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The Capacitance Model of Frenetic



Version History

Version	Date	Changes from	Comments
1.0	June 2025	Richard Schlesinger	First release
1.1	October 2026	Marcos Luna	Added benchmark transformer LB2

01 Model Features and Accuracy

Model features

- Calculates primary open-circuit capacitance to predict first resonant frequency
- FEM-based physical model
- Quasi-3D modelling concept
- Fully flexible regarding geometry
- Assumptions:
 - Based on Inside-Window cross section (see Figure 1)
 - Voltage ratio = turns ratio
 - Linear potential distribution along turns of windings
 - All windings have their last turn on ground potential
 - Relative permittivities of insulation materials assumed based on literature data

Model accuracy

Expected error regarding predicted resonant frequency: below 20%.
Measured error between 7.5% and 19.2% on 3 different magnetics, average 11.6%.

Accuracy disclaimer: Capacitances are subject to many uncertainties and the implicated error of some model assumptions depends on several factors. For details see Sections 03 and 04.

02 Why a Capacitance Model?

Parasitic capacitances lead to several resonances in the frequency spectrum of a magnetic component. A power electronic converter should be operated below the first resonance of the magnetic component to ensure reliable operation. Hence, we calculate the first resonance using the primary open-circuit capacitance and the magnetizing inductance to ensure the converter is operated below that particular frequency.

03 How does it work?

Model concept

The model concept can be summarized in 3 main steps:

1. An assumed voltage of 1V is applied to the primary winding. This results in an electric potential distribution along the turns of all windings based on the assumptions listed in the upcoming section.
2. Electrostatic energies per unit length (p.u.l.) and per unit angle (p.u.a.) are calculated from the basic cross sections that the transformer consists of. The calculations are done using FEM.
3. The previously calculated energies are scaled to the total capacitance value using the Quasi-3D modelling concept based on the Inside-Window (IW) cross section as shown in Figure 1. Detailed information on the Quasi-3D modelling concept can be found in [1].

Electrostatic Potential Distribution

The first model step is to apply 1V to the primary winding. It is assumed that the last turn of each winding is grounded. Hence, the electric potential distribution of the primary winding ranges from 1V (first turn) to 0V (last turn). The distribution of the voltage is assumed to be linear along the turns of the winding. The electric potential of all other windings is then derived from the turns ratio of the primary winding to the particular secondary windings whose last turn is also at ground potential (0V). The potential distribution of the secondary windings is also assumed to be linear.

Electrostatic Energy Calculations

The 3D magnetic is calculated using the Inside-Window (IW) cross section shown in Figure 1. For this cross section, the model acquires the electrostatic energy p.u.l. using

$$W'_{el} = \iint w_{el} dA$$

and the electrostatic energy p.u.a. using

$$W''_{el} = \iint x w_{el} dA$$

with $x=0$ at the spot that represents the center of rotation. In the above equations, w_{el} is the electrostatic energy density, and x is the radial coordinate. All calculations are performed using a fully automated FEM model based on COMSOL Multiphysics.

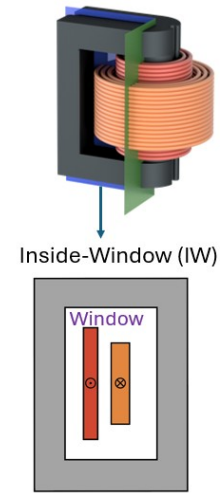


Figure 1 - Quasi-3D Inside-Window Modelling Concept

Obtaining the 3D Capacitance

The 3D magnetic is split up in curved and straight winding sections as shown in Figure 2. The model uses a radial electrostatic energy weighting in curved winding sections to take into account the curvature of the winding section. Straight sections are simply regarded by their section length. The total capacitance according to the Quasi-3D modelling concept is then calculated using

$$C = \frac{2}{V_{ref}^2} (W'_{el} l_{straight} + W''_{el} 2\pi)$$

where V_{ref} is the assumed voltage applied to the primary winding, W'_{el} is the electrostatic energy per unit length (p.u.l.), $l_{straight}$ is the total length of the straight winding sections (if any), and W''_{el} is the electrostatic energy per unit angle (p.u.a.) needed in curved winding sections.

