A MODEL OF ROTATING AND MAGNETIZED MIXING-LENGTH THEORY

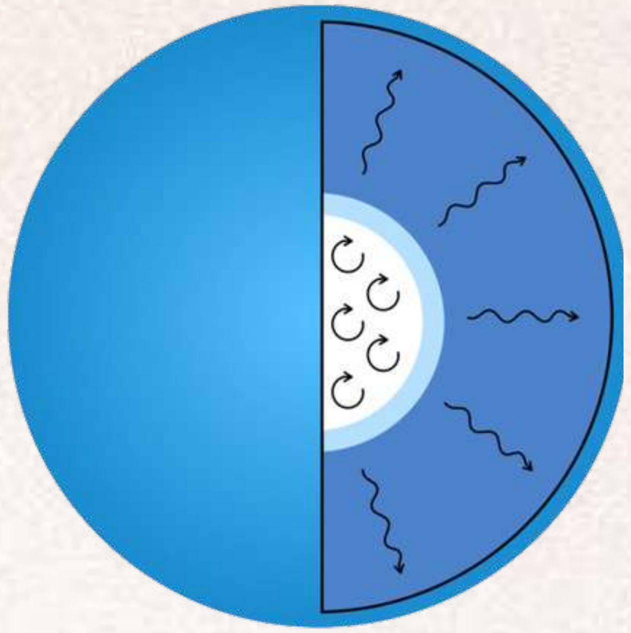
LEÏLA BESSILA, STÉPHANE MATHIS

4D Stars Workshop

Toulouse, January 2025

CONVECTION IN STARS

Low-mass stars

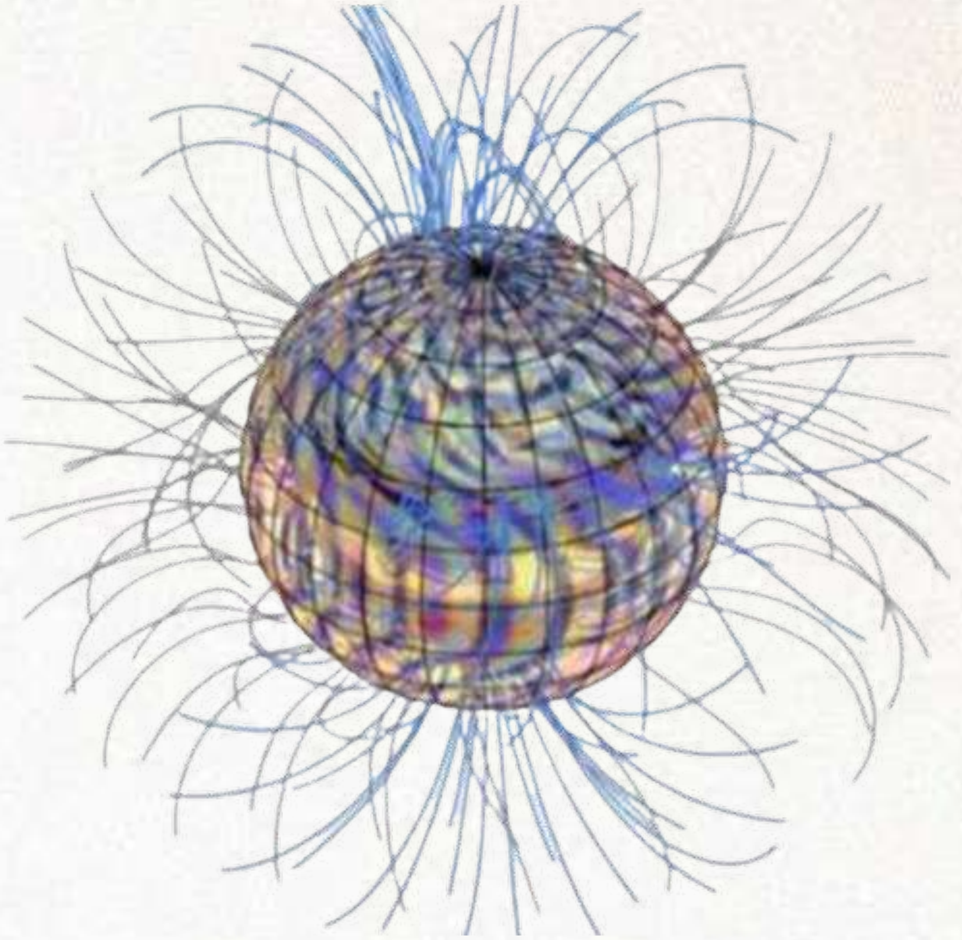


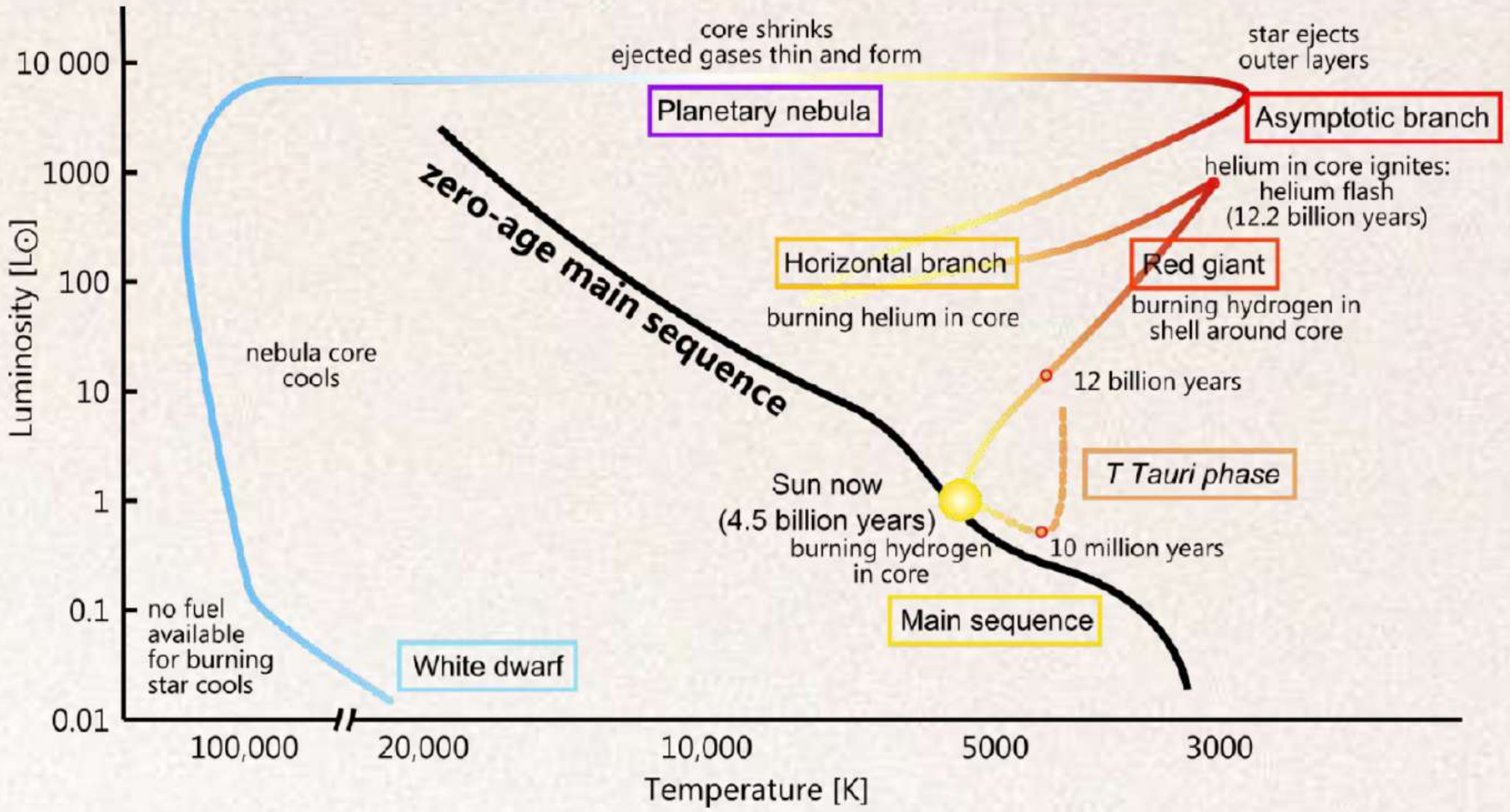
High mass stars

Paramount for **heat transport, mixing of chemicals, magnetic field generation…**

