f'(x) = \lim_{h \to 0} \frac{6x^2 + 12hx + 6h^2 - 5x - 5h - 6x^2 + 5x}{h}$$
$$= \lim_{h \to 0} \frac{h(12x + 6h - 5)}{h}$$
$$= \lim_{h \to 0} (12x + 6h - 5)$$
$$= 12x - 5$$

(5)

(b) (1) $y = 3x^4 - 2\sqrt{x} + 6$

$$= 3x^4 - 2x^{\frac{1}{2}} + 6$$
$$\frac{dy}{dx} = 12x^3 - 2 \cdot \frac{1}{2} x^{-\frac{1}{2}}$$
$$= 12x^3 - \frac{1}{\sqrt{x}}$$

(4)

(2) $y = \frac{x^3 - 2x^{-2}}{3x}$

$$= \frac{x^2}{3} - \frac{2x^{-3}}{3}$$
$$\frac{dy}{dx} = \frac{2x}{3} + 2x^{-4}$$
$$= \frac{2x}{3} + \frac{2}{x^4}$$

(5)

[14]

77 marks

SECTION B

QUESTION 5

(a) $A = 1 + \frac{0,18}{12}$

$= 1,015$

$1 + i^e = A^{12}$

$= 1,195618171$

$i^e = 0,195618171$

$\approx 19,6\%$ (3)

(b) $500\ 000 = \frac{x\left[1 - A^{-60}\right]}{\frac{0,18}{12}}$

$= x\,(39,38026889)$

$x = 12696,71371$

$\approx$ R12 696,71 (4)

(c) O.B. $= 500\ 000A^{24} - \frac{12\,700\left[A^{24} - 1\right]}{\frac{0,18}{12}}$

$= 714\,751,406 - 363\,645,7141$

$= 351\,105,6919$

$\approx$ R351 105,69

ALTERNATIVELY :
R12 700 used in Present Value Formula not valid – Max 3

O.B. $= \frac{12\ 700\left[1 - A^{-36}\right]}{\frac{0,18}{12}}$

$= 351\,290,6907$

$\approx$ R 351 290,69 (4)

(d) $500\,000\left[1 - i\right]^2 = 304\,200$

$(1 - i)^2 = 0,6084$

$1 - i = 0,78$

$-i = -0,22$

$i = 22\%$ (3)

(e) $\dfrac{P(1-0{,}22)^5}{P}$

$= 0{,}2887174368$

$\approx 28{,}9\%$

ALTERNATIVELY :

Reduced market value after 5 years

$= 500\ 000(1-0{,}22)^5$

$= 144\ 358{,}7184$

% of original

$= \dfrac{144\ 358{,}7184}{500\ 000} \times 100$

$= 28{,}87174368$

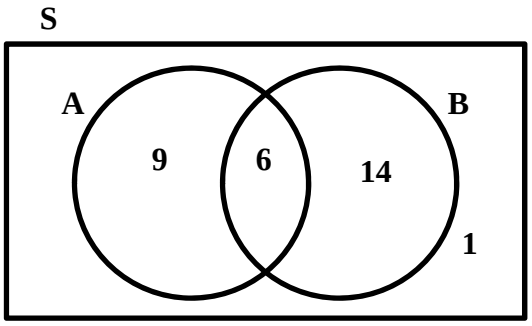
$\approx 28{,}9\%$

(3)

[17]

QUESTION 6

(a) (1)



(3)

(2) $n(\text{A or B}) = 29$

(1)

(1) (i) $P(A \cap B) = \dfrac{6}{30}$

$= 0{,}2$

(1)

(ii) $P(A) \times P(B) = \dfrac{15}{30} \times \dfrac{20}{30}$

$= 0{,}33333333\ldots.$

$P(A \cap B) \neq P(A) \times P(B)$

$\therefore$ A and B are not independent.

