NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

# MATHEMATICS: PAPER II

## MARKING GUIDELINES

Time: 3 hours

150 marks

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**These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.**

**The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.**

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## SECTION A

## QUESTION 1

(a)

$$m_{QR} = \frac{4-8}{-3-1} = 1$$

$$m_{PQ} = \frac{a-4}{2-(-3)} = 1$$

$$a-4 = 5$$

$$a = 9$$

(b) Midpoint of ST (1 ; 7)

$$m_{ST} = \frac{8-6}{4-(-2)} = \frac{1}{3}$$

Line perpendicular to ST

$$y = -3x + c$$

$$7 = -3(1) + c$$

$$c = 10$$

$$y = -3x + 10$$

(c) (1)

$$m_{ED} = \frac{3-1}{7-6} = 2$$

$$m_{AB} = 2$$

Therefore ED//AB and gradients are the same

(2) E (7 ; 3) is the midpoint of CB (proportion theorem ED//AB)

$$C(0\ ;\ 2)$$

$$B(14\ ;\ 4)$$

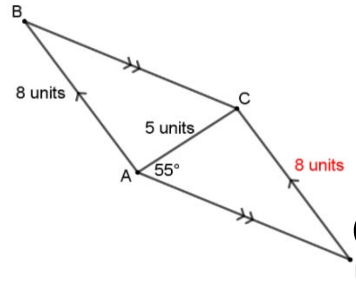
(3)

$$\tan\theta = 2 \qquad \tan\beta = \frac{1}{7}$$

$$\theta = 63{,}43^\circ \qquad \beta = 8{,}13^\circ$$

$$C\hat{B}A = 55{,}3^\circ$$

## QUESTION 2




(a) (1) $\frac{\sin \hat{D}}{5} = \frac{\sin 55°}{8}$

$A\hat{D}C = 30{,}8°$ (For the 8 on diagram)

(2) $A\hat{C}D = 94{,}2°$

Area of $\Delta ADC = \frac{1}{2}(5)(8)\sin 94{,}2°$

Area of $\Delta ADC = 19{,}95$ units$^2$

Therefore the area of parallelogram
ABCD is 39,9 units$^2$

(b) (1) $\cos\theta = \frac{5}{13}$

In quadrant 4:
$x = 5$
$y = -12$
$r = 13$

$\sin\theta$
$= \frac{-12}{13}$ or $-\frac{12}{13}$

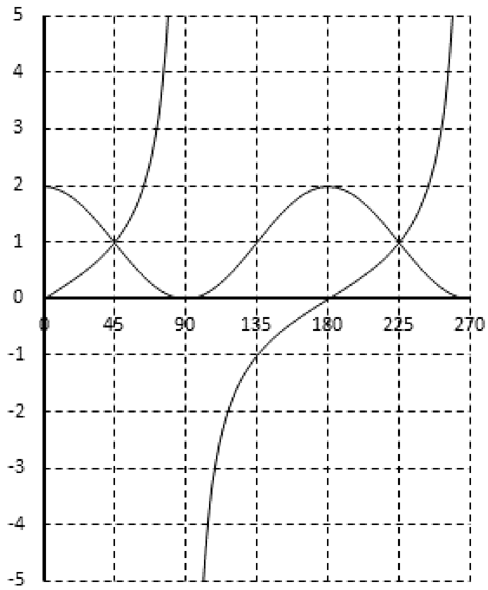
(2) $\cos\theta\cos 45° - \sin\theta\sin 45°$
$= \left(\frac{5}{13}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-12}{13}\right)\left(\frac{\sqrt{2}}{2}\right)$
$= \frac{17\sqrt{2}}{26}$

## QUESTION 3

(a) Period of graph is $180^\circ$

(b) $90^\circ$ and $270^\circ$

(c)




cos shape
cos start and end points
cos turning points
tan asymptotes
tan (45° ; 1);(135° ; –1);(225° ; 1)
tan shape through $0^\circ;180^\circ$

(d) $x = 45^\circ + k.180^\circ$ $\qquad k \in Z$

## QUESTION 4

(a) $\tan\theta = 1$

$\theta = 45^\circ + k.180^\circ$ (Full marks for answer)

