MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 26 pages and an Information Sheet of 2 pages (i–ii). Please check that your paper is complete.

2.   Read the questions carefully.

3.   **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number on the paper.**

4.   Diagrams are not necessarily drawn to scale.

5.   You may use an approved non-programmable and non-graphical calculator, unless stated otherwise.

6.   Round off your answers to **one decimal digit** where necessary, unless stated otherwise.

7.   All necessary working details must be clearly shown.

8.   Ensure that your calculator is in **DEGREE** mode.

9.   It is in your own interest to write legibly and to present your work neatly.
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FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
7	13	26	17	10	9	17	10	10	6	10	15

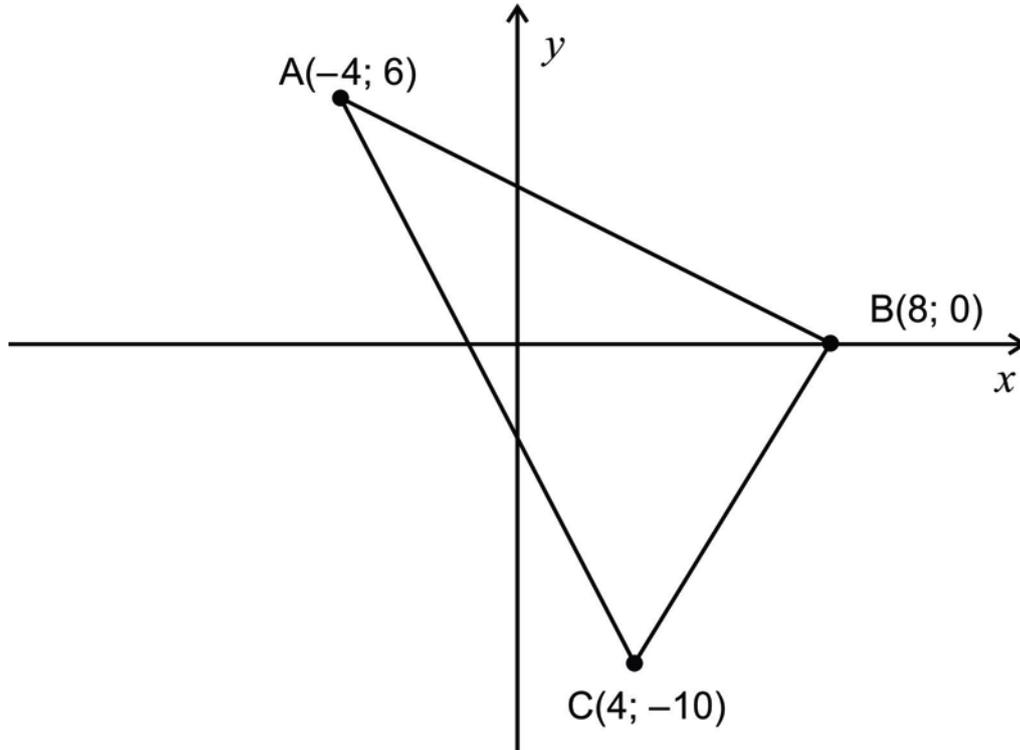| **TOTAL**	**/150**

SECTION A

QUESTION 1

A(– 4; 6); B(8; 0) and C(4; – 10) are three points on a Cartesian plane.

(a)   Calculate the length of the straight line joining A and B. (Leave your answer in surd form.)

