NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2017

MATHEMATICS: PAPER I
MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1)

$$\begin{aligned} 4-\sqrt{2-x} &= 3x \\ -\sqrt{2-x} &= 3x-4 \\ 2-x &= 9x^2-24x+16 \\ 9x^2-23x+14 &= 0 \\ (9x-14)(x-1) &= 0 \\ x &= \frac{14}{9} \text{ or } x=1 \end{aligned}$$

Check: $x=\frac{14}{9}$: $4-\sqrt{2-x}=\frac{10}{3}$

$3x=\frac{14}{3}$

$\therefore x=\frac{14}{9}$ is not a solution (6)

(2)

$$\begin{aligned} 2(5)^{9-x} &= 1\ 250 \\ 5^{9-x} &= 625 \\ 5^{9-x} &= 5^4 \\ 9-x &= 4 \\ x &= 5 \end{aligned}$$

(3)

(b) (1) For A and B: Let $y = 0$

A (–1; 0) B(3; 0)

For C: let $x = 0$

$y = 3\,(1)(–3) = –9$

$\therefore$ C $(0; -9)$

For D: = $x_D = \frac{-1+3}{2} = 1$ for writing as coordinates

$\therefore y = 3(1+1)(1-3)$

$\therefore y = -12$

$\therefore$ D $(1; -12)$ (6)

(2) $x \leq -1$ or $x \geq 3$ (2)

(c)

$x = 6 \quad or \quad x = 3$

$(x-6)(x-3) = 0$

$x^2 - 9x + 18 = 0$

$t = -9$

OR

$x = -6 \quad or \quad x = -3$

$(x+6)(x+3) = 0$

$x^2 + 9x + 18 = 0$

$t = 9$

Alternate: Product of roots = 18

Numbers are 6 and 3 or –6 and –3

Sum of roots = –t

$\therefore$ –t = 6 + 3 = 9 OR –t = –6 –3 = –9

$\therefore$ t = 9 or t = –9

Alternate: Let the smaller root be a.

The other root is *a* + 3.

The equation is:

$$
\begin{aligned}
(x-a)\left[x-(a+3)\right] &= 0 \\
x^2 - \left(a+a+3\right)x + a\left(a+3\right) &= 0 \\
\therefore t &= -2a-3 \\
a\left(a+3\right) &= 18 \\
a^2 + 3a - 18 &= 0 \\
\left(a+6\right)\left(a-3\right) &= 0 \\
a &= -6 \text{ OR } a = 3
\end{aligned}
$$

$\therefore$ t = 9 or t = –9

(5)

(d)

$$
\begin{aligned}
y &= 2-3x \\
(2-3x)^2 &= 2x^2 - 1 \\
4 - 12x + 9x^2 - 2x^2 + 1 &= 0 \\
7x^2 - 12x + 5 &= 0 \\
(7x-5)\left(x-1\right) &= 0 \\
x &= 1 \quad \text{or} \quad x = \frac{5}{7} \\
\therefore y &= -1 \quad \text{or} \quad y = -\frac{1}{7}
\end{aligned}
$$

(7)

[29]

QUESTION 2

(a)

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$2\ 500\ 000 = x\left[\frac{1-\left(1+\frac{6}{400}\right)^{-(5\times 4)}}{\frac{6}{400}}\right]$$

$$x \approx \text{R}145\ 614{,}34$$

(4)

(b)

$$F = x\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$F = 145\ 614{,}34\left[\frac{\left(1+\frac{6}{400}\right)^{(4\times 4)}-1}{\frac{6}{400}}\right]$$

$$F = 2\ 611\ 210{,}19$$

NB: Discrepancies in rounding

$$A = P\left(1+\frac{r}{100t}\right)^{nt}$$

$$A = 2\ 500\ 000\left(1+\frac{6}{400}\right)^{16}$$

$$A = 3\ 172\ 463{,}87$$

Balance of loan = R561 253, 68

ALTERNATE:

$$\text{Balance outstanding} = 145\ 614{,}34\left[\frac{1-\left(1+\frac{6}{400}\right)^{-4}}{\frac{6}{400}}\right] = \text{R}561\ 253{,}67$$

(5)

(c) In four years, the farmer paid: $145\ 614{,}34 \times 4 \times 4 = \text{R}2\ 329\ 829{,}44$

