National College of Business Administration and Economics Lahore

AN ATTRIBUTE NP CONTROL CHART FOR PROCESS MONITORING USING EXPONENTIATED HALF LOGISTIC DISTRIBUTION (EHLD)

BY

AMMARA TANVEER

MASTER OF PHILOSOPHY
IN
STATISTICS

JULY, 2019

NATIONAL COLLEGE OF BUSINESS ADMINISTRATION AND ECONOMICS

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A dissertation submitted to
School of Social Sciences

In Partial Fulfillment of the
Requirements for the Degree of

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بسم الله الرحمن الرحيم

In the name of ALLAH,

The Most Beneficial,

The Most Merciful,

NATIONAL COLLEGE OF BUSINESS ADMINISTRATION AND ECONOMICS LAHORE

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Dissertation Committee:

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Chairman

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Member

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Member

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Rector
National College of Business Administration and Economics

DECLARATION

It is to declare that this research work has not been submitted for obtaining similar degree from any other university/college.

AMMARA TANVEER
July, 2019

Dedicated
to

My Respected Mother,

my late Father Muhammad Tanveer Aslam

and specially my respected Teacher

Dr. Muhammad Azam,

with their kind support

today I able to

submit my thesis.

ACKNOWLEDGEMENT

In the name of Almighty Allah, Who bestowed on me His blessings and gave me courage and vision to accomplish this work successfully. I invoke peace for Holy Prophet Hazrat Muhammad (PBUH) who is forever a symbol of guidance for humanity.

I would like to extend my gratitude to the National College of Business Administration and Economics for providing me such environment and resources, which enabled me to complete this research successfully.

I feel great honor and pleasure in expressing my profound and cordial gratitude to my Supervisor, ***Dr. Muhammad Azam*** for academic support in whole process from the beginning to the end of this thesis. Thanks to all my teachers in the college for giving me the different concepts throughout my studies.

Especially, I have no words to pay gratitude to my mother and siblings whose affection, guidance and continuous encouragement enabled me to complete this research work. In the end, I pray to Almighty Allah to give me wisdom and strength to use this knowledge the way He wants.

RESEARCH COMPLETION CERTIFICATE

Certified that the research work contained in this thesis entitled **"An Attribute NP Control Chart for Process Monitoring using Exponentiated Half Logistic Distribution (EHLD)"** has been carried out and completed by **Ammara Tanveer** under my supervision during her **M.Phil. Statistics** Programme.

(Dr. Muhammad Azam)
Supervisor

SUMMARY

Control charts are efficient techniques for observing the production process in statistical process control (SPC). An advanced checking of the process of production is required to enhance the top quality of the products made, to produce the goods as per the given requirements and to decrease the evaluation cost. Control charts are therefore effective in achieving high service quality. In the current study, we suggested an np control chart based on repetitive group sampling scheme (RGS) and multiple dependent state sampling scheme (MDS) by using Exponentiated Half-Logistic distribution.

The shift in scale, shape and both parameters are obtained under RGS sampling scheme and also find the shift in scale parameter under MDS sampling scheme. An extensive collection of tables are given. A comparison is made between the suggested and existing control charts. A real life example and simulation study is also presented to understand the research work.

LIST OF ABBREVIATIONS

RGS	Repetitive Group Sampling
MDS	Multiple Dependent State Sampling
ARL	Average Run Length
ASN	Average Sample Number
EHLD	Exponentiated Half Logistic Distribution

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CHAPTER 1

INTRODUCTION

This chapter presents introduction of the topic in detail and consist of seven sections. Section 1.1 gives study background. Section 1.2 explains meaning of quality control and its techniques. Section 1.3 shows history of control chart and its classification. In section 1.4 *np* control chart is discussed. In section 1.5 history of repetitive sampling scheme is given. Section 1.6 gives history of multiple dependent state sampling scheme and finally section 1.7 gives objectives of our study.

1.1 STUDY BACKGROUND

A lot of goods are introduced to sell in the marketplace; merely a few of them will be constantly used by customers owed to quality. Some of important quality characteristics of the manufactured goods decide the quality of the product and also producers are responsible for the standard of the products. It is difficult for the producer or manufacturer to get better the quality of the manufactured goods by knowing the key factors involved in commercial growth and success. It is essential that the product should meet the requirement of the consumer. Therefore, statistically quality control plays a fundamental role in the industry growth.

1.2 QUALITY CONTROL

Quality control is array of testing and measures to chase sequentially to make sure that the quality of manufactured goods is retained and enhanced closest to the set of standards and so forth encountered any errors are also eliminated or reduced. The center of attention of quality control is to make sure that the manufactured goods and product mechanized are not only reliable, also fulfill the customer requirements.

In today's world the majority of the organizations comprise a quality control department that provides the set of principles to be following for all products. Quality control depends on testing of goods, as product examination provide a clearer depiction of the quality of end product.

These are two diverse ways of calculating the product quality

- Inspection by 100 percent
- By means of statistical quality control, statistical instruments used to determine the arriving or leaving value of the product when 100% inspection is too costly or impossible.

Different standards are available for quality control but among the most significant supervision tools is the control chart. Control chart is the main instrument for observing the process and in the developing of high-quality products.

1.3 CONTROL CHART

Walter A. Shewhart constructed the control chart in 1920s. Shewhart focused that it is essential to forecast future output and economically handle a process by putting a manufacturing process onto a statistical control state at which variability is only popular-cause and keep it in check.

By carefully designed experiment, Shewhart formed the framework for control chart and the idea of a statistic control situation. Although Shewhart was simply drawing statistical mathematical theories, he recognized that information commencing physical procedures typically generate a "curve of normal distribution." He found that in manufacturing data the observed difference is often not the same for the nature data. Shewhart found that while each method shows variety, some procedures show controlled variations so that they are natural to the process, whereas others show uncontrolled variations so that they are not always present in the process causal scheme.

The charts are frequently built for monitoring quality features that reflect product attributes: c control chart, u control chart, np control chart, *p* control chart and for controlling variables these charts are commonly constructed *x*-bar chart, *r* chart, *s* chart, s^2 chart.

1.4 NP CONTROL CHART

This chart is being used to track the amount of non-conforming sample units. It is an alteration of the *p* chart, used in circumstances where it is better for staff to characterize the process's overall performance of particular unit and numbers to the more conceptual proportion.

1. The control boundaries are $n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$, n is sample size and inter-term practice estimate means during the configuration of the control chart.

2. The non-conforming (*np*) used to plot against the control limits instead of the non-conforming fraction (*p*).

3. *n* is constant in sample size.

1.5 REPETITIVE GROUP SAMPLING SCHEME (RGS)

In 1965 Sherman suggested a Repetitive group sampling plan. Author presented a fresh acceptance sampling plan with a straight forward design and operation procedure that is middle in sample size effectiveness among the single sample plan and the sampling sequential probability ratio plan. In this plan when a sample is taken, the number of defects is counted, then the lot is approved, dismissed or the sample is totally overlooked, and we start again with a fresh sample. This is preceded until we are told either to acknowledge or dismiss the lot by the set criterion. We can’t even continue to follow that how many times our sample has been repeated.

1.6 MULTIPLE DEPENDENT STATE SAMPLING SCHEME (MDS)

Multiple dependent state (MDS) sampling system was created by Wortham and Baker (1976). Conclusion will be made not only on current but also previous lot and upcoming lot. Multiple deferred state sampling is an attribute monitoring procedure inside which one of three choices is dictated by the choice criteria for each lot:

1. Accept the lot

2. Reject the lot

3. Provisionally accept or dismiss the lot on the basis of future associated lots.

It is designed to complement current sampling processes such as chain sampling, dependent phase sampling, exponential smooth sampling and fixed sampling of deferred status. A natural expansion of fixed deferred state sampling plans is the multiple deferred state sampling procedures. In addition to having desirable characteristics with regard to their operating characteristic curves, these processes have been discovered to be appealing to most management staff. This sort of operation focuses on early detection of quality degeneration than on fixed sampling processes for deferred state.

1.7 OBJECTIVES OF THE STUDY

The study's primary goals are as follows

1. Development of an np control chart using exponentiated half-logistic distribution on the basis of a repetitive group sampling scheme.
2. Development of an np control chart using exponentiated half-logistic distribution on the basis of multiple dependent state sampling.
3. Find the variation when scale shape parameter is shifted.
4. Find the variations when shape and scale both parameters are shifted.
5. Program building using R programming for the suggested control charts.
6. Contrast of the suggested control charts with the current control charts.

CHAPTER 2

LITERATURE REVIEW

We are presenting among this section literature review on our study. This chapter is consisting of two sections. Section 2.1 is on repetitive group sampling scheme and section 2.2 is based on multiple dependent state sampling scheme.

2.1 REPETITIVE GROUP SAMPLING SCHEME (RGS)

RGS is suggested by Sherman (1965). RGS is expansion of single sampling plan. Recurring sampling awaits a choice in RGS.

Jeyadurga et al. (2018) developed RGS-based *np* control chart. Author used ideal chart building parameters. The chart is likened to a single control chart for sampling. It is noted that, the suggested chart detects shifts quicker than the single sampling-based chart. To detect the change, the suggested chart requires tiny size of sample. Also analyzed the economic design of the chart based on RGS.

Rao (2018) creates an attribute control chart depending on an exponentiated half-logistic distribution with either the shape parameter known or unknown. The wide range of tables is given for manufacturing use.

Ahmad et al. (2014) suggested a repetitive-sampling $\bar{X}$ control chart. In terms of middling run duration, the suggested chart is likened to the current Shewart type control chart. The author found that the suggested chart performs well in comparison with the current chart.

Rao and Naidu (2015) have suggested a fresh sampling plan for group acceptance for exponentiated half-logistic distribution. Their suggested scheme showed that percentile ratio rises for all parameters and decreases the amount of associations 'g. They showed that as r raises, the amount of associations decreases.

Seo and Kang (2015) extracted unknown parameters from the moment estimators and maximum likelihood estimators of exponentiated half-logistic distribution. Fisher information for exact expression is derived to find confidence intervals.

Aslam and Jun (2015) have suggested a control chart while a Weibull distribution follows the lifetime of the item on the basis of the amount of errors. It determines the coefficients and the duration of the test. The coverage of tables for different shift parameters is defined for the average running lengths out of control. A research is performed to demonstrate the suggested manufacturing use of control chart.

Aslam et al. (2014) introduced a t chart which follows the exponential-distribution based on repetitive sampling. The ARL is computed and linked with the existing chart. This obviously shown that the suggested chart obtained lesser values of ARL than existing chart.