(3)

(b)   Calculate the coordinates of M, the midpoint of AB.
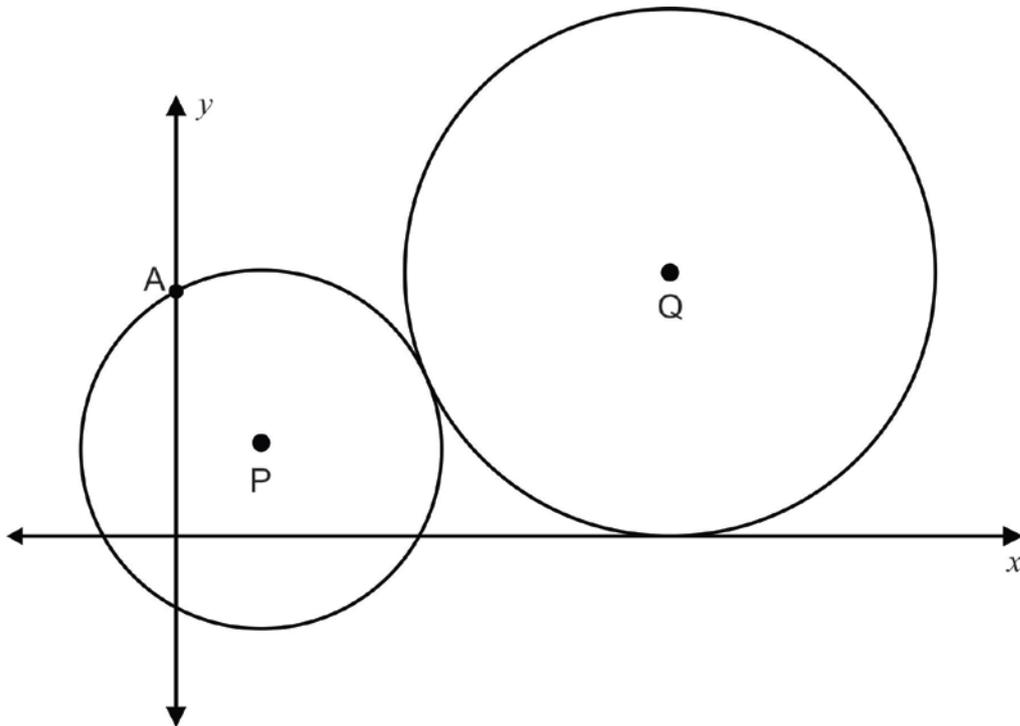
(1)

(c)   Show, using analytical methods, that $A\hat{M}C \neq 90°$.

(3)

[7]

QUESTION 2

In the diagram below:

- Circle P has equation: $(x-3)^2 + (y-3)^2 = 25$.
- Circle Q has equation: $(x-15)^2 + (y-m)^2 = 64$.
- Circle Q touches the x-axis and circle P.
- A is a y-intercept of circle P.

(a)   Determine the value of m in the equation of circle Q.

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_____   (2)

(b)   Write down the length of line PQ.

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_____   (2)

(c)   Calculate the coordinates of point A.

(4)

(d)   Is the tangent to circle centre P at A parallel to the straight line joining P and Q? Show all working.

(5)

[13]

QUESTION 3

(a)   Given: $\sin 34° = p$. With the use of a diagram find the value of $\cos 34°$ in terms of p.

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_____   (3)

(b)   Simplify $\dfrac{-\cos(90° + \theta) - \sin^3\theta}{\sin 2\theta}$ as far as possible.

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_____   (5)

(c)   (1)   If $\sin^2\theta - \cos^2\theta + \sin\theta + 1 = 0,$ show that $2\sin^2\theta + \sin\theta = 0.$

(1)

(2)   Hence, or otherwise, determine the general solution for θ.

(7)

(d)   Prove the following identity:

$\sin 3x = 3\sin x - 4\sin^3 x$

(6)

