NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2022

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) $m_{PQ} = \frac{8}{4} = 2$

(2) $\frac{t-9}{-6-2} = 2$ **OR** $\frac{t-17}{-6-6} = 2$

$t = -7$

(3) $m = -\frac{1}{2}$

Midpoint of PQ is $(4;13)$

$13 = -\frac{1}{2}(4) + c$

$c = 15$

$y = -\frac{1}{2}x + 15$

(4) $y = -\frac{1}{2}(14) + 15$

$y = 8$

$H(14;4)$ and $P(2;9)$

$PH = \sqrt{(14-2)^2 + (4-9)^2}$

$PH = 13$

(b) $\tan\beta = 5$

$\beta = 78,7°$

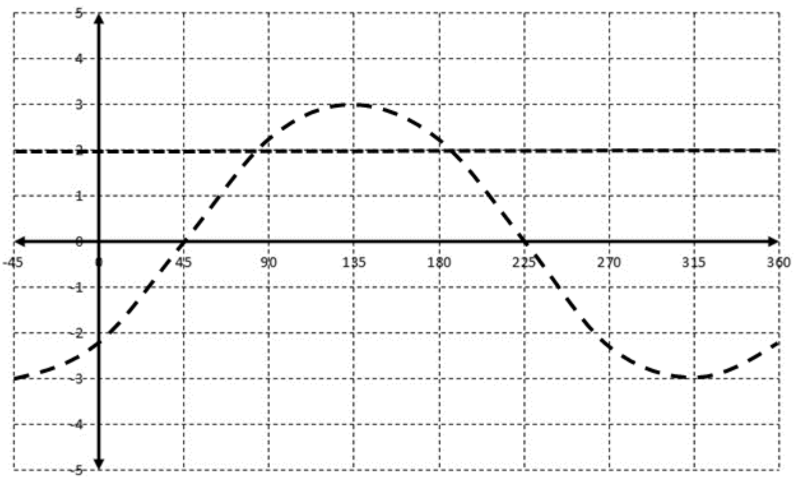
$\tan\alpha = 1/2$

$\alpha = 26,6°$

$\theta = 52,1°$

QUESTION 2

(a)



starting point
y-intercept
turning point
shape

straight line through 2 end points correct

(b) $3\sin(x - 45°) = 2$

$\sin(x - 45°) = 2/3$

Reference angle: $41,8°$

In quadrants 1 and 2

Solution 1:

$(x - 45°) = 41,8° + k.360°$

$x = 86,8° + k.360°$

Solution 2:

$(x - 45°) = 180° - 41,8° + k.360°$

$x = 183,2° + k.360°$

$x = \{86,8°; 183,2°\}$

(c) $x \in (86,8°; 183,2°)$

QUESTION 3

(a) (1) $\frac{\cos 2\theta + 1}{\sin 2\theta} + \tan\theta = \frac{1}{\sin\theta\cos\theta}$

$\frac{2\cos^2\theta - 1 + 1}{2\sin\theta\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$

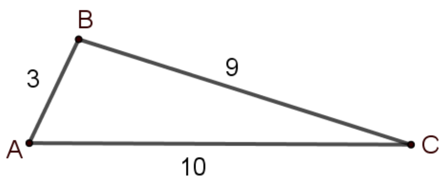
$\frac{1}{\sin\theta\cos\theta}$

(2) $\theta = \{0°; 90°\}$

(b) (1) $9^2 = 3^2 + 10^2 - 2(3)(10)\cos A$

$\cos A = 7/15$

$A = 62,2°$

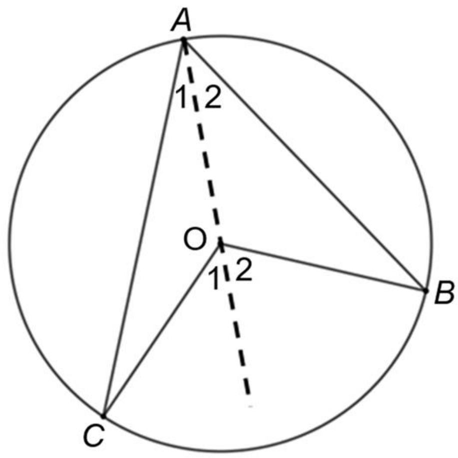


(2) Area $\Delta ABC = \frac{1}{2}(3)(10)\sin 62,2°$

Area $\Delta ABC = 13,3$ units^2

QUESTION 4

(a) Construction A through O



Proof:

$O_1 = A_1 + C$ Exterior angle of triangle

$A_1 = C$ Isos triangle

$O_1 = 2A_1$

Similarly

$O_2 = 2A_2$

Therefore

$O_1 + O_2 = 2(A_1 + A_2)$

$2 \times CAB = COB$

(b) (1) $COD = 100°$ (Angle at centre $= 2\times$ angle at circumference)

$D_2 = OCD = 40°$ (radii, isos Δ)

$y = 18°$

(2) $x + y = 36°$ (Angles in same segment) ✓

Therefore

$x = 18°$

(c) (1) $J_1 + J_2 = 90°$ (Angles in a semi-circle) ✓

$RK = 10$ units

$JK^2 = 10^2 - 5^2$

$JK = \sqrt{75}$

(2) $M_2 = 90°$ Line from centre drawn to midpoint of chord

$OM^2 = 5^2 - \left(\frac{\sqrt{75}}{2}\right)^2$

$OM = \frac{5}{2}$

$ML = \frac{5}{2}$

QUESTION 5

(a) $\frac{EC}{4} = \frac{9}{3}$ (Proportion theorem)

$EC = 12$ units

(b) $\frac{EG}{12} = \frac{2}{3}$

$EG = 8$ units

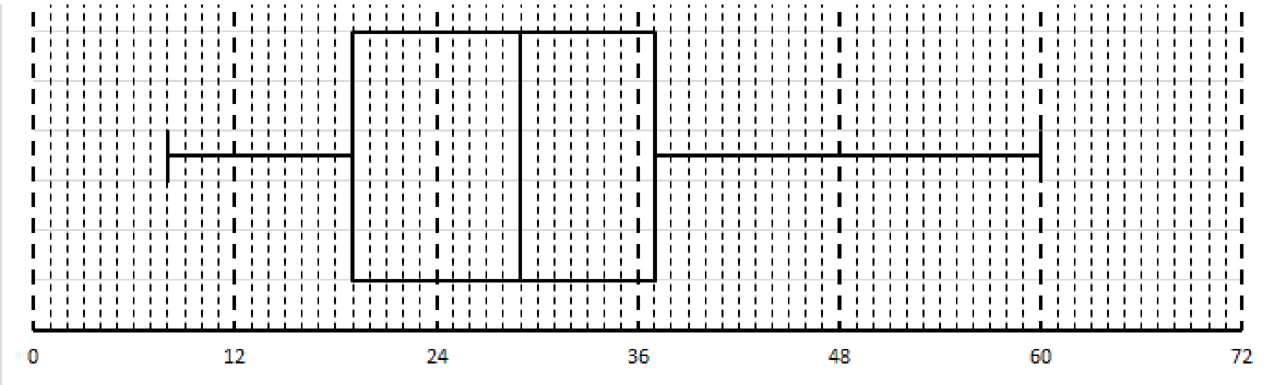
$\frac{FG}{8} = \frac{9}{12}$

$FG = 6$ units

QUESTION 6

(a) (1) $74 - 10 = 64$ people

(b) 8 19 29 37 60 (refer to the box and whisker)



(c) $\frac{21}{80} \times 100 = 26,25\%$

SECTION B

QUESTION 7

(a) Skewed to the left or negatively skewed

(b) Stays the same

(c) $\dfrac{9(5)+16(15)+25k}{25+k}=16,5$

$285+25k=412,5+16,5k$

$8,5k=127,5$

$k=15$

(d) (1) D 0,94

(2) B would increase; as the outlier T is making line less steep

(3) The correlation coefficient would decrease.
This point lies close to the line of best fit.

