NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2018

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) $y = -3x - 1$

$\therefore 9x^2 = 9x^2 + 6x + 1$

$\therefore -6x = 1$

$x = -\frac{1}{6}$

$\therefore 3\left(-\frac{1}{6}\right) + y = -1$

$\therefore y = -\frac{1}{2}$ (5)

(b) $2^{2(x-2)} - 2^{4(3x+4)} = 0$

$2^{2x-4} = 2^{12x+16}$

$2x - 4 = 12x + 16$

$-10x = 20$

$x = -2$ (3)

(c) (1) $\Delta = b^2 - 4ac$

$= (-5)^2 - 4(2)(-3)$

$= 25 + 24$

$= 49$

$\therefore \Delta$ is a perfect square

$\therefore$ Roots are Real and rational

Alternative:

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x-3) = 0$

$x = -\frac{1}{2}$ OR $x = 3$

Hence roots are real and rational

(2) $(2x - 3)(2x + 1) = 0$

$x = \frac{3}{2}$ or $x = -\frac{1}{2}$ (3)

[14]

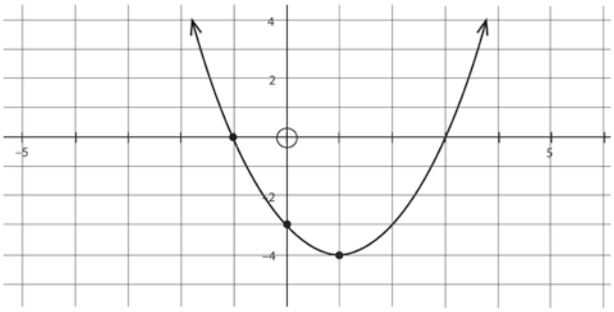
QUESTION 2

(a) $f(x) = x^2 - 2x - 3$

$f(x) = x^2 - 2x + 1 - 1 - 3$

$f(x) = (x - 1)^2 - 4$ (4)

(b)



Shape
x int.
y int.
Tpt.

(4)

(c) Range = $[-4;\infty)$ OR $y \geq -4$ (2)

(d) For gradient, use: (–1; 0) and (0; –3)

$m = \frac{-3-0}{0-(-1)}$ $\therefore$ $m = -3$

$c = -3$

$\therefore$ $y = -3x - 3$ (3)

[13]

QUESTION 3

(a) $p = -3$ and $q = -2$

$$y = \frac{a}{x+3} - 2 \quad \text{substitute } (-4;-4)$$

$$\therefore\ -4 = \frac{a}{-4+3} - 2$$

$$\therefore\ a = 2$$ (4)

(b) $y = x + 1$ and $y = -x - 5$ (2)

[6]

QUESTION 4

(a) $A = P(1+i)^n$

$$7\,024{,}64 = 5\,600\left(1 + \frac{r}{100}\right)^2$$

$$r = 12\%\ \text{p.a.}$$

$$\left(1 + \frac{k}{400}\right)^4 = 1 + \frac{12}{100} \quad \text{where } k \text{ represents the nominal rate}$$

$k = 11{,}49\%$ p.a. compounded quarterly.

Alternative

$$5\,600\left(1 + \frac{i^{(4)}}{4}\right)^8 = 7\,024{,}64$$

$$\therefore\ \left(1 + \frac{i^{(4)}}{4}\right)^8 = 1{,}2544$$

$$\therefore\ 1 + \frac{i^{(4)}}{4} = 1{,}02873\ldots$$

$$\therefore\ i^{(4)} = 11{,}49\%$$ (4)

(b) (1) $$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$850\,000 = x\left[\frac{1-\left(1+\frac{8{,}5}{1200}\right)^{-(20\times 12)}}{\frac{8{,}5}{1200}}\right]$$

$$x = \text{R}7\,376{,}497484$$

$$x \approx \text{R}7\,376{,}50$$ (4)