Balance of loan after four years = R561 253,68

Paid towards original loan: 2 500 000 – 561 253,68 = R1 938 746,32

Interest charges were: 2 329 829,44 – 1 938 746,32 = R391 083,12 (3)

[12]

QUESTION 3

(a) (1) $r = \frac{1}{3}$

$45 + 15 + 5 + ... = \sum_{n=1}^{\infty} 45\left(\frac{1}{3}\right)^{n-1}$ (3)

(2) $-1 < \frac{1}{3} < 1$ (1)

(3) $S_\infty = \frac{45}{1 - \frac{1}{3}} = 67,5$ (2)

(b) (1) $S_n = \frac{n}{2}(7n + 19)$

$4\ 878 = \frac{n}{2}(7n + 19)$

$9\ 756 = 7n^2 + 19n$

$0 = 7n^2 + 19n - 9\ 756$

$n = \frac{-19 \pm \sqrt{19^2 - 4(7)(-9756)}}{14}$

$n = \frac{-19 \pm \sqrt{273\ 529}}{14}$

$\therefore n = 36$ (5)

(2) $T_7 = S_7 - S_6$

$S_6 = \frac{6}{2}(7(6) + 19)$

$S_6 = 183$

$S_7 = \frac{7}{2}(7(7) + 19)$

$S_7 = 238$

$T_7 = 55$

Alternate

$a = T_1$

$S_1 = 13 \quad S_2 = 33$

$\therefore T_2 = 20 \quad d = 7$

$\therefore T_7 = 13 + 6(7) = 55$

Alternate

$d = 2\left(\frac{7}{2}\right) = 7$

$a = S_1 = \frac{1}{2}(7 + 19) = 13$

$T_7 = a + (n-1)d$

$= 13 + 6(7)$

$= 55$ (4)

(c) (1) Volume of each cylinder $= \pi r^2 h$

Ratio of consecutive volumes $= \dfrac{\pi\left(\frac{9}{10}r\right)^2\left(\frac{9}{10}h\right)}{\pi r^2 h}$

$= \left(\dfrac{9}{10}\right)^3$

Which is a constant

$\therefore$ The sequence is geometric.

(3)

(2) Volume = S_{12}

$= \dfrac{a\left(1-r^{12}\right)}{1-r}$

$= \dfrac{\pi(18)^2(64)\left(1-\left[\left(\frac{9}{10}\right)^3\right]^{12}\right)}{1-\left(\frac{9}{10}\right)^3}$

$= 234\ 968{,}5\ \text{cm}^3$

≈ 235 litres

(4)

[22]

QUESTION 4

Quadratic pattern $\therefore\ T_n = an^2 + bn + c$

$a + b + c = -1$

$4a + 2b + c = 4$

$9a + 3b + c = 11$

$\therefore 3a + b = 5$ and $5a + b = 7$

$3a + b = 5 \qquad 5a + b = 7$

$b = 5 - 3a$

Sub into: $5a + b = 7$

$\therefore 5a + 5 - 3a = 7$

$2a = 2$

$a = 1$

$\therefore b = 2$

$\therefore c = -4$

Alternate

$-1 \quad 4 \quad 11$

$5 \quad 7$ First difference

2 Second difference

$\therefore 2a = 2,\ \therefore a = 1$

$\therefore T_n = n^2 + bn + c$

$3a + b = 5$ and Sub. $a = 1$

$3(1) + b = 5,\quad b = 2$

$a + b + c = -1$ Sub. $a = 1$ and $b = 2$

$1 + 2 + c = -1$

$\therefore c = -4$

[6]

QUESTION 5

(a) $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Working

$$f(x) = \frac{1}{2}x^2$$

$$f(x+h) = \frac{1}{2}(x+h)^2$$

$$f(x+h) = \frac{1}{2}(x^2+2xh+h^2)$$

$$f(x+h) = \frac{1}{2}x^2+xh+\frac{1}{2}h^2$$

$$f'(x) = \lim_{h\to 0}\frac{\left[\frac{1}{2}x^2+xh+\frac{1}{2}h^2\right]-\left(\frac{1}{2}x^2\right)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{xh+\frac{1}{2}h^2}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{h\left(x+\frac{1}{2}h\right)}{h}$$

$$f'(x) = x$$

(5)