QUESTION 8

Construction: ED

$EDC = x$ (Tan chord theorem)

Isos triangle (Tangents drawn from common point)

$DCE = 180° - 2x$

$MKJ = 180° - 2x$ (Exterior angle of cyclic quad)

QUESTION 9

(a) $D_3 = E$ (Tan chord theorem)

$D_1 = F_2$ (Alternate angles // lines)

$\Delta FED /// \Delta GDF$ (A.A.A)

(b) $\frac{ED}{DF} = \frac{FD}{GF}$ $\Delta FED /// \Delta GDF$

$FD^2 = ED.GF$

but

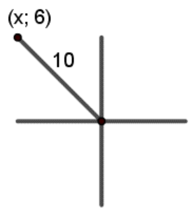
$FD^2 = FH^2 + HD^2$ Pythag

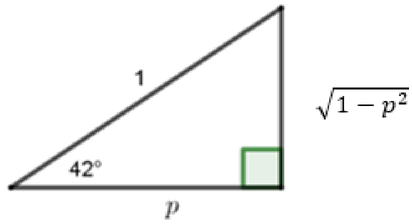
Therefore

$FH^2 + HD^2 = ED.GF$

QUESTION 10

(a)





$x = -8$

$y = 6$

$r = 10$

$\cos(-A - 42^\circ)$

$\cos A \cos 42^\circ - \sin A \sin 42^\circ$

$\frac{-8}{10}p - \frac{6}{10}\sqrt{1-p^2}$

OR

$\frac{-4p - 3\sqrt{1-p^2}}{5}$

(b) $4\sin^2\theta = \sin 2\theta$

$4\sin^2\theta - 2\sin\theta\cos\theta = 0$

$2\sin\theta(2\sin\theta - \cos\theta) = 0$

$2\sin\theta = 0$ OR $2\sin\theta = \cos\theta$

$\theta = k.180^\circ$ $\tan\theta = \frac{1}{2}$

$\theta = 26{,}6^\circ + k.180^\circ$

(c) $\dfrac{BC}{\sin\left(180^\circ-\left(\beta+\theta\right)\right)}=\dfrac{m}{\sin\beta}$

$$BC=\frac{m\sin\left(\beta+\theta\right)}{\sin\beta}$$

$C\hat{B}D=\theta$ (Alt angles // lines)

$$\cos\theta=\frac{p}{BC}$$

$$BC=\frac{p}{\cos\theta}$$

Therefore

$$\frac{m\sin\left(\beta+\theta\right)}{\sin\beta}=\frac{p}{\cos\theta}$$

$$\frac{m\left(\sin\beta\cos\theta+\sin\theta\cos\beta\right)}{\sin\beta}=\frac{p}{\cos\theta}$$

(d) $3^2=x^2+x^2-2x^2\cos30^\circ$ (let $BC=BD=x$) **OR** $\dfrac{BD}{\sin75^\circ}=\dfrac{3}{\sin30^\circ}$

$9=2x^2\left(1-\cos30^\circ\right)$

$x=\sqrt{\dfrac{9}{2\left(1-\cos30^\circ\right)}}$

$x=5{,}8$ units $=BD=BC$

If pole is vertical then:

$\text{Rope}^2=4^2+2^2$

Rope should be $\sqrt{20}=4{,}5$ metres

The ropes should be 1,3 metres shorter.

QUESTION 11

(a) $3(0)+4x=12$

$x=3$

$P(3;0)$

$M(0;4)$

$MP=\sqrt{25}=5$

Therefore $NP=2$

Equation of circle *P*: $(x-3)^2+y^2=4$

(b) $16k^2+9k^2=4$

$k=0,4$

Alternative:

x-coordinate of N is $\frac{3}{5}$ of 3 hence $\frac{9}{5}$ (line ‖ to side of Δ)

y-coordinate from straight line

$N(1,8;1,6)$

$3y+4x=12$

$y=-\frac{4}{3}x+4$

Tangent

$1,6=\frac{3}{4}(1,8)+c$

Equation of tangent through *N*

$y=\frac{3}{4}x+\frac{1}{4}$

The *y*-intercept is $\frac{1}{4}$

QUESTION 12

$$\left(x-\frac{5}{2}\right)^2+\left(y-3\right)^2=\frac{25}{4}$$

$$B\left(\frac{5}{2};3\right)$$

$$m_{BC}=-\frac{4}{3}$$

Gradient for $h(x)$ to be a tangent at C is (tangent perpendicular to radius)

$$m=\frac{3}{4}$$

A tangent at C would have an equation of

$$1=\frac{3}{4}(4)+c$$

$$y=\frac{3}{4}x-2$$

$$h(x)=\frac{1}{4}x+px+t$$

Therefore

$$p=\frac{1}{2}$$

$$t=-2$$

Total: 150 marks