Signaling with Evidence

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- The pharmaceutical company has incentives to hide some information about the advancements of the project to keep the development ongoing in case the project's difficulty changes.
- The government would anticipate the company's behavior and thus requires to receive physical evidence with every report.

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- Requiring physical evidence restricts the messages between the parties.
- *"You cannot show more than what you have, but you can show less."*
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- This scenario applies to many investor-developer problems: the development of a vaccine or drugs, the production of a movie, the manufacturing of an electric car ...
- Verifiability of messages was discussed before in the literature in different forms.

Selection of the Literature

- Voluntary disclosure: Grossman and Hart (1980), Grossman (1981), and Milgrom (1981)
  - Static model with unraveling results
  - This is no longer the case in a dynamic model with our specific verifiability assumption that forces the sender to disclose at most his type
- Dynamic disclosure game: Shin (2003) and Hart, Kremer, and Perry (2017),
  - Considered dynamic voluntary disclosure
  - Both papers discussed truth-learning equilibria.

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- Would a developer always reports the status of the project truthfully?
- What is the optimal strategy of the investor when such a condition on the project and the messages are imposed?

Preview of the findings

1. Under this verifiability assumption, the communication problem of an investor and a developer allows for a simple class of equilibria:
   - The developer sends the least amount of information that keeps the project running.
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1. Under this verifiability assumption, the communication problem of an investor and a developer allows for a simple class of equilibria:
   - The developer sends the least amount of information that keeps the project running.
   - The investor continues if the information is "good enough"
2. Telling the truth in every period is NOT an equilibrium strategy
3. The investor will not start the investment without verifiability.

Environment

- 4 period model - 2 nontrivial stage game
- 2 players: an investor and a developer

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- Strategies:
  - Developer, $\sigma_t : H^t \to \Delta M_t$
  - Investor, $s_t : M^t \to [0, 1]$

H^t is the set of histories that the developer see at time t.

M_t is the set of messages available at time t. M^t the history.

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  M_t is the set of messages available at time t. M^t the history.
- Payoffs:
  - Developer gets $D \in \mathbb{R}_+$ upon project completion only
  - Investor pays continuation cost c_t and gets $R \in \mathbb{R}_+$ upon project completion only

Notation

- $\theta \sim p(\theta)$ the type of the project: how difficult?
- $x_t \sim f_{\theta,t}$ advancement in the project at time t
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- $u(m^t)$ utility of the investor
- $w(h^t)$ utility of the developer

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- θ according to $p(\theta)$
- x_2 realized according to $f_{\theta,2}$
- developer sends m_2 according to $\sigma_2(x_1, m_1, \theta, x_2)$
- investor decides $s_2(m_1, m_2)$ and pays c_2 if continue

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t=3
- x_3 realized according to $f_{\theta,3}$
  - ⋆ If the project in not completed, Both get zero
  - ⋆ If the project in completed, Developer gets D and Investor gets R

Verifiability Assumption

- We discuss the verifiability assumption in a way to capture the idea of requiring physical evidence
- Puts restriction on the message m_t that the developer can send at time t

$$0 \leq m_t \leq \sum_{i=1}^{t} x_i$$

Implications

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- Assume that in period t the investor stops at message m_t that is sent with strictly positive probability by the developer.
- The developer is allowed to assign a positive probability for sending any message below the cumulative advancements in the project.
- If the investor is stopping at m_t but is willing to continue at $m'_t < m_t$, then the developer would send m'_t and assign zero probability for sending m_t.

Formal Proof

Results

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- We use that the developer's utility in the following period is decreasing in the message he sent in the first period.
- Since the investor is willing to continue in both messages, the developer is better off by sending m_1'.
- The most profitable deviation is to send the lowest message in the first period that will make the investor continue.

Class of equilibria

Proposition 2

There exists a class of equilibria where

- *The investor plays a cutoff strategy in every period*
- *The developer*
  - *reports truthfully in the second stage of the game*
  - *sends the lowest message that would make the project continue in the first stage, if possible*

The cutoffs $(\bar{m}_1, \bar{m}_2)$ are characterized using the inventor's utility function between stopping and continuing the project.

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Formally

- Proof: the previous lemma fixes the behavior of the investor on equilibrium. We show that a best response of the developer on equilibrium using backward induction.
- We characterize the continuum of equilibria in this class by characterizing the set of cutoffs of each stage of the game.

Property of the Class of Equilibria

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Implication:

- The developer's expected utility is decreasing in the first cutoff while the investor's utility is increasing.

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- This assumption implies that the information released by the developer is useful.
- Without it, the developer could release no information at all and the project would start.

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Suppose that the developer is allowed to send any message at any history and suppose Assumption 1 holds. Then, the investor does not start the project in equilibrium.

- If there exists a message that makes the investor continue, all developers without a full project would prefer to send that message.
- There cannot be a separation between incomplete projects.
- Requiring evidence allows for projects to be completed with positive probability when they would not be completed otherwise.
- The proposition connects a feature of the product, partially verifiability, to the probability of the project's success.

