NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2019

# MATHEMATICS: PAPER I
# MARKING GUIDELINES

Time: 3 hours 150 marks

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**These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.**

**The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.**

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## SECTION A

## QUESTION 1

(a) (1) $(37-x)-(x+5)=(x+13)-(37-x)$

$37-x-x-5=x+13-37+x$

$-4x=-56$

$x=14$

(2) $T_1=19, \quad T_2=23, \quad T_3=27$

$d=4$

$T_n=19+(n-1)(4)$

$T_n=4n+15 \quad (2)$

(b) $S_3=\dfrac{a(r^n-1)}{r-1} \quad ;r\neq 1$

$91=\dfrac{a(3^3-1)}{3-1}$

First term: $a=7$

**Alternate:**

$a+3a+9a=91$

$a=7$

(c) $S_\infty=\dfrac{a}{1-r} \quad ;-1<r<1$

$\dfrac{375}{4}=\dfrac{a}{1-r}$

$\therefore 4a=375-375r$

$\therefore a=\dfrac{375}{4}(1-r)$ …eq1

$S_2=\dfrac{a(r^n-1)}{r-1} \quad ;r\neq 1$

$90=\dfrac{a(r^2-1)}{r-1}$

$90=\dfrac{a(r-1)(r+1)}{(r-1)}$

$90=a(r+1)$

$90=\dfrac{375}{4}(1-r)(r+1)$

$$90 = -\frac{375}{4}(r-1)(r+1)$$

$$90 = -\frac{375}{4}(r^2-1)$$

$$r^2 = \frac{1}{25}$$

$$\therefore r = \frac{1}{5} \quad \text{or} \quad r = -\frac{1}{5}$$

and $\therefore a = \frac{375}{4}(1-r)$

$$\therefore a = 75 \quad a = \frac{225}{2}$$

**Alternate:**

$$S_\infty = \frac{a}{1-r} \quad ; -1 < r < 1$$

$$\frac{375}{4} = \frac{a}{1-r}$$

$$\therefore 4a = 375 - 375r$$

$$\therefore a = \frac{375}{4}(1-r) \quad \ldots\text{eq1}$$

$$S_2 = a + ar$$

$$S_2 = a(1+r)$$

$$90 = \frac{375}{4}(1-r)(1+r)$$

$$90 = -\frac{375}{4}(r^2-1)$$

$$r^2 = \frac{1}{25}$$

$$\therefore r = \frac{1}{5} \quad \text{or} \quad r = -\frac{1}{5}$$

and $\therefore a = \frac{375}{4}(1-r)$

$$\therefore a = 75 \quad a = \frac{225}{2}$$

(d) (1) Difference test:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 32699 | | 32896 | | 33091 | | 33284 | | 33475 |
| 1st diff. | | 197 | | 195 | | 193 | | 191 | |
| 2nd diff. | | | -2 | | -2 | | -2 | | |

Constant second difference therefore quadratic

(2) $a+b+c=32699$

$3a+b=197$

$2a=-2$

$\therefore a=-1$

$\therefore b=200$

$\therefore c=32500$

$T_n=-n^2+200n+32500$

(3) $0=-2n+200$
$n=100$
On the 100th day

**Alternate**:
$-T_n=n^2-200n+\left(-100\right)^2-32500-\left(-100\right)^2$ ✓
$-T_n=(n-100)^2-42500$
$T_n=-(n-100)^2+42500$
Maximum on the 100th day

## QUESTION 2

(a) $f'(x)=\lim_{h\to 0}\frac{-(x+h)^2+2(x+h)-(-x^2+2x)}{h}$

$f'(x)=\lim_{h\to 0}\frac{-x^2-2xh-h^2+2x+2h+x^2-2x}{h}$

$f'(x)=\lim_{h\to 0}\frac{h(-2x-h+2)}{h}$

$f'(x)=\lim_{h\to 0}(-2x-h+2)$

$f'(x)=-2x+2$

(b) $g'(x)=6x^2+6x$

$g'(-2)=6(-2)^2+6(-2)$

$g'(-2)=12$

For coordinate of the point of contact:

$g(-2)=2(-2)^3+3(-2)^2+1$

$g(-2)=-3$

Substitute: $(-2;-3)$ in $y=12x+c$

$-3=12(-2)+c$

$c=21$

Equation of tangent: $y=12x+21$

(c) (1) $y=\sqrt[3]{x^2}+3x^2-4x$

$y=x^{\frac{2}{3}}+3x^2-4x$

$\frac{dy}{dx}=\frac{2}{3}x^{-\frac{1}{3}}+6x-4$

$\therefore \frac{dy}{dx}=\frac{2}{3\sqrt[3]{x}}+6x-4$

(2) $y = (x+\pi)^{-1}(x^{-1}+\pi^{-1})$

$y = \left(\frac{1}{x+\pi}\right)\left(\frac{1}{x}+\frac{1}{\pi}\right)$

$y = \left(\frac{1}{x+\pi}\right)\left(\frac{\pi+x}{\pi x}\right)$

$y = \left(\frac{1}{\pi x}\right)$

$y = \frac{1}{\pi}x^{-1}$

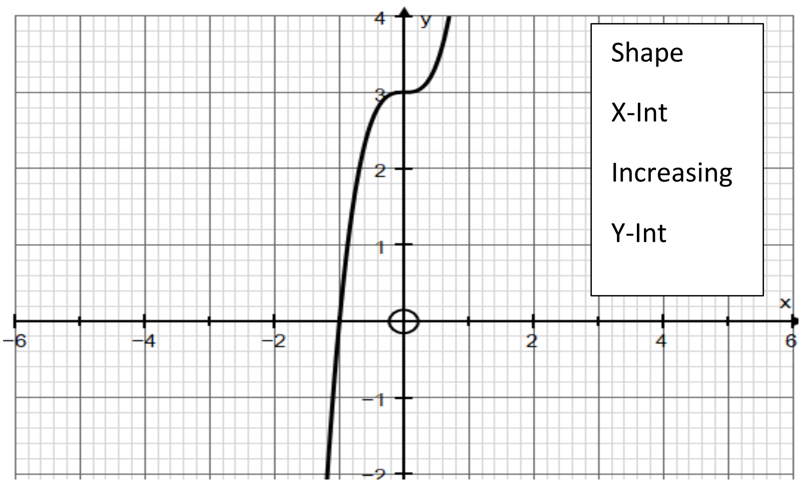
$\frac{dy}{dx} = -\frac{1}{\pi}x^{-2}$

$\frac{dy}{dx} = -\frac{1}{\pi x^2}$

## QUESTION 3

(a) $f'(x) = 9x^2$, therefore a positive gradient for all real values of x.

(b)




(c) $(0;3)$

## QUESTION 4

(a)

$c = 3$

$f(-1) = a(-1)^2 + b(-1) + 3 = 0$

$a = b - 3$ eq. 1

$f(1) = a(1)^2 + b(1) + 3 = 2$

$a + b = -1$ eq. 2

$\therefore b - 3 + b = -1$

$\therefore b = 1$ and $a = -2$

(b)

$f(x) = -2x^2 + x + 3$

$f'(x) = -4x + 1$

For decreasing gradient: $f'(x) < 0$

$-4x + 1 < 0$

$x > \frac{1}{4}$

(c)

$\therefore -3 = -2x^2 + x + 3$

$\therefore 0 = -2x^2 + x + 6$ ✓

$x = 2$ ; $x = -\frac{3}{2}$ (n/v)