(b) (1) $\dfrac{2\sin\theta\cos\theta + 2\cos^2\theta - 1 + 1}{\cos^2\theta - \sin^2\theta}$ $\qquad\qquad \dfrac{2\cos\theta}{\cos\theta - \sin\theta}$

$$\frac{2\cos\theta(\sin\theta + \cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$$

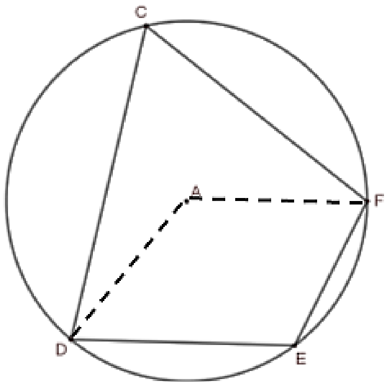
$$\frac{2\cos\theta}{\cos\theta - \sin\theta}$$

Therefore LHS = RHS

(2) $\theta = \{45^\circ\ ;\ 135^\circ\}$

## QUESTION 5

(a) Construction: AD and AF




$\hat{A}_1 = 2\hat{C}$ (Angle at centre = 2 x angle at circumference)
$\hat{A}_2 = 2\hat{E}$ (Angle at centre = 2 x angle at circumference)
$\hat{A}_1 + \hat{A}_2 = 360°$ (Revolution)
$2\hat{C} + 2\hat{E} = 360°$
therefore
$\hat{C} + \hat{E} = 180°$

(b) $\hat{E}_1 + \hat{E}_2 = 105°$ (Opposite angles of a cyclic quad)
$\hat{E}_2 = 73°$ (Tan chord theorem)
$\hat{E}_1 = 32°$

(c) (1) $\hat{D}_2 = 47°$ (Radii, isosceles triangle)

(2) $\hat{E} = 43°$ (Angle at centre = 2 × angle at circumference)

(3) $A\hat{F}E = 90°$ (Line from centre to midpoint of chord)
$\hat{C}_1 = 47°$ (Angles in a triangle)
Therefore DACB is a cyclic quad
(Converse: Exterior angle of a cyclic quad = interior opposite angle)

## QUESTION 6

(a) (1) $r = 1$

(2) Perfect correlation

(3) $y = -13 + 3x$

(4) When the *x* value is way outside of the values used (e.g. 150 or a very small value like 1 or 2). This would be extrapolation.

(b) (1) 40 people

(2) 10 out of 140 = 7,1%

## SECTION B

## QUESTION 7

(a) (1) IQR = 80 – 60 = 20
60 – 1,5*20 = 30
P lies below (60 – 30 = 30) and is therefore an outlier

(2) Skewed to the left as the mean is to the left of the median

(3) Yes. There were 7 from Class A and 6 from Class B

(b) (1) $$\frac{(1\times p)+(3\times 165)+(5\times 290)+(7\times 185)+(9\times 75)}{715+p}=5$$

$$-4p=-340$$
$$p=85$$

(2) $$Std\ \ dev=\sqrt{\frac{6\ 520}{1\,000}}=2{,}55 \text{ or } 2{,}6$$

## QUESTION 8

(a) $$9^2=r^2+r^2-2r^2\cos 110^\circ$$
$$81=2r^2(1-\cos 110^\circ)$$
$$r=5{,}493\ 485\ 649 \text{ units}$$
$$DB^2=2^2+5{,}493\ 485\ 649^2$$
$$DB=5{,}8462 \text{ units}$$ (Correct answer rounded off to 4 decimal places)

**Alternate solution**

$$\frac{AB}{\sin 35^\circ}=\frac{9}{\sin 110^\circ}$$

$AB = 5{,}493485649$ units
$DB^2 = 2^2+5{,}493485649^2$
$DB = 5{,}8462$ units (Correct answer rounded off to 4 decimal places)

(b) $$9^2=(5{,}8462)^2+(5{,}8462)^2-2(5{,}8462)^2\cos C\hat{D}B$$
$$C\hat{D}B=100{,}7^\circ$$

## QUESTION 9

(a) $\hat{E}_2 = \hat{B}$ or $\hat{C}_3 = \hat{B}$ (Tan chord theorem)

