



Please paste the barcoded label here

TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION MAY 2023

MATHEMATICS: PAPER I

EXAMINATION NUMBER

Grid for entering examination number

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

- 1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.
4. Write your answers in the spaces provided.
5. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
6. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

- 7. Diagrams are not necessarily drawn to scale.
8. If necessary, round off answers to ONE decimal place, unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. FOUR blank pages (pages 21-24) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Table with 14 columns: Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, TOTAL. Row 1: 12, 15, 10, 16, 12, 9, 7, 14, 7, 11, 9, 13, 15, /150

SECTION A**QUESTION 1**Solve for x :

(a) $3x(x - 5)(3x - 1) = 0$

(3)

(b) $\sqrt{2x - 1} - 7 = 0$

(2)

(c) Solve for x and y if $2x - y = 5$ and $x^2 - y^2 = 7$

(7)
[12]

QUESTION 2

(a) (1) If $f(x) = 2x^2 + 5$, then by using first principles find $f'(x)$.

(5)

(2) Find the equation of a tangent to $f(x)$ at $x = 3$.

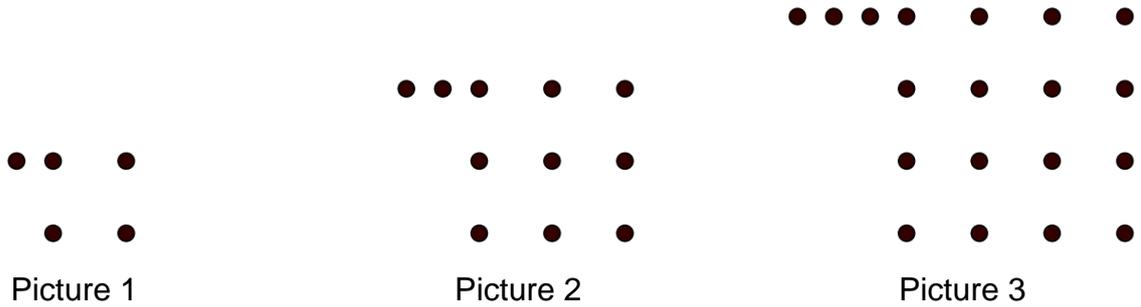
(5)

(b) Determine $g'(x)$ if $g(x) = \sqrt{x} + x + \frac{1}{x}$

(5)
[15]

QUESTION 3

(a) Refer to the pattern below when answering the questions that follow:



(1) How many dots would be required to draw Picture 4?

(1)