Conclusion

- The effect of the verifiability condition on a dynamic model of communication between an investor and a developer.
- A class of equilibria where the developer lies in the early stages of the game and reports truthfully in the second stage.
- The investor builds her beliefs accordingly and is still willing to continue in the game if she gets a message high enough (cutoff strategy).
- Consequences on the utilities of the players
- Truthtelling is not an equilibrium
- Relaxing this condition will cause the development not to start to begin with.

Extension

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- We consider the case where the outside option is private information of the developer (iid over time)
- We found that the model becomes analogous to a model of multidimensional screening, where the investor must screen over both the current status of the project, as well as the value of the outside option.
  - The investor has no way to screen high values of outside option from low values
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  - The investor has no way to screen high values of outside option from low values
  - Any developer of a low outside option value can argue that he is of high outside option value and extracts all potential surplus from the investor
- Future work: outside option depends on the current status of the project (private information of the developer)

Thank You

Class of equilibria

Proposition 4

$\left((\sigma^*_{\bar{m}_1}, \sigma^*_2); (1, s^*_{\bar{m}_1}, s^*_{\bar{m}_2}), (\lambda^*_1, \lambda^*_2)\right)$, where $\bar{m}_1 < \bar{x}$ and $\bar{m}_2 < \bar{x}$ satisfy

$$\begin{cases} u_1(m_1, c) \geq u_1(m_1, b) & \forall m_1 \geq \bar{m}_1 \\ u_1(m_1, c) < u_1(m_1, b) & \forall m_1 < \bar{m}_1 \\ u_2(\bar{m}_1, \bar{m}_2, c) = u_2(\bar{m}_1, \bar{m}_2, b) \end{cases}$$

is an equilibrium of the above game.

$$\sigma^*_2(\cdot|x_1, m_1, \theta, x_2) = \delta_{x_1 + x_2}$$

$$\sigma^*_{\bar{m}_1}(\cdot|x_1) = \begin{cases} \delta_{x_1} & \text{if } x_1 < \bar{m}_1 \text{ or } x_1 \geq \bar{x} \\ \delta_{\bar{m}_1} & \text{if } x_1 \in [\bar{m}_1, \bar{x}). \end{cases}$$

$$s^*_{\bar{m}_1}(m_1) = \begin{cases} 0 & \text{if } m_1 \in [0, \bar{m}_1) \cup [\bar{x}, k_1] \\ 1 & \text{if } m_1 \in [\bar{m}_1, \bar{x}) \end{cases}$$

$$s^*_{\bar{m}_2}(m_2) = \begin{cases} 0 & \text{if } m_2 \in [0, \bar{m}_2) \cup [\bar{x}, k_1 + k_2] \\ 1 & \text{if } m_2 \in [\bar{m}_2, \bar{x}) \end{cases}$$

Backward Induction

Claim 2

Suppose that the investor starts the development in period $t = 0$ and plays $s^*_{\bar{m}_1}$ for a fixed $\bar{m}_1$. Then σ^*_2 is the developer's best response for $s^*_{\bar{m}_2}$.

Claim 3

Suppose that the investor's strategy is $(1, s^*_{\bar{m}_1}, s^*_{\bar{m}_2})$ where $\bar{m}_1 \in M_1$ is fixed. Suppose also that the developer plays according to σ^*_2 in period $t = 2$. Then, $\sigma^*_{\bar{m}_1}$ is a developer's best response for $s^*_{\bar{m}_1}$ in period $t = 1$.

return

Proof of Lemma 1

- For a fixed history $h^2 = (x_1, \theta, x_2)$, assume that on equilibrium, the developer sends message m_2' with positive probability, i.e., $\sigma^*(m_2'|h^2) > 0$. By the verifiability assumption, $x_1 + x_2 \geq m_2'$.
- WLOG, assume $x_1 + x_2 < \bar{x}$.[1] For any $m_2 \in [0, x_1 + x_2]$, if $s^*(m_1, m_2) = 0$, then $w_2(h^2, m_2) = 0$. Otherwise, if $s^*(m_1, m_2) = 1$, then $w_2(h^2, m_2) > 0$.
- Assume $s^*(m_1, m_2') = 0$. If $\exists\ m_2'' \in [0, m_2')$ such that $s^*(m_1, m_2'') = 1$. Notice that by the verifiability assumption, the developer can send m_2'' since it is less than $x_1 + x_2$.
- However, if $\sigma^*(m_2''|h^2) > 0$, then it has to be that $\sigma^*(m_2'|h^2) = 0$. Which is a contradiction with $\sigma^*(m_2'|h^2) > 0$.
- Thus, in equilibrium $\exists\ m_2^*$, such that,

$$s^*(m_2) = \begin{cases} 0 & \text{if } m_2 \in [m_2^*, \bar{x}) \\ 1 & \text{if } m_2 \notin [m_2^*, \bar{x}) \end{cases}$$

Return

[1] Otherwise, the developer would report truthfully and the investor would stop.