(3)

(b)(1) $26^2 \times 10^2$

$= 67\ 600$

(2)

(2) $26^2 \times 10^n \geq 3 \times 10^6$

$10^n \geq 4437{,}869822$

$n \geq \log 4437{,}869822$

$\geq 3{,}64717\ldots$

i.e. 4 digits. **OR** 26. 26. 10. 10. 10 … 4 digits. Trial and Error

(4)

[14]

QUESTION 7

(a) $f(0) = 10$
i.e. 10 m (1)

(b) $-5t^2 + 30t + 10 = 0$

$t^2 - 6t - 2 = 0$

$t = \frac{6 \pm \sqrt{36+8}}{2}$

$= 3 \pm \sqrt{11}$

$\therefore$ After 6,3 sec. (3)

(c) $t_{TP} = \frac{-30}{2(-5)}$

$= 3$

$f(3) = -5 \times 3^2 + 30 \times 3 + 10$

$= 55$ m (4)

(d) $h(t) = a(t-4)^2 + 60$

$(0;10): 10 = a(-4)^2 + 60$

$-50 = 16a$

$a = \frac{-25}{8}$

$h(t) = \frac{-25}{8}(t-4)^2 + 60$

$= \frac{-25}{8}(t^2 - 8t + 16) + 60$

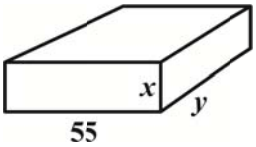
$= \frac{-25t^2}{8} + 25t - 50 + 60$

$= \frac{-25t^2}{8} + 25t + 10$ (6)

[14]

QUESTION 8

(a) (1)



$$x + y + 55 = 115$$
$$y = 60 - x$$
$$V = xy \times 55$$
$$= 55x(60 - x)$$
$$= -55x^2 + 3300x$$

(4)

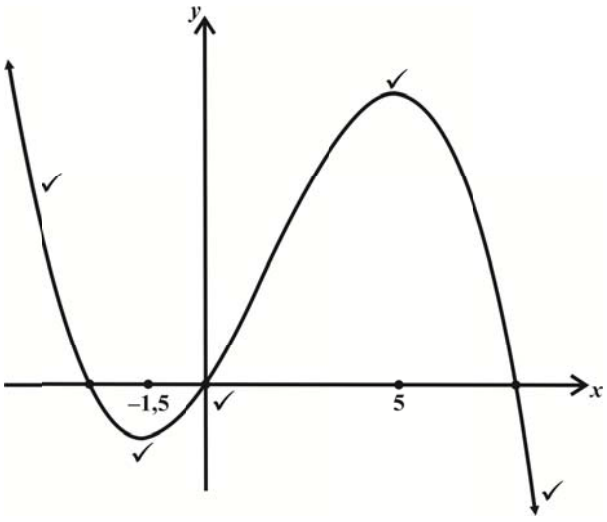
(2)
$$\frac{dV}{dx} = -110x + 3300 = 0$$
$$-110x = -3\,300$$
$$x = 30$$
$$y = 30$$
$$\text{Max. Vol.} = 55 \times 30^2$$
$$= 49\,500 \text{ cm}^3$$

ALTERNATIVELY

$$\text{Max. Vol.} = -55 \times 30^2 + 3300 \times 30$$
$$= 49500$$

(6)

(b) $f(x) = ax^3 + bx^2 + cx$



(5)

(2) (i) $x < -1,5$ or $x > 5$

(2)

(ii) $f''(x) > 0$

Point of inflection at $x = \frac{-1,5+5}{2} = 1,75$

$\therefore \quad x < 1,75$

Alternatively

$f(x) = ax^3 + bx^2 + cx$

$f'(x) = 3ax^2 + 2bx + c = 3a\,(x + 1,5)(x - 5)$

$= 3ax^2 - 10,5ax - 22,5a$

$f''(x) = 6ax + 2b > 0$ (concave up)

$a < o$ 

$\therefore\ 6ax > -2b$

$x < \frac{-b}{3a}$

$x < \frac{-(-5,25a)}{3a} \qquad (2b = -10,5a)$

$x < 1,75$

(3)

[20]

QUESTION 9

(a) $S_\infty = p$

$$\frac{a}{1-r} = p$$
$$a = p - pr$$
$$pr = p - a$$
$$r = 1 - \frac{a}{p}$$

For convergence $-1 < r < 1$

$$-1 < 1 - \frac{a}{p} < 1$$
$$-2 < -\frac{a}{p} < 0$$
$$-2p < -a < 0$$
$$2p > a > 0$$

i.e. $0 < a < 2p$ (5)

(b) $a = \frac{p}{4}$

$$\frac{p}{4} \div (1-r) = p$$
$$\frac{p}{4} = p - pr$$
$$pr = p - \frac{p}{4}$$
$$= \frac{3p}{4}$$
$$r = \frac{3}{4}$$

(3)
[8]

73 marks

Total: 150 marks