(b) $y = \dfrac{2x^3}{x}-\dfrac{x^{\frac{1}{2}}}{x}$

$$y = 2x^2 - x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 4x+\frac{1}{2}x^{-\frac{3}{2}}$$

$$\frac{dy}{dx} = 4x+\frac{1}{2x^{3/2}}$$

(5)

[10]

79 marks

SECTION B

QUESTION 6

(a)

$$f(x)=2\left(x^2-10x+\frac{47}{2}\right)$$
$$f(x)=2\left[x^2-10x+(-5)^2-(-5)^2+\frac{47}{2}\right]$$
$$f(x)=2\left[(x-5)^2-25+\frac{47}{2}\right]$$
$$f(x)=2(x-5)^2-3$$

Alternate

$$-\frac{b}{2a}=\frac{20}{4}=5 \qquad \therefore \frac{b}{2a}=-5$$
$$-\frac{\Delta}{4a}=\frac{(-20)^2-4(2)(47)}{8}$$
$$\frac{\Delta}{4a}=-3$$
$$\therefore f(x)=a\left(x+\frac{b}{2a}\right)^2+\frac{\Delta}{4a}$$
$$=2(x-5)^2-3$$

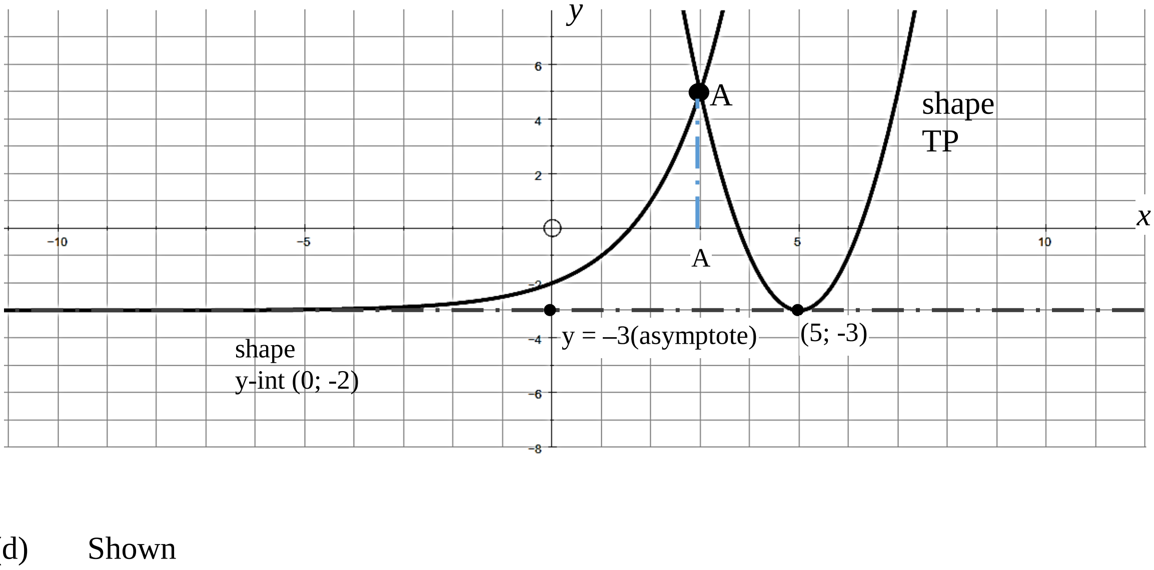
(4)

(b)

$$h(x)=b^x-q \quad \text{sub.: } (0;-2)$$
$$-2=b^0+q$$
$$q=-3$$
$$h(x)=b^x-3 \quad \text{sub.: } (1;-1)$$
$$-1=b^1-3$$
$$b=2$$
$$h(x)=2^x-3$$

(3)

(c)



(5)

(d) Shown (1)

(e)

$$x=2^y-3$$
$$y=\log_2(x+3)$$

(3)

(f) Domain of h^{-1} = Range of h = $(-3;\ \infty)$ (2)

[18]

QUESTION 7

$$\therefore\ f(x)=\frac{-2(x+1)+4}{x+1}$$
$$=\frac{4}{x+1}-2$$

Asymptotes are $x = -1$ and $y = -2$

Alternate 1

$$\frac{a}{x+1}+q=\frac{-2x+2}{x+1}$$
$$a+q(x+1)=-2x+2$$
$$qx+(a+q)=-2x+2$$
$$q=-2,\quad a+q=2,\quad a=4$$
$$f(x)=\frac{4}{x+1}-2$$