(2)

$$A = 850\,000\left(1+\frac{8,5}{1\,200}\right)^{144}$$

$$A = 2\,348\,755,326$$

$$F = 7376,497484\left[\frac{\left(1+\frac{8,5}{1\,200}\right)^{144}-1}{\frac{8,5}{1\,200}}\right]$$

$$F = 1836\,218,39$$

$$A - F = \text{R}512\,536,9355$$

$$\therefore \text{ Balance} \approx \text{R}512\,536,94$$

Alternative (1)

$$A = 850\,000\left(1+\frac{8,5}{1\,200}\right)^{144}$$

$$A = 2\,348\,755,326$$
$$\approx 2\,348\,755,33$$

$$F = 7376,50\left[\frac{\left(1+\frac{8,5}{1\,200}\right)^{144}-1}{\frac{8,5}{1\,200}}\right]$$

$$F = 1836\,218,39$$

$$A - F = \text{R}512\,536,94$$

Alternative (2)

$$P = 7\,376,497484\left[\frac{1-\left(1+\frac{8,5}{1\,200}\right)^{-(8\times 12)}}{\frac{8,5}{1\,200}}\right]$$

$$P = 512\,536,9358$$

$$\therefore \text{ Balance} \approx \text{R}512\,536,94$$ (4)

(3) Total amount paid in 12 years = 7 376,497484 × 144
Total amount paid in 12 years = R1 062 215,638

$\text{Balance} \approx \text{R}512\,536,9355$

Amount that went towards paying original loan = 850 000 – 512 536,9355 = 337 463,0645

% of 144 payments gone towards reducing the amount outstanding =
$$\frac{337\,463,0645}{1062\,215,638}\times 100$$
= 31,77% (3)

[15]

QUESTION 5

(a) $f(x) = 2x^2 + x + 7$

$$f'(x) = \lim_{h\to 0}\frac{2(x+h)^2+(x+h)+7-(2x^2+x+7)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2x^2+4xh+2h^2+x+h+7-2x^2-x-7}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{h(4x+2h+1)}{h}$$ notations

$f'(x) = 4x + 1$ (5)

(b) $$f(x) = \frac{x(x^2-2x-3)}{x-3}$$

$$f(x) = \frac{x(x-3)(x+1)}{(x-3)}$$

$f(x) = x^2 + x$

$f'(x) = 2x + 1$

$f'(2) = 5$ (5)

(c) $y = 4x^{-1} - 5x^{\frac{1}{2}}$

$$\frac{dy}{dx} = -4x^{-2} - \frac{5}{2}x^{-\frac{1}{2}}$$ (3)

(d) $g(x) = \frac{1}{2}x + 5 \quad \therefore\ m_{\tan} = -2$

For the point of contact: $f'(x) = 2x - 3$

$2x - 3 = -2$

$x = \frac{1}{2} \quad \therefore\ y = -\frac{29}{4}$

Substitute $\left(\frac{1}{2}; -\frac{29}{4}\right)$ in $y = -2x + c$

$$\frac{-29}{4} = -2\left(\frac{1}{2}\right) + c$$

$$c = -\frac{25}{4}$$

$y = -2x - \frac{25}{4}$ i.e. $4y = -8x - 25$ (5)

[18]

QUESTION 6

(a) $[2(3)^2 - 3(3) + 1] + [2(4)^2 - 3(4) + 1] + [2(5)^2 - 3(5) + 1]$

$10 + 21 + 36$

$= 67$ (3)

(b) $T_3 = ar^2 = 36$

$T_3 = ar^5 = 7\ 776$

$\frac{ar^5}{ar^2} = \frac{7\ 776}{36}$ m

$r^3 = 216$

$r = 6$

$\therefore a = 1$ (6)

[9]

75 marks

SECTION B

QUESTION 7

(a) First and last = (5 × 2) × 2 = 20

Middle 18 = 18 × 4 × 2 = 144

$\therefore$ Perimeter = 164 (4)

(b) (1) Converging areas: $\pi \cdot 16^2; \pi \cdot \frac{16^2}{7}; \pi \cdot \frac{16^2}{7^2}$