(2) Determine the expression for Picture n in the form of $T_n = an^2 + bn + c$

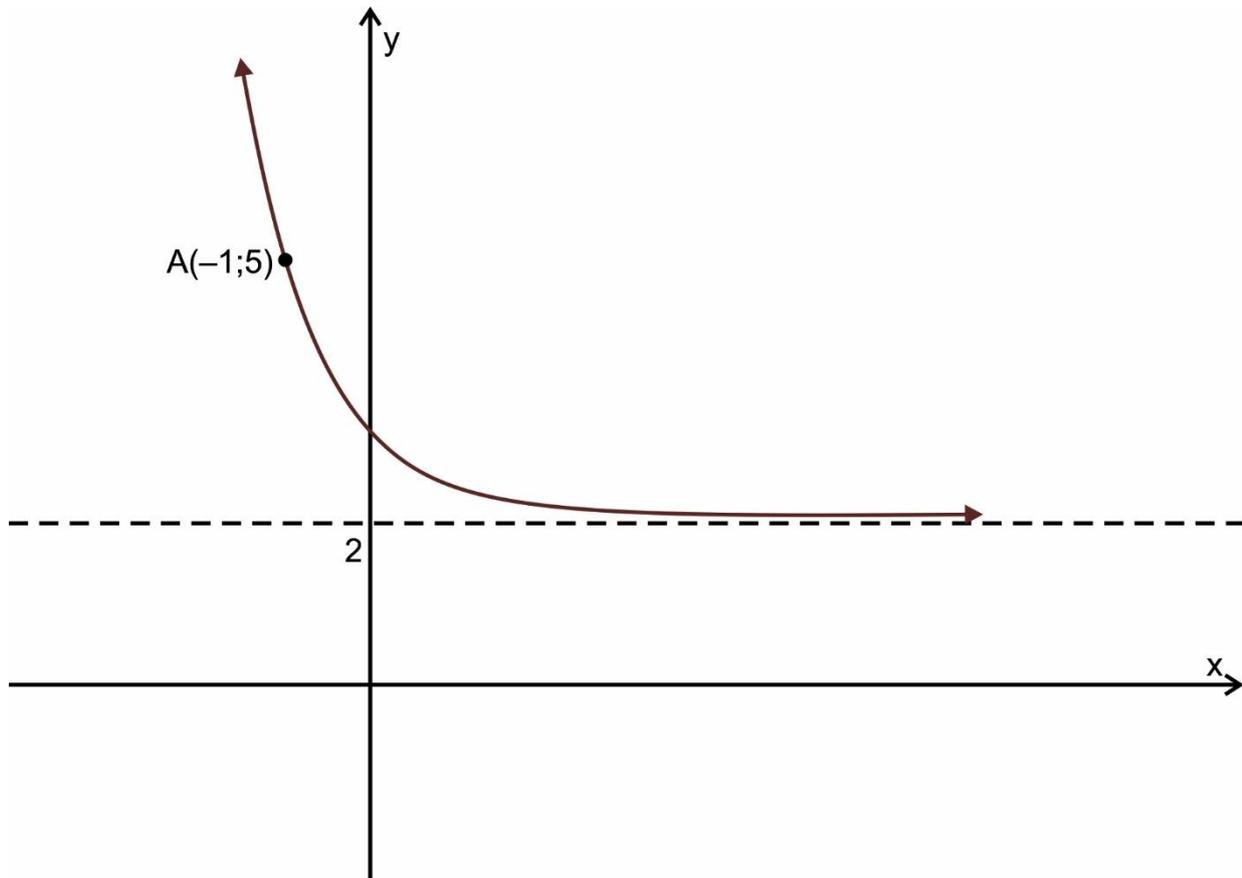
(5)

- (b) The fourth term of an arithmetic sequence is 29 and the fourteenth term is 44. Determine the first term.

(4)
[10]

QUESTION 4

(a) In the diagram below $g(x) = a^x + q$ has been drawn with A (-1;5) a point on g .



(1) Determine the values of a and q .

(2) Determine the co-ordinates of the y -intercept of g . (2)

(1)

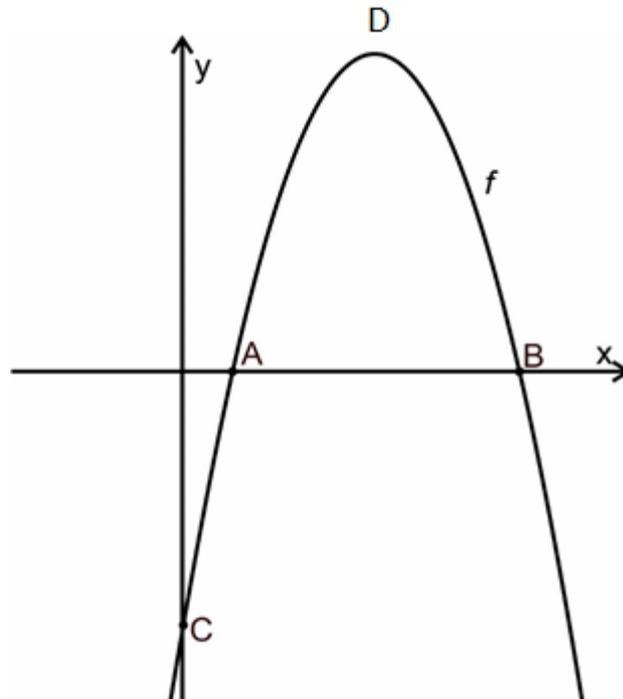
- (3) Sketch the graph of $g^{-1}(x)$ on the set of axes on page 6.
(Label all asymptotes and intercepts with the axes)

(3)

- (4) For what value of x is $g^{-1}(x) > 0$?

