MATHEMATICS: PAPER I

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 8 pages and an Information Sheet of 2 pages (i–ii). Please check that your paper is complete.

2.   Read the questions carefully.

3.   Answer all the questions.

4.   Number your answers exactly as the questions are numbered.

5.   You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6.   All necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.

7.   Diagrams are not necessarily drawn to scale.

8.   It is in your own interest to write legibly and to present your work neatly.

SECTION A

QUESTION 1

(a)   Solve for x and y: $3x + y = -1$

$$9x^2 = y^2$$ (5)

(b)   Solve for x:

$4^{x-2} - 16^{3x+4} = 0$ (3)

(c)   (1)   Show that the roots of $2x^2 - 5x = 3$ are real and rational.   (3)

        (2)   Determine the value that must be added to the negative root so that
                the roots are equal.   (3)

                                                                                          [14]

QUESTION 2

Given: $f(x) = (x - 3)(x + 1)$

(a)   Rewrite $f(x)$ in the form $f(x) = a(x + p)^2 + q$.   (4)

(b)   Sketch the graph of $f(x)$. Show the turning point and the intercepts with the
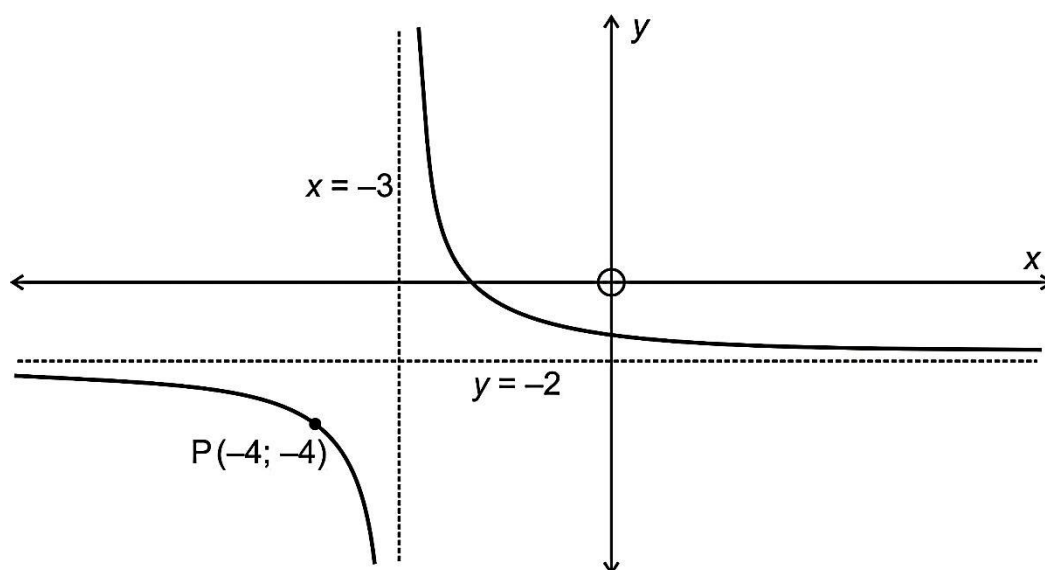   axes.   (4)

(c)   Write down the range of $f(x)$.   (2)

(d)   Determine the equation of the line that passes through the negative
        x-intercept and the y-intercept.   (3)

                                                                                      [13]

QUESTION 3

The graph of $y = \dfrac{a}{x+p} + q$ is sketched.

(a)   Determine the values of *a*, *p* and *q*.   (4)

(b)   Determine the equation(s) of the line(s) of symmetry of the graph sketched.   (2)

[6]

QUESTION 4

Round off your answers to 2 decimal digits where necessary.

(a)   A savings account grows to R5 600 after 1 year and to R7 024,64 after a further 2 years, with compound interest that is calculated quarterly.

Determine the nominal rate compounded quarterly.   (4)

(b)   Mrs. Smith pays fixed monthly instalments towards paying off a home loan. A loan of R850 000 was granted 12 years ago to be repaid over a period of 20 years. The interest is charged at 8,5% per annum compounded monthly.

