NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2018

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (3; 1)

(b) $m_{AB} = \dfrac{3-(-1)}{5-1} = 1$

$y = -x + c$

$1 = -(3) + c$

$c = 4$

$y = -x + 4$

(c) $AB = \sqrt{(3-(-1))^2 + (5-1)^2}$

$AB = \sqrt{32}$ or 5,66 units

(d) $(x-3)^2 + (y-1)^2 = 8$

$x^2 + y^2 - 6x - 2y + 2 = 0$

(e) $(4-3)^2 + (y-1)^2 = 8$

$(y-1)^2 = 7$

$y = 1 \pm \sqrt{7}$

(f) M is 3 units away from the *y*-axis. The radius of the circle is $\sqrt{8}$ units. Therefore the shortest distance of the circle from the *y*-axis is $3 - \sqrt{8}$ units.

QUESTION 2

(a) $O\hat{M}N = 90^{\circ}$
(Tangent perpendicular to line from centre)

(b) (1) $x^2 - 8x + 16 + (y+4)^2 = 9 + 16$

$O(4;-4)$

(2) $(x-4)^2 + (y+4)^2 = 25$

5 units

(c) $MN^2 + OM^2 = ON^2$

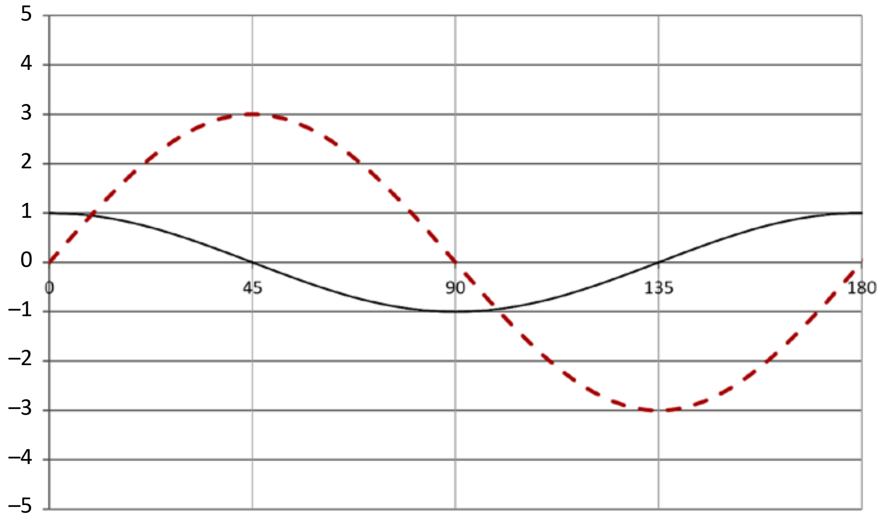
$ON = \sqrt{(11-4)^2 + (-5-(-4))^2}$ $\therefore ON = \sqrt{50}$

$MN = \sqrt{50}^2 - 5^2$

$MN = 5$ units

QUESTION 3

(a)



Starting point $(0^\circ;0^\circ)$ and Finishing point $(180^\circ;0^\circ)$
Both Turning points correct $(45^\circ;3)$ and $(135^\circ;-3)$
X intercept of $(90^\circ;0)$
Shape of the graph

(b)

$$3\sin 2x = \cos 2x$$
$$\tan 2x = \frac{1}{3}$$

Reference angle = $18{,}43^\circ$

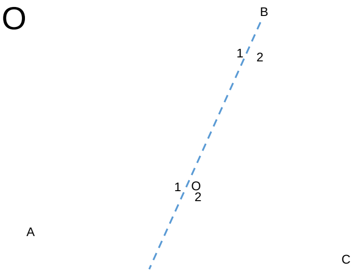
$$2x = 18{,}43^\circ + k180^\circ$$
$$x = 9{,}22^\circ + k90^\circ$$
$$x = \{9{,}22^\circ;99{,}22^\circ\}$$

(c)

$$\cos 2x = 0$$
$$x = \{45^\circ;135^\circ\}$$

QUESTION 4

(a) Look for construction on diagram or labelled BO



$\hat{O}_1 = \hat{A} + \hat{B}_1$ Exterior angle of triangle

$\hat{A} = \hat{B}_1$ Isos triangle **OR** Radii

Similarly in other triangle

$\hat{O}_1 = 2 \times \hat{B}_1$

$\hat{O}_2 = 2 \times \hat{B}_2$

Therefore

$A\hat{O}C = 2 \times A\hat{B}C$

(b) (1) $\hat{B}_1 = 73^\circ$ Opposite angles of cyclic quad

(2) $O\hat{M}B = 40^\circ$ (isos triangle)

$M\hat{O}B = 100^\circ$ (angles of a triangle)

$\hat{T}_2 = 50^\circ$ (Angle at centre = two times angle at circumference)

