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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 32 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Three blank pages (pages 30 to 32) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Ensure that your calculator is in **DEGREE** mode.
8. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
9. It is in your own interest to write legibly and to present your work neatly.
10. Round off to one decimal place unless otherwise stated.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	TOTAL
19	**13**	**11**	**9**	**16**	**8**	**12**	**9**	**8**	**12**	**12**	**10**	**11**	**/150**

SECTION A

QUESTION 1

(a) $P(2\,;a)$; $Q(-3\,;4)$ and $R(1;8)$ are on the same straight line.

Determine the value of *a.*

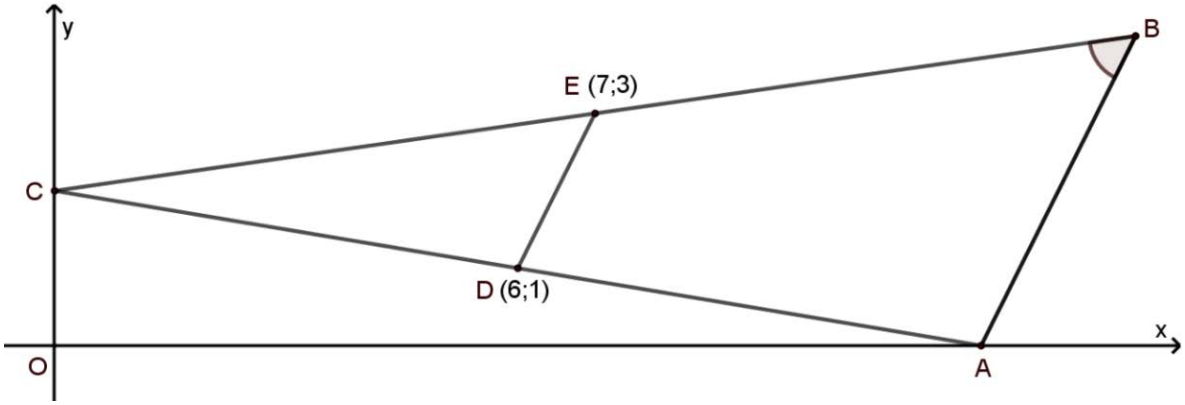
(3)

(b) $S(4\,;8)$ and $T(-2\,;6)$ are joined by a straight line. Determine the equation of a line that is perpendicular to ST and goes through the midpoint between *S* and *T*.

(4)

(c) In the diagram below, ΔCBA is drawn with C on the *y*-axis and A on the *x*-axis.

- Point $D(6\,;1)$ lies on *CA* and point $E(7\,;3)$ lies on *CB.*
- The equation of *AB* is $y-2x+24=0$.
- The equation of *CB* is $7y=x+14$.



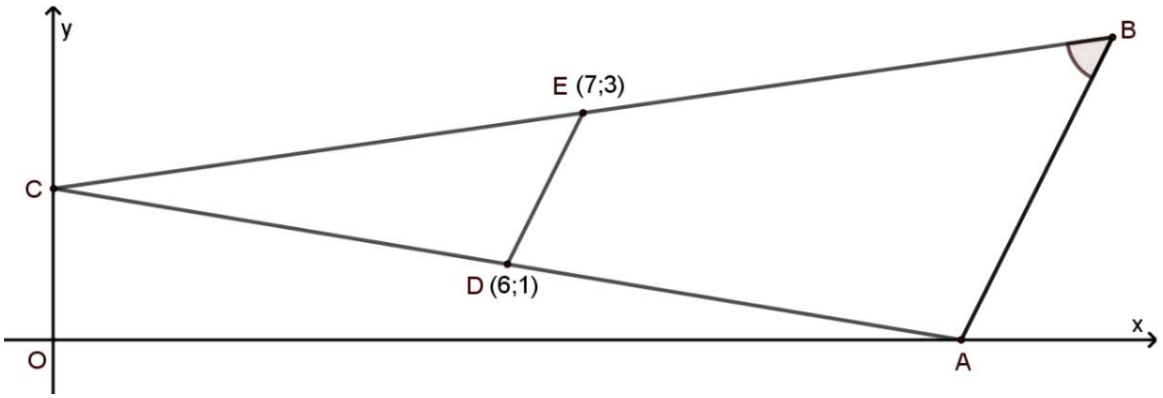
(1) Prove that $ED \,//\, AB$.

(3)

(2) If *D* is the midpoint of *CA*, then determine the coordinates of *B*.

(4)

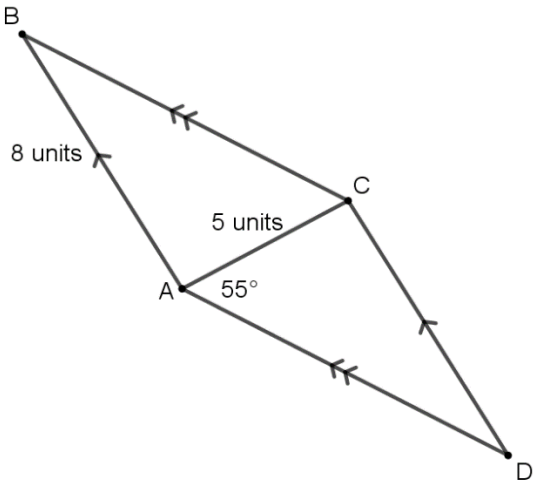
(3) Calculate the size of $C\hat{B}A$.



(5)
[19]

QUESTION 2

(a) A parallelogram ABCD is such that the diagonal *AC = 5*; *AB = 8* units and $C\hat{A}D = 55°$.



(1) Calculate the size of angle $A\hat{D}C$.