We need to model convection over **evolutionary timescales**

MODELLING STARS ALONG THEIR EVOLUTION

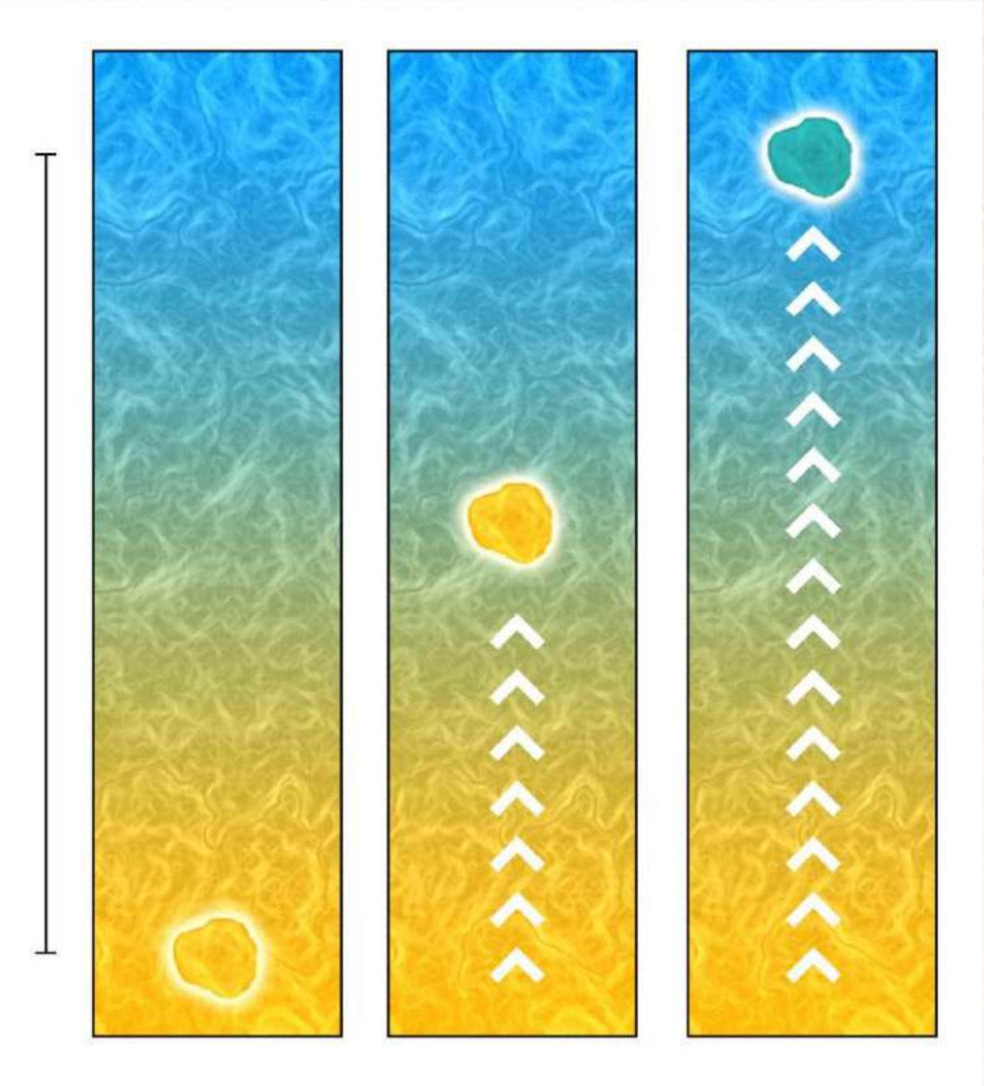




3D Numerical simulations
Hundreds of years

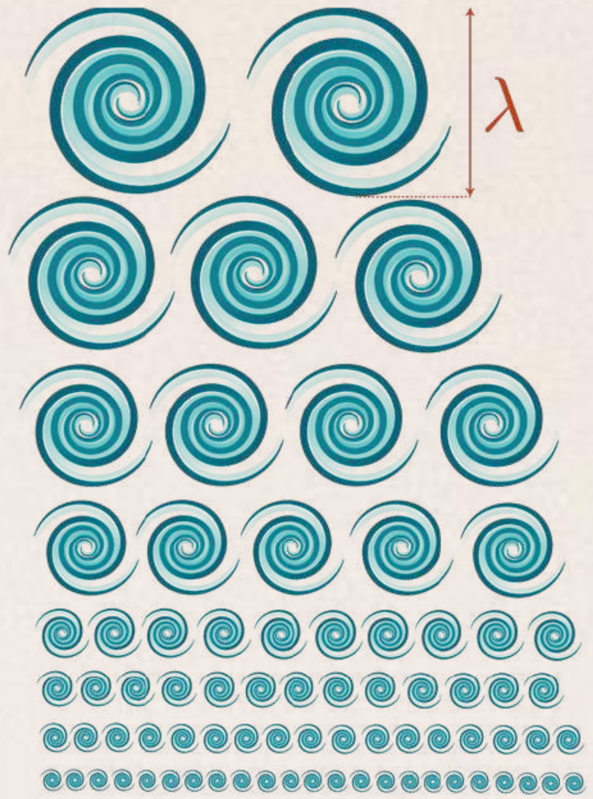
1D, 2D or 3D
Stellar evolution codes
Several billions years

MIXING-LENGTH THEORY (MLT)



Mixing-Length Theory

TURBULENT CASCADE



Various scales

MLT

Reduced to the most energetic eddy

MLT in stellar evolution codes is **without rotation and magnetism**

HEAT FLUX MAXIMIZED CONVECTION

Convection is dominated by the linear mode **that carries the most heat**

[Malkus 1954]

Stevenson 1979

Turbulent Thermal Convection in the Presence of Rotation and a Magnetic Field: A Heuristic Theory

DAVID J. STEVENSON†

Ω B

Rotation or magnetic field

- Asymptotic prescriptions (rapid rotation/low rotation)
- No viscous/heat diffusion