(1)   Determine the monthly instalment that Mrs. Smith has been paying.   (4)

(2)   Determine the current outstanding balance on the loan, i.e. the outstanding balance immediately after the 144th payment.   (4)

(3)   What percentage of the 144 payments made has gone towards reducing the amount outstanding?   (3)

[15]

QUESTION 5

(a)   Given $f(x) = 2x^2 + x + 7$, determine $f'(x)$ from first principles.   (5)

(b)   Determine $f'(2)$ if $f(x) = \dfrac{x^3 - 2x^2 - 3x}{x - 3}$.   (5)

(c)   Differentiate with respect to x: $y = \dfrac{4}{x} - 5\sqrt{x}$.   (3)

(d)   Given $f(x) = x^2 - 3x - 6$.

        Determine the equation of the tangent to f which is perpendicular to the line

        $g(x) = \dfrac{1}{2}x + 5$.   (5)

                                                                        [18]

QUESTION 6

(a)   Evaluate: $\displaystyle\sum_{3}^{5}(2x^2 - 3x + 1)$ (3)

(b)   The third term of a geometric series is 36 and the sixth term is 7 776.
        Determine the first term and the common ratio.   (6)

                                                                        [9]

                                                            | **75 marks** |

SECTION B

QUESTION 7

(a)
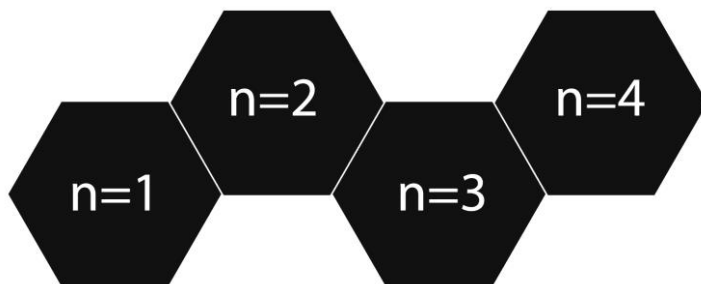
Congruent regular hexagons are arranged as shown in the diagram. The perimeter of one hexagon is 12 units.

Determine the perimeter of the shape formed by 20 hexagons arranged in this manner. (4)

(b)   The diagram shows a number of concentric circles. The radii of the largest three circles are given by 16 ; $\dfrac{16}{\sqrt{7}}$ and $\dfrac{16}{7}$ respectively.
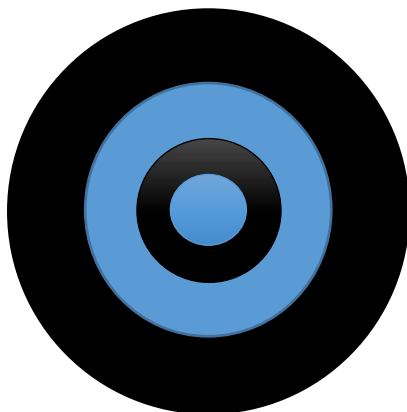
If smaller concentric circles are drawn following the above pattern:

(1)   Show that the areas of the circles form a converging geometric series. (4)

(2)   Determine the sum to infinity of this series.
       Round off your answer to the nearest whole number. (3)

(c)   Consider the sequence: 12; 9; 4; –3; –12 …
       Determine the nth term of the sequence. (6)

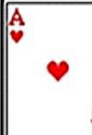**[17]**

QUESTION 8

(a)   Two fair dice are rolled. Determine the probability that the sum of the scores of the two dice is a prime number.   (5)

(b)   Kia and Rae are given one attempt at shooting at the target in a game of archery.