(4)

(2) Calculate the area of parallelogram *ABCD*.

(4)

(b) If $13\cos\theta = 5$ and $\theta \in [180° ; 360°]$, then, without the use of a calculator, determine the values of the following:

(1) $\sin\theta$

(2)

(2) $\cos(\theta + 45°)$

(3)

[13]

QUESTION 3

Given $f(x) = \cos 2x + 1$ and $g(x) = \tan x$ for $x \in [0° ; 270°]$.

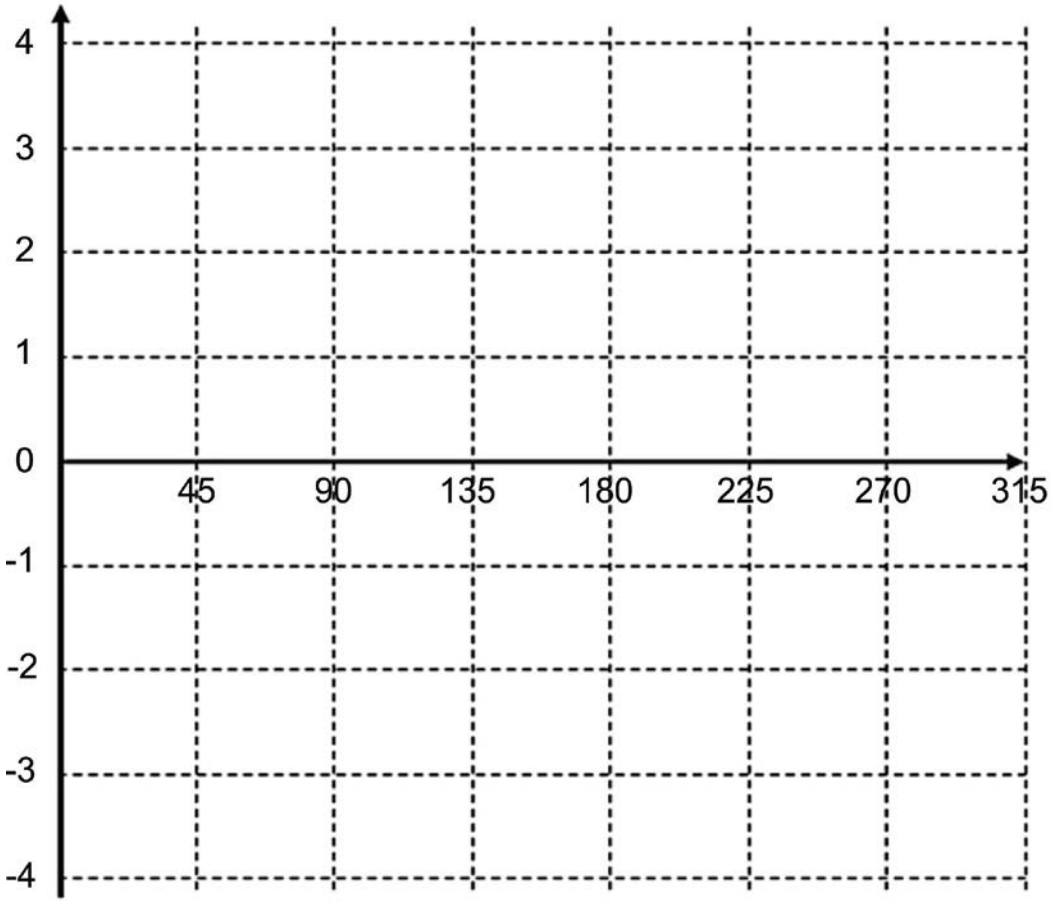
(a) Write down the period of *f.*

(1)

(b) Write down the two asymptotes of *g.*

(2)

(c) Sketch the graphs of *f* and *g* on the same set of axes below. Label all intercepts with axes, asymptotes, turning points and end points.



(6)

(d) Use your graph to write down the general solution of *x* for the equation $\cos 2x + 1 = \tan x$.

(2)
[11]

QUESTION 4

(a) Determine the general solution for $\cos\theta - \sin\theta = 0$.

(3)

(b) (1) Prove the identity $\frac{\sin 2\theta + \cos 2\theta + 1}{\cos 2\theta} = \frac{2\cos\theta}{\cos\theta - \sin\theta}$.

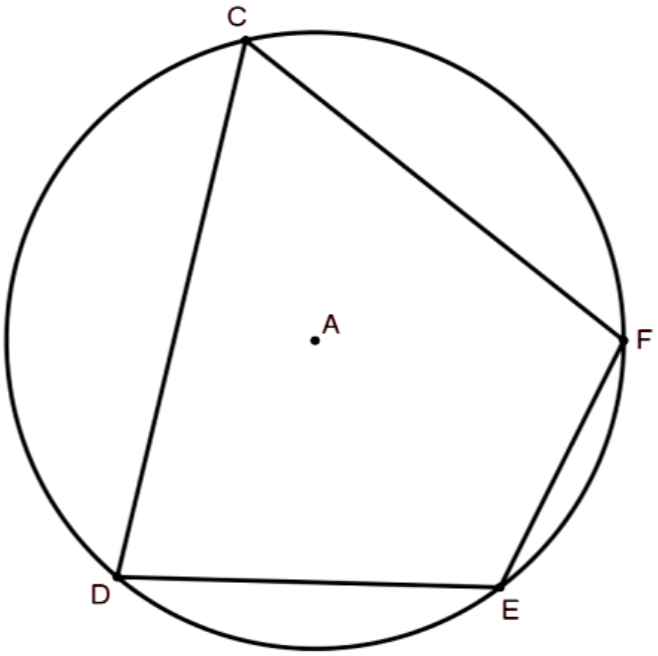
(4)

(2) Write down all the values for θ in the interval $[0° ; 180°]$ for which the identity is not valid.