Augustson & Mathis 2019

A Model of Rotating Convection in Stellar and Planetary Interiors. I. Convective Penetration

K. C. Augustson and S. Mathis

AIM, CEA, CNRS, Université Paris-Saclay, Université Paris Diderot, Sarbonne Paris Cité, F-91191 Gif-sur-Yvette Cedex, France; kyle.augustson@cea.fr

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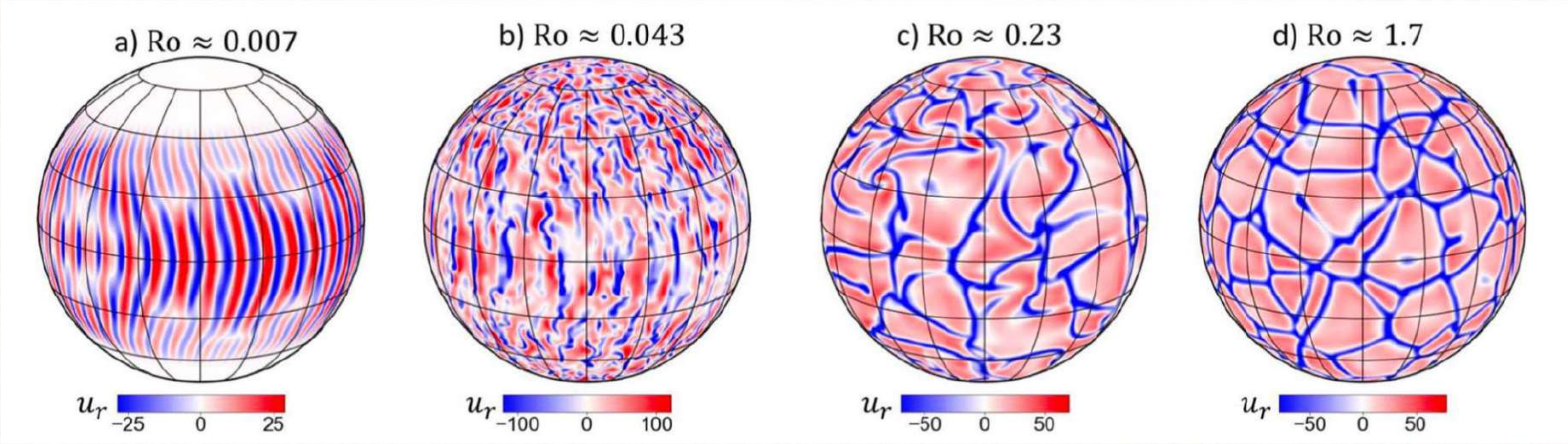
Ω

Rotation only

- Not asymptotic
- Viscous and heat diffusion

ROTATING CONVECTION



Ω

ROSSBY NUMBER

$$\mathcal{R}o \equiv \frac{u_0}{2\Omega \ell_0}$$

Advection/Coriolis

ROTATING CONVECTION

Stevenson 1979, Augustson & Mathis 2019

$$\frac{\partial \boldsymbol{v}}{\partial t} + 2\Omega \times \boldsymbol{v} = \frac{-1}{\rho}\nabla p - \boldsymbol{g}\alpha\theta$$

$$\nabla \cdot \boldsymbol{v} = 0$$

$$\frac{\partial \theta}{\partial t} = -\boldsymbol{\beta} \cdot \boldsymbol{v}$$