$\therefore$ Asymptotes are: $x=-1$ and $y=-2$

Alternate 2

$$x+1\overline{)-2x+2}\quad -2$$
$$\underline{-2x-2}$$
$$4$$
$$\therefore -2x+2=-2(x+1)+4$$
$$\frac{-2x+2}{x+1}=-2+\frac{4}{x+1}$$

$\therefore$ Asymptotes are: $x=-1$ *and* $y-2$

Alternate 3

$$\frac{-2x+2}{x+1}=\frac{-2(x+1)+q}{x+1}$$
$$-2+q=2$$
$$\therefore q=4$$
$$\therefore\ \frac{-2(x+1)+4}{x+1}$$
$$=\frac{-2+4}{x+1}$$

$\therefore x=-1;\quad y=-2 \quad \}$ Asymptotes

[6]

QUESTION 8

$$f(x)=2x+\frac{1}{2x}$$
$$f(x)=2x+\frac{1}{2}x^{-1}$$
$$f'(x)=2-\frac{1}{2x^2}$$
$$f'(-1)=2-\frac{1}{2}$$
$$=\frac{3}{2}$$

Equation of tangent is:

$$y=\frac{3}{2}x+c$$
$$\frac{-5}{2}=\frac{3}{2}(-1)+c$$
$$c=-1$$
$$\therefore Eq: y=\frac{3}{2}x-1$$

[6]

QUESTION 9

Eq. AB : $y = mx + 9$ sub (12;0)

$0 = 12m + 9$

$m = \dfrac{-3}{4}$

$\therefore$ length CD $= -\dfrac{3}{4}x + 9$

Area ΔCOD $= \dfrac{1}{2}xy = \dfrac{1}{2}x\left(-\dfrac{3}{4}x+9\right)$

$= -\dfrac{3}{8}x^2 + \dfrac{9}{2}x$

$\dfrac{dA}{dx} = -\dfrac{3}{4}x + \dfrac{9}{2}$

$0 = -\dfrac{3}{4}x + \dfrac{9}{2}$

$x = 6$

Alternate

$x = \dfrac{-b}{2a}$

$= \left(\dfrac{-9}{2}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{-8}{3}\right)$

$= 6$

[8]

QUESTION 10

(a)

$f(x) = ax^3 + bx^2 - 3$

$f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$

$f''(-2) = 6a(-2) + 2b$

$0 = -12a + 2b$

$b = 6a$

Sub: $(-3\,;\,6)$ in $f(x)$

$f(-3) = a(-3)^3 + b(-3)^2 - 3 = 6$

$-27a + 9b - 9 = 0$

$-27a + 9(6a) - 9 = 0$

$27a = 9$

$a = \dfrac{1}{3}$ $\therefore\ b = 6\left(\dfrac{1}{3}\right) = 2$ (7)

(b) (1) $f(x) = \dfrac{1}{3}x^3 + 2x^2 - 3$

$f'(x) = x^2 + 4x$

$\dfrac{f'(x)}{x} = x + 4$ and $x \neq 0$

$\dfrac{f'(x)}{x} \geq 0,$ when $x \geq -4$ $x \neq 0$

OR $x \in [-4;0)$ *or* $(0;\infty)$ (4)

(2) *f* is concave up when

$f''(x) > 0$

$f''(x) = 6ax + 2b = 2x + 4$

$2x + 4 > 0$

$x > -2$ (3)

[14]

