NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2014

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION A

QUESTION 1

(a) (1) $\frac{x+2}{2} = 4$ a $\therefore x = 6$ a

$\frac{8+0}{2} = y$ a $\therefore y = 4$ (3)

(2) $(1;2)$, $(p-1;3)$ and $(-3;-6)$ are collinear points.

$\frac{3-2}{p-1-1}$ a $= \frac{-6-3}{-3-(p-1)}$ a m

$\therefore \frac{1}{p-2} = \frac{-9}{-2-p}$

$\therefore -2-p = -9p+18$

$\therefore 8p = 20$

$\therefore p = \frac{5}{2}$ a

OR

$\frac{-6-2}{-3-1}$ a $= \frac{-6-3}{-3-(p-1)}$ a m

$\therefore \quad 2 = \frac{-9}{-2-p}$

$\therefore -4-2p = -9$

$\therefore \quad -2p = -5$

$\therefore \quad p = \frac{5}{2}$ a

OR

$m = \frac{2+6}{1+3} = 2$ a

$\therefore y = 2x + c$ m

$\therefore$ subs $(1;2)$

$2 = 2(1) + c$

$0 = c$

$\therefore y = 2x$ a

$\therefore 3 = 2(p-1)$

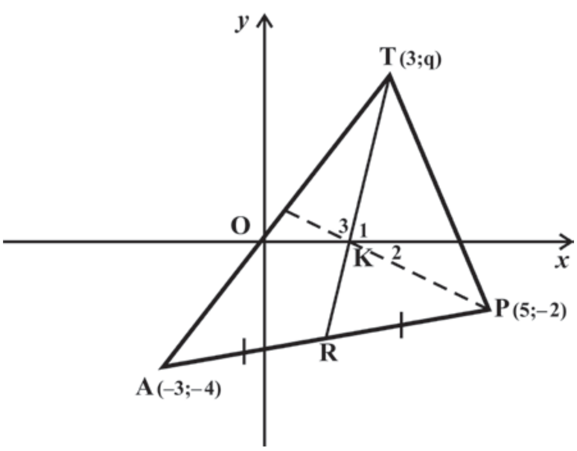
$\therefore 3 = 2p-2$

$\therefore -2p = -5$

$\therefore p = \frac{5}{2}$ a (4)

(b) (1) $m_{OA} = \frac{0-(-4)}{0-(-3)} = \frac{4}{3}$ a Eqn of

$q = \frac{4}{3}$ a $\times 3 = 4$ (3)



Alternative:

$m_{OA} = m_{OT}$ a

a $\frac{4}{3} = \frac{q}{3}$ a

$\therefore q = 4$

(2) k is the x-intercept of TR

$R\left(\frac{5-3}{2};\frac{-2-4}{2}\right)=(1;-3)$ m a a

$m_{TR}=\frac{4-(-3)}{3-1}=\frac{7}{2}$ a

Eqn of TR: $y+3=\frac{7}{2}(x-1)$ a

For K: $0+3=\frac{7}{2}(x-1)$ m

$\therefore 6=7x-7$

$\therefore x=\frac{13}{7}$ (6)

(3) (i) $m_{TK}=\tan\hat{K}_1=\frac{7}{2}$ m $\therefore \hat{K}_1=74,1°$ a

(ii) $m_{KP}=\frac{0-(-2)}{\frac{13}{7}-5}=-\frac{7}{11}$ m a $180-\hat{K}_2=147,5°$ a $\therefore \hat{K}_2=32,5°$ a (6)

[22]

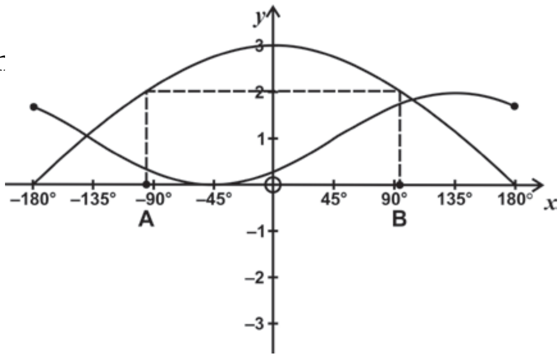
QUESTION 2

(a) (1) $a = 3$ a $b = \frac{1}{2}$ a (2)