(2)

- (b) In the diagram below $f(x) = -x^2 + 5x - 4$. D is the turning point of f and A, B and C the intercepts with the respective axes.



- (1) Determine the coordinates of points A, B, C and D.

(7)

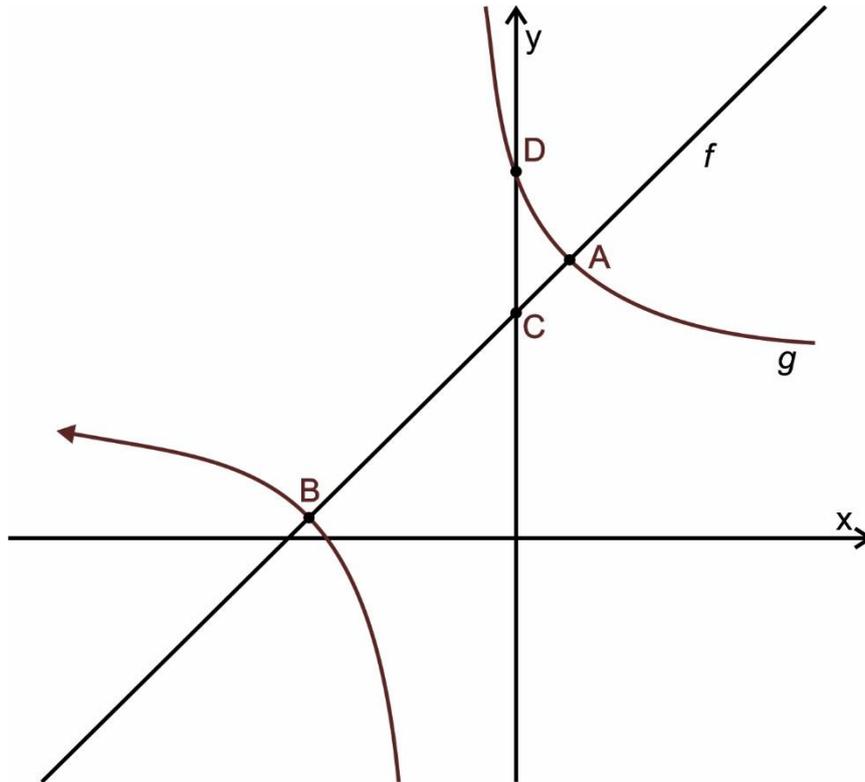
- (2) If $h(x) = f(x) + t$, then for what value(s) of t will h have roots that are real and equal?

(1)
[16]

QUESTION 5

In the diagram below the graphs of f and g have been drawn.

- f is an **axis of symmetry** for the graph of g .
- C and D lie on the y -axis.



- (a) If $g(x) = \frac{3}{x+1} + q$ and $f(x) = x + 3$; then determine the equation of the horizontal asymptote for g .

(1)

(b) If $q = 2$ then determine,

(1) the length of CD.

(1)

(2) the coordinates of A and B if f and g intersect at A and B. (*Give answers in simplest surd form.*)

(c) For what values of x is $f(x) \geq g(x)$?

(7)

(3)
[12]

QUESTION 6

You take out a home loan of R820 000 from the bank. The fixed interest rate you are given by the bank is 10% per annum compounded monthly.

- (a) If you are required to pay off the loan over a period of 20 years; then if the first payment is made at the end of the first month, what are the minimum monthly payments required to pay off the loan?