Boussinesq equations

Neglecting all diffusive processes

$$\mathcal{F}_\Omega = \frac{\rho c_p}{\alpha g}\frac{N_\star^3}{k^2}\hat{s}^3$$

growthrate

wavenumber

CONVECTIVE HEAT-FLUX
to maximise

$$\hat{s}^2 + \mathcal{O}^2 - \frac{(z^3 - 1)}{z^3} = 0$$

DISPERSION RELATION

ROTATION PARAMETER

$$\mathcal{O}^2 = \frac{4\Omega^2}{N_*^2}$$

$$N_*^2 = |g\alpha\beta|,$$

$$\hat{s} = \frac{s}{N_*},$$

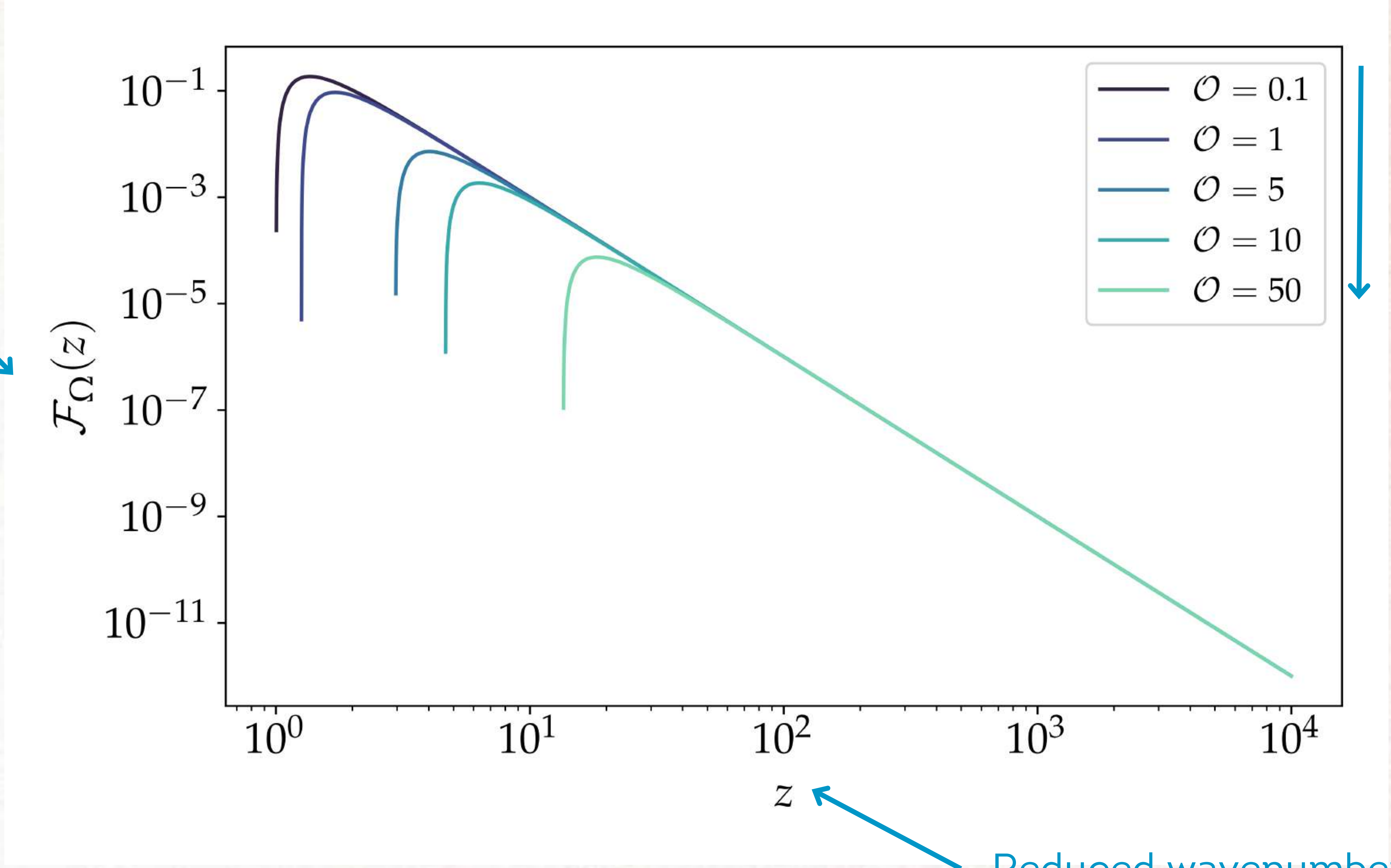
$$z^3 = 1 + a^2 = \frac{k^2}{k_z^2},$$