(e)   In the diagram below, the graph of $f(x) = a\cos bx$ is sketched for $x \in [0°; 360°]$.
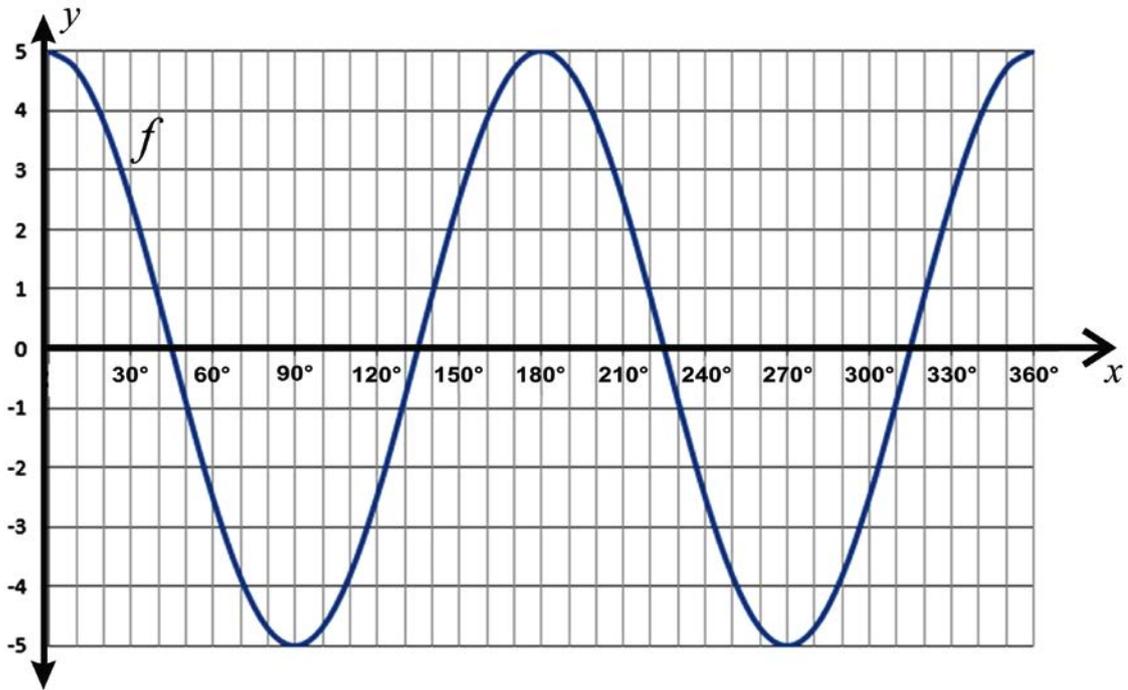
(1)   Determine the values of a and b.

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_____   (2)

(2)   If $g(x) = q$, then for what value(s) of q will the equation $f(x) = g(x)$ have no real solution?

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_____   (2)
                                                          [26]

QUESTION 4

(a)   In the diagram below:

- P, R and S lie on the circle with the centre O.
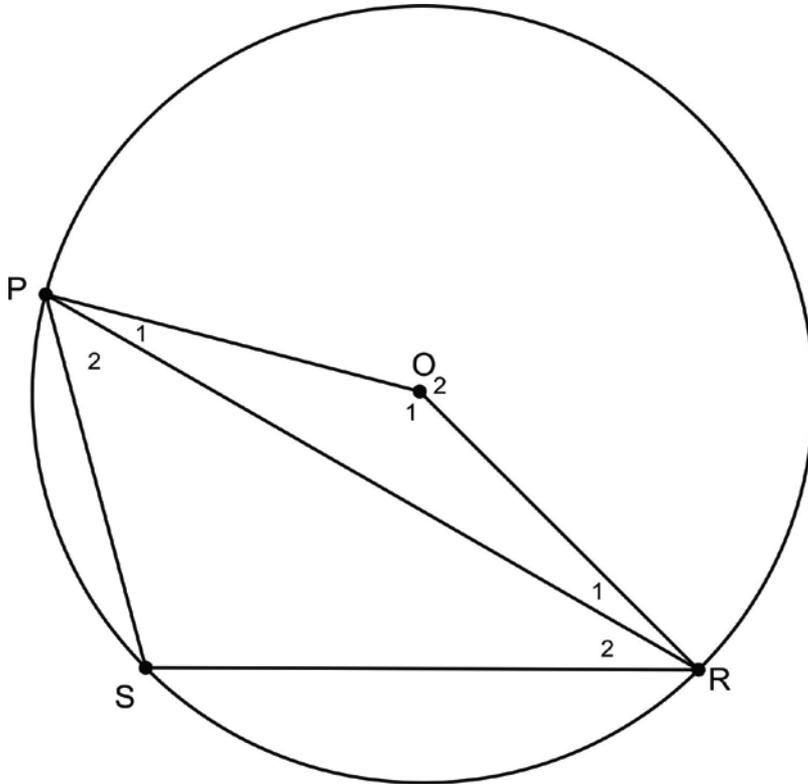
(1)   If $P\hat{S}R = 102°$, determine the size of $\hat{O}_1$, giving reasons.