(4)

- (b) If, instead of the minimum payment, you pay R20 000 per month, then how long would it take for you to pay off the loan? (*Round up to the nearest month*)

(5)
[9]

74 marks

SECTION B**QUESTION 7**

You place R750 into an investment at the end of each month. The investment has an interest rate of 15% per annum compounded monthly. After you make the 36th payment you reduce your monthly payments to R500 per month for two years. After these two years you stop making monthly payments and leave your investment to grow. What is your investment worth at the end of the tenth year?

[7]

QUESTION 8

- (a) If $3^{x+1} + 3^{x-1} = 20$ is simplified, then $x = \log_w t$. Find a possible value for w and t .

(4)

- (b) Given: $\frac{5^{-1} \cdot 75^{2x}}{15^{4x} \cdot 3^{-3x}} = k \times 3^x$ where k is a constant.

Determine the value of k .

(5)

- (c) The equation $3^{x^2 - px + 1} = 27$
Prove that the equation has real and unequal roots for all real values of p .

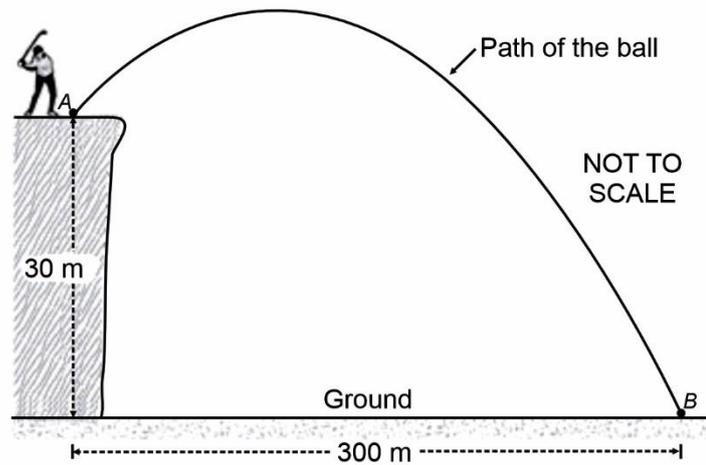
(5)
[14]

QUESTION 9

A golf ball is hit from point A to point B.

- Point A is 30 metres vertically above the ground.
- Point B is 300 metres away from A on ground level.
- The path of the golf ball is modelled by $y = ax^2 + bx + c$.
- The ball passes through the point (200;30).

The path of the flight of the golf ball is placed in the Cartesian plane so that A is on the y axis and B is on the x axis.



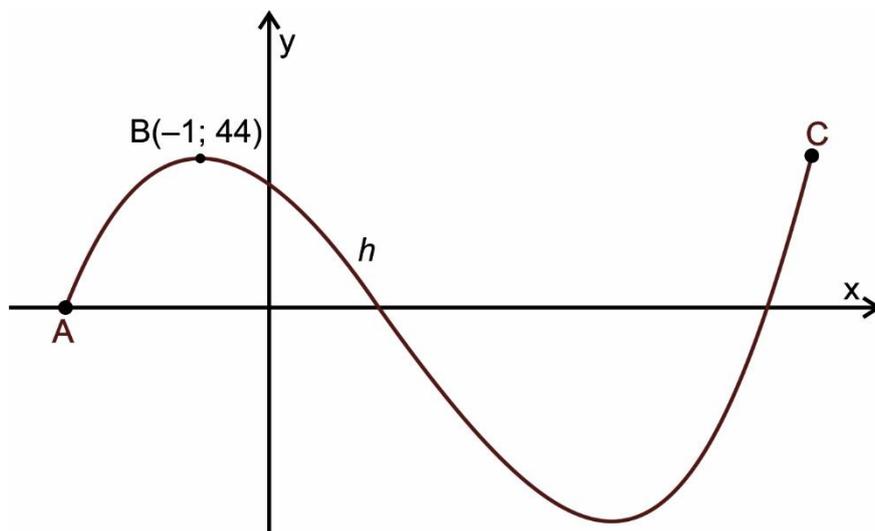
Determine the maximum height the ball reaches above the ground.