Common ratio = $\frac{\pi \cdot 16^2}{7} \div \pi \cdot 16^2$

$= \frac{1}{7}$

Since –1 < *r* < 1, the series converges (4)

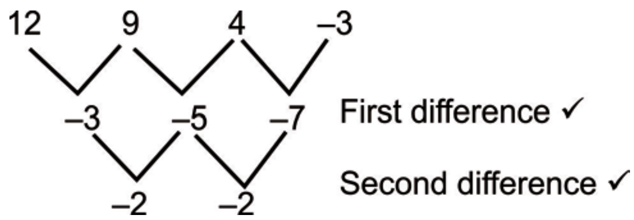
(2) $S\infty = \frac{a}{1-r}$

$S\infty = \frac{16^2 \cdot \pi}{1-\frac{1}{7}}$

$S\infty = 938{,}289$

≈ 938 (3)

(c)



$T_n = an^2 + bn + c$

$a + b + c = 12$

$4a + 2b + c = 9$

$9a + 3b + c = 4$

$3a + b = –3$ and $5a + b = –5$

$\therefore b = –3 – 3a$

Substitute $b = –3 –3a$ into $5a + b = –5$

$5a – 3 – 3a = –5$

$a = –1$

$\therefore b = 0$ and $c = 13$

$\therefore T_n = –n^2 + 13$ (6)

[17]

QUESTION 8

(a) The sums that are prime are: {2 ; 3 ; 5 ; 7 ; 11}

2: (1 ; 1)

3: (1 ; 2), (2 ; 1)

5: (1 ; 4), (2 ; 3), (3 ; 2), (4 ; 1)

7: (1 ; 6), (2 ; 5), (3 ; 4), (4 ; 3), (5 ; 2), (6 ;1)

11: (5 ; 6), (6 ; 5)

Total favourable outcomes =15

$$P(\text{sum is prime}) = \frac{15}{36}$$

$$= \frac{5}{12}$$ (5)

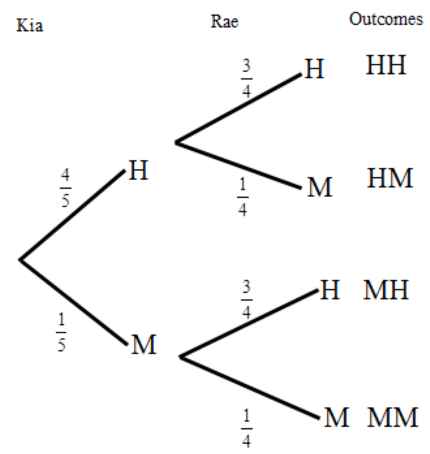
(b) P(Kia will miss) = 1 – P(Kia will hit)

$$= 1 - \frac{4}{5}$$

$$= \frac{1}{5}$$

Similarly, $P(\text{Rae will miss}) = 1 - \frac{3}{4}$

$$= \frac{1}{4}$$



KEY: H represents HIT M represents MISS

P(target will be missed by only one of them) = P(HM) + P(MH)

$$= \frac{4}{5}\times\frac{1}{4} + \frac{1}{5}\times\frac{3}{4}$$

$$= \frac{7}{20}$$ (6)

(c) (1) $8!$

$= 40\ 320$ (2)

(2) $2^3 \times 4!$

$= 192$

$$P(\text{same suit}) = \frac{192}{40\,320}$$

$$= \frac{1}{210}$$ (4)

[17]

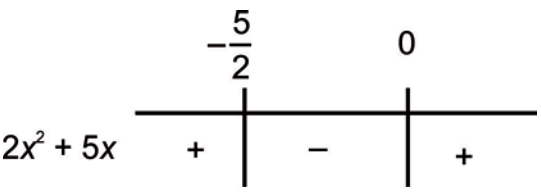
QUESTION 9

(a) $(x + 1)(2x + 3) < 3$

$2x^2 + 5x < 0$

$x(2x + 5) < 0$

Critical Values: $0; -\frac{5}{2}$



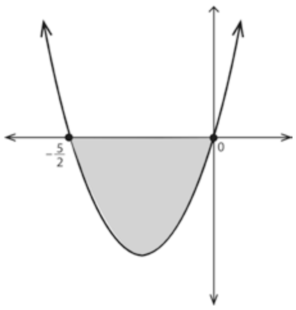
Solution: $\left\{x : -\frac{5}{2} < x < 0\right\}$