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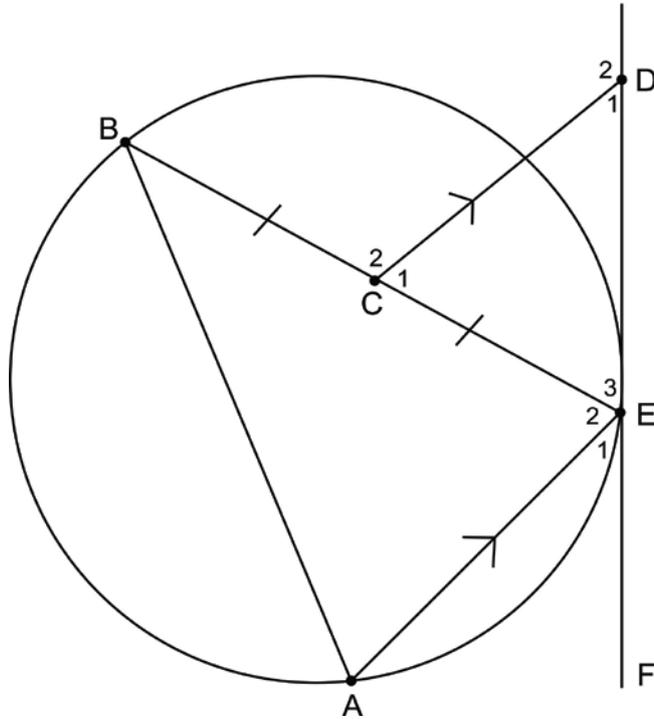
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_____   (3)

(2)   Calculate the radius, *r*, of the circle if PR = 10 units.

(3)

(b)   In the diagram below:

- FD is a tangent to the circle at point E.
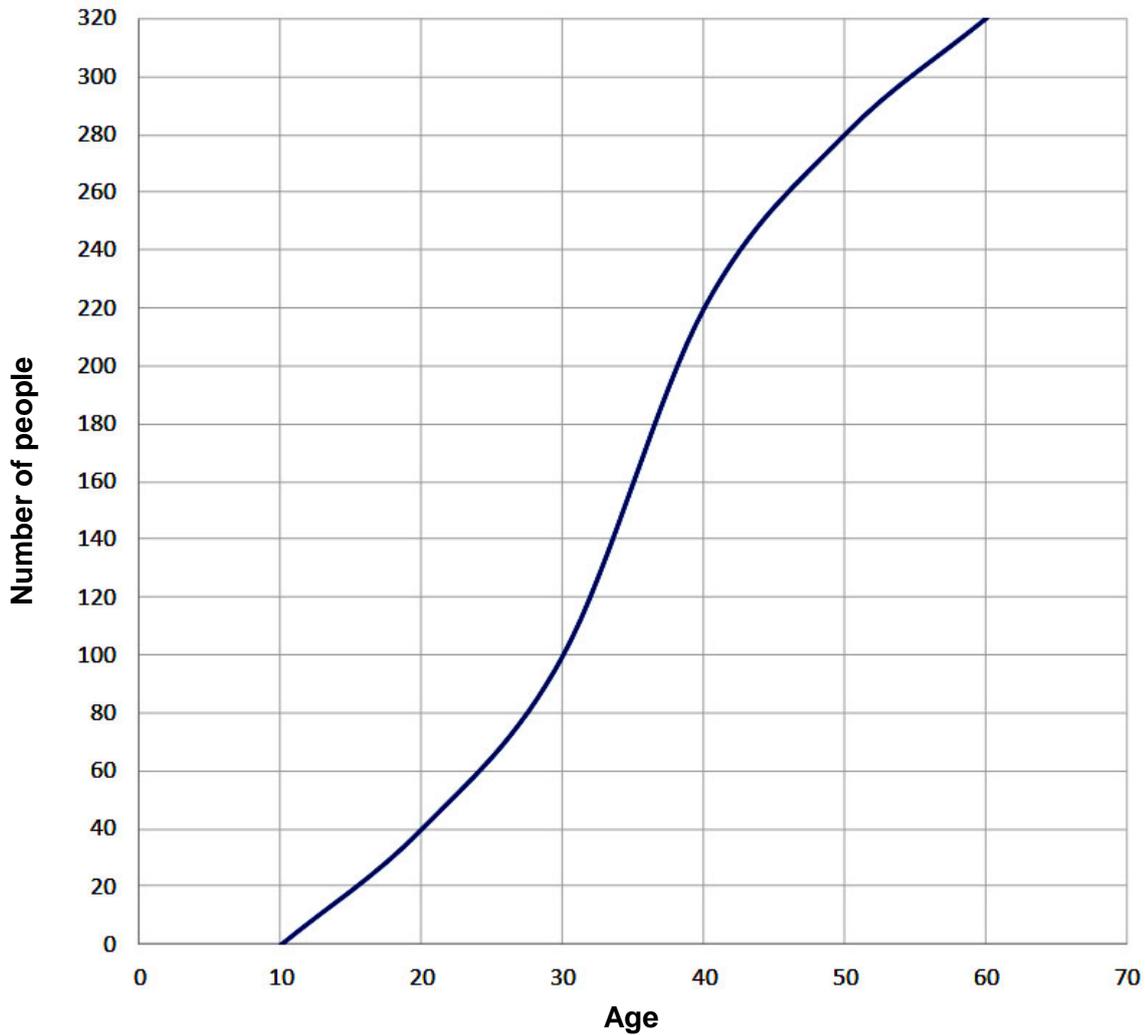- AE//CD.
- A, B and E lie on the circle.
- BC = CE.

(1)   Prove that $\hat{D}_1 = \hat{B}$.

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_____   (4)

(2)   Prove that $\Delta ABE \; /// \; \Delta EDC$.

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(3)

(3)   Show that $2EC^2 = AE \cdot DC$.

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(4)

[17]

QUESTION 5

In the diagram below, the cumulative frequency curve represents the age of people who attended a music concert.

(a)   Write down the median age of people who attended the music concert. On the graph, show where you would read off the answer.

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_____   (2)

(b)   What could be the minimum age of someone in the upper quartile? On the graph, show where you would read off the answer.

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_____   (2)

(c)   What percentage of people were younger than 30?

(2)

(d)   (1)   On the same set of axes, draw the curve if a group of twenty people between the ages of 50 and 60 left the concert and were not included.

(3)

       (2)   What effect would this have on the lower quartile?

(1)

[10]

73 marks

SECTION B

QUESTION 6

You gather information from students who live on their own and ask them to fill in a questionnaire. Below is a summary of your data.

Income of student per month in rand	2 000	4 500	3 200	8 000	3 700	5 300	8 700	9 200	4 900	6 150
Number of loaves of bread bought per month	32	21	21	14	25	18	13	13	20	12

(a) Determine the correlation coefficient and comment on the strength of the relationship between the income of a student and the number of loaves of bread they buy per month.

