NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2018

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1) 0,973 (2)

(2) Strong, positive (2)

(b) $y = 0,1 + 0,6x$ (2)

(c) $y = 0,0981 + 0,0057(120)$
$y = 0,7821$ **OR** 0,7780 (calculator) (2)

(d) No, as this would be extrapolation. (2)
[10]

QUESTION 2

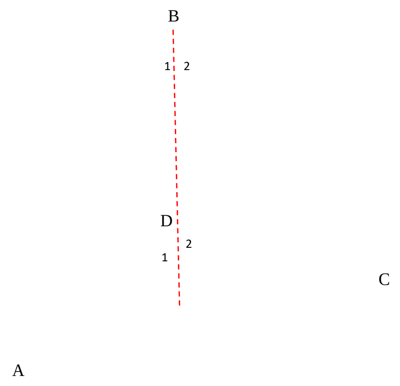
(a) $6y + 5(0) = 30$
$y = 5$
T(0; 5) (2)

(b) Area = 9 × 5
N(9; 0) (2)

(c) $m_{MN} = \frac{3-0}{0-9} = \frac{1}{3}$
$\therefore y = -\frac{1}{3}x + 3$ (3)

(d) $-\frac{1}{3}x + 3 = -\frac{5}{6}x + 5$
$-2x + 18 = -5x + 30$
$3x = 12$
$x = 4$
$S\left(4; \frac{5}{3}\right)$
For R: 6(0) + 5*x* = 30
$R(6;0)$
$Area\ \Delta RSN = \frac{1}{2} \times (9-6) \times \frac{5}{3}$
$Area\ \Delta RSN = \frac{5}{2}\ units^2$ (8)
[15]

QUESTION 3



(a) Construction: refer to the diagram (1)

R.T.P: $2 \times A\hat{B}C = A\hat{D}C$ (1)

Proof:

$\hat{D}_1 = \hat{B}_1 + \hat{A}$ (Ext angle of triangle)

$\hat{D}_2 = \hat{B}_2 + \hat{C}$ (Ext angle of triangle)

but

$\hat{B}_1 = \hat{A}$ and $\hat{B}_2 = \hat{C}$ (Isos triangle radii)

Therefore

$\hat{D}_1 + \hat{D}_2 = 2\hat{B}_1 + 2\hat{B}_2$

$2 \times A\hat{B}C = A\hat{D}C$ (4)

(b) $OM = ON$ (radii)

$O\hat{M}N = 55°$ (Δs in an isos Δ)

$M\hat{O}N = 70°$

$\hat{S}_1 = 35°$ (Angle at centre is twice the angle at the circumference)

$\hat{S}_1 = S\hat{T}R = 35°$ (tan chord theorem) (6)

[12]

QUESTION 4

(a) $a = 5$ and $b = 1$ (2)

(b) $y = 5\sin(x - 30°) - 2$ (2)

(c) max of $g = 4$

$\therefore \min = \frac{8}{4} = 2$ (2)

(d) $k > 5$ or $k < -5$ (2)

(e) $5\sin x = 4\cos x$

$\tan x = \frac{4}{5}$

$x = 38{,}66 + k \cdot 180°$

$A(-141{,}34°; -3{,}12)$ (5)

[13]

QUESTION 5

(a) $2\sin\theta\cos\theta + \cos\theta = 0$

$\cos\theta(2\sin\theta + 1) = 0$

$\cos\theta = 0$ or $\sin\theta = -\frac{1}{2}$

$\theta = 90° + k \cdot 360°$ or $\theta = 270° + k \cdot 360°$

$\theta = 210° + k \cdot 360°$ or $\theta = 330° + k \cdot 360°$

Alt: $\theta = -90° + 360k$

Alternative:

$\sin 2\theta = \sin(\theta - 90°)$

$\therefore 2\theta = \theta - 90° + 360k$ or $2\theta = 180 - (\theta - 90°) + 360k$

$\therefore \theta = -90° + 360k$ $\therefore 3\theta = 270 + 360k$

$\therefore \theta = 90 + 120k$

Alternative:

$\sin 2\theta = -\cos\theta$

$\sin 2\theta = -\sin(90 - \theta)$

$\therefore\ 2\theta = 180 + 90 - \theta + 360k$

$3\theta = 270 + 360k$

$\theta = 90 + 120k$

OR

$2\theta = 360 - (90 - \theta) + 360k$

$\theta = 270 + 360k$

$k \in \mathbb{Z}$

(8)