$\hat{C}_3 = \hat{F}_2$ (Tangents drawn from common point)
$\hat{C}_1 = \hat{B}$ (Radii)

Therefore

$\Delta ABC /// \Delta DEC$ (A.A.A)

(b) $\frac{AB}{DE} = \frac{BC}{EC}$

AB.EC = BC.DE

But AB = AE (Radii)

Therefore AE.EC = BC.DE

## QUESTION 10

(a) (1) $\frac{4}{9}$ (Prop theorem)

(2) $\frac{HC}{5k} = \frac{3}{8}$
$HC = \frac{15k}{8}$

$\frac{HC}{AF} = \frac{\frac{15k}{8}}{4k} = \frac{15}{32}$

(b) $\hat{A}_1 = 2\hat{B}_1$ (Angle at centre = 2 x angle on circle)
$\hat{E}_1 = 90° - \hat{E}_2$ (Angle in semi-circle)
$\hat{A}_1 = \hat{E}_1 + \hat{B}_1$ ($\hat{E}_1 = \hat{B}_1$ angles in same segment)
Thus $\hat{A}_1 = 90° - \hat{E}_2 + \hat{B}_1$

**Alternate solution**
$\hat{E}_2 = 90° - \hat{E}_1$ (Angles in semi-circle)
$B_1 = \hat{E}_1$ (Angles in same segment)
$\hat{E}_2 = 90° - \hat{B}_1$
$\hat{A}_2 = 180° - 2\hat{B}_1$
$\hat{B}_1 = 90° - \hat{E}_2$
$\hat{A}_2 = 180° - \hat{B}_1 - \hat{B}_1$
$\hat{A}_2 = 180° - \hat{B}_1 - 90° + \hat{E}_2$
$\hat{A}_1 = 180° - 90° - \hat{E}_2 + \hat{B}_1$
$\hat{A}_1 = 90° - \hat{E}_2 + \hat{B}_1$

## QUESTION 11

(a) $\hat{C}_1 + \hat{C}_2 + \hat{F}_1 + \hat{F}_2 = 180°$ (Opposite angles of cyclic quad)

$\hat{B}_1 + \hat{B}_2 + \hat{F}_2 + \hat{F}_3 = 180°$ (Opposite angles of cyclic quad)

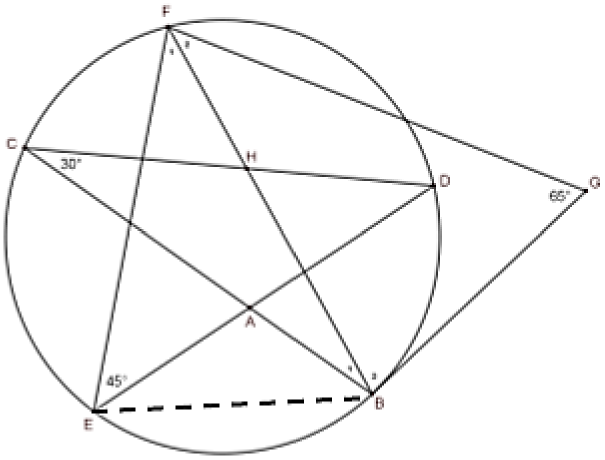
But

$\hat{F}_1 = \hat{F}_3$ (Angles in same segment)

Therefore

$\hat{C}_1 + \hat{C}_2 = \hat{B}_1 + \hat{B}_2$

(b)




Construction EB or CF or BD

$B\hat{E}D = 30°$ (Angles in same segment)

$\hat{B}_2 = 75°$ (Tan chord theorem)

$\hat{F}_2 = 40°$ (Angles in a triangle)

## QUESTION 12

(a) Gradient of radius $\frac{1}{2}$

Sub in point P(2 ; 2)

$2 = \frac{1}{2}(2) + c$

$c = 1$

(b) $0 = -2x + 6$

$x = 3$

B(3 ; 0) (Note: If assumption is made then max 2 for final two marks)