(2)
[9]

QUESTION 5

(a) Prove the theorem that states; "The opposite angles of a cyclic quadrilateral are supplementary".



R.T.P: $\hat{C} + \hat{E} = 180°$

Construction:

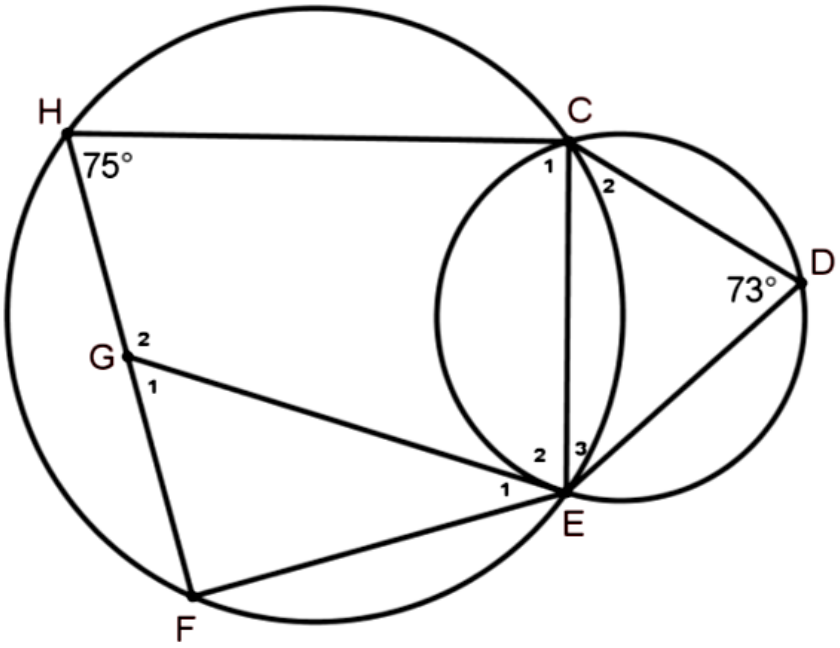
(1)

Proof:

(4)

(b) In the diagram below:

- The two circles intersect at *C* and *E.*
- *G* is a point on chord *HF* in the bigger circle.
- *GE* is a tangent to the smaller circle at *E.*
- $F\hat{H}C = 75°$.
- *D* is a point on the smaller circle with $C\hat{D}E = 73°$.

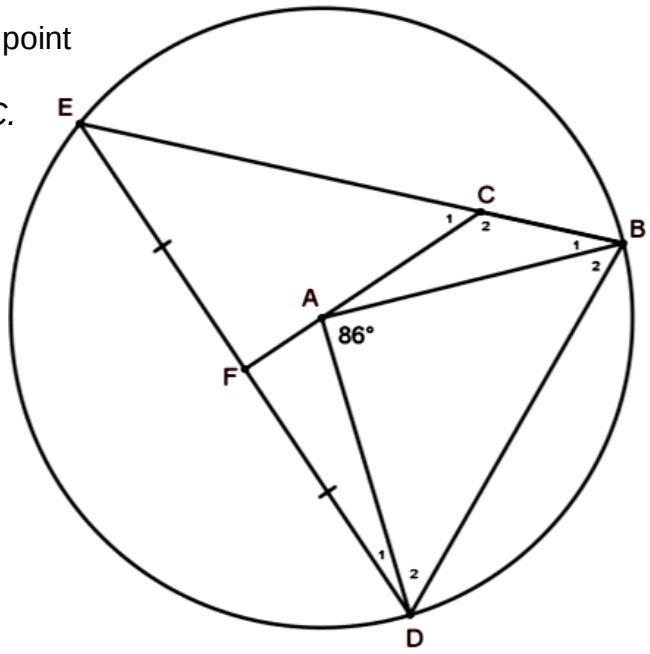


Giving reasons where required, determine the size of $\hat{E}_1$.

(5)

(c) In the diagram below:

- C is a point on chord *BE* and *F* is a point on chord *DE* with *EF = FD.*
- A, the centre of the circle lies on *FC.*
- $D\hat{A}B = 86°$.



(1) Determine the size of $\hat{D}_2$.

(1)

(2) Determine the size of $D\hat{E}B$ and give a reason for your answer.

(1)

(3) Prove that *DACB* is a cyclic quad.

(4)
[16]

QUESTION 6

(a) Refer to the table below to answer the questions that follow:

x	10	16	11	12	32	24	21	19
y	17	35	20	23	83	59	50	44

(1) Calculate the correlation coefficient for the table above.

(1)

(2) What can you infer from your answer to (1) above?

(1)