$$a^2 = \frac{k_x^2}{k_z^2} + \frac{k_y^2}{k_z^2} = a_x^2 + a_y^2.$$

ROTATING CONVECTION

HEAT-FLUX MAXIMIZATION

Convective heat-flux

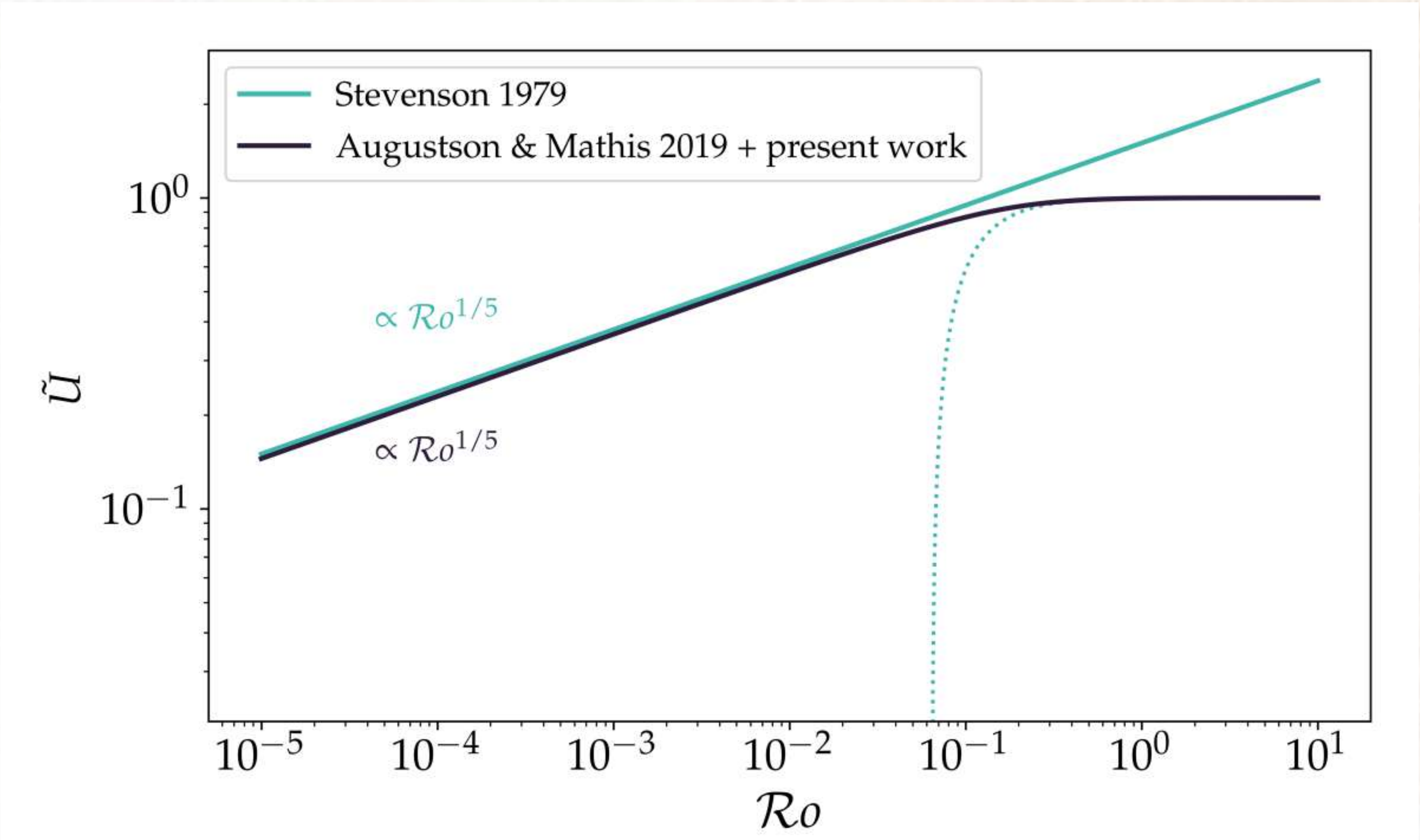


Rotation increases

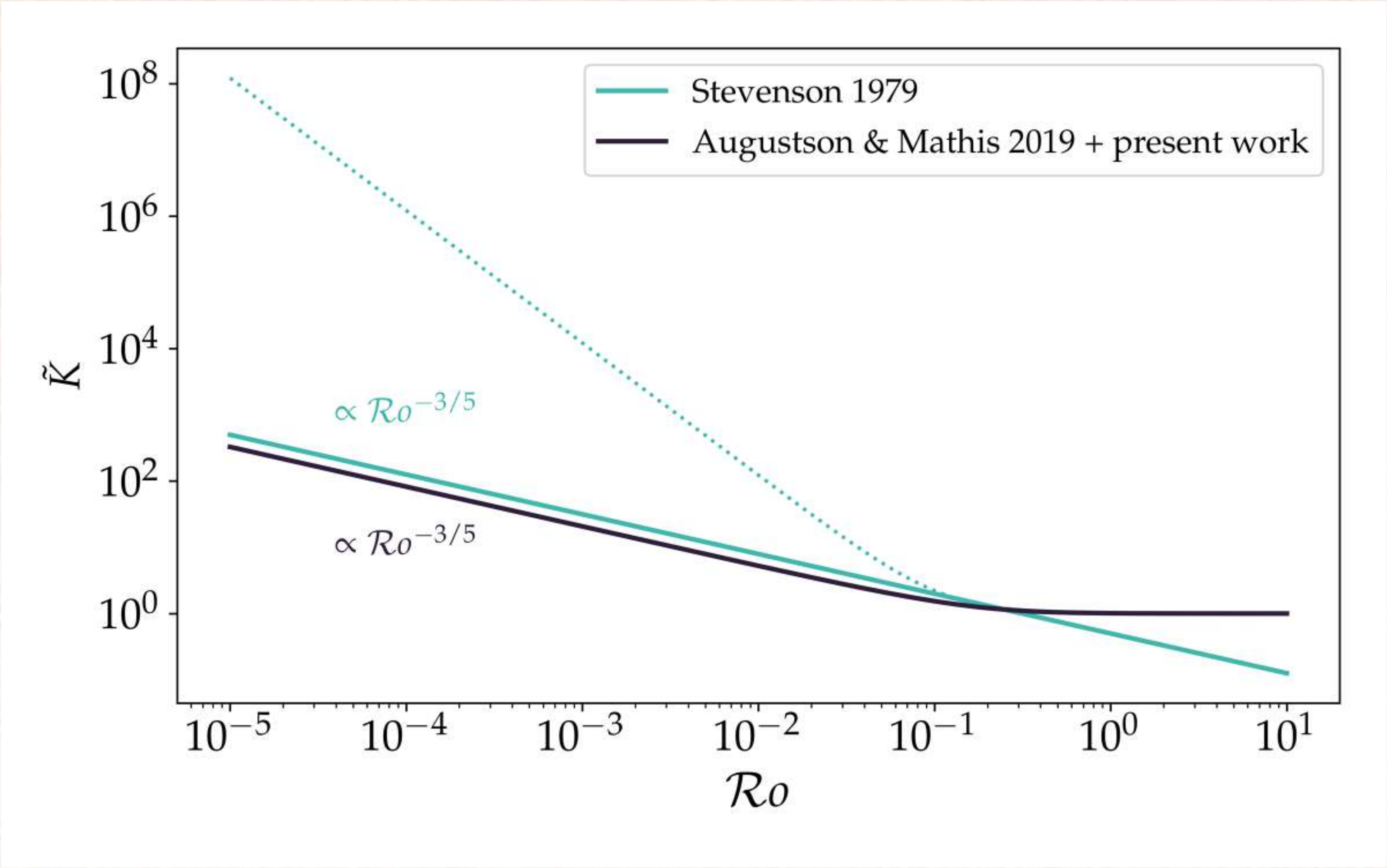
Reduced wavenumber

ROTATING CONVECTION

Velocity modulation $\dfrac{v_\Omega}{v_0}$



Wavenumber modulation $\dfrac{k_\Omega}{k_0}$



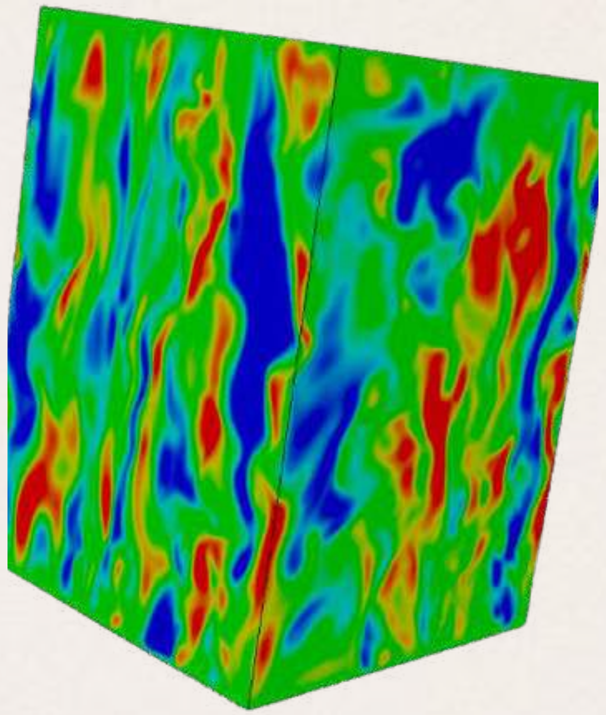
ROSSBY NUMBER

$$\mathcal{R}o \equiv \frac{u_0}{2\Omega \ell_0}$$