Aslam et al. (2014) by using RGS introduced the attributes and variables control charts. For fixed values of sample size they originate the control limits coefficients and fraction of non-conforming.

Samina than and Mahalingam (2017) presented a new mixed-repetitive-group sampling scheme and developed the OC function and ASN. Authors obtain tables for both the symmetric and the asymmetric fraction nonconforming. The study proves that mixed RGS plan requires a smaller amount sampling when the examination is on the basis of attribute and variables quality characteristics.

Aslam et al. (2012) constructed repetitive-group sampling plan for Weibull and generalized exponential distribution. Life time of the product follows median as quality parameter. The parameters are obtained based on acceptable and limiting quality level. The study proves that the RGS plan performed well than variable single sampling plan.

Aslam et al. (2014) offered repetitive-group-sampling plan for Weibull distribution. They indicated the condition of optimization to achieve the plan's n condition and recognition. Tables are given comprising the optimal parameters. Planned chart is compared with single sampling plan, resulting that the planned chart requires lesser ASN as compared with existing chart.

2.2 MULTIPLE DEPENDENT STATE SAMPLING SCHEME (MDS)

Wortham and Baker (1976) establish multiple-dependent state (MDS) sampling scheme. Decision will be made not only on current but also previous lot and upcoming lot.

Arshad et al. (2017) use various dependent state sampling to create a control-chart to monitor process variation. The S^2control chart is consisting of two control boundaries. The ARL is calculated against the current Shewhart S^2 chart, Zhang et al., Guo et al. and Aslam et al. Also provided is an instance of simulation-based information and real-life information set.

Aslam et al. (2014) suggested a chart consisting of two pair of MDS-based control limits. Considering the targeting in-control ARL, the parameters and the limits are determined. Compared to the current $\overline{X}$ Control Chart, this chart shows better efficiency than the current control chart. A simulation research is also conducted to verify the control chart's efficiency.

Aslam et al. (2014) suggested a t chart based on MDS sampling which follows an exponential distribution. The tables for ARL's are calculated and compared with Santiago and Smith (2013) by comparing the tables, in conclusion, the planned chart is superior to the current chart in discovering the change.

Aslam et al. (2014) suggested process loss consideration MDS variable sampling plan. The planned parameters of the plan are set up by fulfilling the seller and consumer risks at a variety of acceptable quality level and limiting quality level. They compared the plan with single variable plan and Aslam et al. (2012). They select same values of L_{AQL}, L_{LTPD}, consumer risk and producer risk. By comparing with both plans, it is observed that the planned chart gives less sample size.

Aslam et al. (2016) discovered a new np control chart which follows Pareto distribution of second kind. The decreasing trend had been observed in ARL's values.

The MDS scheme is suggested by Balamurali and Jun (2007) offers better security with a lower sample size. The suggested scheme is built on a straightforward value and can therefore be applied more easily than double and multiple sampling plans.

Aslam (2017) suggested MDS control chart with process mean changed for Weibull distribution. The chart's efficiency is calculated as an ARL. Aslam and Jun (2015) are likened to the suggested chart. It is noted that the suggested chart is rapid in the finding of the out-of-control process since the amount of dependent states rises.

Balamurali et al. (2017) planned a multiple-deferred state sampling scheme on the basis of information of present and consecutive lot samples. Author measured the quality of the product by its median life and life time follows a generalized inverted exponential distribution by satisfying the both consumer and producer risk. Optimal parameters are obtained and operating characteristic is performed. The planned chart is also compared with existing plans.

CHAPTER 3

METHODOLOGY AND DISCUSSIONS

In this chapter, we have suggested repetitive-group sampling scheme and multiple-dependent sampling scheme based on exponentiated half-logistic distribution. The mathematical formulas are obtained in order to calculate parameters. Section 3.1 gives the overview of the exponentiated half-logistic distribution. In segment 3.2 we design the suggested control chart based on repetitive group sampling scheme. Section 3.2.1 provides shift in scale parameter. Section 3.2.2 gives shift in shape parameter. Section 3.2.3 provides shift in shape and scale parameter. In section 3.3 optimization conditions are given and discussed the tables which provide optimal parameters. In section 3.4 comparisons is made between suggested and existing control charts. Section 3.5 designed suggested control chart based on MDS formulas are obtained to find the parameters. Segment 3.5.1 discussed performance measure of the MDS chart, in this section tables are provided with parameters. In section 3.6 comparisons is made between suggested and existing control charts. Section 3.7 gives the real life example and final section 3.8 gives the simulation of suggested study for the better understanding of the study.

3.1 THE EXPONENTIATED HALF-LOGISTIC DISTRIBUTION (EHLD)

Exponentiated half-logistic distribution (EHLD) is a simplification of half logistic distribution as suggested by Mudholkar and Srivastava (1993). Cordeiro et al. (2014) studied mathematical properties of the exponentiated half-logistic family. Seo and Kang (2015) extracted unknown parameters from the moment estimators and maximum likelihood estimators of exponentiated half-logistic distribution. An entropy estimator is also derived for the distribution. ELGARHY et al. (2017) suggested “A New Exponentiated Extended G family of distributions”. They study its mathematical properties. Usman et al. (2017) studied the Kumaraswamy half logistic distribution. They explore the properties and applications of observe model. Anwar and Bibi (2018) suggested half-logistic generalized Weibull (HLGW) distribution. The properties are investigated by them.

The probability density function (pdf) and the cumulative distribution function (cdf) of EHLD are respectively given as

$$f(t) = \frac{2e^{-t/\sigma}\alpha\,(1 - e^{-t/\sigma})^{\alpha-1}}{\sigma\,(1 + e^{-t/\sigma})^{\alpha+1}}\ ; t \geq 0, \alpha > 0, \sigma > 0 \tag{1}$$

$$F\,(t) = \left[\frac{(1 - e^{-t/\sigma})}{(1 + e^{-t/\sigma})}\right]^{\alpha}\ ; t \geq 0, \alpha > 0, \sigma > 0 \tag{2}$$

Here α is the parameter of shape; σ is the parameter of scale. The average of EHLD distribution is specified by

$$\mu = \sigma\left[ln\left(\frac{1 + 0.5^{1/\alpha}}{1 - 0.5^{1/\alpha}}\right)\right] \tag{3}$$

Let

$$\eta = ln\left(\frac{1 + 0.5^{1/\alpha}}{1 - 0.5^{1/\alpha}}\right)$$

$$\mu = \sigma\eta \tag{3a}$$

$$\sigma = \frac{\mu}{\eta} \tag{3b}$$

3.2 DESIGN OF SUGGESTED CONTROL CHART ON THE BASIS OF RGS

Following are the steps of np control chart on the basis of repetitive group sampling scheme.

Step 1: Choose from the manufacturing process subgroup a random sample of size n and place it on the life test for the particular time t_0. List before t_0 the number of failures and represent it as D.

Step 2: If $LCL_1 \leq$ D $\leq UCL_1$ then the process is said to be in- control. If either D $\geq UCL_2$ or D $\leq LCL_2$ then the process is said to be out of control

Step 3: Repeat Step 1 and 2 if $LCL_2 <$ D $< LCL_1$ or $UCL_1 <$ D $< UCL_2$, until a decision is made.

The control limits for an np control chart are as illustrated

$$UCL_1 = np_0 + k_1\sqrt{np_0(1 - p_0)} \tag{4a}$$

$$LCL_1 = \max[0, np_0 - k_1\sqrt{np_0(1 - p_0)}] \tag{4b}$$

$$UCL_2 = np_0 + k_2\sqrt{np_0(1 - p_0)} \tag{4c}$$

$$LCL_2 = max[0, np_0 - k_2\sqrt{np_0(1 - p_0)}] \tag{4d}$$

The likelihood p_0 is generally unknown, so, the control limits for the use in practical application will be (see Montgomery 2013)

$$UCL_1 = \bar{d} + k_1\sqrt{\bar{d}(1 - \bar{d}/n)} \tag{5a}$$

$$LCL_1 = max[0, \bar{d} - k_1\sqrt{\bar{d}(1 - \bar{d}/n)}] \tag{5b}$$

$$UCL_2 = \bar{d} + k_2\sqrt{\bar{d}(1 - \bar{d}/n)} \tag{5c}$$

$$LCL_2 = max[0, \bar{d} - k_2\sqrt{\bar{d}(1 - \bar{d}/n)}] \tag{5d}$$

While $\bar{d}$ is the average amount of mistakes over an opening sample in a subgroup. We will regard as the control boundaries in the structure of Eq. (4) to examine the efficiency of the suggested chart.

The method is stated to be controlled when $t = a\mu_0$, $\mu = \mu_0$ (the parameter of scale is $\sigma = \sigma_0$ and the parameter of shape is $\alpha = \alpha_0$). If the method is in-control the failure likelihood of an item p_0 is gained from Equation (2) by

$$p = \left[\frac{(1 - e^{-t/\sigma})}{(1 + e^{-t/\sigma})}\right]^{\alpha}$$

$$p_0 = \left[\frac{(1 - e^{-a\mu_0/\sigma_0})}{(1 + e^{-a\mu_0/\sigma_0})}\right]^{\alpha_0}$$

$$p_0 = \left[\frac{(1 - e^{-a\sigma_0\eta_0/\sigma_0})}{(1 + e^{-a\sigma_0\eta_0/\sigma_0})}\right]^{\alpha_0}$$

$$sp_0 = \left[\frac{(1 - e^{-a\eta_0})}{(1 + e^{-a\eta_0})}\right]^{\alpha_0} \quad (6)$$

3.2.1 Shift in Scale Parameter

We suppose, the parameter of the scale is shifted to $\sigma_1 = c\,\sigma_0$, where c is a shift constant (or scale parameter is $\sigma = \sigma_1$). So, if the process is un-controlled the failure likelihood of an item p_1 is specified as

$$p = \left[\frac{(1 - e^{-t/\sigma})}{(1 + e^{-t/\sigma})}\right]^{\alpha}$$

$$p_1 = \left[\frac{(1 - e^{-a\mu_0/\sigma_1})}{(1 + e^{-a\mu_0/\sigma_1})}\right]^{\alpha}$$

$$p_1 = \left[\frac{(1 - e^{-a\sigma_0\eta_0/c\sigma_0})}{(1 + e^{-a\sigma_0\eta_0/c\sigma_0})}\right]^{\alpha}$$

$$p_1 = \left[\frac{(1 - e^{-a\eta_0/c})}{(1 + e^{-a\eta_0/c})}\right]^{\alpha} \quad (7)$$