Alternative

$2x^2 + 5x < 0$

Sketch $y = 2x^2 + 5x$

x-int: $x = 0$ OR $x = -\frac{5}{2}$



Solution: $\left\{x : -\frac{5}{2} < x < 0\right\}$

(4)

(b) $$\sqrt{\frac{5 - x^2}{1 + 2x^2}} = \frac{1}{3}$$

$$\frac{5 - x^2}{1 + 2x^2} = \frac{1}{9}$$

$9(5 - x^2) = (1 + 2x^2)$

$11x^2 - 44 = 0$

$x = \pm 2$ (5)

[9]

QUESTION 10

(a) $y = \log_{\frac{1}{p}} x$ substitute $(3; -1)$

$-1 = \log_{\frac{1}{p}} 3$

$\left(\frac{1}{p}\right)^{-1} = 3$

$p = 3$

$\therefore f(x) = -x^3 + mx^2 + nx + 3$ substitute $(-1; 0)$

$0 = -(-1)^3 + m(-1)^2 + n(-1) + 3$

$n = m + 4$

$f'(x) = -3x^2 + 2mx + n$ substitute $x = -1$ when $m = 0$

$0 = -3(-1)^2 + 2m(-1) + n$

$0 = -3 - 2m + n$ substitute $n = m + 4$

$0 = -3 - 2m + m + 4$

$m = 1$

$\therefore n = 5$ (6)

(b) $f'(x) = -3x^2 + 2x + 5$

$f''(x) = -6x + 2$

g is concave up for all x > 0

$\therefore$ Both are concave up if $f''(x) > 0$

$\therefore -6x + 2 > 0$

$\therefore x < \frac{1}{3}$ (4)

(c) $g(x) = \log_{\frac{1}{3}} x$

For $g^{-1}(x)$: $x = \log_{\frac{1}{3}} y$

$y = \left(\frac{1}{3}\right)^x$ (3)

(d) Domain of $g^{-1}(x)$: $x \in \mathbb{R}$ (2)

(e) x intercept of $f : x^3 - x^2 - 5x - 3 = 0$

$(x + 1)(x + 1)(x - 3) = 0$

$x = 3$ or $x = -1$

x intercept of g : $\log_{\frac{1}{3}} x = 0$

$\left(\frac{1}{3}\right)^0 = x \quad \therefore \; x = 1$

$\therefore$ solution for x if $\frac{f(x)}{g(x)} \leq 0$ is: $1 < x \leq 3$ (7)

(f) $k > 1$ (2)

[24]

QUESTION 11

Let the amount increased/decreased by be x

$\therefore$ Length of square $= 3 + x$

Perpendicular height = $9 - x$

$$V = \frac{1}{3}(3+x)^2(9-x)$$
$$V = \frac{1}{3}\left(9+6x+x^2\right)(9-x)$$
$$V = \frac{1}{3}\left(81+54x+9x^2-9x-6x^2-x^3\right)$$
$$V = \frac{1}{3}\left(-x^3+3x^2+45x+81\right)$$
$$V = -\frac{1}{3}x^3+x^2+15x+27$$

$\frac{dV}{dx} = -x^2+2x+15$ for min / max let $\frac{dV}{dx} = 0$

$x^2 - 2x - 15 = 0$

$x = 5$ or $x = -3$

Maximum for $x = 5$

$\therefore$ New length $= 3 + 5 = 8$

$\therefore$ New perp. height $= 9 - 5 = 4$

$\therefore \; \frac{\text{New length}}{\text{New perp. length}} = \frac{8}{4} = 2$

Volume is a maximum when the perp. height is half that of the length of the square. (8)

[8]

75 marks

Total: 150 marks