Advection/Coriolis

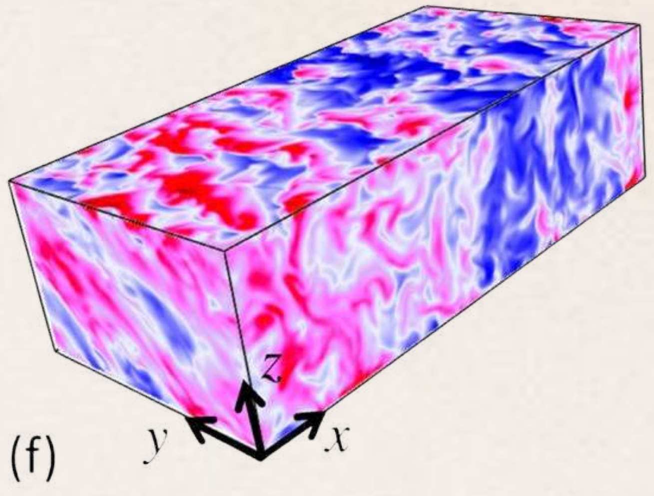
The **higher the rotation rate**, the **lower the convective velocity** and the **higher the convective wavenumber**

COMPARISON WITH SIMULATIONS OF ROTATING CONVECTION



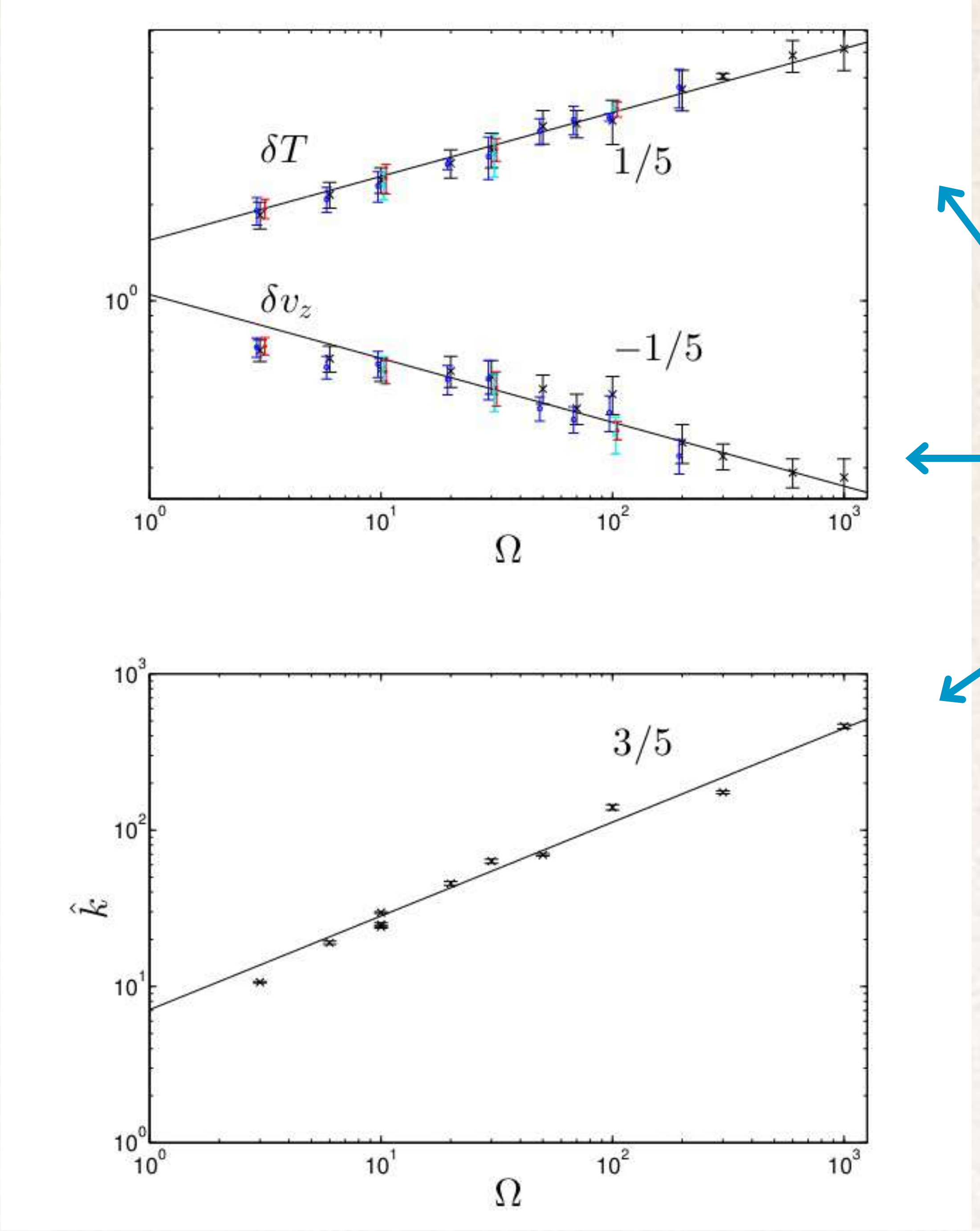
[Barker et al. 2014]

Rotation and gravity aligned



[Currie et al. 2020]

Misaligned gravity and rotation



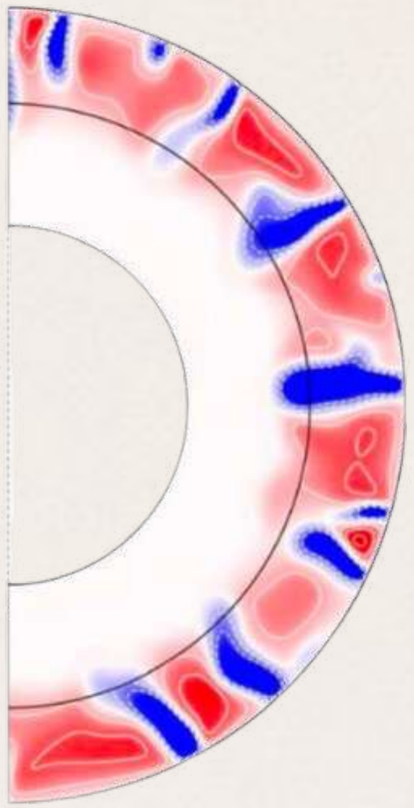
Lines : Stevenson 1979 prescriptions

[Barker et al. 2014]