3.2.2 Shift in Shape Parameter

It's suppose that the parameter of the shape is shifted to $\alpha_1 = f\,\alpha_0$(or the shape parameter is $\alpha = \alpha_1$). The likelihood that an item is failed p_2 is specified as

$$p = \left[\frac{(1 - e^{-t/\sigma})}{(1 + e^{-t/\sigma})}\right]^{\alpha}$$

$$p_2 = \left[\frac{(1 - e^{-a\mu_0/\sigma_0})}{(1 + e^{-a\mu_0/\sigma_0})}\right]^{\alpha_1}$$

$$p_2 = \left[\frac{(1 - e^{-\mathrm{a}\sigma_0\eta_1/\sigma_0})}{(1 + e^{-\mathrm{a}\sigma_0\eta_1/\sigma_0})}\right]^{f\alpha_0}$$

$$p_2 = \left[\frac{(1 - e^{-\mathrm{a}\eta_1})}{(1 + e^{-\mathrm{a}\eta_1})}\right]^{f\alpha_0} \quad (8)$$

Let

$$\eta_1 = ln\left(\frac{1 + 0.5^{1/f\alpha_0}}{1 - 0.5^{1/f\alpha_0}}\right).$$

3.2.3 Shift in Shape and Scale Parameters

Let's suppose that the shape and scale both parameter are shifted to $\alpha_1 = \mathrm{f}\alpha_0$ and $\sigma_1 = \mathrm{c}\,\sigma_0$ (the parameter of shape is $\alpha = \alpha_1$ and parameter of scale is $\sigma = \sigma_1$). The likelihood that an item is failed p_3 is specified as

$$p = \left[\frac{(1 - e^{-t/\sigma})}{(1 + e^{-t/\sigma})}\right]^{\alpha}$$

$$p_3 = \left[\frac{(1 - e^{-\mathrm{a}\mu_0/\sigma_1})}{(1 + e^{-\mathrm{a}\mu_0/\sigma_1})}\right]^{\alpha_1}$$

$$p_3 = \left[\frac{(1 - e^{-\mathrm{a}\mu_0/\mathrm{c}\sigma_0})}{(1 + e^{-\mathrm{a}\mu_0/\mathrm{c}\sigma_0})}\right]^{f\alpha_0}$$

$$p_3 = \left[\frac{(1 - e^{-\mathrm{a}\sigma_0\eta_1/\mathrm{c}\sigma_0})}{(1 + e^{-\mathrm{a}\sigma_0\eta_1/\mathrm{c}\sigma_0})}\right]^{f\alpha_0}$$

$$p_3 = \left[\frac{(1 - e^{-\mathrm{a}\eta_1/\mathrm{c}})}{(1 + e^{-\mathrm{a}\eta_1/\mathrm{c}})}\right]^{f\alpha_0} \quad (9)$$

Let

$$\eta_1 = ln\left(\frac{1 + 0.5^{1/f\alpha_0}}{1 - 0.5^{1/f\alpha_0}}\right).$$

The process is likely to be stated in-controlled L_1(p) is specified as

$$L_1(\text{p}) = \text{P}\ (LCL_1 \leq \text{d} \leq UCL_1) \qquad d = 0,1,2, \ldots\ldots \text{n}$$

$$L_1(\text{p}) = \text{P}\ (\text{d} \geq LCL_1) + \text{P}\ (\text{d} \leq UCL_1)$$

$$L_1(\text{p}) = \sum_{d=[LCL_1]+1}^{n} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=0}^{[UCL_1]} \binom{n}{d} p^d (1-p)^{n-d}$$

$$L_1(\text{p}) = \sum_{d=[LCL_1]+1}^{[UCL_1]} \binom{n}{d} p^d (1-p)^{n-d} \quad d = 0,1,2, \ldots\ldots \text{n} \tag{10}$$

The process is likely to be stated out of control L_2(p) is specified as

$$L_2(\text{p}) = \text{P}\ (\text{d} > UCL_2) + \text{P}\ (\text{d} < LCL_2)$$

$$L_2(\text{p}) = \sum_{d=[\ UCL_2]+1}^{n} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=0}^{[LCL_2]} \binom{n}{d} p^d (1-p)^{n-d}$$

$$L_2(\text{p}) = \sum_{d=[\ UCL_2]+1}^{[LCL_2]} \binom{n}{d} p^d (1-p)^{n-d} \tag{11}$$

The likelihood of repeating the sampling until the choice is made L_3(p) is as illustrated

$$L_3(\text{p}) = \text{P}\ (LCL_2 \leq \text{d} < LCL_1) + \text{P}\ (UCL_1 \leq \text{d} < UCL_2)$$

$$L_3(\text{p}) = [\text{P}\ (\text{d} \geq LCL_2) + \text{P}\ (\text{d} < LCL_1)] + [\text{P}\ (\text{d} \geq UCL_1) + \text{P}\ (\text{d} < UCL_2)]$$

$$L_3(\text{p}) = \left[\sum_{d=[LCL_2]+1}^{n} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=0}^{[\ LCL_1]} \binom{n}{d} p^d (1-p)^{n-d} \right] + \left[\sum_{d=[\ UCL_1]+1}^{n} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=0}^{[UCL_2]} \binom{n}{d} p^d (1-p)^{n-d} \right]$$

$$L_3(\mathrm{p}) = \sum_{d=[LCL2]+1}^{[LCL1]} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=[UCL_1]+1}^{[UCL_2]} \binom{n}{d} p^d (1-p)^{n-d} \quad (12)$$

The likelihood that the process will be controlled under RGS is attained as

$$P_{in}(p) = \frac{L_1(p)}{1 - L_3(p)} \quad (13)$$

The likelihood that the process will be controlled when it is controlled under RGS is attained as

$$P_{in}(p_0) = \frac{L_1(p_0)}{1 - L_3(p_0)} \quad (13a)$$

The likelihood that the process will be controlled, but it is effectively out of control under RGS is acquired as

$$P_{in}(p_1) = \frac{L_1(p_1)}{1 - L_3(p_1)} \quad (13b)$$

The likelihood that the process will be controlled, but it is effectively out of control under RGS is acquired as

$$P_{in}(p_2) = \frac{L_1(p_2)}{1 - L_3(p_2)} \quad (13c)$$

The likelihood that the process will be controlled, but it is effectively out of control under RGS is acquired as

$$P_{in}(p_3) = \frac{L_1(p_3)}{1 - L_3(p_3)} \quad (13d)$$

The ASN is acquired for the suggested RGS chart as

$$ASN(p) = \frac{n}{1 - L_3(p)} \quad (14)$$

The ASN for the suggested chart is acquired as follows when the method is in control

$$ASN(p_0) = \frac{n}{1 - L_3(p_0)} \tag{14a}$$

The ASN for the suggested chart is acquired as follows when the method is out of control

$$ASN(p_1) = \frac{n}{1 - L_3(p_1)} \tag{14b}$$

The ASN for the suggested chart is acquired as follows when the method is out of control

$$ASN(p_2) = \frac{n}{1 - L_3(p_2)} \tag{14c}$$

The ASN for the suggested chart is acquired as follows when the method is out of control

$$ASN(p_3) = \frac{n}{1 - L_3(p_3)} \tag{14d}$$

The ARL for the suggested chart under RGS is acquired as

$$ARL = \frac{1}{1 - P_{in}(P)} \tag{15}$$

The controlled ARL of the suggested control chart under RGS is acquired as

$$ARL_0 = \frac{1}{1 - P_{in}(p_0)} \tag{15a}$$

The out-of-control ARL of the suggested RGS-based control chart is acquired as

$$ARL_1 = \frac{1}{1 - P_{in}(p_1)} \tag{15b}$$

The out-of-control ARL of the suggested RGS-based control chart is acquired as

$$ARL_2 = \frac{1}{1 - P_{in}(p_2)} \tag{15c}$$

The out-of-control ARL of the suggested RGS-based control chart is acquired as

$$ARL_3 = \frac{1}{1 - P_{in}(p_3)} \tag{15d}$$

3.3 OPTIMIZATION CONDITIONS

The following are the optimization conditions

Minimize ASN (p_0)

Subject to

$ARL_0 \geq r_0$

$k_2 > k_1$

Tables 1-5 present optimal parameters for developing the control chart, such as control coefficients k_2 and k_1 for fixed sample size (n). These parameters are calculated to be as close to the given ARL values, as the in control ARL = 200, 300 and 370 in that order. It also determines the optimal parameters for the parameter of shape α = 1.5, 2.0, 2.5, 3.0. Shift constant ‘f’ changes from 1.0 to 0.1. We can examine from tables that the ARL is decreasing if the shift constant is decreasing. ASN value is also decreasing. The sample sizes n = 10, 15, 20, 25 are fixed.

Tables 6-8 present the optimal parameters for the scale parameter at α = 1.5, 2.0, 2.5, 3.0. The consistent ' c ' change is shifting from 1.0 to 0.1.

Tables 9 give the optimal parameters for the shape and scale both parameters at α = 1.5, 2.0, 2.5, 3.0. The consistent 'c' change is shifting from 1.0 to 0.1.

Table 1: *ARL's* while the parameter of the shape is shifted for specified sample sizes as r_0 = 200

	n = 10, α = 1.5		n = 15, α = 2		n = 20, α = 2.5		n = 25, α = 3	
	a = 0.5512680 k_1= 0.05528892, k_2 = 0.502351		a = 0.7989646 k_1= 0.6462318, k_2 = 0.876014		a = 0.7397315 k_1= 0.7845172, k_2 = 0.870943		a = 0.9177807 k_1= 0.3937093, k_2 = 0.8649713	
	UCL_1 = 3.09, LCL_1 = 2.93 UCL_2 = 3.74, LCL_2 = 2.28		UCL_1 = 7.26, LCL_1 = 4.81 UCL_2 = 7.70, LCL_2 = 4.37		UCL_1 = 8.22, LCL_1 = 4.92 UCL_2 = 8.39, LCL_2 = 4.74		UCL_1 = 9.55, LCL_1 = 7.68 UCL_2 =10.67, LCL_2 = 6.55	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	19.95	200.42	21.52	200.79	26.32	200.22	53.62	200.83
0.9	19.19	12.47	21.43	42.05	26.25	37.21	53.29	53.94
0.8	18.32	5.95	21.32	21.46	26.17	18.37	52.92	29.86
0.7	17.35	3.77	21.19	13.58	26.07	11.37	52.51	20.09
0.6	16.33	2.72	21.04	9.52	25.95	7.84	52.08	14.86
0.5	15.30	2.13	20.87	7.11	25.80	5.78	51.62	11.63
0.4	14.31	1.76	20.67	5.55	25.62	4.47	51.14	9.46
0.3	13.41	1.52	20.46	4.48	25.40	3.58	50.63	7.91
0.2	12.61	1.35	20.22	3.72	25.14	2.95	50.10	6.76
0.1	11.93	1.24	19.96	3.15	24.84	2.49	49.55	5.88