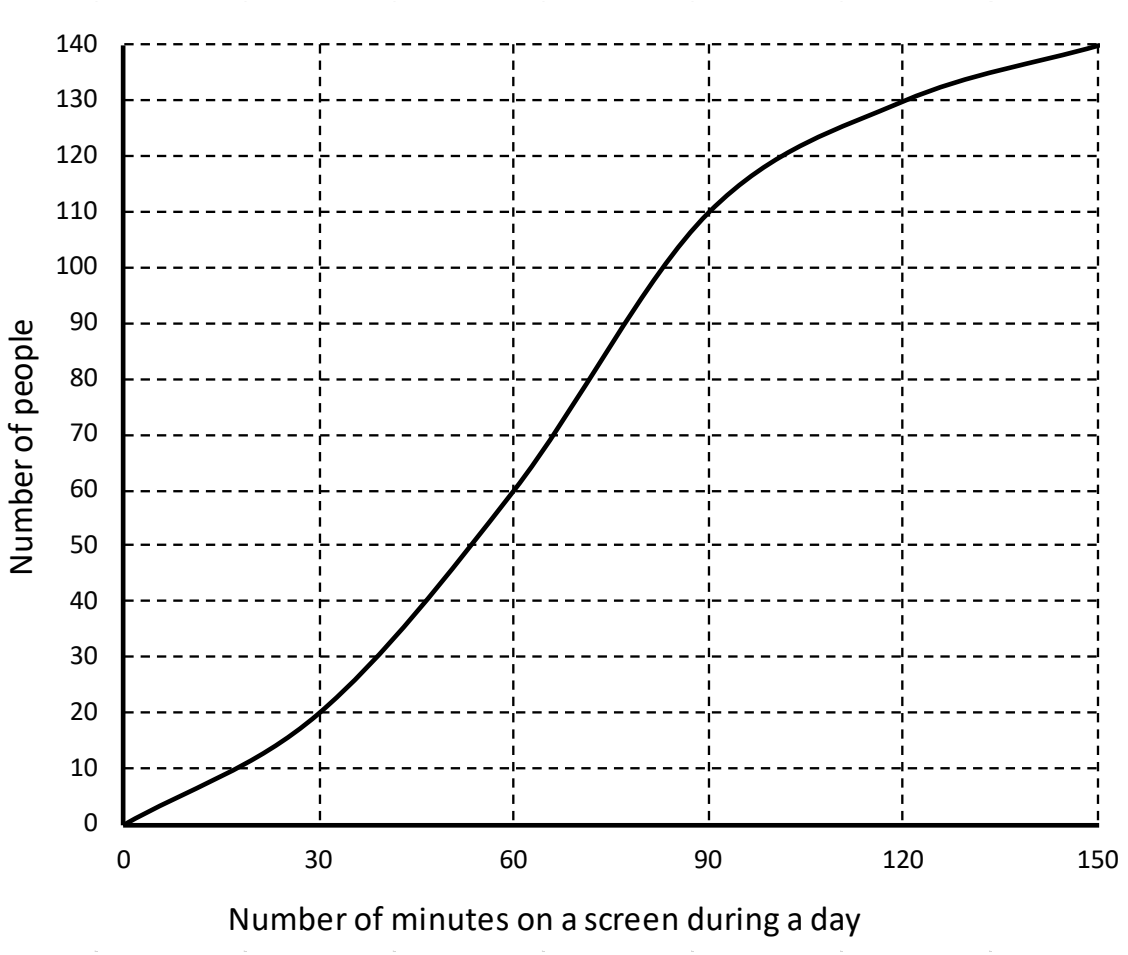
(3) Use your calculator to determine the equation for the line of best fit in the form $y = A + Bx$.

(2)

(4) Give a positive value for x where it might not be a wise decision to predict a value for y using the line of best fit. (*Explain your answer*.)

(1)

(b) Below is a cumulative frequency curve that represents the amount of time people spend looking at a screen each day.



(1) How many people spend between 30 and 60 minutes on a screen during the day?

(1)

(2) What percentage of people spends more than 120 minutes on a screen during the day?

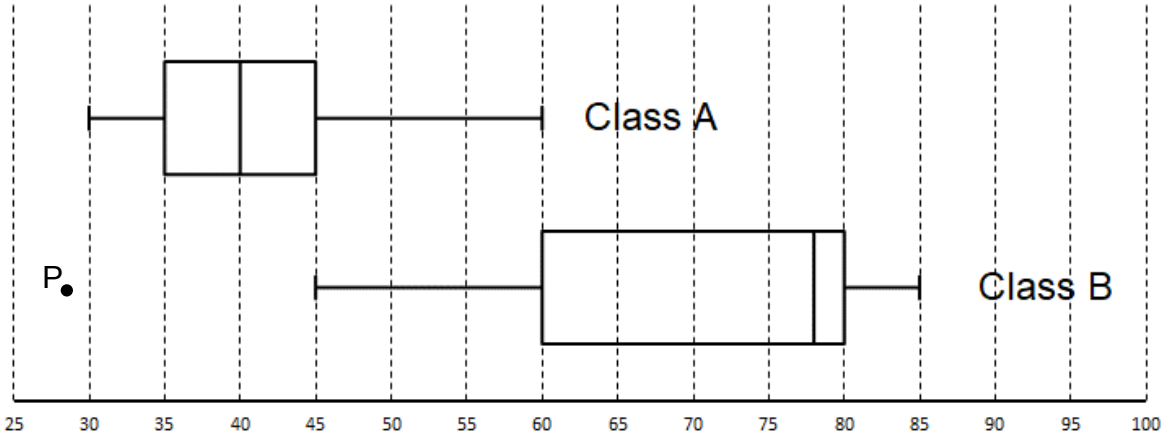
(2)
[8]

76 marks

SECTION B

QUESTION 7

(a) The box and whisker plot below is a summary of the results of students who wrote the same maths test. In Class A there are 28 students and in Class B there are 25 students *including the student represented by the point at P*.



(1) A data value is an outlier if it is less than $Q_1 - 1{,}5 * IQR$ or greater than $Q_3 + 1{,}5 * IQR$. Show that P is an outlier.

(3)

(2) The teacher of Class B says that their data is skewed. State whether the data is skewed to the left or to the right. (*Explain your answer*.)

(2)

(3) One of the teachers states that 13 students in the grade got between 45% and 60% for the test. Could they be correct? (*Explain your answer*.)

(2)

(b) The table below shows the height of trees in a forest.