OTHER INTERESTING RESULTS WITH ROTATING MLT

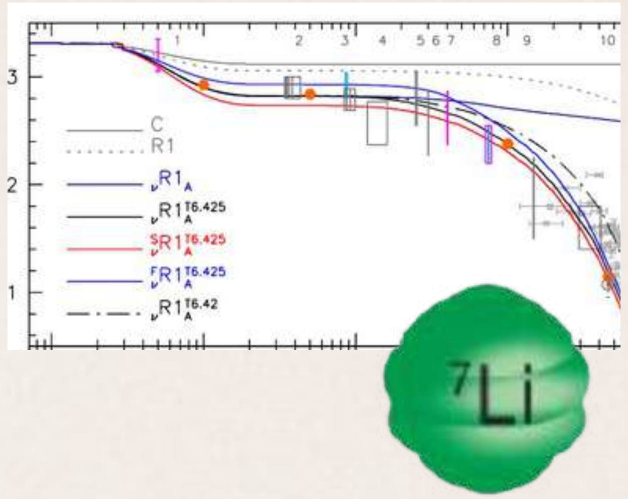
Despite its simplicity

OVERSHOOT



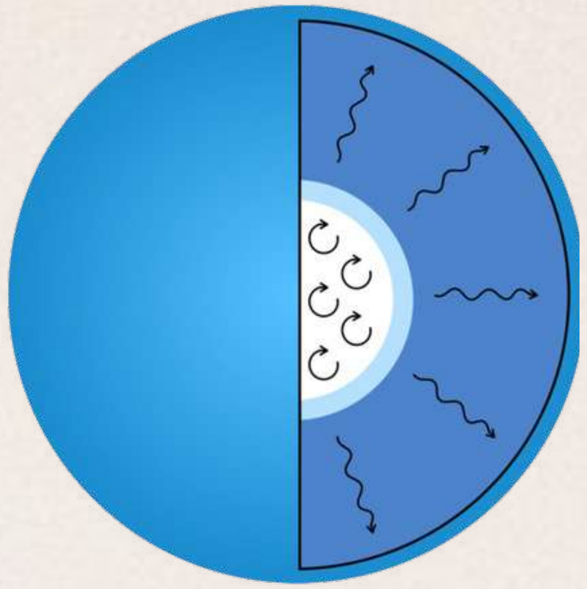
[Korre et al. 2020]

LITHIUM DEPLETION & TRANSPORT



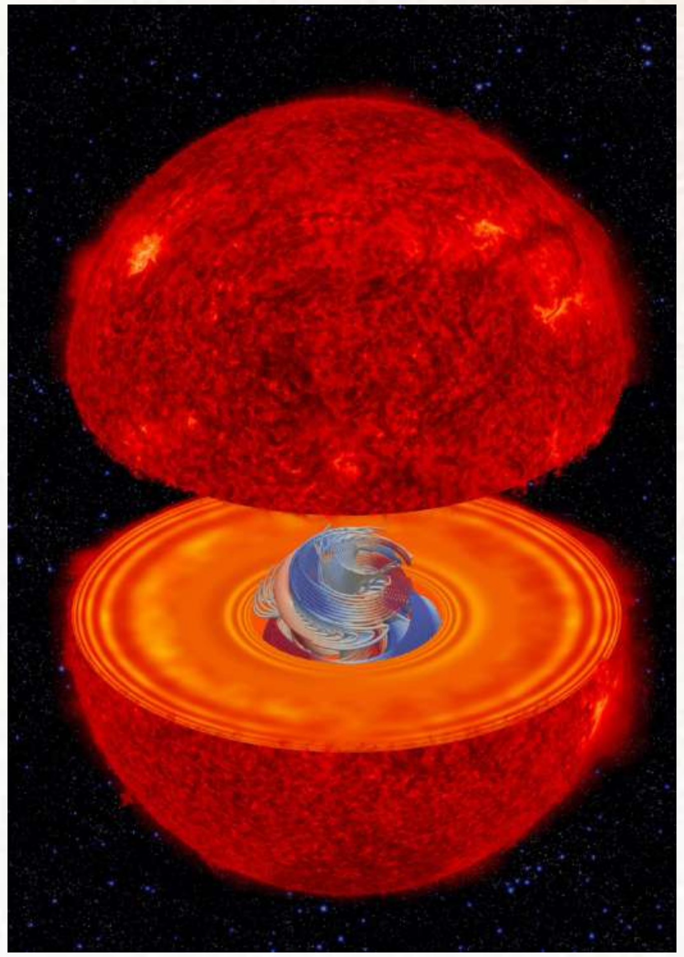
[Dumont et al. 2020]