$m_{BT} = \frac{1}{2}$

Gradient of line through centre = –2

$y = -2x + c$

$1 = -2(5) + c$

$c = 11$

$y = -2x + 11$

$-2x + 11 = \frac{1}{2}x + 1$

$x = 4$

$y = 3$

Radius of circle

$r = \sqrt{(3-2)^2 + (4-2)^2}$

$r = \sqrt{5}$

Minimum distance from the *x*-axis

$3 - \sqrt{5}$ OR 0,76

## QUESTION 13

(a) $x^2 + 2x + 1 + y^2 - 2y + 1 = 1$

$(x+1)^2 + (y-1)^2 = 1$

Distance between centres

$d = \sqrt{(4-1)^2 + (3-(-1))^2}$

$d = \sqrt{25}$

$d = 5$

Radius 1 + Radius 2

1 + 2 = 3

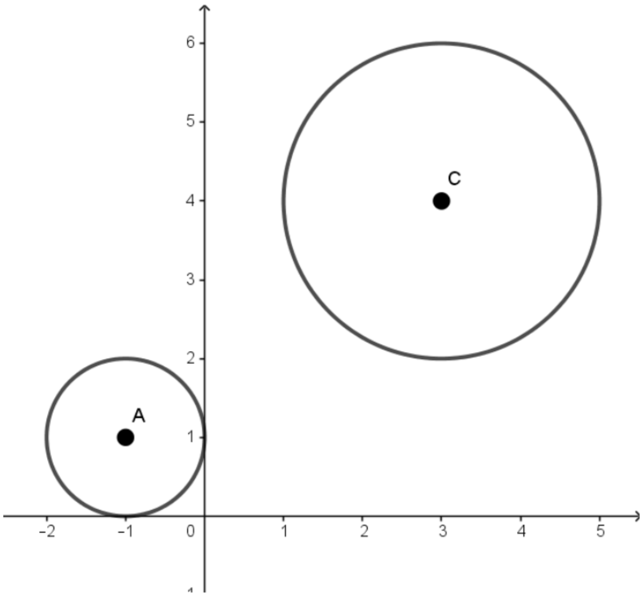
Distance between the centres – (Radius 1 + Radius 2) = 2 metres

Therefore

The areas will never intersect

**Alternate Solution**

$(x+1)^2 + (y-1)^2 = 1$




Circle centre (4;3) + radius

Circle centre (-1; 1) + radius

Radii not going to cross the y axis

Radius 1 and radius 2 will not touch

**Alternate Solution**

$x^2 + y^2 + 2x - 2y + 1 = (x-3)^2 + (y-4)^2 - 4$

$8x + 6y = 20$

$x^2 + (-\frac{4}{3}x + \frac{10}{3})^2 + 2x - 2(-\frac{4}{3}x + \frac{10}{3}) + 1 = 0$

$25x^2 - 38x + 49 = 0$

No real solutions for the quadratic therefore no intersection

(b) Equation of quarter circle

$$\frac{\pi r^2}{4} = 8\pi$$

Radius of quarter circle $= \sqrt{32}$

Equation of circle centre A

$$(x-2)^2 + (y-2)^2 = 4$$

$$FA = \sqrt{(-1-2)^2 + (5-2)^2}$$
$$FA = \sqrt{18}$$

$$AB = \sqrt{32} - \sqrt{18}$$
$$AB = \sqrt{2}$$

$$BC^2 = 11 - 2$$
$$BC = 3$$

Therefore

Perimeter of ABCD $= 6 + 2\sqrt{2}$

**Alternative Solution**

Equation of FB

$$m_{FB} = -1$$
$$2 = -1(2) + c$$
$$c = 4$$

Coordinates of B

$$y = -x + 4$$
$$(x-5)^2 + (-x+4+1)^2 = 32$$
$$x^2 - 10x + 25 + x^2 - 10x + 25 = 32$$
$$2x^2 - 20x + 18 = 0$$
$$x^2 - 10x + 9 = 0$$
$$(x-9)(x-1) = 0$$
$$x = 9 \text{ or } x = 1$$

Coordinates B(1 ; 3)

AB $= \sqrt{2}$ (Use of the distance formula)

$$BC^2 = 11 - 2$$
$$BC = 3$$

Therefore

Perimeter of ABCD $= 6 + 2\sqrt{2}$

**Total: 150 marks**