Height of trees in metres	Frequency
$0 < x \leq 2$	$\boldsymbol{p}$
$2 < x \leq 4$	165
$4 < x \leq 6$	290
$6 < x \leq 8$	185
$8 < x \leq 10$	75

(1) If the approximate mean for the height of trees is 5 metres, calculate the value of $\boldsymbol{p}$.

(3)

(2) 200 new trees with a height of 10 cm were planted. Once these have been recorded, it is given that:

$$\sum_{i=1}^{915+p} \left(x_i - \overline{x}\right)^2 = 6\,520$$

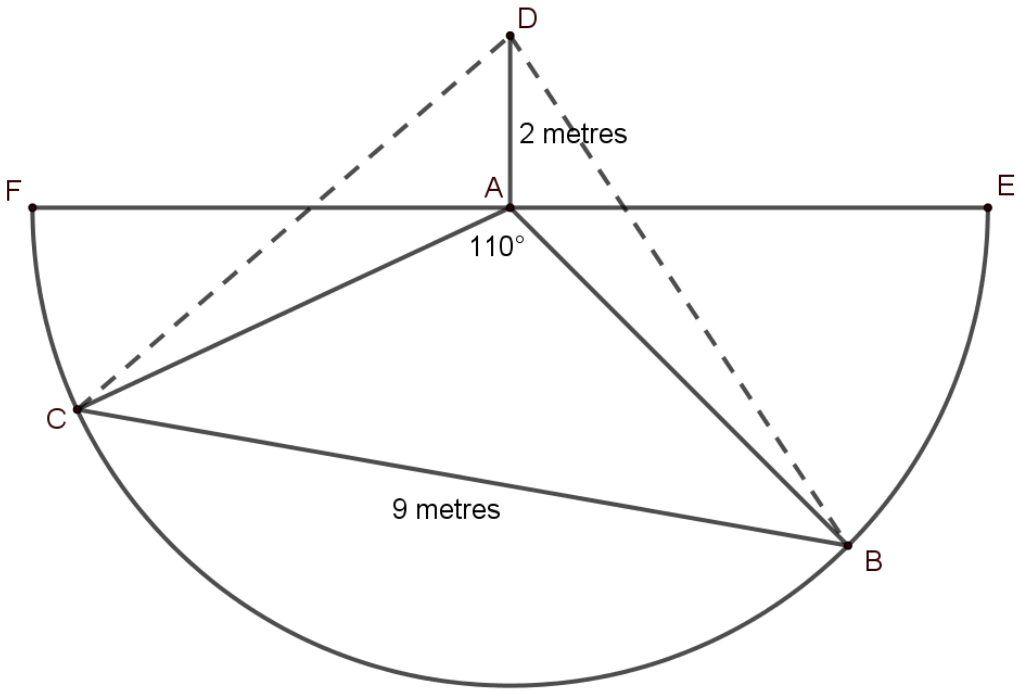
Determine the new approximate standard deviation.

(2)
[12]

QUESTION 8

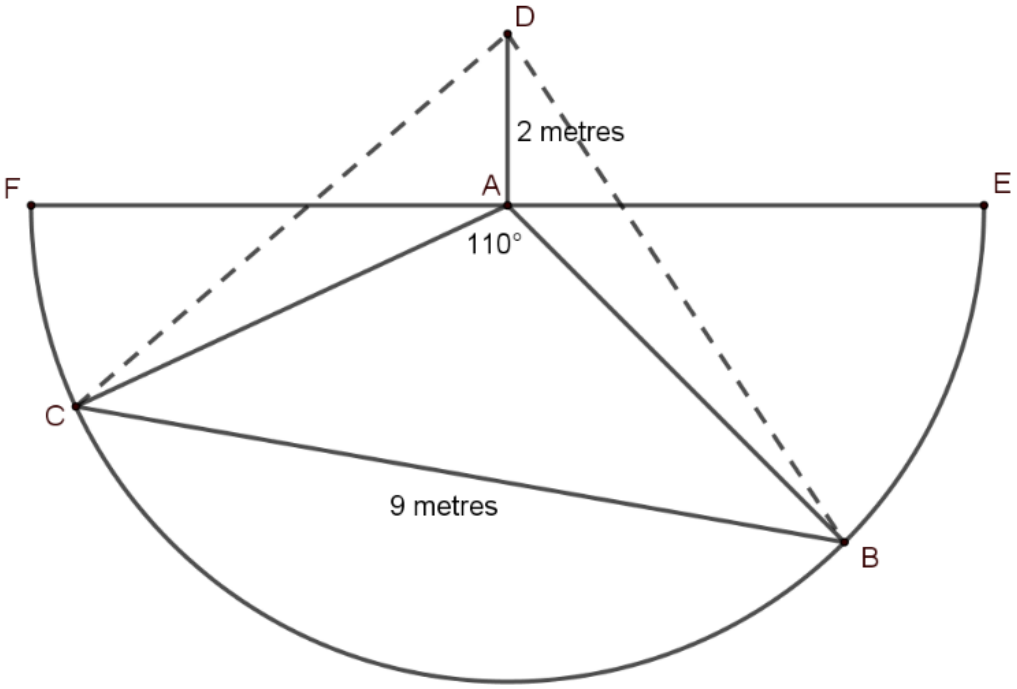
In the diagram below:

- The semi-circle with centre *A* and diameter *FAE* going through *B* and *C* lie on a horizontal plane.
- *CB* = *9* metres and $C\hat{A}B = 110°$.
- *AD* is a vertical pole to the plane and is 2 metres in height.