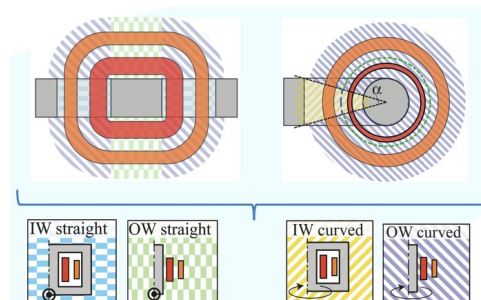


Figure 2 - Winding Sections

Model Assumptions & Limitations

Capacitance modelling and the present model as such are subject to several assumptions, limitations, and difficulties to match the model result with reality. In particular, the following circumstances affect and limit the accuracy of the capacitance model:

- Uncertainties
 - o Permittivity: the permittivity of insulation material and PCB material is only given in ranges and depends on several factors such as frequency [2] and temperature [3]. Since the permittivity has a big impact on the electrostatic energy stored between the turns, its effect on capacitance is significant.
 - o Geometry: as always, geometric uncertainties lead to a difference between the real geometry and the idealized geometry in the model. Further factors such as flying leads and their geometrical arrangement add up to this.
- Voltages on secondaries assumed with turns ratio: The assumption of equating voltage ratios with turns ratios is very good for well-coupled transformers but decreases in quality for devices whose windings are coupled to a lesser extent [4].
- The electrostatic potential is assumed to be distributed linearly along the turns of the same winding. Since inter-turn capacitances within a winding are not all the same, this is only an approximation [5].
- Only the Inside-Window cross section shown in Figure 1 is taken into account. The Outside-Window cross section is neglected for simplicity and because typically, most electric energy is stored between the winding turns and the core has less of an influence, which is also supported by the results in Section 04.
- The strands of a Litz wire are modelled as one equivalent round conductor. The total outer diameter of the Litz wire is calculated using a formula from the standard DIN EN 60317-11 [6]. The equivalent round conductor diameter can then be calculated by subtracting the outermost insulation from the outer diameter. Note that the resulting values typically do not perfectly coincide with reality. Hence, predicted effective outer insulation layer thicknesses tend to differ from reality, which affects the modelled electrostatic energy and resulting capacitance.

Benchmark Transformers

The model has been verified with three transformer prototypes shown in Figure 3 and Figure 4. The transformers cover very different winding configurations. In particular, we compare two transformers with Litz wire (LB1 and LB2) and a planar transformer (PB1). Transformer LB1 is a 2-chamber design, in which the windings are quite far apart, transformer LB2 is an interleaved design with the secondary layers placed on both sides of the primary (S-P-P-S), and transformer PB1 consists of windings that are traces embedded in PCB material. The details of the transformers are given in Table 1.

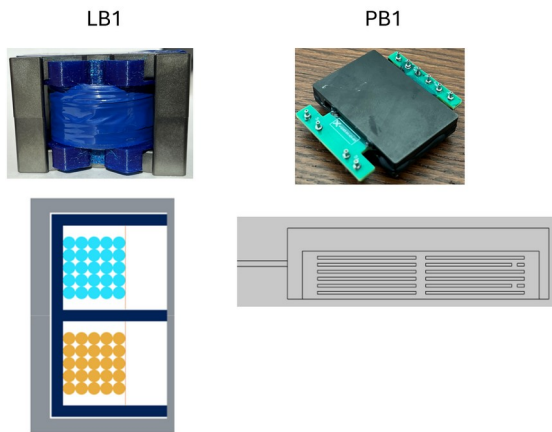


Figure 3 - Benchmark transformer prototypes



Figure 4 - Benchmark transformer LB2

Table 1 - Transformer prototype data. Litz wire of LB1 according to [7]; permittivity of silk from [2] and of FR4 from [8].

Trafo	Core	Winding type	Conductors	Insulation, ϵ_r	N1/N2/N3
LB1	PQ65/44	2-chamber	Litz 240×0.07 mm	Silk, 4.5	25/25/-
PB1	EQ38/8/25	Planar interleaved	PCB copper traces	FR4, 4	8/4/2
LB2	PQ35/35	Interleaved S-P-P-S	Pri.: Litz 105×0.05 mm Sec.: Litz 225×0.05 mm	Silk, 4.5	30/20/-

Verification Results

With the primary open-circuit capacitance model, we put a focus on predicting the first resonant frequency of the transformer. The transformer's first resonant frequency is calculated according to

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}.$$

Here, the capacitance affects the resonant frequency with the exponent of $\frac{1}{2}$, which reduces the error of the predicted first resonant frequency

compared to the error of the capacitance. Table 2 shows the predicted and measured first resonant frequencies both for floating secondary windings and for secondary windings that have their last turn on ground as assumed in the model.