Table 2: ***ARL's*** **while the parameter of the shape is shifted as r_0 = 300 and α = 2**

	n = 10		n = 15		n = 20		n = 25	
	a = 0.5937813 k_1= 0.097755169, k_2 = 0.399326		a = 0.5822424 k_1 = 0.88124085, k_2 = 0.8820722		a = 0.6472815 k_1 = 1.0221351, k_2 = 1.0785852		a = 0.6912889 k_1= 1.23808879, k_2 = 1.2789334	
	UCL_1 = 3.15, LCL_1 = 2.87 UCL_2=3.58, LCL_2 = 2.43		UCL_1 = 5.99, LCL_1 = 2.87 UCL_2 =5.99, LCL_2 = 2.87		UCL_1 = 8.70, LCL_1 = 4.40 UCL_2 = 8.82, LCL_2 = 4.29		UCL_1 = 11.69, LCL_1 = 5.79 UCL_2 =11.79, LCL_2 = 5.69	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	19.97	300.32	21.44	300.51	26.32	300.92	29.55	300.48
0.9	19.27	13.68	21.17	15.05	26.23	29.03	29.64	41.37
0.8	18.46	6.50	20.83	7.13	26.10	13.05	29.76	19.05
0.7	17.57	4.12	20.43	4.46	25.94	7.65	29.88	11.16
0.6	16.65	2.98	19.96	3.17	25.72	5.08	29.10	7.31
0.5	15.71	2.33	19.44	2.43	25.42	3.66	30.07	5.14
0.4	14.81	1.93	18.86	1.96	25.04	2.79	30.06	3.80
0.3	13.96	1.66	18.26	1.66	24.57	2.22	29.95	2.93
0.2	13.19	1.47	17.66	1.44	24.04	1.85	29.72	2.35
0.1	12.51	1.34	17.09	1.30	23.46	1.58	29.36	1.94

Table 3: *ARL's* while the parameter of the shape is shifted as r_0 = 370 and α = 2

	n = 10		n = 15		n = 20		n = 25	
	a = 0.9424639 k_1 = 0.52002428, k_2 = 0.559912		a = 0.7967768 k_1 = 0.71566975, k_2 = 0.968518		a = 0.6466634 k_1 = 0.9532185, k_2 = 1.048110		a = 0.7581569 k_1 = 1.2570453, k_2 =1.3277907	
	UCL_1 = 5.54, LCL_1 = 3.90 UCL_2 = 5.61, LCL_2 = 3.84		UCL_1 = 7.38, LCL_1 = 4.66 UCL_2 = 7.86, LCL_2 = 4.18		UCL_1 = 8.55, LCL_1 = 4.55 UCL_2 = 8.75, LCL_2 = 4.35		UCL_1 = 12.62, LCL_1 = 6.51 UCL_2 = 12.79, LCL_2 = 6.34	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	16.29	370.21	21.53	370.01	26.32	370.56	29.59	370.24
0.9	16.25	89.99	21.44	46.93	26.23	29.70	29.64	63.08
0.8	16.19	49.86	21.33	22.72	26.11	13.20	29.71	29.64
0.7	16.15	33.86	21.21	14.07	25.94	7.71	29.79	17.50
0.6	16.09	25.29	21.06	9.75	25.72	5.11	29.89	11.53
0.5	16.03	19.99	20.88	7.23	25.43	3.67	29.98	8.11
0.4	15.97	16.42	20.68	5.61	25.05	2.79	30.05	5.99
0.3	15.90	13.86	20.46	4.51	24.58	2.23	30.08	4.59
0.2	15.84	11.95	20.22	3.73	24.05	1.85	30.05	3.63
0.1	15.78	10.48	19.96	3.16	23.46	1.58	29.95	2.95

Table 4: ARL's while the parameter of the shape is shifted as r_0 = 370 and α = 3

	n = 10		n = 15		n = 20		n = 25	
	a = 0.8008269 k_1 = 0.18693262, k_2 = 0.5685758		a = 0.4902479 k_1 = 0.20165683, k_2 = 0.2444121		a = 0.6632994 k_1 = 0.738043, k_2 = 0.8295848		a = 0.7582928 k_1 = 1.2195140, k_2 = 1.2490227	
	UCL_1 = 3.28, LCL_1 = 2.74 UCL_2 =3.83, LCL_2 = 2.18		UCL_1 = 3.07, LCL_1= 2.47 UCL_2 =3.14, LCL_2 = 2.40		UCL_1 = 6.42, LCL_1 = 3.56 UCL_2 = 6.59, LCL_2 = 3.38		UCL_1 = 9.88, LCL_1 = 4.37 UCL_2 = 9.94, LCL_2 = 4.31	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	ARL	*ASN*	*ARL*
1	19.97	370.47	29.96	370.16	28.69	370.92	30.90	370.52
0.9	19.67	30.48	29.07	16.16	28.45	27.79	30.89	49.91
0.8	19.34	15.25	27.84	6.95	28.15	12.80	30.89	24.16
0.7	18.99	9.94	26.36	4.12	27.80	7.77	30.89	14.92
0.6	18.62	7.26	24.75	2.86	27.39	5.34	30.88	10.30
0.5	18.23	5.67	23.11	2.18	26.94	3.97	30.86	7.59
0.4	17.84	4.63	21.56	1.78	26.43	3.11	30.83	5.86
0.3	17.44	3.89	20.14	1.52	25.88	2.54	30.76	4.67
0.2	17.03	3.36	18.90	1.35	25.29	2.13	30.65	3.82
0.1	16.62	2.96	17.86	1.23	24.69	1.85	30.51	3.2 0

Table 5: *ARL's* while the parameter of the shape is shifted as r_0 = 300 and α = 3

	n = 10		n = 15		n = 20		n = 25	
	a = 0.8011999 k_1 = 0.43761073, k_2 = 0.65635916		a = 0.6557643 k_1 = 0.5344432, k_2 = 0.7220106		a = 0.8727061 k_1= 1.0366778, k_2 = 1.155178		a = 0.6611102 k_1 = 0.8529688, k_2 = 0.959549	
	UCL_1 = 3.64, LCL_1 = 2.38 UCL_2 =3.96, LCL_2 = 2.06		UCL_1 = 4.59, LCL_1 = 2.81 UCL_2 = 4.91, LCL_2 = 2.49		UCL_1 = 8.73, LCL_1 = 4.38 UCL_2 = 8.98, LCL_2 = 4.13		UCL_1 = 8.06, LCL_1= 4.37 UCL_2 = 8.29, LCL_2 = 4.14	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	19.97	300.76	24.39	300.84	26.32	300.02	32.98	300.03
0.9	19.67	29.92	23.91	17.53	26.29	79.29	32.86	44.87
0.8	19.34	15.12	23.39	8.52	26.26	43.24	32.72	19.65
0.7	18.99	9.88	22.81	5.46	26.23	28.65	32.54	11.17
0.6	18.62	7.24	22.19	3.96	26.19	20.86	32.31	7.24
0.5	18.23	5.66	21.56	3.08	26.15	16.07	32.03	5.09
0.4	17.84	4.62	20.91	2.52	26.10	12.86	31.68	3.81
0.3	17.43	3.89	20.26	2.13	26.05	10.58	31.27	2.98
0.2	17.03	3.36	19.63	1.86	25.99	8.89	30.78	2.42
0.1	16.62	2.96	19.01	1.65	25.94	7.61	30.23	2.02

Table 6: *ARL's* while the parameter of the scale is shifted for specified sample sizes as $r_0 = 200$

	n = 10, α = 1.5		n = 15, α = 2		n = 20, α = 2.5		n = 25, α = 3	
	a = 0.9500074 k_1 = 0.47716305, k_2 = 0.750015		a = 0.7703821 k_1 = 0.6959164, k_2 = 0.918496		a = 0.7660026 k_1= 0.8074456, k_2 = 1.0804421		a = 0.9847301 k_1 = 1.16254630, k_2 = 1.269290	
	UCL_1 = 5.48, LCL_1 = 3.98 UCL_2 = 5.91, LCL_2 = 3.55		UCL_1 = 6.52, LCL_1 = 3.95 UCL_2 = 6.93, LCL_2 = 3.54		UCL_1 = 8.27, LCL_1 = 4.87 UCL_2 = 8.84, LCL_2 = 4.30		UCL_1 = 15.12, LCL_1 = 9.30 UCL_2 = 15.38, LCL_2 = 9.04	
c	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	16.28	200.31	21.49	200.19	26.32	200.00	29.67	200.08
0.9	14.99	4.33	20.39	4.30	25.61	4.43	30.17	5.75
0.8	13.36	1.97	18.60	1.82	23.71	1.68	28.95	1.65
0.7	11.78	1.32	16.65	1.22	21.33	1.13	26.07	1.08
0.6	10.66	1.09	15.40	1.04	20.17	1.01	25.06	1.00
0.5	10.13	1.01	15.03	1.00	20.00	1.00	25.00	1.00
0.4	10.01	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00

Table 7: *ARL's* while the parameter of the scale is shifted as $r_0 = 300$

	n = 10, α = 1.5		n = 15, α = 2		n = 20, α = 2.5		n = 25, α = 3	
	a = 0.6523552 k_1 = 0.30734632, k_2 = 0.4879256		a = 0.9315071 k_1 = 0.8401192, k_2 = 0.9259327		a = 0.8560764 k_1 = 0.5869916, k_2 = 0.9236466		a = 0.9948439 k_1 = 1.3612363, k_2 = 1.4057785	
	UCL_1 = 3.46, LCL_1 = 2.56 UCL_2 =3.72, LCL_2 = 2.30		UCL_1 = 8.47, LCL_1 = 5.23 UCL_2 = 8.63, LCL_2 = 5.06		UCL_1 = 9.20, LCL_1 = 6.63 UCL_2 = 9.94, LCL_2 = 5.89		UCL_1 = 15.81, LCL_1 = 8.99 UCL_2 = 15.92, LCL_2 = 8.89	
c	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	19.97	303.99	21.55	300.35	31.45	300.19	28.68	300.89
0.9	17.79	4.53	20.69	5.49	27.20	3.05	29.85	4.84
0.8	15.18	2.08	18.88	1.97	23.30	1.41	28.68	1.56
0.7	12.82	1.39	16.76	1.23	20.85	1.07	25.92	1.07
0.6	11.14	1.13	15.41	1.04	20.07	1.00	25.05	1.00
0.5	10.28	1.03	15.03	1.00	20.00	1.00	25.00	1.00
0.4	10.02	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00