(2) $720°$ a (1)

(3) Indicated at A a and B a (2)

(4) See graph
Vertical shift and am
Horizontal shift a
Shape a (3)



(b) $\frac{\cos 2\beta}{\sin(\beta - 45°)}$

$= \frac{\cos^2\beta - \sin^2\beta}{\sin\beta.\cos 45° - \sin 45°.\cos\beta}$ a a

$= \frac{(\cos\beta - \sin\beta)(\cos\beta + \sin\beta)}{\frac{\sqrt{2}}{2}\sin\beta - \frac{\sqrt{2}}{2}\cos\beta}$ a

$= \frac{(\cos\beta - \sin\beta)(\cos\beta + \sin\beta)}{-\frac{\sqrt{2}}{2}(\cos\beta - \sin\beta)}$ a

$= -\sqrt{2}(\cos\beta + \sin\beta)$

$= -\sqrt{2}T$ or $-\frac{2}{\sqrt{2}}T$ a (5)

(c) (1) $\tan A = \tan 135°$
$A = 315°$ a a
(Answer only: full marks)

Alternate: $\tan A = -1$ a
$\therefore A = 360 - 45$
$= 315$ a (2)

(2) $A = 495°$ a or $A = 675°$ a (2)

(d) (1) $OP^2 = 5^2 + 4^2 = 41 \quad \therefore OP = \sqrt{41}$ m

$\cos(90° + \theta) = -\sin\theta = -\frac{4}{\sqrt{41}}$ a a (3)

(2) $\frac{4}{5} = \frac{5}{-a} \therefore -4a = 25 \quad \therefore a = \frac{-25}{4}$ a a (2)

[22]

QUESTION 3

(a) (1) 35±3 [a] (1)
(2) 55±3 [a] (1)
(3) 65 ±3[a] (1)

(b) 700 [a] (1)

(c) 150±3 [a] (1)

(d) Upper limit $= Q_3 + 1,5 \times IQR = 65 + 1,5 \times 30 = 110$ [a] [a]
Lower limit $= Q_1 - 1,5 \times IQR = 35 - 1,5 \times 30 = -10$ [a] [a]
From the cumulative frequency graph, marks of all learners were between 0 and 90. [a]
Therefore, no isolated values. (5)

[10]

QUESTION 4

(a) (1) $\hat{B}_2 = x$; [a] tan/ chord thm [a] (2)
(2) $\hat{C}_4 = x$; [a] tan/ chord thm [a] (2)
(3) $\therefore \hat{T} = 180 - 2x$ [a] [a] [a] ; <'s of aΔ [a] [a] (4)
(4) $\hat{A} = 180 - y$; opp <'s of cyclic quad [a] (2)
(5) $\hat{B}_1 = 180 - y$ [a] ; angles in same segment [a] (2)

(b) (1) Co-int. angles ; DE//PQ [a] (1)

(2) $\hat{S}_2 = \hat{Q}$; Ext. angle of cyclic quad [a] (1)

(3) (i) $\hat{R}_1 = \hat{R}_1$; common [a]
(ii) $\hat{S}_2 = \hat{E}_1 + \hat{E}_2$; both 90° [a] (2)

(4) $\frac{ER}{SR} = \frac{DR}{ER}$ [a] [a]; $(\Delta DER /// \Delta ESR)$
$\therefore \frac{6}{SR} = \frac{10}{6}$ [a]
$\therefore SR = 3,6$ [a]
$\therefore DS = 10 - 3,6$
$= 6,4$ [a] (5)

[21]

75 marks

SECTION B

QUESTION 5

(a) (1) LHS

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$
$$= \frac{2\times\dfrac{\sin\theta}{\cos\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$$
$$= \frac{2\sin\theta}{\cos\theta} \div \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$$
$$= \frac{2\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{1}$$
$$= 2\sin\theta\cos\theta$$
$$= \sin 2\theta$$
$$= \text{RHS}$$

(4)

(2) $\frac{(1+\tan\theta)^2}{1+\tan^2\theta} = \frac{1+\tan^2\theta}{1+\tan^2\theta}$ a $+ \frac{2\tan\theta}{1+\tan^2\theta} = 1+\sin 2\theta$ a

Therefore the maximum is 2 a

(3)