(3)

(b) Determine the gradient of the line of best fit, round your answer off to five decimal places.
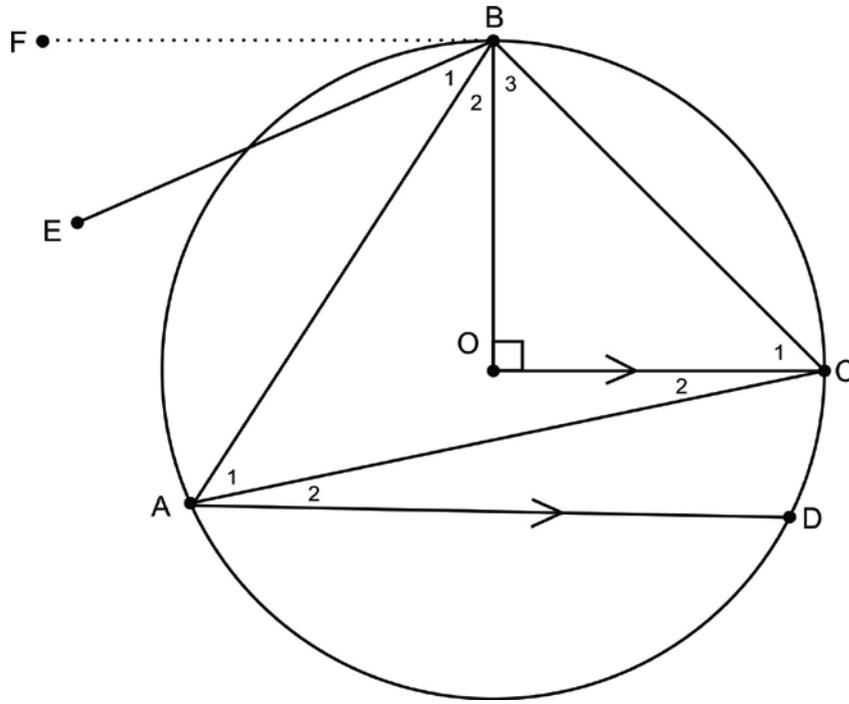
(2)

(c) A student has an income of R7 500 per month. Use the line of best fit to determine the approximate number of loaves the student will buy per month. Comment on your answer.

(4)

[9]

QUESTION 7

(a)   Use the diagram to prove the theorem that states that the angle subtended by a chord at the centre of a circle is twice the angle subtended at the circle.

(6)

(b)   In the diagram below:

- A, B, C and D lie on the circle with centre O.
- BO $\perp$ OC.
- AD//OC.

(1)   If $\hat{B}_1 = 43°$, prove that EB is not parallel to AC.
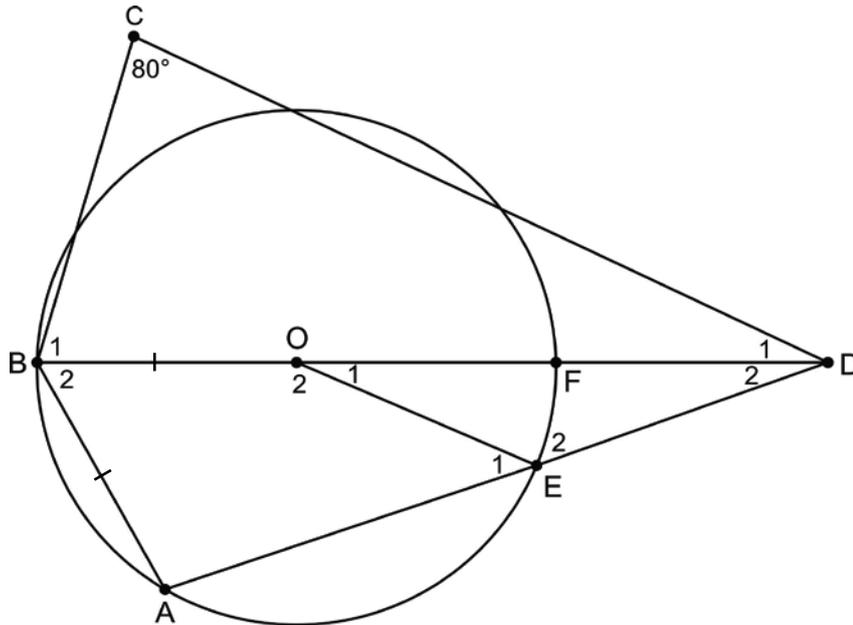