Table 8: *ARL's* while the parameter of the scale is shifted as $r_0 = 370$

	n = 10, α = 1.5		n = 15, α = 2		n = 20, α = 2.5		n = 25, α = 3	
	a = 0.6521354 k_1 = 0.0482576, k_2 = 0.59935838		a = 0.8480571 k_1 = 0.7641154, k_2 = 0.9392819		a = 0.6590831 k_1 = 0.5823619, k_2 = 0.7271074		a = 0.7644676 k_1 = 1.06173089, k_2 = 1.131890	
	UCL_1 = 3.08, LCL_1 = 2.94 UCL_2 =3.88, LCL_2 = 2.14		UCL_1 = 7.47, LCL_1 = 4.57 UCL_2 = 7.81, LCL_2 = 4.24		UCL_1 = 6.62, LCL_1 = 3.86 UCL_2 = 6.39, LCL_2 = 3.58		UCL_1 = 10.28, LCL_1 = 5.35 UCL_2 = 10.44, LCL_2 = 5.19	
c	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	19.97	370.12	21.53	370.57	28.69	370.69	30.99	370.60
0.9	17.79	4.54	20.54	4.85	27.02	4.16	30.75	4.81
0.8	15.19	2.08	18.73	1.89	24.22	1.67	28.85	1.59
0.7	12.82	1.39	16.69	1.23	21.51	1.14	26.03	1.08
0.6	11.15	1.13	15.39	1.04	20.22	1.02	25.06	1.00
0.5	10.28	1.03	15.03	1.00	20.01	1.00	25.00	1.00
0.4	10.02	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	10.00	1.00	15.00	1.00	20.00	1.00	250.00	1.00

Table 9: *ARL's* while the parameters of shape and scale are shifted as r_0= 200

		n = 10, α = 1.5		n = 15, α = 2		n = 20, α = 2.5		n = 25, α = 3	
		a = 0.9500074 k_1 = 0.47716305, k_2 = 0.750015		a = 0.7703821 k_1 = 0.6959164, k_2 = 0.918496		a = 0.7660026 k_1 = 0.8074456, k_2 = 1.080442		a = 0.9847301 k_1 = 1.16254630, k_2 = 1.269290	
		UCL_1 = 5.48, LCL_1 = 3.98 UCL_2 = 5.91, LCL_2 = 3.55		UCL_1 = 6.52, LCL_1 = 3.95 UCL_2 = 6.93, LCL_2 = 3.54		UCL_1 = 8.27, LCL_1 = 4.87 UCL_2 = 8.84, LCL_2 = 4.30		UCL_1 = 15.12, LCL_1 = 9.30 UCL_2 = 15.38, LCL_2 = 9.04	
f	c	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	1	16.28	200.31	21.49	200.19	26.32	200.00	29.67	200.08
0.9	0.9	14.99	4.33	20.39	4.30	25.61	4.43	30.17	5.75
0.8	0.8	13.36	1.97	18.60	1.82	23.71	1.68	28.95	1.65
0.7	0.7	11.78	1.32	16.65	1.22	21.33	1.13	26.07	1.08
0.6	0.6	10.66	1.09	15.40	1.04	20.17	1.01	25.06	1.00
0.5	0.5	10.13	1.01	15.03	1.00	20.00	1.00	25.00	1.00
0.4	0.4	10.01	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.3	0.3	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.2	0.2	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00
0.1	0.1	10.00	1.00	15.00	1.00	20.00	1.00	25.00	1.00

Table 10: ARL's of the suggested control chart with Existing control chart while parameter of the shape is shifted as r_0 = 300 and α = 2

	Suggested		Existing		Suggested		Existing	
	a = 0.5822424, n = 15 k_1 = 0.88124085, k_2 = 0.8820722		a = 0.3389691, n = 15 k_1= 1.369041, k_2= 2.790004		a = 0.6912889, n = 25 k_1=1.23808879, k_2= 1.2789334		a = 0.618004, n = 25 k_1=1.872718, k_2= 2.695981	
	UCL_1 =5.99, LCL_1 = 2.87 UCL_2 =5.99, LCL_2 = 2.87		UCL_1 = 9, LCL_1 = 3 UCL_2 =12, LCL_2 = 1		UCL_1 =11.69, LCL_1 = 5.79 UCL_2 =11.79, LCL_2 = 5.69		UCL_1 = 20, LCL_1 = 10 UCL_2 =22, LCL_2 = 8	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	21.44	300.51	16.92	302.19	29.55	300.48	25.88	300.01
0.9	21.17	15.05	19.95	190.22	29.64	41.37	26.35	116.59
0.8	20.83	7.13	17.46	94.14	29.76	19.05	27.72	37.70
0.7	20.43	4.46	18.62	43.09	29.88	11.16	30.37	12.56
0.6	19.96	3.17	20.70	19.02	29.10	7.31	34.19	4.57
0.5	19.44	2.43	23.98	8.19	30.07	5.14	26.69	2.01
0.4	18.86	1.96	28.21	3.57	30.06	3.80	33.90	1.24
0.3	18.26	1.66	30.54	1.76	29.95	2.93	28.45	1.04
0.2	17.66	1.44	26.29	1.15	29.72	2.35	25.50	1.00
0.1	17.09	1.30	18.73	1.01	29.36	1.94	25.01	1.00

Table 11: *ARL's* of the suggested control chart with Existing control chart while parameter of the shape is shifted as r_0 = 370 and α = 3

	Suggested		Existing		Suggested		Existing	
	a = 0.4902479, n = 15 k_1= 0.20165683, k_2= 0.24441216		a =1.029771, n = 15 k_1=1.890701, k_2 = 3.0315		a = 0.6632994, n = 20 k_1 = 0.738043, k_2 = 0.8295848		a = 0.512036, n = 20 k_1= 1.830755, k_2 = 2.725221	
	UCL_1 = 3.07, LCL_1 = 2.47 UCL_2 =3.14, LCL_2 = 2.40		UCL_1 =14, LCL_1 = 7 UCL_2 =16, LCL_2 = 5		UCL_1 = 6.42, LCL_1 =3.56 UCL_2 =6.59, LCL_2 = 3.38		UCL_1 =14, LCL_1 = 3 UCL_2 =16, LCL_2 = 3	
f	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*	*ASN*	*ARL*
1	29.96	370.16	15.68	370.06	28.69	370.92	20.81	370.01
0.9	29.07	16.16	16.20	130.42	28.45	27.79	21.06	223.80
0.8	27.84	6.95	17.23	46.69	28.15	12.80	21.82	92.05
0.7	26.36	4.12	19.03	17.00	27.80	7.77	23.33	35.32
0.6	24.75	2.86	21.74	6.43	27.39	5.34	25.97	13.51
0.5	23.11	2.18	24.41	2.70	26.94	3.97	29.87	5.30
0.4	21.56	1.78	24.10	1.46	26.43	3.11	33.38	2.31
0.3	20.14	1.52	20.06	1.09	25.88	2.54	31.85	1.32
0.2	18.90	1.35	16.37	1.00	25.29	2.13	25.41	1.05
0.1	17.86	1.23	15.07	1.00	24.69	1.85	20.79	1.00

Table 12. ***ARL's*** **of the suggested control chart with Existing control chart while parameter of the shape is shifted as r_0 = 370 and α = 23**

	Suggested (RGS)	Existing (Single sampling plan)	Suggested (RGS)	Existing (Single sampling plan)
	a = 0.6466634, n = 20 k_1= 0.9532185, k_2= 1.048110	a = 01.491, n = 38 k = 3.1600	a = 0.7581569, n = 25 k_1 = 1.2570453, k_2 = 1.3277907	a = 0.703, n = 47 k = 3.0885
	UCL_1 = 8.55, LCL_1 = 4.55 UCL_2 = 8.75 LCL_2 = 4.35	UCL = 36, LCL = 20	UCL_1 = 12.62, LCL_1 = 6.51 UCL_2 = 12.79, LCL_2 = 6.34	UCL = 23, LCL = 4
f	*ARL*	*ARL*	*ARL*	*ARL*
1	370.56	370.04	370.24	370.01
0.9	29.70	238.54	63.08	247.22
0.8	13.20	144.36	29.64	154.76
0.7	7.71	83.31	17.50	93.13
0.6	5.11	46.17	11.53	54.52
0.5	3.67	24.68	8.11	31.22
0.4	2.79	12.80	5.99	17.54
0.3	2.23	6.56	4.59	9.73
0.2	1.85	3.47	3.63	5.41
0.1	1.58	2.05	2.95	3.13

Table 13: *ARL's* of the suggested control chart with Existing control chart while parameter of the scale is shifted as r_0 = 370

	Suggested (RGS)	Existing (Single sampling plan)	Suggested (RGS)	Existing (Single sampling plan)
	a = 0.6590831, n = 20, α = 2.5 k_1 = 0.5823619, k_2 = 0.7271074	a = 0.841, n = 23, α = 2.5 k = 3.0645	a = 0.7644676, n = 23, α = 3 k_1 = 1.06173089, k_2 = 1.131890	a = 0.957, n = 46, α = 3 k = 3.0965
	UCL_1 = 6.62, LCL_1 = 3.86 UCL_2 = 6.39, LCL_2 = 3.58	UCL = 15, LCL = 1	UCL_1 = 10.28, LCL_1 = 5.35 UCL_2 = 10.44, LCL_2 = 5.19	UCL = 31, LCL = 11
c	*ARL*	*ARL*	*ARL*	*ARL*
1	370.69	37.02	370.60	370.00
0.9	4.16	59.92	4.81	34.96
0.8	1.67	11.16	1.59	4.01
0.7	1.14	2.99	1.08	1.30
0.6	1.02	1.34	1.00	1.01
0.5	1.00	1.03	1.00	1.00
0.4	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

3.4 CONTRAST OF THE SUGGESTED CONTROL CHART

Among this segment, the control chart performance in RGS based on exponentiated half logistic distribution is contrasted with the current chart Jeyadurga et al. (2018). Table 10 and 11 demonstrate the optimum parameters n, a, k_1, k_2, higher and lower control boundaries and also demonstrate the respective control chart ARL and ASN values by repetitive group sampling for the parameter of the shape α = 2 and 3 and r_0 = 300 and 370. Since these tables shows, so as to the suggested chart identify the change quicker than the existing chart For example, when shape parameter r_0 = 300, α = 2 and n = 25 the suggested chart identify the shift at 41th observation and ASN = 29.64, however, the shift is identify by the existing chart at 116th observation. Thus, the suggested chart is efficient in identifying the process shift instead of existing chart.