(3) $\hat{M}_3 = 17^\circ$ (Angles in a triangle)

$\hat{M}_2 = 23^\circ$ (40 – 17)

(c) (1) $\hat{P}_1 = 56^\circ$ (Tan chord theorem)

$\hat{P}_2 = 54^\circ$ (Angles on straight line)

$\hat{S} = 54^\circ$ (Angles in same segment)

(2) $\hat{R}_1 = 37^\circ$ (Angles in same segment)

Therefore

$Q\hat{R}S = 93^\circ$

But for QS to be the diameter the size of $Q\hat{R}S = 90^\circ$

Hence QS is not the diameter

QUESTION 5

(a) $A\hat{C}E = x$ (Angles in same segment)

$A\hat{F}E = x + y$ (Ext angle of a triangle)

$A\hat{F}E = C\hat{D}E$

Therefor

FCDE is a cyclic quadrilateral (Converse: ext angle of cyclic quad)

(b) $A\hat{E}B = x$ (Isos triangle)

$A\hat{C}E = x$ (Angles in same segment)

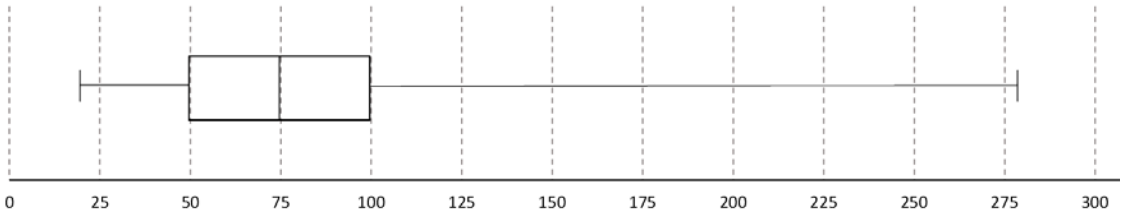
$A\hat{E}B = A\hat{C}E$

Therefore

AE is a tangent (Converse: tan chord theorem)

QUESTION 6

(a)



LV + HV Q1 Q2 Q3 Diagram

(b) 2000 households

(c) Skewed to the right or positively skewed

(d) (1) Standard deviation would decrease as the households using more than 100 litres would lower their consumption and hence the values will be closer to the mean.

(2) The data would become more symmetrical as the mean will move much closer to the median. OR the data would become less positively skewed

SECTION B

QUESTION 7

(a) Pamphlets; the correlation coefficient is closer to 1

(b) (1) It would increase as the values would be closer to line of best fit

(2) Pamphlets: Gradient would increase
Television: Gradient would decrease

(c) Television; as the gradient is steeper and although the correlation coefficient is lower, the outliers fall on the high side (Low expenditure; High sales), so you can predict that there will be an increase in sales.

QUESTION 8

(a) (1) $\cos 334^\circ .\ \sin 244^\circ = \cos 26^\circ .(-\sin 64^\circ)$

$\sin 64^\circ .(-\sin 64^\circ)$

$= -p^2$

(2) $8\sin 16^\circ .\cos 16^\circ .\cos 32^\circ$
$= 4\sin 32^\circ .\cos 32^\circ$
$= 2\sin 64^\circ = 2p$

(b) $\sin 43^\circ = \cos(90^\circ - k)\cos 23^\circ + \cos 246^\circ \sin 23^\circ$

$\sin 43^\circ = \cos(90^\circ - k)\cos 23^\circ - \cos 66^\circ \sin 23^\circ$

$\sin(66^\circ - 23^\circ) = \sin k \cos 23^\circ - \cos 66^\circ \sin 23^\circ$

Therefore

$k = 66^\circ$

(c) $$\frac{2\cos 2\theta . cos\theta}{\cos^2\theta - \sin^2\theta} + 2\tan\theta . \sin\theta = \frac{2}{\cos\theta}$$

$$\frac{2\cos 2\theta . cos\theta}{\cos 2\theta} + \frac{2\sin^2\theta}{\cos\theta}$$

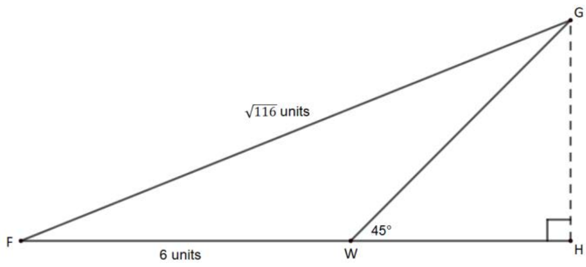
$$\frac{2\cos^2\theta}{\cos\theta} + \frac{2\sin^2\theta}{\cos\theta}$$