$g(x) = \frac{d}{x} - 2$ … substitute $(2;-3)$

$-3 = \frac{d}{2} - 2$

$d = -2$

## QUESTION 5

(a) $1116 = 558(1{,}08)^x$

$(1{,}08)^x = 2$

$x = \log_{1{,}08} 2$

$x \approx 9$ years

NB: For trial and improvement, full marks may be awarded

(b) $x = 558(1{,}08)^y$

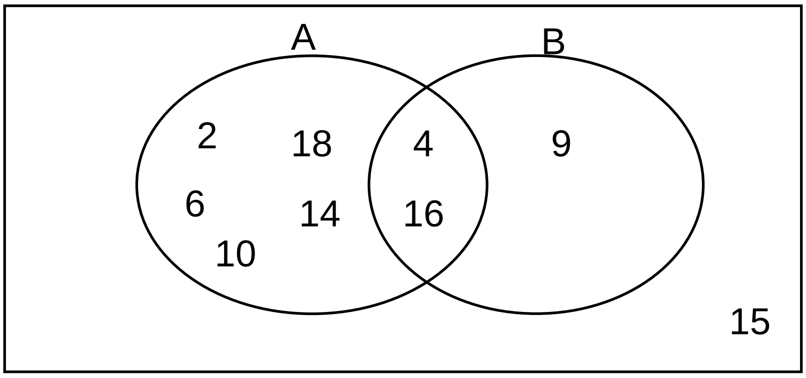
$\frac{x}{558} = (1{,}08)^y$

$y = \log_{1{,}08}\left(\frac{x}{558}\right)$

$\therefore f^{-1}(x) = \log_{1{,}08}\left(\frac{x}{558}\right)$

## QUESTION 6

(a)




(b) $P(A \text{ or } B) = \frac{8}{9}$

(c) $P(A \text{ and } B) = \frac{2}{9}$

(d) $P(A' \text{ and } B') = \frac{1}{9}$

## SECTION B

## QUESTION 7

(a) (1) $\text{P(all will pass)} = \frac{4}{5} \times \frac{5}{7} \times \frac{2}{3}$

$\therefore \text{P(all will pass)} = \frac{8}{21}$

(2) $\text{P(all will fail)} = \left(1 - \frac{4}{5}\right) \times \left(1 - \frac{5}{7}\right) \times \left(1 - \frac{2}{3}\right)$

$\text{P(all will fail)} = \frac{1}{5} \times \frac{2}{7} \times \frac{1}{3}$

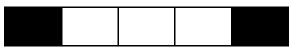
$\text{P(all will fail)} = \frac{2}{105}$

(3) $\text{P(at least one will pass)} = 1 - \text{P(all will fail)}$

$= 1 - \frac{2}{105}$

$= \frac{103}{105}$

(b)



(1) 26 Letters in the alphabet

Number of possible passwords: $26 \times 25 \times 24 \times 23 = 358\,800$

**Alternate:** $\frac{26!}{(26-4)!} = 358\,800$

(2) $P\left(\text{unlock on first attempt}\right) = \frac{1}{358800}$

(3) $P\left(\text{he will be locked out}\right) = 1 - P\left(\text{he will not be locked out}\right)$

$= 1 - \left(\frac{1}{358\,800} \times \frac{1}{358\,799} \times \frac{1}{358\,798}\right)$

$\approx 1$

## QUESTION 8

(a) $A = P(1-i)^n$

$$A = 325000\left(1-\frac{7}{100}\right)^9$$

$$A = 169\,133,602$$

$$\%\text{Depreciation} = \frac{169\,133,602}{325\,000}\times 100$$

$$\%\text{Depreciation} \approx 52\%$$

(b) $P = x\left[\dfrac{1-(1+i)^{-n}}{i}\right]$

$$1\,825\,000 = x\left[\frac{1-\left(1+\frac{9,5}{100(12)}\right)^{-25\times 12}}{\left(\frac{9,5}{100(12)}\right)}\right]$$

$$x = 15\,944,96406$$

**OPTION 1 AMOUNT ACCESSED:**

$= (15\,944,96406\times 12\times 9)\times 75\%$

$= 1\ 291\,542,089$

**OPTION 2 AMOUNT ACCESSED:**

Balance of the loan $=$ A $-$ $F_v$

$$A = 1\,825\,000\left(1+\frac{9,5}{100(12)}\right)^{9\times 12}$$

$$A = 4\,276\,835,507$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right]$$

$$F = 15\,944,96406\left[\frac{\left(1+\frac{9,5}{100(12)}\right)^{(9\times 12)} - 1}{\frac{9,5}{100(12)}}\right]$$

$$F = 2\,705\,886,942$$

Balance of the loan $= 4\,276\,835,507 - 2\,705\,886,942$

Balance of the loan $= 1\,570\,948,565$

**Alternate for OPTION 2:**

$$\text{Balance of the loan} = 15\ 944{,}96406\left[\frac{1-\left(1+\frac{9{,}5}{1200}\right)^{-16\times 12}}{\frac{9{,}5}{1200}}\right]$$

