NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2019

# MATHEMATICS: PAPER II

## MARKING GUIDELINES

Time: 3 hours | 150 marks

**These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.**

**The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.**

## SECTION A

## QUESTION 1

(a) k = 1

(b) $AB = \sqrt{(5-1)^2 + (5-2)^2}$
$AB = 5$

(c) Midpoint $\left(\frac{11}{2};1\right)$
$m = \frac{5-1}{5-5,5} = -8$
$5 = -8(5) + c$
$c = 45$
$y = -8x + 45$

(d) $\tan C\hat{A}B = \frac{4}{3}$ $\quad C\hat{A}B = 53,13°$

(e) Area $\Delta ABC = \frac{1}{2}(5)(7) \sin 53,13°$
Area $\Delta ABC = 14$ units$^2$

(f) $(x-5)^2 + (y-5)^2 = 25$

(g) Yes, as AB is 5 units, the same length as the radius.

## QUESTION 2

(a) OM = 3 units

(b) $2x^2 = 9$
$x = \sqrt{\frac{9}{2}}$ and $y = \sqrt{\frac{9}{2}}$

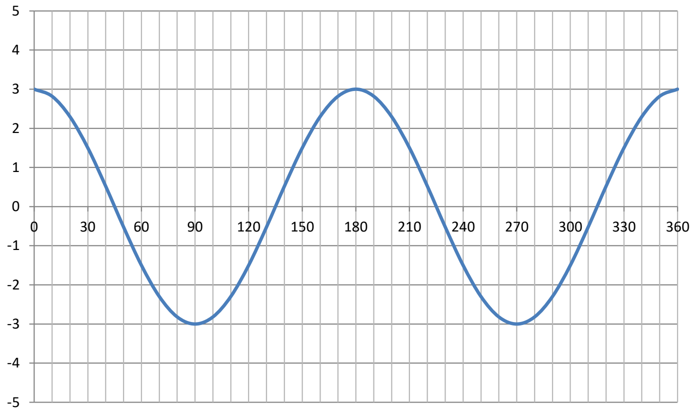
(c) $y = -x + c$
Sub in point $M\left(\frac{3}{\sqrt{2}};\frac{3}{\sqrt{2}}\right)$
$\frac{3}{\sqrt{2}} = -\frac{3}{\sqrt{2}} + c$

$c = 3\sqrt{2}$ or 4,24
$y = -x + 3\sqrt{2}$

(d) $Area\ \Delta OPN = \frac{1}{2}\left(3\sqrt{2}\right)\left(3\sqrt{2}\right) = 9$ units$^2$

## QUESTION 3

(a)




Starting point (0; 3)
Turning Point (90°; –3)
Turning Point (180°; 3)
End Point (360°; 3)

(b) $3\cos 2x = 2$

$\cos 2x = \dfrac{2}{3}$

Ref angle = 48,19°

$2x = 48{,}19° + k.360°$
$x = 24{,}1° + k.180°$

Alternate: $2x = \pm\, 48{,}19° + k.360°$
$x = \pm\, 24{,}1° + k.180°$

$2x = 311{,}81° + k.360°$
$x = 155{,}91° + k.180°$
$\therefore x \in \{24{,}1°;\ 155{,}9°; 204{,}1°; 335{,}9°\}$

(c) Max value = 2 – (–3) = 5 units

## QUESTION 4

(a) (1) $\dfrac{\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\sin\theta}{\cos\theta+\sin\theta}=\tan 2\theta$

$\dfrac{\sin\theta\cos\theta+\sin^2\theta+\sin\theta\cos\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}$

$\dfrac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$

$\dfrac{\sin 2\theta}{\cos 2\theta}$

$=\tan 2\theta$

(2) $\tan 2\theta = -5$

Reference angle = 78,7°

$2\theta = 101{,}3° + k.180°$

$\theta = 50{,}7° + k.90°$

Alternate:

$2\theta = -78{,}7° + k.180°$

$\theta = -39{,}4° + k.90°$

(b) (1) Method for finding missing side

$=\sqrt{1-m^2}$

(2) $\sin^2 63°$

$=\cos^2 27°$

$=1-m^2$

(3) $=\sin(45° + 27°)$

$=\sin 45°\cos 27° + \cos 45°\sin 27°$

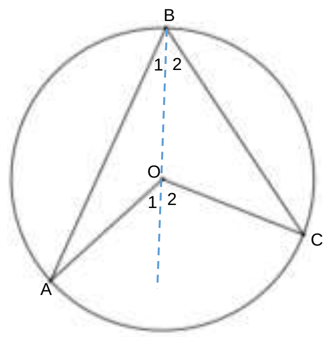
$=\dfrac{\sqrt{2}\left(\sqrt{1-m^2}+m\right)}{2}$ or any form of this answer