[Source: <https://img.clipartfest.com>]

If the probability that Kia will hit the target is $\dfrac{4}{5}$, and the probability that Rae will hit the same target is $\dfrac{3}{4}$, find the probability that the target will be missed by only one of them.   (6)

(c)   Consider the following 8 cards taken from four different suits:

[Source: <http://www.milefoot.com>]

If these cards are randomly arranged:

   (1)   In how many different ways can they be arranged?   (2)

   (2)   Determine the probability that the cards are arranged so that the first card will be a king and the cards of the same suit are together.   (4)
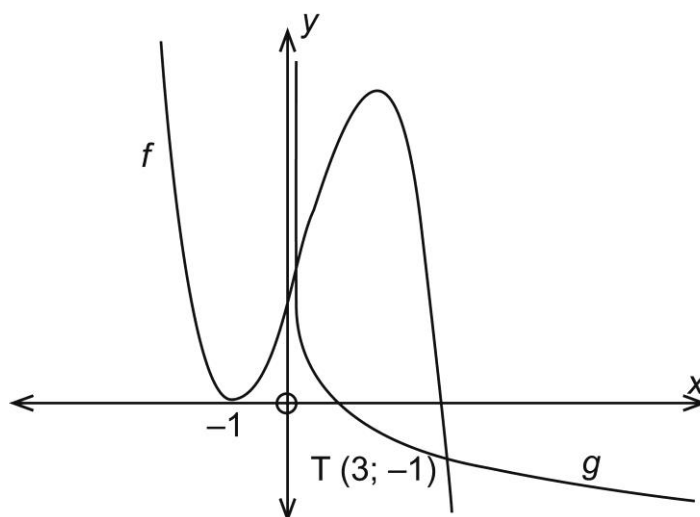
[17]

QUESTION 9

(a)   Solve for x if $(x+1)(2x+3)<3$.   (4)

(b)   Solve for x if $\sqrt{\dfrac{5-x^2}{1+2x^2}}=\dfrac{1}{3}$.   (5)

                                                                                                    [9]

QUESTION 10

The graph of a cubic function f and a logarithmic function g is drawn on the set of axes.

The equations of the functions are given as $f(x) = -x^3 + mx^2 + nx + p$ and $g(x)=\log_{\frac{1}{p}} x$. The graph of g passes through T(3; –1) and the graph of f has a turning point at (–1; 0).

(a)   Show that $p = 3$, $m = 1$ and $n = 5$.   (6)

(b)   Determine the interval where $f(x)$ and $g(x)$ are both concave up.   (4)

(c)   Determine g^{-1}, the inverse of g in the form $y = \dots$ (3)

(d)   State the domain of g^{-1}.   (2)

(e)   Determine the x intercepts of f and g and hence solve for f if $\dfrac{f(x)}{g(x)}\leq 0.$ (7)

(f)   Determine the value(s) of k for which the root of $\log_{\frac{1}{p}}(x+k)=0$ is negative.   (2)

                                                                                                    [24]

QUESTION 11

The diagram represents an inverted right square pyramid.
The length of each side of the square is 3 cm and the perpendicular height of the pyramid is 9 cm. A new pyramid is to be constructed so that the volume of the pyramid is maximised.
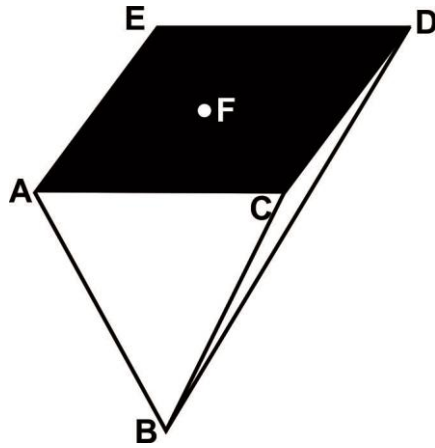
The new pyramid is constructed according to the following rules:
- The pyramid must remain a right square pyramid.
- The perpendicular height of the pyramid must be decreased by the same amount that the length of each side of the square is increased.

Find the ratio of the length of the side of the square to the perpendicular height when the volume is a maximum.

Useful formula: Volume of a Pyramid $= \dfrac{1}{3}$ A × H   **[8]**

75 marks

Total: 150 marks