(a) Calculate the length of DB. (*Round off to four decimal places.*)

(6)



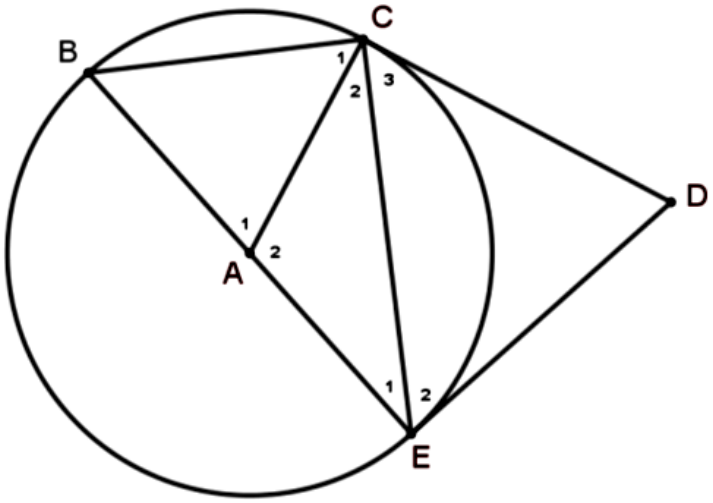
(b) Calculate the size of $C\hat{D}B$. (*Round off to one decimal place.)*

(3)
[9]

QUESTION 9

In the diagram below:

- *DC* and *DE* are tangents to the circle at *C* and *E* respectively.
- *A* is the centre of the circle.
- *B* lies on the circle and *BAE* is a straight line.



(a) Prove that $\Delta ABC \,///\, \Delta DEC$.

(5)

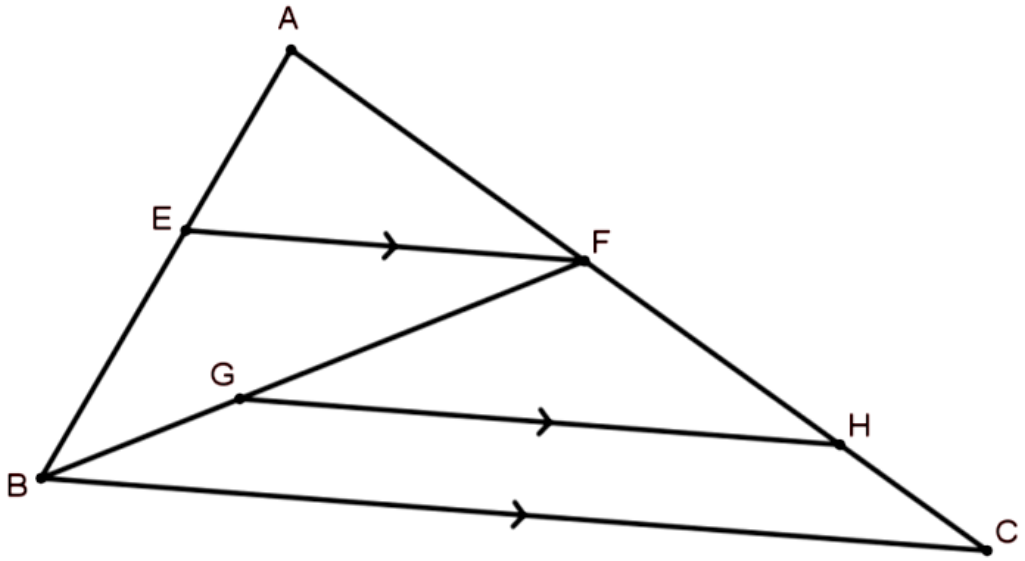
(b) Hence, show that $AE.EC = BC.DE$.

(3)
[8]

QUESTION 10

(a) In the diagram below:

- E lies on AB and F on AC in ΔABC with $EF \parallel BC$.
- G lies on FB and H on FC in ΔFBC with $GH \parallel BC$.
- $\frac{AE}{EB} = \frac{4}{5}$ and $\frac{FG}{FB} = \frac{5}{8}$.



(1) Calculate the value of $\frac{AF}{AC}$.

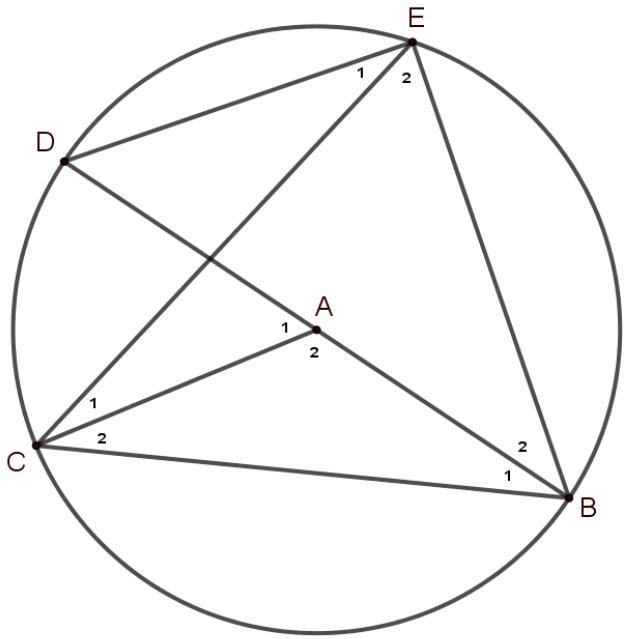
(2)

(2) Calculate the value of $\frac{HC}{AF}$.

(4)

(b) In the diagram below:

- *A* is the centre of the circle.
- *B, C, D* and *E* lie on the circumference of the circle.
- *DAB* is a diameter.