(4) $=\tan 63° = \dfrac{\sin 63°}{\cos 63°} = \dfrac{\cos 27°}{\sin 27°}$

$=\dfrac{\sqrt{1-m^2}}{m}$

## QUESTION 5

(a)




RTP: $A\hat{O}C = 2 \times A\hat{B}C$

Look for construction on diagram or labelled BO

$\hat{O}_1 = \hat{A} + \hat{B}_1$ Exterior angle of triangle

$\hat{A} = \hat{B}_1$ Isos triangle OR Radii

Similarly in other triangle

$\hat{O}_1 = 2 \times \hat{B}_1$

$\hat{O}_2 = 2 \times \hat{B}_2$

Therefore

$A\hat{O}C = 2 \times A\hat{B}C$

(b) $\hat{S}_2 = 55°$ (Radii; isos triangle)

$\hat{S}_1 + \hat{S}_2 = 90°$ (Angle in semicircle)

$\hat{S}_1 = 35°$

$\hat{T}_2 = 35°$ (Tan chord theorem)

(c) $A\hat{B}D = 80°$ (Tan chord theorem)

$D\hat{B}C = 32°$ (Angles in same segment)

$A\hat{G}E = 112°$ (Ext angle of cyclic quad)

## QUESTION 6

(a) It would strengthen the correlation coefficient.

(b) Closer to negative 1 as the relationship is indirect or negative gradient.

(c) No; as the correlation is not equal to –1.

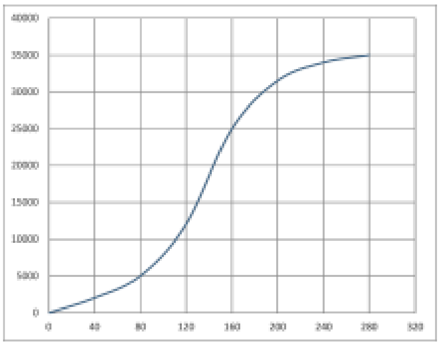
(d) He is saying that the value for x is outside of the lowest and highest values used to find the line of best fit so he cannot be sure it is accurate.

## SECTION B

## QUESTION 7

(a) Method mark for workings.
$\bar{x} = 134,86$ (Full marks for correct answer)

(b)




Starting point (0; 0)
End Point (280; 35 000)
Accuracy on the (80; 5 000)
Smooth curve drawn

(c) Finding position $\frac{35\,000}{2}$
Any value from 125 to 150

(d) Getting a number from 33 000 to 34 500
35 000 – number = ± 1 800

(e) (1) Median would stay the same as top half affected.
(2) Decrease; values would squeeze closer to the mean.
(3) Skewed more to the left, top half of data would be squeezed closer to the median.

## QUESTION 8

(a) $\dfrac{EC}{BE} = \dfrac{k}{2k} = \dfrac{1}{2}$

(b) $\dfrac{\frac{1}{2}(k)(2m)\sin\hat{B}}{\frac{1}{2}(3k)(3m)\sin\hat{B}}$

$= \dfrac{2}{9}$

(c) $\Delta ABC /// \Delta DBE$ (AAA)

$\dfrac{AC}{DE} = \dfrac{AB}{DB} = \dfrac{3k}{2k}$

$\dfrac{AC}{17} = \dfrac{3}{2}$

$AC = 25{,}5$ units

## QUESTION 9

(a) $M\hat{A}N = M\hat{B}N = 90°$ (Tangent perpendicular to radius)

$B\hat{N}A = 44°$ (Angles in a quadrilateral)

$C\hat{N}D = 44°$ (Angle at centre = 2 x angle at circumference)

$N\hat{C}D = N\hat{D}C$ (Radii; isos triangle)

$B\hat{A}N = N\hat{B}A$ (Tangents drawn from common point)