Balance of the loan = 1 570 948,565

**Amount accessed** $= 5\times(1\ 825\ 000 - 1\ 570\ 948{,}565)$

**Amount accessed** $= 1\ 270\ 257{,}175$

**Therefore OPTION 1 would yield the largest amount.**

## QUESTION 9

(a) $2(x^2 - x) = a$

$x^2 - x = \frac{a}{2}$

$x^2 - x + \left(-\frac{1}{2}\right)^2 = \frac{a}{2} + \left(-\frac{1}{2}\right)^2$

$\left(x - \frac{1}{2}\right)^2 = \frac{a}{2} + \frac{1}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{2a+1}{4}}$

$x = \frac{1}{2} \pm \frac{\sqrt{2a+1}}{2}$

$x = \frac{1 \pm \sqrt{2a+1}}{2}$

(b) (1) $\log_p \left[x.\left(x+p\right)\right] = 0$

$p^0 = x.(x+p)$

$1 = x^2 + xp$

$x^2 + xp - 1 = 0$

$x = \frac{-p \pm \sqrt{p^2 + 4}}{2}$

(2) $(x - p + 3)^2 = 4$

$x - p + 3 = \pm 2$

$x - p + 3 = 2$ or $x - p + 3 = -2$

$x = p - 1$ or $x = p - 5$

(c) (1) $\sqrt{x+4} = \dfrac{4}{\sqrt{x-2}}$

$\left(\sqrt{x+4}\right)\left(\sqrt{x-2}\right) = 4$

$\sqrt{x^2+2x-8} = 4$

$x^2+2x-24=0$

$x = 4$ or $x = -6$ (n/v)

(2) $\left(2^x\right)^4 - 8.2^x = 0$ let $2^x = k$

$k^4 - 8k = 0$

$k(k^3-8)=0$

$k(k-2)(k^2+2k+4)=0$

$2^x = 0$ or $2^x = 2$ or $\left(2^x\right)^2 + 2(2^x) + 4 = 0$

$\therefore x = 1$

**Alternate:**

$2^{4x} - 8.2^x = 0$

$2^{4x} = 8.2^x$

$\dfrac{2^{4x}}{2^x} = 8$

$2^{3x} = 8$

$2^{3x} = 2^3$

$\therefore x = 1$

(3) $3^x\left(x^2-3x+2\right) \leq 0$

$3^x > 0$ hence 0 is not a critical value

Critical Values: 1 ; 2

$1 \leq x \leq 2$

(d) $9x^2 - 12px + 4p^2 = 0$

$\Delta = \left(-12p\right)^2 - 4(9)(4p^2)$

$\Delta = 144p^2 - 144p^2$

For equal roots: $\Delta = 0$

$\therefore 0 = 0$, hence all real values of $p$ will result in a perfect square and equal roots.

**Alternate:**

$9x^2 - 12px + 4p^2 = 0$

$(3x-2p)(3x-2p) = 0$

All Real values of $p$ will result in a perfect square and equal roots.