(b) $3\sin\theta.\sin 22° = 3\cos\theta.\cos 22° + 1$

$3\sin\theta.\sin 22° - 3\cos\theta.\cos 22° = 1$ a

$-3\left(\cos(\theta + 22°)\right) = 1$ a

$\cos\left(\theta + 22°\right) = -\frac{1}{3}$ a

$\theta + 22° = \pm 109{,}5° + k.360°;\ k \in Z$ a

$\theta = 87{,}5° + k.360°;\ k \in Z$ or $\theta = -131{,}5° + k.360°;\ k \in Z$ a a

OR

$\left.\begin{array}{l}\theta + 22 = 109{,}5 + k.360 \\ \theta + 22 = 250{,}5 + k.360\end{array}\right\}$ a

$\left.\begin{array}{l}\therefore \theta = 87{,}5 + k.360 \\ \text{or } \theta = 228{,}5 + k.360;\end{array}\right\}$ a $k \in \mathbb{Z}$ a

(6)

(c) $4^2 = 5^2 + 6^2 - 2(5)(6)\cos Y$ a

$\therefore 16 = 25 + 36 - 60\cos Y$

$\therefore \cos Y = \frac{45}{60}$

$\therefore \cos Y = \frac{3}{4}$ a

$6^2 = 4^2 + 5^2 - 2(4)(5)\cos Z$ a

$\therefore \cos Z = \frac{5}{40}$

$\cos Z = \frac{1}{8}$ a

$\therefore \cos\hat{Y} + \cos\hat{Z} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ a (5)

(d) $V_{cylinder} = \pi r^2 h$

$V_{pyramid} = \frac{1}{3} \times \text{Area of base} \times \perp \text{height}$

$\therefore 3\ 000 = \frac{1}{3} \times \left(\frac{1}{2} \times 20 \times 20 \times \sin 60°\right) \times \perp h$ a a

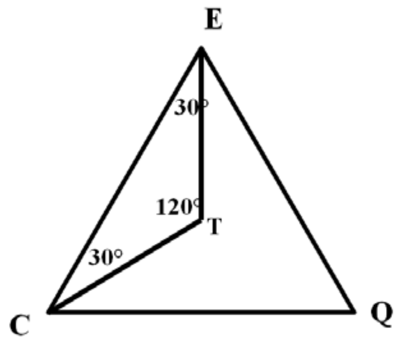
$\therefore \perp h = 51{,}961524\ ...$ a

$\frac{r}{\sin 30°} = \frac{20}{\sin 120°}$ m

$\therefore r = 11{,}54700\ ...$ a

$V_{cylinder} = \pi r^2 h = \pi \times 11{,}547\ ...^2 \times 51{,}9615\ ... = 21765{,}592\ ...$ a

$V_{remaining} = 18766\ cm^3$ a (7)



[25]

QUESTION 6

(a) (1) $M\left(\frac{-1-1}{2};\frac{5-1}{2}\right)=\left(\overset{a}{-1};\overset{a}{\frac{5}{2}}\right)$ (2)

(2) $M_{QA}=\frac{6-2}{-1-(-3)}=\frac{4}{2}=2$ [a]

$\therefore M_{OA}=-\frac{1}{2}$ [a] $y=\frac{5}{2}$

$\therefore y-6=-\frac{1}{2}(x+1)$ [a] $\perp$ bisector of AC passes through centre:

$\therefore \frac{5}{2}-6=-\frac{1}{2}(x+1)$ [a]

Therefore, $\therefore x=6$

Therefore, $\left(6;\frac{5}{2}\right)$ is the centre of the circle. [a] (5)

(b) (1) $m_{GH}=\frac{7-5}{3-1}=1$ [a]

$m_{HI}=\frac{2-7}{8-3}=\frac{-5}{5}=-1$ [a]

$\therefore m_{GH}\times m_{HI}=-1$ [a]

$\therefore G\hat{H}I=90°$

OR

$GH=\sqrt{8}$

$HI=\sqrt{50}$ [a a]

$GI=\sqrt{58}$

$\therefore GI^2=GH^2+HI^2$

$\therefore G\hat{H}I=90°$ [a] (3)

(2) $GH^2=(1-3)^2+(5-7)^2=8$ [a]