The control chart performance in RGS is also contrasted with the current chart Rao (2018) based on exponentiated half logistic distribution. Table 12 and 13 demonstrate the optimum parameters, higher and lower control boundaries and also demonstrate the respective control chart ARL values by repetitive group sampling and single sampling for the parameter of the shape α = 2, r_0 = 370 and for the parameter of the scale α = 2.5, 3 and r_0 = 370. These tables shows, so as to the suggested chart identify the change quicker than the existing chart For example, when shape parameter r_0 = 370, α = 2 and n = 25 the suggested chart identify the shift at 63th observation, however, the shift is identify by the existing chart based on single sampling at 247th observation. Thus, the suggested chart is efficient in identifying the process shift instead of existing chart. Since the parameter of the scale r_0 = 370, α = 2.5 and n = 20 the suggested chart identify the shift at 4th observation, however, the shift is identify by the existing chart based on single sampling at 59th observation. Thus, the suggested chart is efficient in identifying the process shift instead of existing chart.

3.5 DESIGN OF SUGGESTED CONTROL CHART ON THE BASIS OF MDS

Following are the steps of np control chart based on multiple dependent state sampling.

Step 1: Choose from the production process a random sample of size n. Count the amount of failed items D by the specified time t = aμ_0 where μ_0 is the target mean when the process is in control and ‘a’ is constant.

Step 2: If $LCL_2 \leq D \leq UCL_2$ and if the process is not repeated until the i^{th} sample choice is completed, declare the process as in control. If $D > UCL_1$ or $D < LCL_1$ declare the process as out of control.

The cumulative distribution function (cdf) of EHLD is given as

$$F\,(t) = \left[\frac{(1\, -\, e^{-t/\sigma})}{(1\, +\, e^{-t/\sigma})}\right]^{\alpha} \;\; ; t\, \geq\, 0, \alpha > 0, \sigma > 0 \qquad (1)$$

$$\mu\, =\, \sigma\left[ln\left(\frac{1 + 0.5^{1/\alpha}}{1 - 0.5^{1/\alpha}}\right)\right] \qquad (3)$$

Let

$$\eta = ln\left(\frac{1 + 0.5^{1/\alpha}}{1 - 0.5^{1/\alpha}}\right)$$

$$\mu\, =\, \sigma\eta \qquad (3a)$$

$$\sigma = \frac{\mu}{\eta} \qquad (3b)$$

We are designing a chart for monitoring the mean shift. The method is stated to be controlled when $t = a\mu_0$, $\mu = \mu_0$ and we express the parameter of scale σ in terms of μ using Eq. (3b), then Eq. (1) can be re written as

$$p = \left[\frac{(1\, -\, e^{-t/\sigma})}{(1\, +\, e^{-t/\sigma})}\right]^{\alpha}$$

$$p_0 = \left[\frac{(1\, -\, e^{-a\mu_0/\mu/\eta}\,)}{(1\, +\, e^{-a\mu_0/\mu/\eta}\,)}\right]^{\alpha}$$

$$p_0 = \left[\frac{(1\, -\, e^{-a\mu_0/\mu_0/\eta})}{(1\, + e^{-a\mu_0/\mu_0/\eta}\,)}\right]^{\alpha}$$

$$p_0 = \left[\frac{(1\, -\, e^{-a\eta})}{(1\, +\, e^{-a\eta})}\right]^{\alpha} \qquad (16)$$

Assume that the process mean is shifted from $\mu = \mu_1$ and $\mu_1 = c\mu_0$ then Eq. (1) becomes

$$p_1 = \left[\frac{(1\, -\, e^{-a\mu_0/\mu/\eta}\,)}{(1\, +\, e^{-a\mu_0/\mu/\eta}\,)}\right]^{\alpha}$$

$$p_1 = \left[\frac{(1 - e^{-\mathrm{a}\mu_0/\mu_1/\eta})}{(1 + e^{-\mathrm{a}\mu_0/\mu_1/\eta})}\right]^{\alpha}$$

$$p_1 = \left[\frac{(1 - e^{-\mathrm{a}\mu_0/\mathrm{c}\mu_0/\eta})}{(1 + e^{-\mathrm{a}\mu_0/c\mu_0/\eta})}\right]^{\alpha}$$

$$p_1 = \left[\frac{(1 - e^{-\mathrm{a}\eta/c})}{(1 + e^{-\mathrm{a}\eta/c})}\right]^{\alpha} \tag{17}$$

The probability that the process will be stated in control at probability p_0 if the process is actually in-controlled

$$P_{in} = P(LCL_2 \le \mathrm{D} \le UCL_2) + [P(LCL_1 \le \mathrm{D} \le LCL_2) + P(UCL_2 \le \mathrm{D} \le UCL_1)] \times [\, P(LCL_2 \le \mathrm{D} \le UCL_2)]^{i}$$

$$P_{in}^0/P_0 = P(\mathrm{d} \ge LCL_2) + \mathrm{P}\ (\mathrm{d} \le UCL_2) + [\mathrm{P}(\mathrm{d} \ge LCL_1) + \mathrm{P}\ (\mathrm{d} \le LCL_2) + \mathrm{P}(\mathrm{d} \ge UCL_2) + \mathrm{P}(\mathrm{d} \le UCL_1)] \times [\, P(\mathrm{d} \ge LCL_2) + \mathrm{P}\ (\mathrm{d} \le UCL_2)]^{i}$$

$$\begin{aligned} P_{in}^0/P_0 = & \sum_{d=[LCL_2]+1}^{n} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} + \sum_{d=0}^{[UCL_2]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} \\ & + \left[\sum_{d=[LCL_1]+1}^{n} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} + \sum_{d=0}^{[LCL_2]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} \right. \\ & \left. + \sum_{d=[UCL_2]+1}^{n} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} + \sum_{d=0}^{[UCL_1]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} \right] \\ & \times \left[\sum_{d=[LCL_2]+1}^{n} \binom{n}{d} {P_0}^d (1-p)^{n-d} + \sum_{d=0}^{[UCL_2]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d}] \right]^{\mathrm{i}} \end{aligned}$$

$$\begin{aligned} \frac{P_{in}^0}{P_0} = & \sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} \\ & + \left[\sum_{d=[LCL_1]+1}^{[LCL_2]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} + \sum_{d=[UCL_2]+1}^{[UCL_1]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} \right] \\ & \times \left[\sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} {P_0}^d (1-P_0)^{n-d} \right]^{i} \end{aligned} \tag{18}$$

Likewise, the probability that the process will be stated in control when the process is shifted to μ_1 is specified as

$$\frac{P_{in}^{1}}{P_1} = \sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} {P_1}^{d}(1-P_1)^{n-d} + \left[\sum_{d=[LCL_1]+1}^{[LCL_2]} \binom{n}{d} {P_1}^{d}(1-P_1)^{n-d} + \sum_{d=[UCL_2]+1}^{[UCL_1]} \binom{n}{d} {P_1}^{d}(1-P_1)^{n-d}\right] \times \left[\sum_{d=[LCL_2]+1}^{[UCL_2]} \binom{n}{d} {P_1}^{d}(1-P_1)^{n-d}\right]^{i} \quad (19)$$

3.5.1 Performance Measure of the Suggested Control Chart

The average run length (ARL) is used to evaluate the efficiency of a control chart. The ARL in control is specified as

$$ARL_0 = \frac{1}{1-P_{in}^{0}} \quad (20)$$

Whereas the out of control ARL is given as

$$ARL_1 = \frac{1}{1-P_{in}^{1}} \quad (21)$$

Table 14–16 shows ARLs values for n = 30, ARL_0 = 200, 300 and 370 with i = 1, 2 and 3 for various shift levels. From these tables it can be seen that the ARL's values are decreases as the shift size reduces (c = 1 to 0.1).

Table 14
ARL Values of Suggested Control Chart for ARL_0 = 200 and n = 30

i = 1				i = 2				i = 3			
k_1	1.21	3.83	3.91	k_1	3.13	1.72	2.87	k_1	3.24	3.44	2.81
k_2	1.15	3.49	2.94	k_2	2.91	1.55	2.42	k_2	2.73	2.75	2.35
α	1	2	3	α	1	2	3	α	1	2	3
a	0.35	0.27	0.72	a	0.29	0.73	0.96	a	0.21	0.96	0.87
c	**ARL**			**c**	**ARL**			**c**	**ARL**		
1	200.17	200.04	200.05	1	200.01	200.03	200.07	1	200.01	200.05	200.02
0.99	114.62	182.13	152.94	0.99	185.17	96.87	124.76	0.99	187.19	151.90	128.95
0.95	36.50	124.64	59.90	0.95	135.48	23.40	40.02	0.95	142.99	63.27	42.76
0.93	25.48	102.85	39.09	0.93	115.61	14.85	26.13	0.93	124.68	43.35	28.01
0.91	18.88	84.75	5.99	0.91	98.50	10.11	17.71	0.91	108.52	30.33	18.99
0.9	16.51	76.89	21.33	0.9	90.87	8.50	14.73	0.9	101.18	25.54	15.79
0.88	12.92	63.22	14.58	0.88	77.24	6.20	10.37	0.88	87.84	18.33	11.09
0.85	9.34	47.03	8.55	0.85	60.37	4.11	6.40	0.85	70.83	11.52	6.81
0.8	5.90	28.59	3.97	0.8	39.75	2.39	3.23	0.8	49.06	5.86	3.39
0.75	3.99	17.33	2.21	0.75	25.98	1.63	1.92	0.75	33.63	3.39	1.98
0.7	2.85	10.54	1.51	0.7	16.88	1.27	1.35	0.7	22.83	2.19	1.37
0.6	1.67	4.06	1.07	0.6	7.10	1.03	1.02	0.6	10.26	1.20	1.03
0.5	1.19	1.82	1.00	0.5	3.10	1.00	1.00	0.5	4.57	1.01	1.00
0.4	1.02	1.12	1.00	0.4	1.56	1.00	1.00	0.4	2.14	1.00	1.00
0.3	1.00	1.00	1.00	0.3	1.06	1.00	1.00	0.3	1.22	1.00	1.00
0.2	1.00	1.00	1.00	0.2	1.00	1.00	1.00	0.2	1.00	1.00	1.00
0.1	1.00	1.00	1.00	0.1	1.00	1.00	1.00	0.1	1.00	1.00	1.00