(3)

(2)   If FB is a tangent to the circle at B and $\hat{A}_2 = 12°$, calculate the size of the acute angle $\hat{FBE}$.

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(8)

[17]

QUESTION 8

In the diagram below:

- O is the centre of the given circle.
- ABCD is a cyclic quad.
- AB = BO.
- $\hat{C} = 80°$.
- A, B, E and F are points on the circle.

(a)   Calculate the size of $\hat{D}_2$.

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_____   (6)

(b)   Prove that EO = ED.

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_____   (4)

[10]

QUESTION 9

In the diagram below:

- AB is 16 units in length.
- AB//CD//EF.
- EF = 10 units.
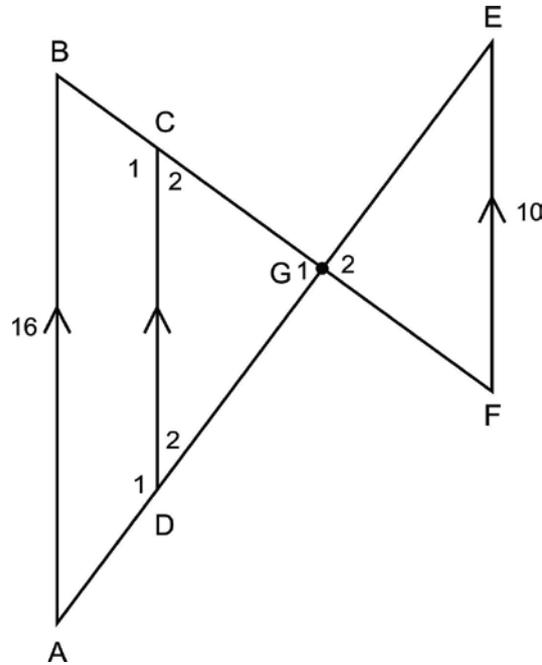
(a)   Explain why $\Delta GCD///\Delta GBA$.

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_____   (2)

(b)   If $\dfrac{GC}{CB} = \dfrac{5}{3}$, determine the length of CD.

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_____   (4)

(c)   Prove that G is the midpoint of ED.

(4)
[10]

QUESTION 10

In the diagram below:

- E (0; 12) is a point on the *y*-axis and D is a point on the *x*-axis.
- C is a point on line EB and B lies on the *x*-axis.
- $E\hat{C}D = 79,29°$.
- The gradient of DC is 0,35.

Calculate the *x* coordinate of point B.