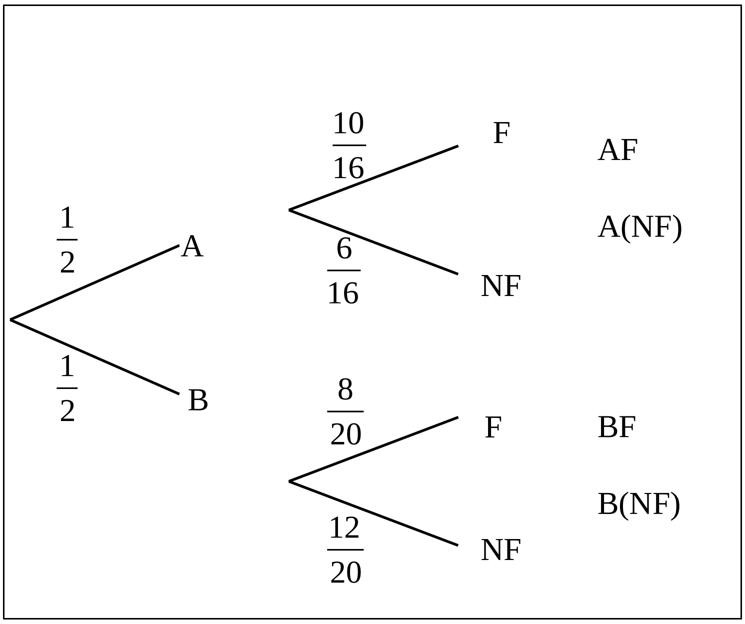
QUESTION 11

(a) (1) Number of arrangements is 5! = 120 (2)

(2) $\frac{6!}{2!} = 360$ (2)

(3) $\frac{120}{360} = \frac{1}{3}$ (1)

(b) (1)



P (picking up a fiction book)

= P ({Shelf A and fiction book} or {Shelf B and fiction book})

= P (AF) + P (BF)

$= \frac{1}{2}\left(\frac{10}{16}\right) + \frac{1}{2}\left(\frac{8}{20}\right)$

$= \frac{41}{80}$ (4)

(2) $1 - \left(\frac{1}{2}\right)\left(\frac{6}{16}\right) = \frac{13}{16}$ **Alternate:** $\left(\frac{1}{2}\right)\left(\frac{10}{16}\right) + \left(\frac{1}{2}\right)\left(\frac{8}{20}\right) + \frac{1}{2}\left(\frac{12}{20}\right) = \frac{13}{16}$ (3)

[12]

QUESTION 12

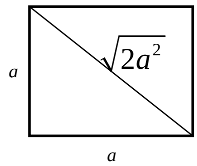
The full sphere touches all eight pieces. Let the side of the cube be a. The radius of the sphere is x. The diagonal of the cube is: $x + x + 2x = 4x$.

The diagonal of each face of the cube has length $\sqrt{2a^2}$ (by Pythag.)

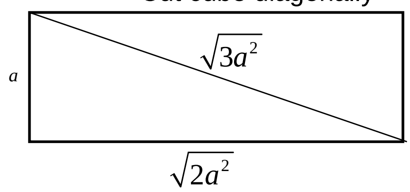
$\therefore$ The diagonal of the cube has length:

$$\sqrt{\left(\sqrt{2a^2}\right)^2 + a^2} = \sqrt{3a^2} \text{ (by Pythag.)}$$

Face of cube



Cut cube diagonally



But: $\sqrt{3a^2} = 4x$

$$\sqrt{3}a = 4x$$

$$a = \frac{4}{\sqrt{3}}x$$

Volume liquid = $a^3 - 2\left(\frac{4}{3}\pi x^3\right) = 116,8$

$$\left(\frac{4}{\sqrt{3}}x\right)^3 - 2\left(\frac{4}{3}\pi x^3\right) = 116,8$$

$$\left(\frac{4}{\sqrt{3}}x\right)^3 \cdot x^3 - \frac{8}{3}\pi x^3 = 116,8$$

$$x^3 = \frac{116,8}{\left(\frac{4}{\sqrt{3}}\right)^3 - \frac{8}{3}\pi}$$

$$x = 3,095 \approx 3,1 \text{ cm}$$

ALTERNATE: Let the side of the cube be a …

$$\sqrt{3a^2} = 4x$$

$$x = \frac{\sqrt{3}a}{4}$$

Volume of liquid = $a^3 - 2\left(\frac{4\pi}{3}x^3\right)$

$$116,8 = a^3 - \left(\frac{8\pi}{3}\right)\left(\frac{\sqrt{3}}{4}\right)^3 a^3$$

$$a^3 = \frac{116,8}{1 - \left(\frac{8\pi}{3}\right)\left(\frac{\sqrt{3}}{4}\right)^3}$$

$$\therefore x = \frac{\sqrt{3}a}{4} = \frac{\sqrt{3}}{4} \times \sqrt[3]{\frac{116,8}{1 - \left(\frac{8\pi}{3}\right)\left(\frac{\sqrt{3}}{4}\right)^3}} \approx 3,1 \text{ cm}$$

Use of volume of cube
Indication of volume of 2 spheres
Subtracting volumes

[7]

71 marks

Total: 150 marks