Table 15

ARL Values of Suggested Control Chart for ARL_0 = 300 and n = 30

i = 1				**i = 2**				**i = 3**			
$\mathbf{k_1}$	3.38	3.09	$\mathbf{k_1}$	3.38	3.09	$\mathbf{k_1}$	3.38	3.09	$\mathbf{k_1}$	3.38	3.09
$\mathbf{k_2}$	0.12	2.82	$\mathbf{k_2}$	0.12	2.82	$\mathbf{k_2}$	0.12	2.82	$\mathbf{k_2}$	0.12	2.82
$\boldsymbol{\alpha}$	1	2	$\boldsymbol{\alpha}$	1	2	$\boldsymbol{\alpha}$	1	2	$\boldsymbol{\alpha}$	1	2
a	0.02	0.77	**a**	0.02	0.77	**a**	0.02	0.77	**a**	0.02	0.77
c		**ARL**	**c**	**c**		**c**		**ARL**		**c**	
1	300.10	300.17	1	300.10	300.17	1	300.10	300.17	1	300.10	300.17
0.99	100.42	221.96	0.99	100.42	221.96	0.99	100.42	221.96	0.99	100.42	221.96
0.95	26.82	90.37	0.95	26.82	90.37	0.95	26.82	90.37	0.95	26.82	90.37
0.93	19.43	62.16	0.93	19.43	62.16	0.93	19.43	62.16	0.93	19.43	62.16
0.91	15.15	43.67	0.91	15.15	43.67	0.91	15.15	43.67	0.91	15.15	43.67
0.9	13.62	36.81	0.9	13.62	36.81	0.9	13.62	36.81	0.9	13.62	36.81
0.88	11.29	26.39	0.88	11.29	26.39	0.88	11.29	26.39	0.88	11.29	26.39
0.85	8.93	16.38	0.85	8.93	16.38	0.85	8.93	16.38	0.85	8.93	16.38
0.8	6.54	7.84	0.8	6.54	7.84	0.8	6.54	7.84	0.8	6.54	7.84
0.75	5.08	4.11	0.75	5.08	4.11	0.75	5.08	4.11	0.75	5.08	4.11
0.7	4.10	2.40	0.7	4.10	2.40	0.7	4.10	2.40	0.7	4.10	2.40
0.6	2.88	1.22	0.6	2.88	1.22	0.6	2.88	1.22	0.6	2.88	1.22
0.5	2.16	1.01	0.5	2.16	1.01	0.5	2.16	1.01	0.5	2.16	1.01
0.4	1.68	1.00	0.4	1.68	1.00	0.4	1.68	1.00	0.4	1.68	1.00
0.3	1.37	1.00	0.3	1.37	1.00	0.3	1.37	1.00	0.3	1.37	1.00
0.2	1.15	1.00	0.2	1.15	1.00	0.2	1.15	1.00	0.2	1.15	1.00
0.1	1.02	1.00	0.1	1.02	1.00	0.1	1.02	1.00	0.1	1.02	1.00

Table 16
ARL Values of Suggested Control Chart for ARL_0 = 370 and n = 30

i = 1				i = 2				i = 3			
$\mathbf{k_1}$	2.95	1.48	$\mathbf{k_1}$	2.95	1.48	$\mathbf{k_1}$	2.95	1.48	$\mathbf{k_1}$	2.95	1.48
$\mathbf{k_2}$	2.43	1.32	$\mathbf{k_2}$	2.43	1.32	$\mathbf{k_2}$	2.43	1.32	$\mathbf{k_2}$	2.43	1.32
$\boldsymbol{\alpha}$	1	2	$\boldsymbol{\alpha}$	1	2	$\boldsymbol{\alpha}$	1	2	$\boldsymbol{\alpha}$	1	2
a	0.69	0.84	**a**	0.69	0.84	**a**	0.69	0.84	**a**	0.69	0.84
c	**ARL**			**c**	**c**			**ARL**	**c**		
1	370.37	370.26	1	370.37	370.26	1	370.37	370.26	1	370.37	370.26
0.99	262.69	92.25	0.99	262.69	92.25	0.99	262.69	92.25	0.99	262.69	92.25
0.95	111.27	17.17	0.95	111.27	17.17	0.95	111.27	17.17	0.95	111.27	17.17
0.93	81.88	10.84	0.93	81.88	10.84	0.93	81.88	10.84	0.93	81.88	10.84
0.91	62.34	7.46	0.91	62.34	7.46	0.91	62.34	7.46	0.91	62.34	7.46
0.9	54.85	6.32	0.9	54.85	6.32	0.9	54.85	6.32	0.9	54.85	6.32
0.88	42.95	4.69	0.88	42.95	4.69	0.88	42.95	4.69	0.88	42.95	4.69
0.85	30.32	3.22	0.85	30.32	3.22	0.85	30.32	3.22	0.85	30.32	3.22
0.8	17.45	1.98	0.8	17.45	1.98	0.8	17.45	1.98	0.8	17.45	1.98
0.75	10.29	1.42	0.75	10.29	1.42	0.75	10.29	1.42	0.75	10.29	1.42
0.7	6.21	1.16	0.7	6.21	1.16	0.7	6.21	1.16	0.7	6.21	1.16
0.6	2.55	1.01	0.6	2.55	1.01	0.6	2.55	1.01	0.6	2.55	1.01
0.5	1.35	1.00	0.5	1.35	1.00	0.5	1.35	1.00	0.5	1.35	1.00
0.4	1.03	1.00	0.4	1.03	1.00	0.4	1.03	1.00	0.4	1.03	1.00
0.3	1.00	1.00	0.3	1.00	1.00	0.3	1.00	1.00	0.3	1.00	1.00
0.2	1.00	1.00	0.2	1.00	1.00	0.2	1.00	1.00	0.2	1.00	1.00
0.1	1.00	1.00	0.1	1.00	1.00	0.1	1.00	1.00	0.1	1.00	1.00

Table 17. Comparison in *ARL's* of suggested control chart and existing chart at ARL_0 = 300, 370, i = 1 and n = 30

	$ARL_0 = 300$						$ARL_0 = 370$					
	α = 1		α = 2		α = 3		α = 1		α = 2		α = 3	
c	Sug	Exi	Sug	Exi	Sug	Exi	Sug	Exi	Sug	Exi	Sug	Exi
1	300.10	300.12	300.17	300.36	300.22	300.03	370.37	370.00	370.26	370.00	370.28	370.03
0.99	100.42	280.62	221.96	255.62	133.85	243.42	262.69	351.75	92.25	326.03	264.98	322.11
0.95	26.82	204.05	90.37	131.96	34.42	88.14	111.27	260.99	17.17	166.03	94.12	158.25
0.93	19.43	170.13	62.16	94.21	22.29	51.12	81.88	214.54	10.84	112.28	60.28	106.07
0.91	15.15	140.41	43.67	67.13	15.18	29.75	62.34	172.37	7.46	74.90	39.50	70.40
0.9	13.62	127.17	36.81	56.65	12.69	22.80	54.85	153.46	6.32	61.04	32.20	57.27
0.88	11.29	103.85	26.39	40.37	9.04	13.62	42.95	120.34	4.69	40.52	21.70	37.91
0.85	8.93	75.97	16.38	24.40	5.69	6.67	30.32	81.96	3.22	22.10	12.44	20.56
0.8	6.54	44.50	7.84	10.86	2.96	2.54	17.45	42.06	1.98	8.52	5.50	7.80
0.75	5.08	25.84	4.11	5.18	1.81	1.39	10.29	21.46	1.42	3.73	2.87	3.35
0.7	4.10	14.99	2.40	2.75	1.30	1.07	6.21	11.16	1.16	1.97	1.79	1.77
0.6	2.88	5.22	1.22	1.24	1.01	1.00	2.55	3.49	1.01	1.08	1.10	1.04
0.5	2.16	2.12	1.01	1.01	1.00	1.00	1.35	1.53	1.00	1.00	1.00	1.00
0.4	1.68	1.20	1.00	1.00	1.00	1.00	1.03	1.06	1.00	1.00	1.00	1.00
0.3	1.37	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: Sug shows Suggested control chart and Exi shows existing control chart

Table 18. Comparison in ARL's of suggested control chart and existing chart (Single sampling plan) at ARL_0 = 200, 300 and 370

	$ARL_0 = 200$, i = 1				$ARL_0 = 300$, i = 2				$ARL_0 = 370$, i = 3			
	α = 1		α = 3		α = 1		α = 3		α = 2		α = 3	
c	Sug	Exi	Sug	Exi	Sug	Exi	Sug	Exi	Sug	Exi	Sug	Exi
1	200.17	200.01	200.05	200.00	300.26	300.00	300.01	300.00	370.16	370.00	370.14	370.00
0.9	16.51	64.48	21.33	23.28	17.99	175.23	18.96	21.66	40.11	60.94	13.19	34.96
0.8	5.90	15.02	3.97	3.16	6.18	45.01	3.84	2.92	8.35	11.01	3.03	4.01
0.7	2.85	4.21	1.51	1.21	2.89	11.51	1.45	1.16	2.49	2.87	1.32	1.30
0.6	1.67	1.68	1.07	1.00	1.68	3.51	1.03	1.00	1.24	1.29	1.02	1.01
0.5	1.19	1.08	1.00	1.00	1.19	1.50	1.00	1.00	1.01	1.01	1.00	1.00
0.4	1.02	1.00	1.00	1.00	1.03	1.04	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: Sug shows Suggested control chart and Exi shows existing control chart

3.6 CONTRAST OF THE SUGGESTED CONTROL CHART

Table 17 shows the suggested chart is compared with existing by Aslam (2017) at ARL_0 = 300, 370, i = 1 and n = 30. When c = 0.99, α = 1 and $ARL_0 = 300$, from Table 17 ARL value of suggested control chart is 100.42 while it is 280.62 from Aslam Chart. So, the suggested control is more responsive to identify a shift in the process as instead to existing control chart.

Table 18 shows the suggested chart is compared with existing chart by Rao (2018) at ARL_0 = 200, 300, 370, i = 1, 2, 3. When c = 0.9, α = 1 and $ARL_0 = 200$, from Table 18 ARL value of suggested control chart is 16.51 while it is 64.48 from Rao Chart. So, the suggested control is more responsive to identify a small shift in the process as instead to existing control chart.

3.7 ILLUSTRATIVE EXAMPLE

We consider Hinkley's (1977) real dataset. This dataset for the Minneapolis /St. Paul is 30 consecutive rainfall values (in inches) in March over a period of 30 years. The observed values are shown below:

0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10,0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05.