## QUESTION 10

(a) $f'(x) = -3x^2 + 2bx + c$

$f'(-x) = -3(-x)^2 + 2b(-x) + c$

$f'(-x) = -3x^2 - 2bx + c$

For: $\boldsymbol{f'(x) = f'(-x)}$

$-3x^2 + 2bx + c = -3x^2 - 2bx + c$

$-4bx = 0 \quad \therefore \quad b = 0$

$g(x) = 2x + d$

$g'(x) = 2$

$g''(x) = 0$

$f'(-1) = -3(-1)^2 + 2b(-1) + c$

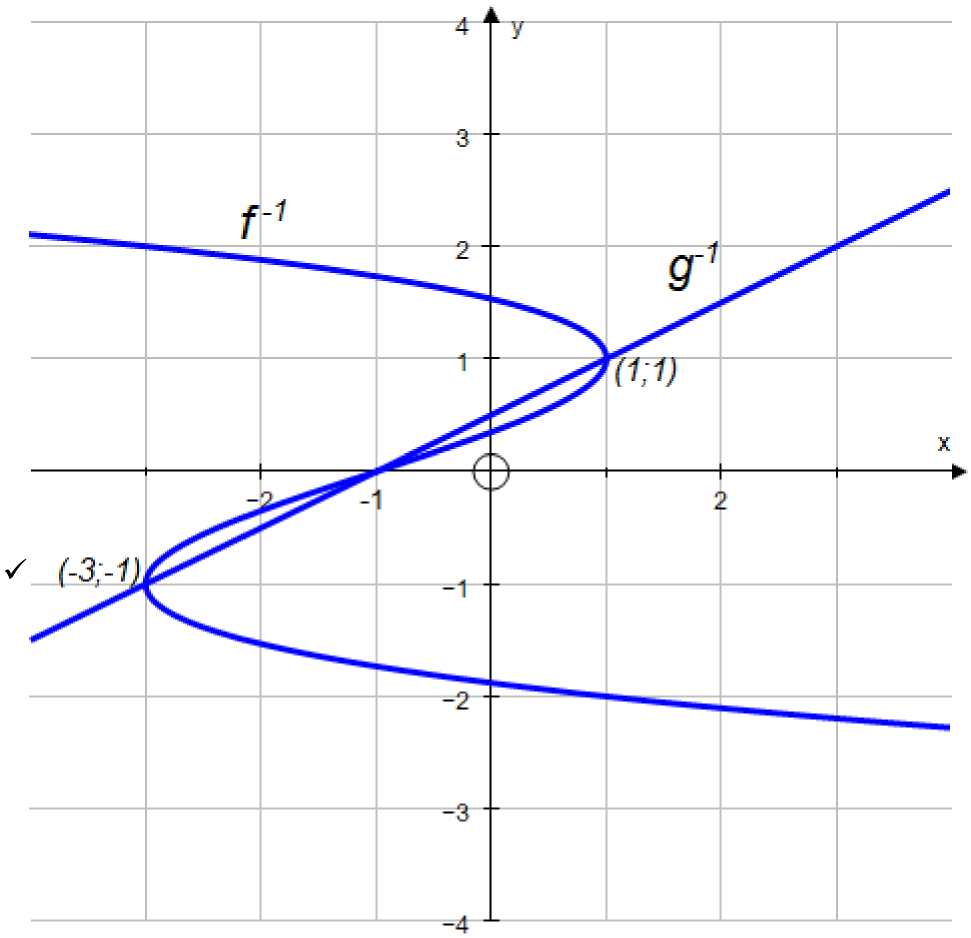
$f'(-1) = -3 - 2b + c$

From: $f'(-1) = g''(2)$

$-3 - 2b + c = 0$ substitute $b = 0$

$c = 3$

(b)



## QUESTION 11

$(AB)^2 = \left(10\sqrt{5}\right)^2 - \left[\left(475 - 10x - x^2\right)^{\frac{1}{2}}\right]^2$ … Pythagoras

$(AB)^2 = 500 - 475 + 10x + x^2$

$AB = \sqrt{25 + 10x + x^2}$

$AB = \sqrt{(5 + x)^2}$

$AB = 5 + x$

Volume of the cone $= \frac{1}{3}\pi\left[\left(475 - 10x - x^2\right)^{\frac{1}{2}}\right]^2 (x + 5)$ ✓

Volume of the cone $= \frac{1}{3}\pi(475 - 10x - x^2)(x + 5)$

Volume of the cone $= \frac{1}{3}\pi\left(475x - 10x^2 - x^3 + 2375 - 50x - 5x^2\right)$

Volume of the cone $= \frac{1}{3}\pi\left(-x^3 - 15x^2 + 425x + 2375\right)$

Volume of the cone $= -\frac{1}{3}\pi x^3 - 5\pi x^2 + \frac{425}{3}\pi x + \frac{2375}{3}\pi$

$\frac{dV}{dx} = -\pi x^2 - 10\pi x + \frac{425}{3}\pi$

**For maximum:**

$-\pi\left(x^2 + 10x - \frac{425}{3}\right) = 0$

$x \approx 7{,}9$ or $x \approx -17{,}9$

Volume of the cone $= -\frac{1}{3}\pi(7{,}9)^3 - 5\pi(7{,}9)^2 + \frac{425}{3}\pi(7{,}9) + \frac{2375}{3}\pi$

Volume of the cone $\approx 4506{,}42$ cm$^3$

**QUESTION 12**

(a) Equation of the line: $y = mx + 1$

$m = \frac{-1-1}{1-0} = -2$

For X: intercept, let y = 0

$0 = -2x + 1$

$x = \frac{1}{2}$

Negative gradient of second derivative graph, therefore the curve is concave down for: $x > \frac{1}{2}$

(b)


**Total: 150 marks**