$\therefore GH=\sqrt{8}$ [a]

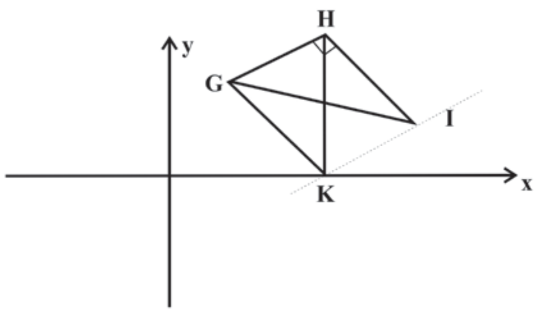
$HI^2=(3-8)^2+(7-2)^2=50$ [a]

$\therefore HI=\sqrt{50}$ [a]

$\therefore$ Area of $\Delta GHI=\frac{1}{2}\times\sqrt{8}\times\sqrt{50}$ [a]

$=10$ (5)

(3)



K lies on line || to GH through I [a]

Eq KI : $y-2=1(x-8)$ [a]

$\therefore 0-2=x-8$ [a]

$x=6$ [a] (4)

[19]

QUESTION 7

(a) (1) True. [a] The point is not part of the trend. [a] (2)

(2) True. [a] The point is not part of the trend. [a] (points closer to line) (2)

(3) True. [a] The line of best fit will be less steep. [a] (2)

(b) (1) A. [a] The values are clustered around the mean. [a] (2)

(2) Mean = 7 [a] and standard deviation is 2,4. [a] (2)

(3) Mean = p, [a] standard deviation = q [a] (2)

[12]

QUESTION 8

(a) (1)

STATEMENT	REASON	
$\hat{A}_2 = \hat{A}_3$	Given	[a]
$\hat{A}_2 = \hat{C}_1$	Alt. angles AD//CE	[a]
$\hat{E} = \hat{A}_3$	Corres angles. AD//CE	[a]

(3)

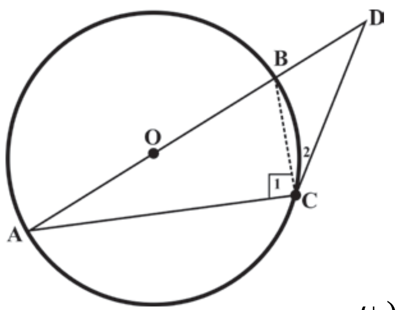
(2) ΔCAE is isosceles. [a] $\hat{C}_1 = \hat{E}$ [a] (2)

(3) $\frac{BD}{DC} = \frac{AB}{AE}$; line // to one side of Δ [a]

But AE = AC ; isos triangle [a]

$\therefore \frac{BD}{DC} = \frac{AB}{AE}$ (2)

(b) Join B to C



$\hat{C}_1 = 90°$; angle in semi circle. [a]

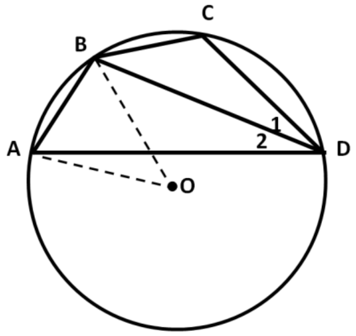
$\hat{C}_2 = \hat{A}$; tan/chord thm [a]

$\hat{D} = \hat{A}$; isos. ΔDAC [a]

$\therefore \hat{A} + \hat{A} + 90 + \hat{A} = 180°$ angles of a triangle [a]

$\therefore \hat{A} = 30°$ [a] (5)

(c) (1) Join O to B and O to A. [a]



In ΔBOA, $2^2 = 3^2 + 3^2 - 2.3.3.\cos\hat{O}$ [a]

$\cos\hat{O} = \dfrac{3^2 + 3^2 - 2^2}{2\times3\times3} = \dfrac{7}{9}$ [a]

$\therefore \hat{O} = 38,9°$

$\hat{D}_2 = 19,5°$; [a] angle at centre. (4)

(2) $\hat{D}_1 = \hat{D}_2$; [a] equal chords; angles in same segment [a]

$A\hat{B}C = 180° - 2\hat{D}_2 = 141,1°$; [a] (3)

[19]

75 marks

Total: 150 marks