Seo et al. (2012), Azimi & Sarikhanbaglu (2014), Torabi & Bagheri (2010), Seo & Kang (2015) and so on evaluated this dataset. The exponentiated half-logistic distribution suits the information well, according to Seo & Kang (2015).We assume that n = 30, α = 2, i = 1, a = 0.84 and r_0 = 370. The value of p_0 from eq. (6) is 0.624. The value of μ from eq. (3) is 1.7423 with test duration t_0 = a μ_0 = (0.84)(1.7423) = 1.4635. The control limits under MDS are calculated from eq. (4a-4d) are UCL_1 = 3.12, $UCL_2 =$ 2.96, $LCL_1 = 0.24$, $LCL_2 = 0.39$ for the parameters k_1 = 1.48, k_2 = 1.32.

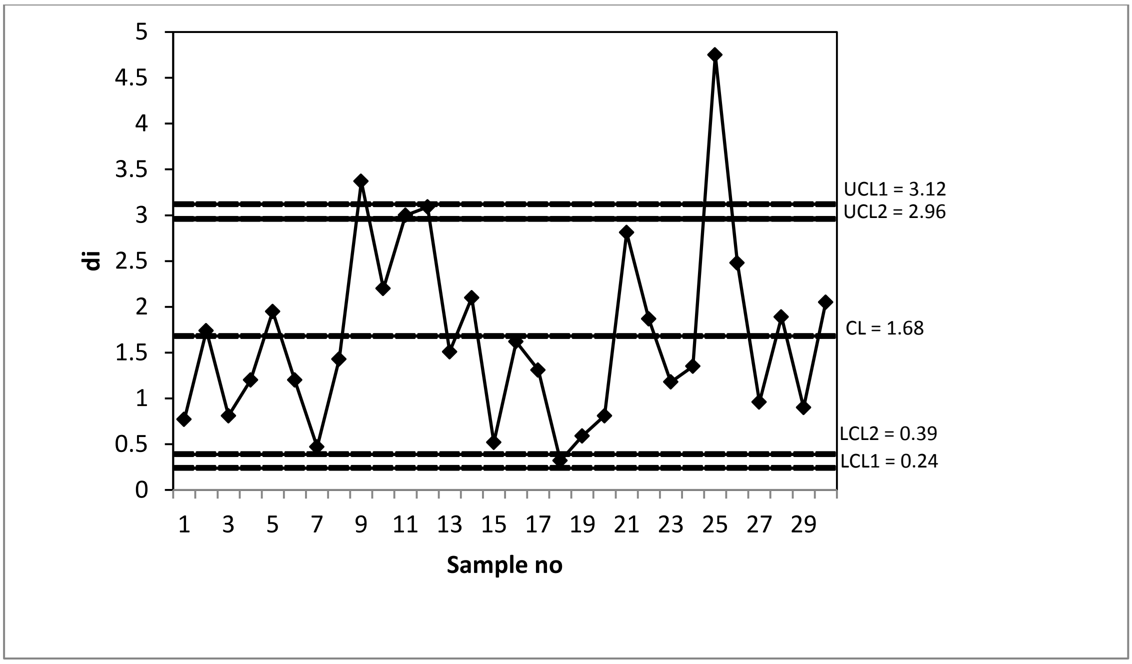

Figure 1: Suggested Control Chart for Real Data

It can be seen from figure 1 that while the service time process is in control state, some points are close to control limits and some points are within repetitive region.

3.8 SIMULATION STUDY

In this chapter, using simulated information, we address the implementation of the proposed chart. The information will be produced from an exponential half-logistic distribution when α = 3 and σ = 1. Let n = 30, a = 0.88 and r_0 = 370. The process is declared to be in control when α = 3 and σ = 1. The value of p_0 from Eq. (6) is 0.4092. Let the value of μ = 2.16, the test duration is therefore calculated as t_0 = a μ_0 = (0.88) (2.16) = 1.90. We find out the control limits coefficients k_1 = 2.735302 and $k_2 = 2.324929$ and control limits are UCL_1 = 19.64, $UCL_2 = 18.54$, $LCL_1 = 4.91$, $LCL_2 = 6.01$, the amount of failures per subgroup is reported before the specified time.

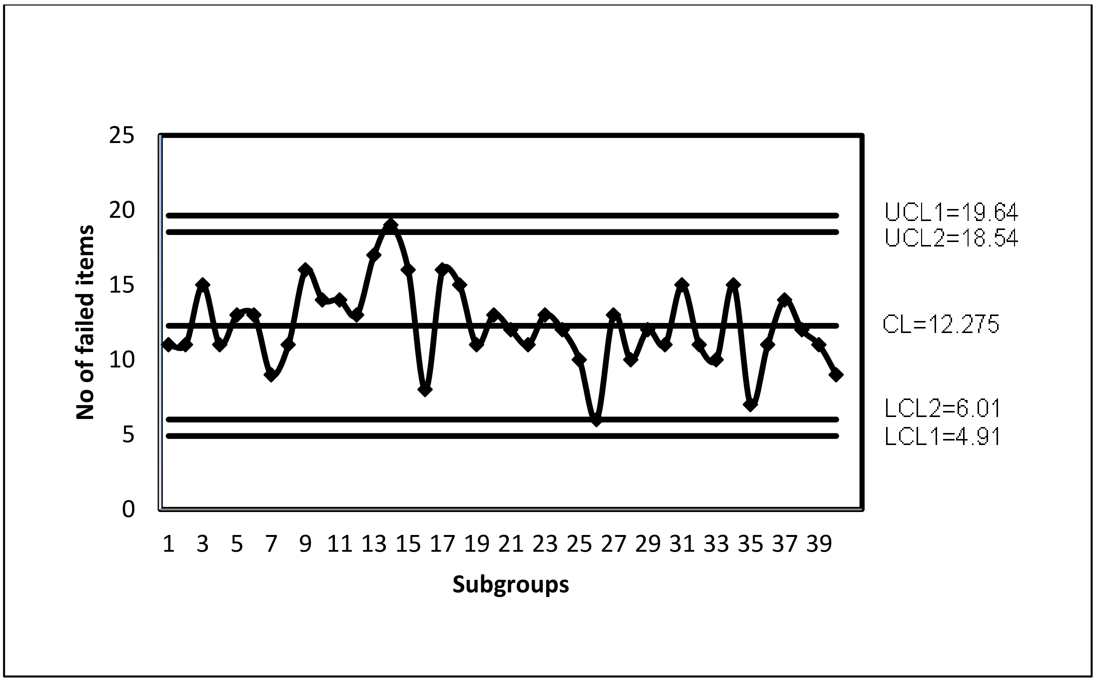

Figure 2: Suggested Control Chart for Simulated Data

Table 19
The Simulated Data

Subgroups	d	Subgroups	d
1	11	21	13
2	11	22	12
3	15	23	11
4	11	24	13
5	13	25	12
6	13	26	10
7	9	27	6
8	11	28	13
9	16	29	10
10	14	30	12
11	14	31	11
12	13	32	15
13	17	33	11
14	19	34	10
15	16	35	15
16	8	36	7
17	16	37	11
18	15	38	14
19	11	39	12
20	11	40	11

We can see this from Figure 2 if the number of fail items in the subgroup is 7,8,9,10,11,12,13,14,15,16,17 or 18, the process is said to be in control. The sampling will be repeated if the subgroup includes 5 or 6 or 19 failed items. If the number of failures is less than or equal to 4 the process is stated to be out of control

CHAPTER 4

CONCLUSION

In this study, an Attribute np control chart for process monitoring using exponentiated half-logistic distribution (EHLD) is provided based on repetitive group sampling (RGS) and multiple dependent state sampling (MDS). We will compare the both charts by existing control charts.

RGS chart is linked with the current chart on the basis of exponentiated half-logistic distribution provided by Jeyadurga, Blamurali, and Aslam (2018). We acquire optimal parameters, control boundaries and also show the respective ARL and ASN values of the control chart built using repetitive group sampling. We compare the table values for the parameters of the shape $\alpha = 2$ and 3 and $r_0 = 300$ and 370 respectively is the particular ARL. We can analyze from these tables that the suggested chart identifies the process changes faster than the current chart. The suggested chart therefore identifies the process shift instead of the existing chart very effectively. The control chart performance in RGS is also contrasted with the current chart Rao (2018) based on exponentiated half logistic distribution. Tables demonstrate the respective control chart ARL values by repetitive group sampling and single sampling for the parameter of the shape $\alpha = 2$, $r_0 = 370$ and for the parameter of the scale $\alpha = 2.5$, 3 and $r_0 = 370$. These tables shows, so as to the suggested chart identify the change quicker than the existing chart.

The efficiency of the suggested MDS control chart is likened to Aslam's (2017) results. We receive the optimal parameters, control boundaries and also present the respective ARL's of the control chart built using various dependent state sampling. Table indicates that the suggested control chart is more responsive than the existing control chart to recognize a shift in the process. The efficiency of the suggested MDS control chart is also likened to Rao (2018) results. Tables shows the suggested chart is compared with existing chart by Rao (2018) at $ARL_0 = 200$, 300, 370, i = 1,2,3. So, the suggested control is more responsive to identify a shift in the process as instead to existing control chart.

Moreover in this paper we find the ARL's for not only when scale and shape parameters are shifted but also find the ARL for both shifted parameters at once under RGS sampling plan. For future work, the suggested control chart might be applied to other distributions.

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ALGORITHM

Step 1: Denote the values of α_0 (shape parameter), a (test termination ratio) and r_0 (in-control target ARL) and set the greatest value for ASN and specify it as ASN_{m}.

Step 2: Put the values of n, k_1 and k_2.

Step 3: Put the values of a, α_0 in Equation (6) find the failure probability (p_0).

Step 4: Discover the control limits of the suggested control chart using the values of n, p_0, k_1 and k_2 in Equations(4a)–(4d).

Step 5: Find the likelihood that the process will be declared in-control when the process is in-control by replacing the values of p_0 and control limits in Equations (10), (12) and (13a) also use the Equation (15a) to locate ARL_0. By using the Equation (14a) compute the ASN at p_0 .

Step 6: Check parameters n, k_1 and k_2 such that the in-control ARL_0 is as near as to the final value r_0. Make the comparison of ASN (p_0) value with ASN_{m}. If ASN (p_0) $\leq$ ASN_{m}, then set ASN_{m} = ASN (p_0).

Step 7: Continue Steps 4, 5 and 6 for various combinations of n, k_1 and k_2 until get an ARL_0 as near as to r_0 and obtain the least value for ASN_{m}.

Step 8: Replace the values (n, k_1 and k_2) in Equations (7), (8), (9), (13b), (13c), (13d), (15b), (15c) and (15d) for discovering the out-of-control ARL_1 for various shift values. Use the Equations (14b), (14c) and (14d) to discover the respective ASN.