Table 2 - Predicted and measured first resonant frequencies

Trafo	Model	Floating Secondaries		Grounded Secondaries	
		Measured	Error	Measured	Error
LB1	387.4	350.2	11%	352.4	10%
PB1	2134.4	2640.0	-19%	2308.2	-8%
LB2	547.8	613.0	-11%	-	-

The error of the predicted resonant frequencies for floating secondaries is slightly higher than for grounded secondaries but remains below the targeted 20% for the given transformer set. To some extent, this higher error results from the fact that the model assumes the last turn of the secondary windings to be grounded. This assumption is used to achieve a short calculation time because it avoids nested loops for all secondaries to determine the reference potential of the last turn of the secondaries, which is not known a priori and would have to be determined such that the transformer reaches a state of minimum energy.

Table 3 shows the calculated primary open-circuit capacitances and the measured open-circuit capacitances. The table shows that the model error mainly results from the assumption of a grounded last secondary turn. For LB1 and PB1, the calculated primary open-circuit capacitances with grounded secondaries are in better agreement with the measurement than those for floating secondaries.

Table 3 - Predicted and measured primary open-circuit capacitances and measured interwinding capacitances

Trafo	Model	Open-Circuit Capacitance (pF)				Interwinding Capacitance (pF)
		Floating Secondaries		Grounded Secondaries		
		Measured	Error	Measured	Error	
LB1	24.8	30.4	-18%	30.0	-17%	15.0
PB1	102.6	67.1	53%	87.9	17%	296.5
LB2	27.9	21.4	30%	-	-	-

The table shows that the planar transformer PB1 has a much higher deviation in grounded vs floating secondary windings (53% vs 17%) than the Litz transformer LB1 (-18% vs -17%). The reason for this is the higher capacitive coupling of the windings manifesting in a higher interwinding capacitance of 296.5 pF and 15 pF, respectively. The higher capacitive

coupling of the windings leads to an energy minimum that differs more from the energy resulting from floating secondary windings.

The higher capacitive interwinding coupling is expected because the winding geometry of transformer PB1 consists of wide traces that are close to each other embedded in a material of high permittivity. This leads to a relatively high amount of electric energy stored between the windings. In contrast, the Litz wire transformer LB1 has a smaller interwinding capacitance because the distance between primary and secondary is much bigger and, although the insulating material (silk) has a high permittivity, part of the space between the windings is filled with air of lower permittivity.

For transformer LB2, the predicted first resonant frequency deviates by -11% from the measurement and therefore stays within the targeted 20%, although the open-circuit capacitance is overestimated by 30%. This illustrates the reduced sensitivity of the resonant frequency to the capacitance error due to the exponent of $1/2$. Part of the deviation is attributed to the modelling of the Litz wire as an equivalent round conductor, whose effective outer insulation thickness differs from reality (see Section 03), and to the terminals and flying leads of the sample, whose parasitic capacitance is not included in the model.

Measurement Setup

The measurement setup is shown in Figure 5. Flying leads (if any) were placed as far apart as possible to minimize their added capacitance. Interwinding capacitances are easy to measure because the capacitance can be taken directly from the measurement. Open-circuit capacitances have to be measured indirectly because the capacitance to-be-measured is in parallel to the ohmic-inductive impedance of the coil. Hence, the open-circuit capacitance C_{oc} has to be extracted from the measured open-circuit inductance (magnetizing inductance) and the measured resonant frequency according to

$$C_{oc} = \frac{1}{4\pi^2 f_{res}^2 L_{oc}}$$



Figure 5 - Measurement setup

The measurements were performed with an Omicron Lab Bode 100. This device measures the S-parameters and calculates the impedance of the device under test from that. The measurements have been performed using the OnePort mode, which consists of connecting the device under test with a single BNC cable. The OnePort measurement mode connects the shell of the connecting BNC cable with ground. This allows all terminals that are to be grounded to be grounded reliably, as opposed to some other measurement modes, in which the BNC shell of the connecting cable is not on ground and may lead to unwanted capacitive currents that are not captured.

Enjoy predicting resonant frequencies and capacitances using Frenetic!

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