(b) (1) Compound angle

$\cos 42° = p$

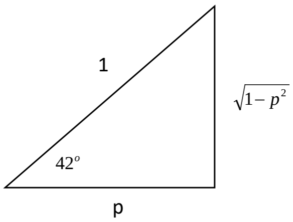
$\cos 42° + 7$

$p + 7$

(3)

(2) $\sin^2 42° + \cos^2 42° = 1$

$\sin 42° = \sqrt{1 - p^2}$



(3)

(c) $\frac{\sin\theta}{1 + \cos\theta} - \frac{(-\sin\theta)}{1 - \cos\theta}$

$= \frac{\sin\theta(1 - \cos\theta) + \sin\theta(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$

$= \frac{2\sin\theta}{1 - \cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta}$

$= \frac{2}{\sin\theta}$

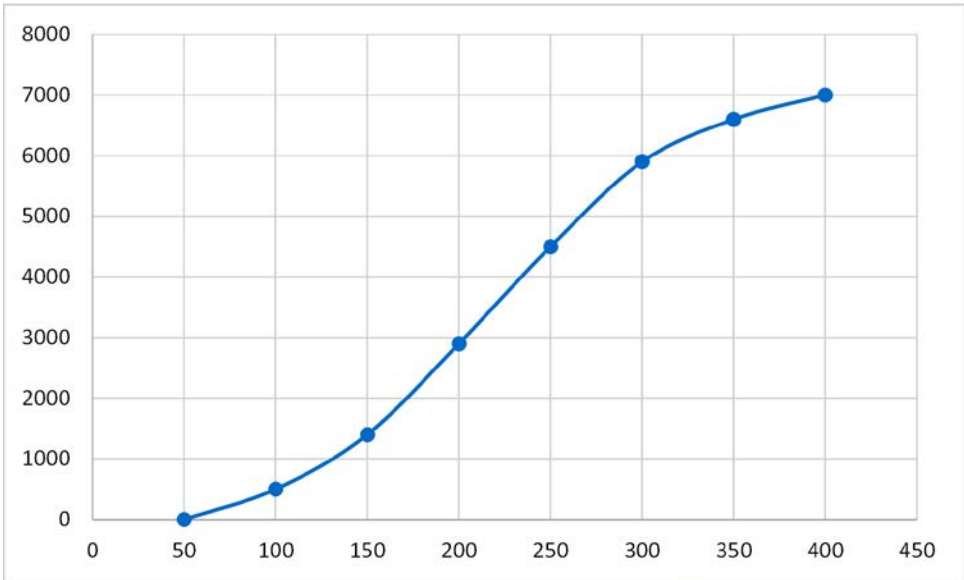
(6)

[20]

QUESTION 6

(a) 219,29 (2)

(b)



mark for the starting point
shape
mark for the curve going through the points and finishing at correct place (3)

(c) $\dfrac{6\ 000}{7\ 000} \times 100$ $\dfrac{6\ 050}{7\ 000} \times 100$ $\dfrac{5\ 950}{7\ 000} \times 100$

$= 85{,}71\%$ $= 86{,}43\%$ $= 85\%$

(Any answer between 85% and 86,5%) (2)

(d) Decrease
The difference between the new mean and new data is reduced. (2)
[9]

79 marks

SECTION B

QUESTION 7

$x^2 - 20x + 100 + y^2 = -p + 100$

$(x-10)^2 + y^2 = -p + 100$

Centre (10; 0)

Equation of line from centre through M is:

$y = -\frac{1}{2}x + c$

$0 = -\frac{1}{2}(10) + c$

$y = -\frac{1}{2}x + 5$

$2x = -\frac{1}{2}x + 5$

$M(2; 4)$

[8]

QUESTION 8

(a) False
Only one diagonal bisects the interior angles. (2)

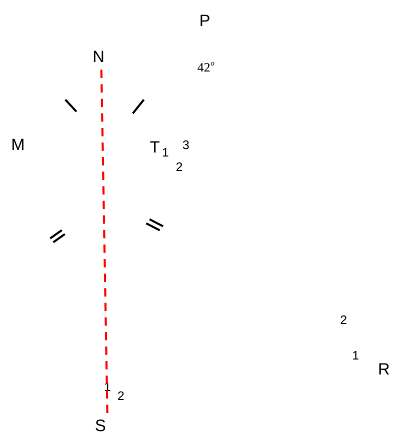
(b) Construction NS

$S\hat{N}T = 42°$; angles in same segment

but $S\hat{M}N = S\hat{N}T$,

Diagonal of kite NMST bisects $M\hat{N}T$

$M\hat{N}T = 84°$



Construct NS

$S\hat{N}R = S\hat{P}R = 42°$; angles in same segment

but $S\hat{N}M = S\hat{N}T$; diagonals of kite

$\therefore M\hat{N}T = 84°$ (5)