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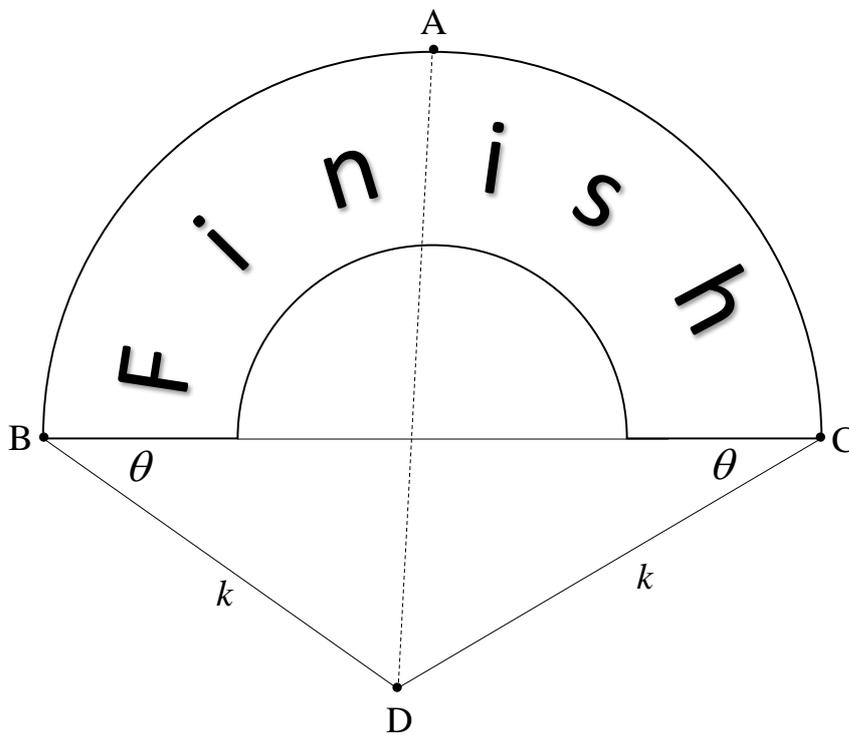_____

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[6]

QUESTION 11

At the end of a marathon there is a semi-circular banner with the word 'Finish' written on it. It stands perpendicular to the horizontal ground.
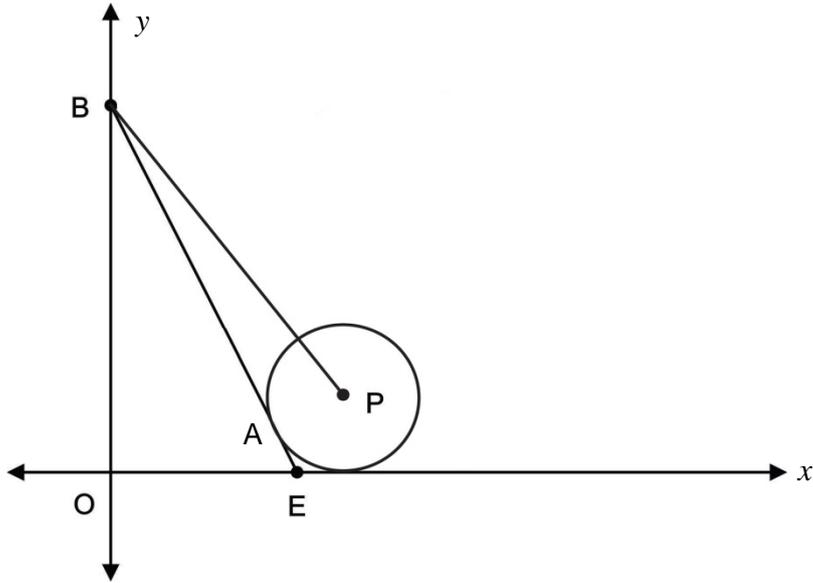
In the diagram below:

- B, C and D are points on the horizontal ground.
- The highest point A on the vertical banner is connected to D by a straight rope AD.
- $\hat{CBD} = \hat{BCD} = \theta$.
- DC = DB = k.

(a)   Show that $\text{BC} = 2k\cos\theta$.

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(b)   Prove that $\text{AD} = k$.

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                                                                                                (6)
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QUESTION 12

In the diagram below, circle centre P touches the *x*-axis.

- BO = 2OE.
- A(10; $\sqrt{5}$ – 1) is on the circle.
- The radius of the circle is $\sqrt{5}$ units.
- BAE is the tangent at A, with E on the *x*-axis.

(a) Determine the equation of the straight line going through A and P.

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(4)

(b)   Hence, calculate the coordinates of P.

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(3)

(c)   Determine the size of AB̂P.

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(8)

[15]

77 marks

Total: 150 marks