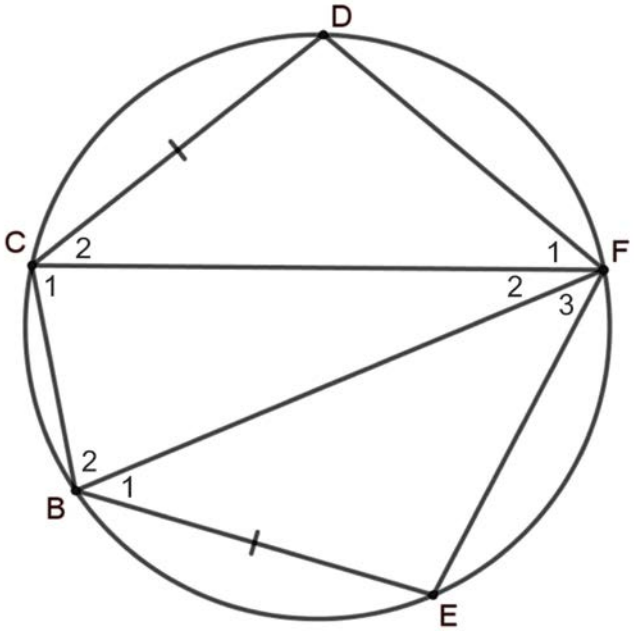
Prove that $\hat{A}_1 = 90° - \hat{E}_2 + \hat{B}_1$.

(6)
[12]

QUESTION 11

(a) In the diagram below:

- *CD* and *BE* are chords with *CD* = *BE.*
- *F* is also a point on the circle.
- $\hat{F}_3 = x$.

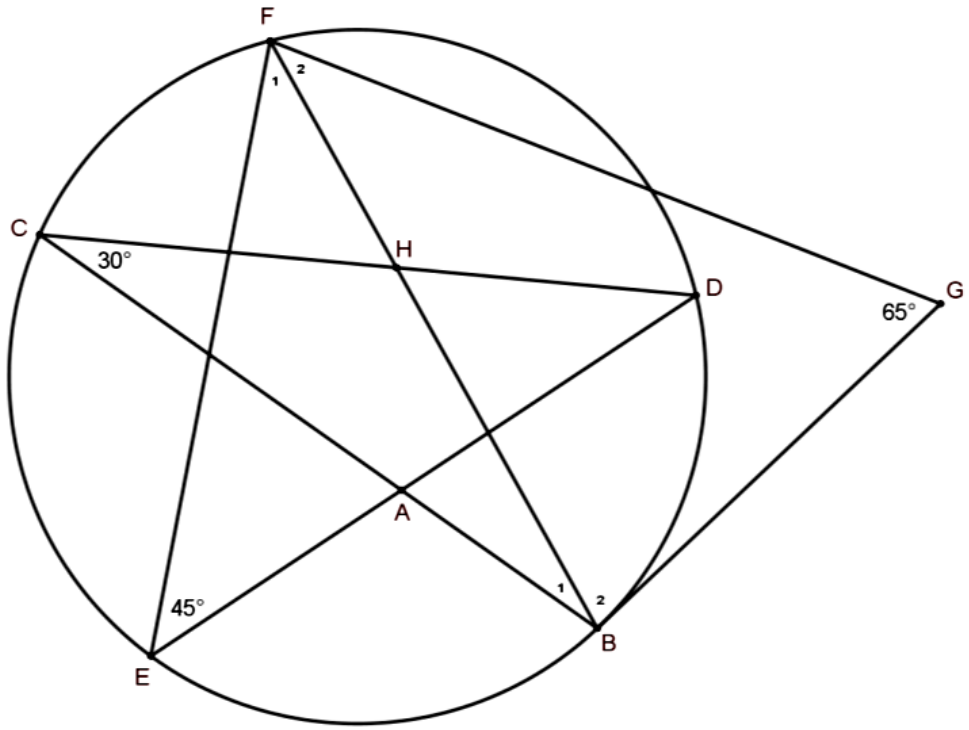


Prove that $\hat{C}_1 + \hat{C}_2 = \hat{B}_1 + \hat{B}_2$.

(6)

(b) In the diagram below:

- *FB* and *CD* are chords that intersect at *H.*
- *CB* and *ED* are chords that intersect at *A.*
- $B\hat{C}D = 30°$.
- $D\hat{E}F = 45°$.
- *GB* is a tangent to the circle at *B* and $F\hat{G}B = 65°$.



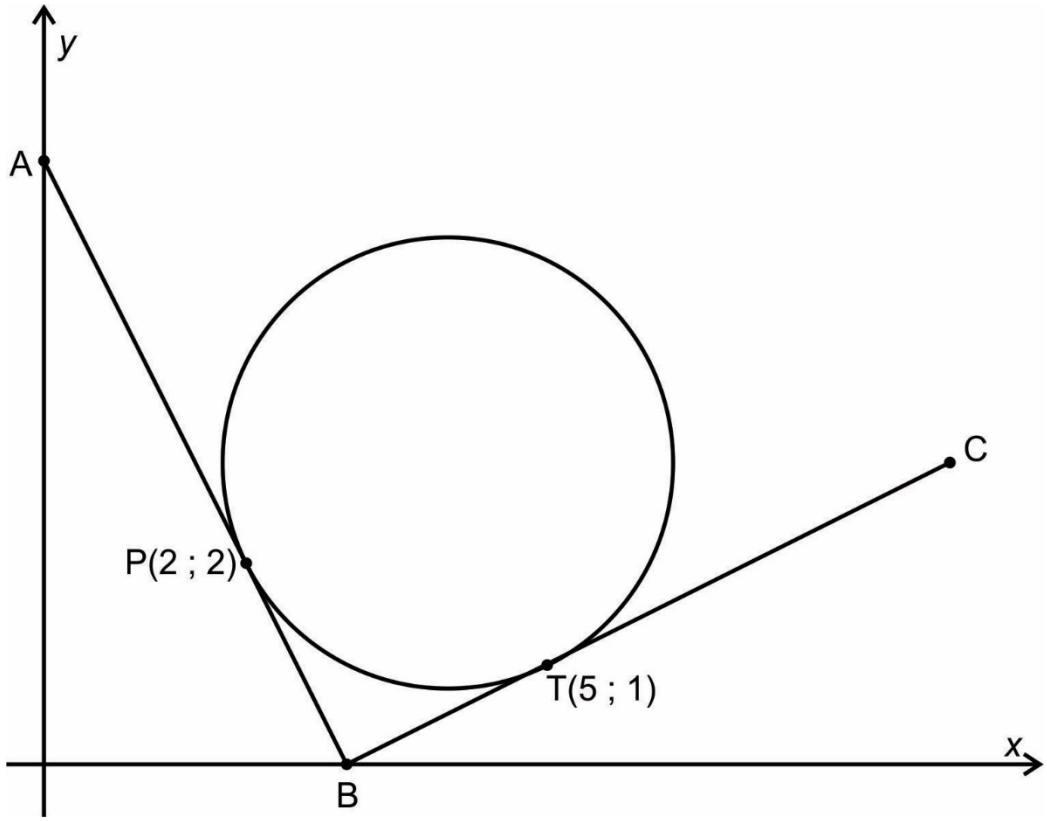
Determine the size of $\hat{F}_2$.