(b) $CD^2 = 9^2 + 6{,}2^2 - 2(9)(6{,}2)\cos 22°$

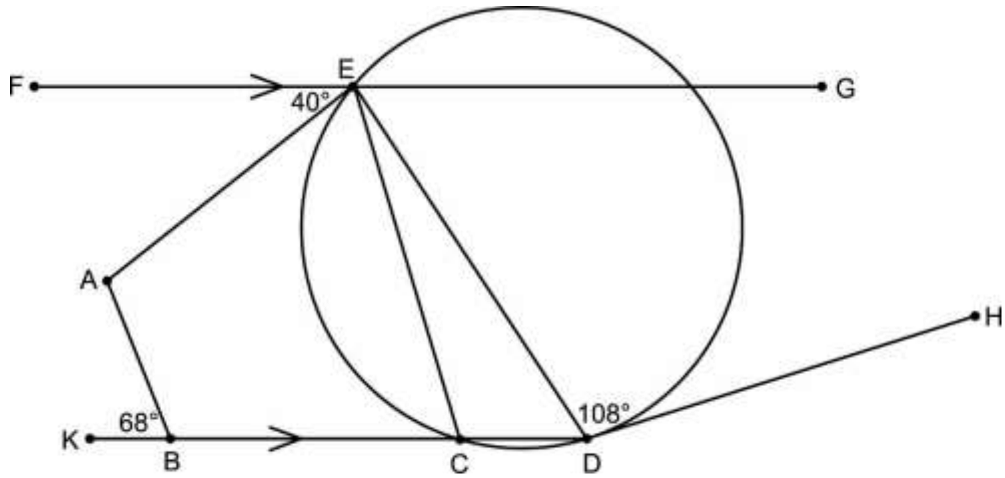
$CD = 4$ units

$\dfrac{DN}{AN} = \dfrac{CD}{AB}$

$\dfrac{27}{25} = \dfrac{4}{AB}$

AB = 3,7 units

**QUESTION 10**




Mark for construction

$B\hat{A}E = 40° + 68° = 108°$ (Alternate angles)

$E\hat{C}D = 108°$ (Tan chord theorem)

$B\hat{A}E = E\hat{C}D$

Therefore

AECB is a cyclic quad (Converse: exterior angle equal to interior opposite angle)

## QUESTION 11

(a) $x^2 - 10x + 25 + y^2 + 2y + 1 = -22 + 26$

$(x-5)^2 + (y+1)^2 = 4$

Centre of small circle (5; –1)

Radius = 2 units

(b) $x^2 + (13-4x)^2 - 10x + 2(13-4x) = -22$

$x^2 + 169 - 104x + 16x^2 - 10x + 26 - 8x = -22$

$17x^2 - 122x + 217 = 0$

$x = 3{,}26$ units

y = 0,02 (Therefore not on the $x$-axis)

(c) $m_{CD} = -4$

Evidence of a perpendicular bisector on diagram, words or labelled

Gradient of perp bisector $\frac{1}{4}$

Sub in centre of small circle (5; –1)

$-1 = \frac{1}{4}(5) + c$

$c = -\frac{9}{4}$

Equate line AB and perpendicular bisector

$-\frac{4}{3}x - \frac{2}{3} = \frac{1}{4}x - \frac{9}{4}$

$-16x - 8 = 3x - 27$

$-19x = -19$

$x = 1$

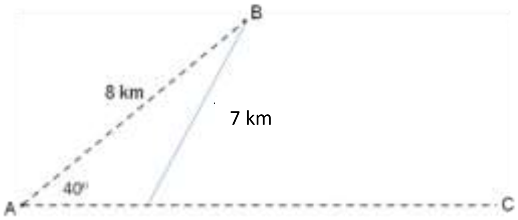
$4(1) + 3y = -2$

$3y = -6$

$y = -2$

Midpoint of line AB is (1; –2)

## QUESTION 12




A mark for a construction that engages with the question

$$\frac{\sin\theta}{8} = \frac{\sin 40^\circ}{7}$$

$\theta = 47{,}27°$

Obtuse angle = 132,73°

$\hat{B} = 180° - 132{,}73° - 40°$
$\hat{B} = 7{,}27°$

$$\frac{\text{A to base of building}}{\sin 7{,}27^\circ} = \frac{7}{\sin 40^\circ}$$

A to base of building = 1,38 km

$$\tan\beta = \frac{160}{1\ 380}$$

Angle of elevation = 6,61°

**Total: 150 marks**