$$\frac{2\cos^2\theta + 2\sin^2\theta}{\cos\theta}$$

$$\frac{2(\cos^2\theta + \sin^2\theta)}{\cos\theta}$$

$$\frac{2}{\cos\theta}$$

(d) $F\hat{W}G = 135^{o}$



$$\frac{\sin F\hat{G}W}{6} = \frac{\sin 135^{o}}{\sqrt{116}}$$

$F\hat{G}W = 23{,}2^{o}$

$W\hat{F}G = 21{,}8^{o}$

$$\sin 21{,}8^{o} = \frac{GH}{\sqrt{116}}$$

$GH = 4$ units

QUESTION 9

(a) $CB = 10$ units
$BA = 8$ units

(b) $$\frac{\text{Area of } \Delta BDC}{\text{Area of } \Delta BED}. = \frac{\frac{1}{2} \times CD \times perp.ht}{\frac{1}{2} \times DE \times perp.ht}$$

Triangles have same perp height

$$= \frac{5}{4}$$

(c) Split of CE into two 5/9 or 5:4 or 4/9

DE/CE = 4/9

DE/CF = 20/81

DE = 11.11 or $\frac{100}{9}$

QUESTION 10

(a) $\hat{F}_1 = \hat{S}_3$ (Tangents drawn from common point)

$\hat{F}_1 = \hat{S}_2$ (Alternate angles DH//SG)

$\hat{S}_2 = \hat{H}_1$ (Angles in same segment)

$\hat{H}_1 = \hat{G}_1 + \hat{G}_2$ (Radii; isos triangle)

$\Delta DSF \; /// \; \Delta OHG$ (A.A.A) or $\hat{D} = \hat{O}_1$ (Angles in a triangle)

(b) $\dfrac{DF}{OG} = \dfrac{SF}{HG}$ (prop theorem)

$$DF = \frac{OG.SF}{HG}$$

But

$$OG = \frac{FH}{2}$$

Therefore

$$2 \times DF = \frac{SF \times FH}{HG}$$

QUESTION 11

(a) AE = 4 units (Line from centre perpendicular to chord)
OE = 0,8 units
OA = 4,08 units
EK = 4,08 + 0,8 = 4,88 units

(b) $\cos B\hat{A}C = \dfrac{29 - 13 - 64}{-2 \times \sqrt{13} \times 8}$

$$B\hat{A}C = 33,69^{\circ}$$

$$\sin 33,69^{\circ} = \frac{\text{height of } B}{\sqrt{13}}$$

height of $B = 2$ units

$$\cos 33,69^{\circ} = \frac{\text{point below } B}{\sqrt{13}}$$

Point below B is 3 units

Distance from K to new point

$\sqrt{1^2 + 4,88^2}$

= 4,98 units

Therefore

Distance from B to K once the fold has been made is

$\sqrt{2^2 + 4,98^2}$

= 5,37 units

QUESTION 12

(a) Line AD $y = x + c$

$-2 = 1 + c$

$c = -3$

x intercept

$0 = x - 3$

$x = 3$

$D(3;0)$

OR

Create a triangle by dropping a perpendicular from point D.
Side lengths are 2.
Therefore x coordinate of D is 1 + 2 = 3

$D(3;0)$

(b) $m_{AB} = \dfrac{1+2}{-2-1}$ $m_{AB} = -1$

$m_{BC} = 1$

$A\hat{B}C = 90^\circ$ since $m_{AB} \times m_{BC} = -1$

(c) $C\hat{D}A = 90^\circ$ (Opp angle of cyclic quad)

$m_{CD} = \dfrac{5}{2-x}$

$m_{AD} = \dfrac{-2}{1-x}$

$\dfrac{5}{2-x} \times \dfrac{-2}{1-x} = -1$

$x^2 - 3x - 8 = 0$

$x = 4,7$ or $x = -1,7$

D needs to move 1,7 units to the right

Alternate Solution

AC is a diameter (Converse: angle in semi-circle)

Midpoint $\left(\frac{3}{2};\frac{3}{2}\right)$

Radius of circle $=\frac{\sqrt{7^2+1^2}}{2}=\frac{\sqrt{50}}{2}$

Third side of triangle

$$\sqrt{\left(\frac{\sqrt{50}}{2}\right)^2-\left(\frac{3}{2}\right)^2}$$

= 3,2 units

3,2 + 1,5 = 4,7 units

D must have coordinates of $(4,7;0)$

Therefore it must move 1,7 units to the right

Alternate Solution

AC is a diameter (Converse: angle in semi-circle)

Midpoint $\left(\frac{3}{2};\frac{3}{2}\right)$

Radius of circle $=\frac{\sqrt{7^2+1^2}}{2}=\frac{\sqrt{50}}{2}$

Circles equation is

$$\left(x-\frac{3}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{50}{4}$$

Find the *x* intercepts

$$\left(x-\frac{3}{2}\right)^2+\left(0-\frac{3}{2}\right)^2=\frac{50}{4}$$

$$x=4,7$$

D must move 1,7 units to the right

Total: 150 marks