(6)
[12]

QUESTION 12

In the diagram below:

- *A* and *B* are the *y* and *x* intercepts respectively of the line $y = -2x + 6$.
- *AB* and *BC* are tangents to the circle at $P(2\,;\,2)$ and $T(5\,;\,1)$ respectively.



(a) Determine the equation of the line passing through *P* and the centre of the circle. (3)

(b) Calculate the **minimum** distance from the circle's circumference to the x-axis.

(7)
[10]

QUESTION 13

(a) A teacher creates two circular areas in her classroom for toddler A and toddler B.

- The first area is for toddler A and is modelled by $x^2 + y^2 + 2x - 2y + 1 = 0$.
- The second area is for toddler B and is modelled by $(x-3)^2 + (y-4)^2 = 4$.

The teacher claims that the playing areas will never touch or intersect. Is the teacher's claim correct?

(5)

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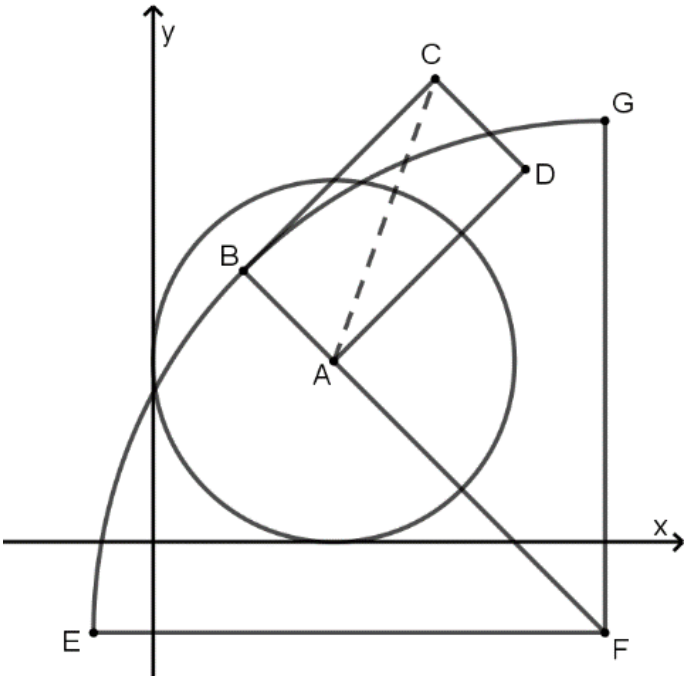
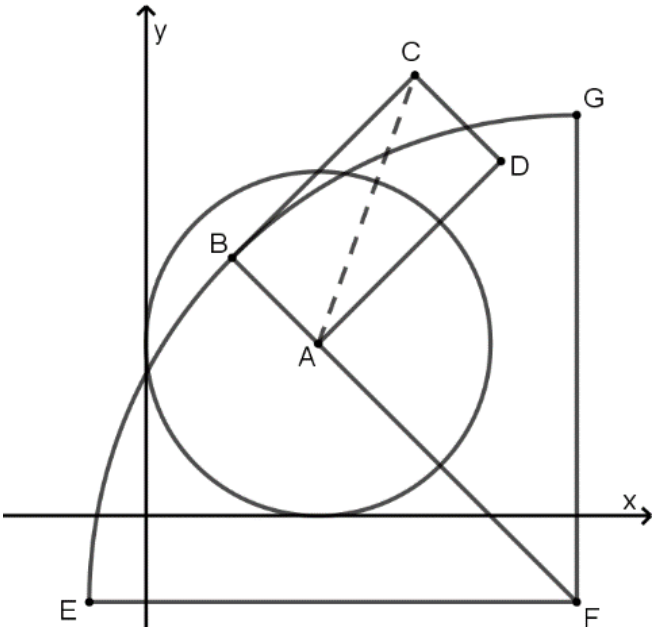
(b) In the diagram below:

- Circle with centre *A* has a radius of 2 units and touches the *x* and *y*-axes.
- *BC* is a tangent to the quarter circle with centre *F*(5 ; –1).
- *FAB* is a straight line.
- The length of *AC* is $\sqrt{11}$ units.



If the area of the quarter circle FEG is 8π units^2, calculate the perimeter of the rectangular space *ABCD.* (*Show all of your workings*.)



(6)
[11]

74 marks

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