QUESTION 10

(a) If $g(x) = -x^3 + x^2 + 21x - 45$ then determine the x intercepts of g .

(3)

(b) In the diagram below; B is one of the turning points of the cubic function h .

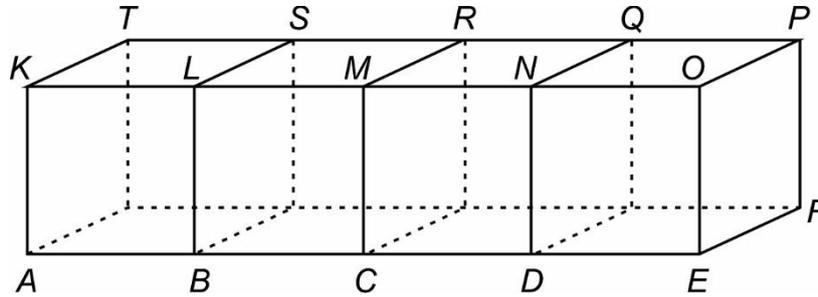


Find the values of a and b if $h(x) = x^3 + ax^2 + bx + 36$

(8)
[11]

QUESTION 11

The diagram shows a steel structure that is made up of four equal rectangular prisms placed next to each other. **The base of each prism is a square.**



The amount of steel available to build the structure above is 10 metres.

Note: Let $AB = x$ and $BL = y$

(a) Show that the volume of the structure is given by $V = 4x^2 - 10,4x^3$

- (b) Determine the dimensions of each rectangular prism if the volume is maximised.
(Give your answers correct to two decimal places.)

(4)
[9]

QUESTION 12

- (a) In a geometric sequence $S_5 = \frac{605}{243}$ and $T_n = w \cdot 3^{-n}$.

Determine the value w .

(6)

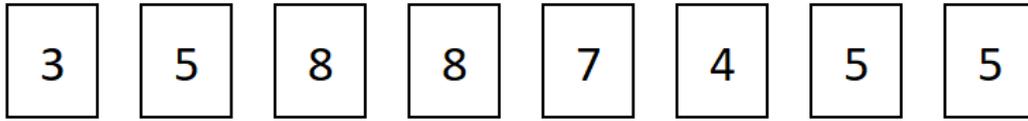
- (b) Determine the value of p if:

$$\sum_{n=1}^{\infty} (2^{-n}) + \sum_{n=1}^p (2n + 1) = 484$$

(7)
[13]

QUESTION 13

(a) You are given 8 cards. Each card has a number on it as shown below.



(1) How many unique eight-digit numbers can be created with the cards above?

(2)

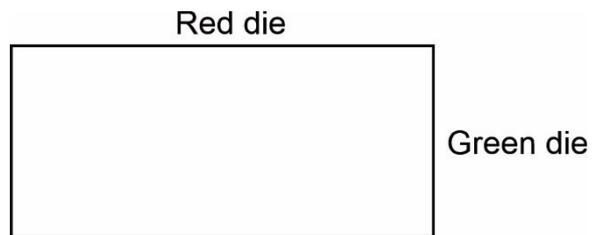
(2) What is the probability that an eight-digit number created will be greater than 80 000 000 and divisible by 5? (Explain your calculations)

(3)

(3) If a ninth numbered card is added to the above set of eight and 15 120 unique 9-digit numbers are created, then what number(s) from 0 to 9 could be on this card?

(4)

- (b) The length of the sides of the rectangular shape below are determined by throwing a regular six-sided red die and a regular six-sided green die.



Determine the probability that the area of the rectangular shape will be greater than 12 units^2 ; have a perimeter less than 22 units and never be a square.

(6)
[15]

76 marks

Total: 150 marks

ADDITIONAL SPACE (ALL QUESTIONS)

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE
ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**