[7]

QUESTION 9

(a) $\hat{E}_2 = \hat{F}$ (Angles in same segment)

$\hat{A}_1 = \hat{F}$ (Alt angles AB//DF)

Therefore

$\hat{E}_2 = \hat{A}_1$

$\hat{C}_1 = \hat{C}_3$; given

$\therefore \Delta CBA \, ||| \, \Delta CDE$ $(A.A.A)$ (4)

(b) $\hat{B} = \hat{D}_2 + \hat{D}_3$ $(\Delta CBA /// \Delta CDE)$

Therefore

ABCD is a cyclic quad (Converse: Ext angle of cyclic quad = interior opp angle) (3)

(c) $\hat{E}_3 = \hat{D}_2 + \hat{D}_3$ (tan chord theorem)

$\hat{A}_2 + \hat{C}_2 = \hat{D}_2 + \hat{D}_3$ (Ext angle of triangle)

Therefore

$\hat{E}_3 = \hat{A}_2 + \hat{C}_2$ (4)

[11]

QUESTION 10

(a) $R\hat{P}S = 90^\circ$ (Line from centre drawn to midpoint of chord MS)

$R\hat{V}T = 90^\circ$ (Line from centre perpendicular to tangent) (4)

(b) $\frac{10}{7} = \frac{RN}{6}$ (Prop Theorem)

$RN = \frac{60}{7}$

$NK = 10 - \frac{60}{7}$

$\therefore NK = \frac{10}{7}$ (5)

(c) $WV^2 = \left(\frac{60}{7} + 6\right)^2 - 10^2$

$WV = 10{,}598211$

$\frac{PN}{WV} = \frac{RN}{RW}$ (ΔRPN///ΔRVW)

$\frac{PN}{10{,}598211} = \frac{\frac{60}{7}}{\frac{60}{7} + 6}$

$PN = 6{,}23$

Alternative

$\frac{RP}{RV} = \frac{RS}{RT}$ (SP//TV)

$\therefore \frac{RP}{10} = \frac{10}{17}$

$\therefore RP = \frac{100}{17}$

$\therefore RN = 10 - \frac{10}{7} = \frac{60}{7}$

$\therefore PN^2 = RN^2 - RP^2$

$= \left(\frac{60}{7}\right)^2 - \left(\frac{100}{17}\right)^2$

$= 38{,}8673$

$\therefore PN = 6{,}23$ (8)

[17]

QUESTION 11

A(3; 2) and B(9; –1)
AB = $\sqrt{45}$ = 6,71
Sum of radii = 3 + 3 = 6
AB > sum of radii $\therefore$ circles do not intersect **[5]**

QUESTION 12

$(x+r)^2+(y-r)^2=r^2$
subs (–2 ; 4)
$(-2+r)^2+(4-r)^2=r^2$
$\therefore 4-4r+r^2+16-8r+r^2=r^2$
$\therefore r^2-12r+20=0$
$\therefore (r-10)(r-2)=0$
$\therefore r=10$ or $\therefore r=2$
$\therefore (x+10)^2+(y-10)^2=100$
And $(x+2)^2+(y-2)^2=4$ **[8]**

QUESTION 13

(a) Coordinates of point B
$OB^2 = 4^2 + 4^2 - 2(4)(4)\cos 120°$
$OB = \sqrt{48}$ or $4\sqrt{3}$
$C(2;\ 4\sqrt{3})$ (5)

(b) Base of ΔOCG
$OG = 4 + 4\cos 60°$
OG = 6 units
Area of $\Delta OCG = \frac{1}{2} \times 6 \times 4\sqrt{3}$
$\Delta OCG = 12\sqrt{3}$ units^2 (5)

(c) $m_{OC} = 2\sqrt{3}$
$m_{CG} = \frac{4\sqrt{3}-0}{2-6}$
$m_{CG} = -\sqrt{3}$
$\tan\alpha = 2\sqrt{3}$
$\alpha = 73,9°$
$\tan\beta = -\sqrt{3}$
$\beta = 120°$
$O\hat{C}G = 120° - 73,9°$
$O\hat{C}G = 46,1°$ (5)
[15]

71 marks

Total: 150 marks