SEISMIC MEASUREMENTS OF THERMAL STRUCTURE IN MASSIVE STARS



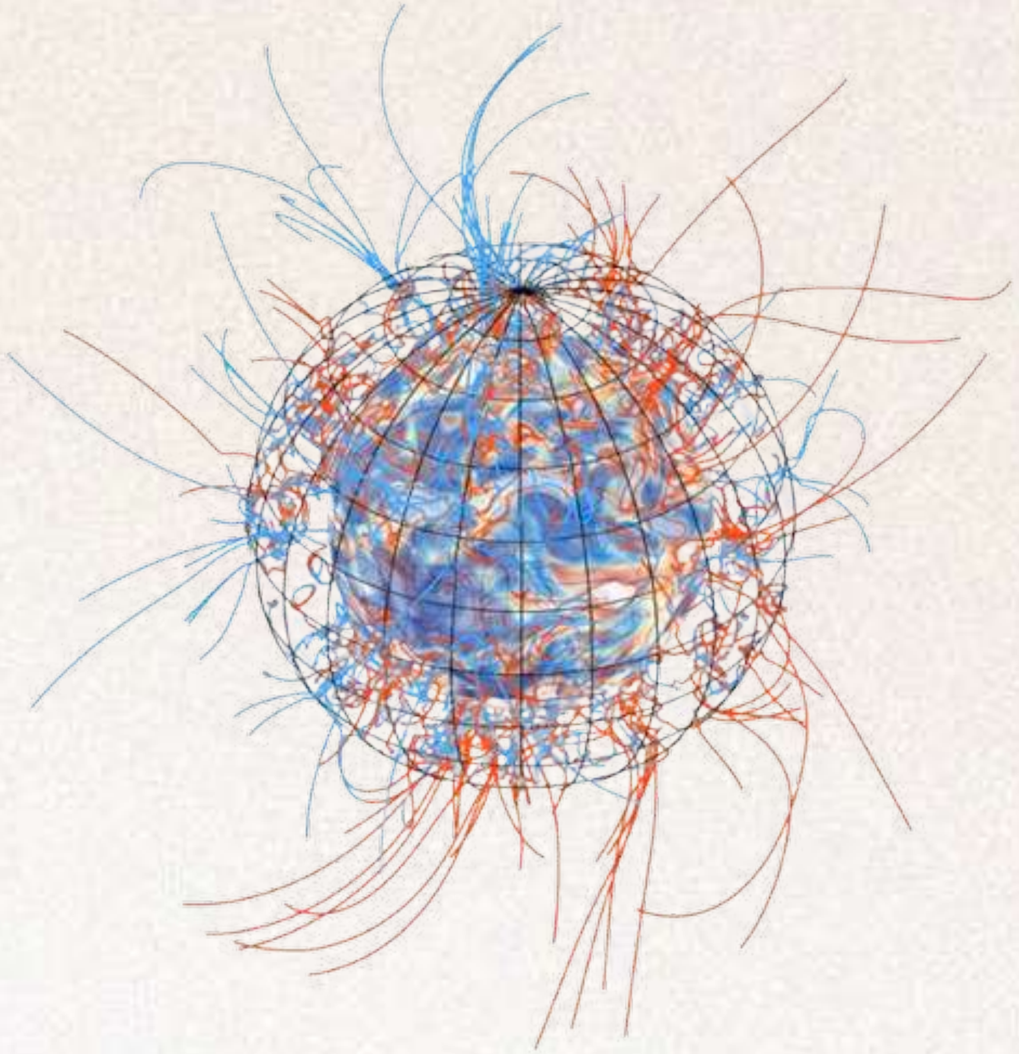
[Michielsen et al. 2019]

WHAT ABOUT THE MAGNETIC FIELD ?



[Li et al. 2022]

Core of Red Giant stars
10-100kG



[Browning et al. 2006]

Sun surface magnetic field
4-8 kG

THE ONSET OF MAGNETISED CONVECTION

STABILITY CRITERION

Schwarzschild criterion

$$\nabla - \nabla_{ad} < 0$$

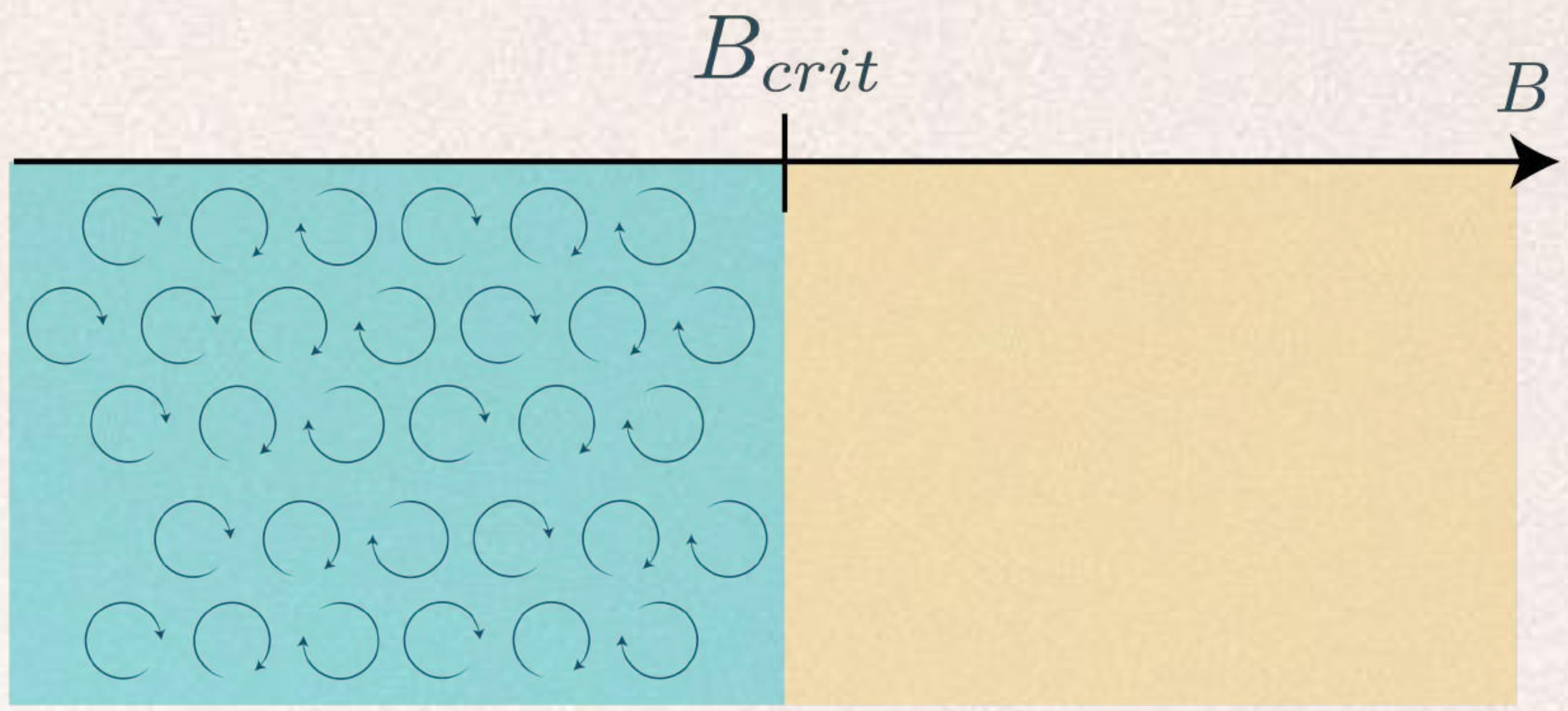
B Gough & Tayler 1966 criterion

$$\nabla - \nabla_{ad} < \delta,$$

$$\delta = \frac{v_a^2}{v_a^2 + c_s^2}$$

Alfvén velocity

Sound speed



CONVECTION :

FROZEN CONVECTION :

CRITICAL MAGNETIC FIELD

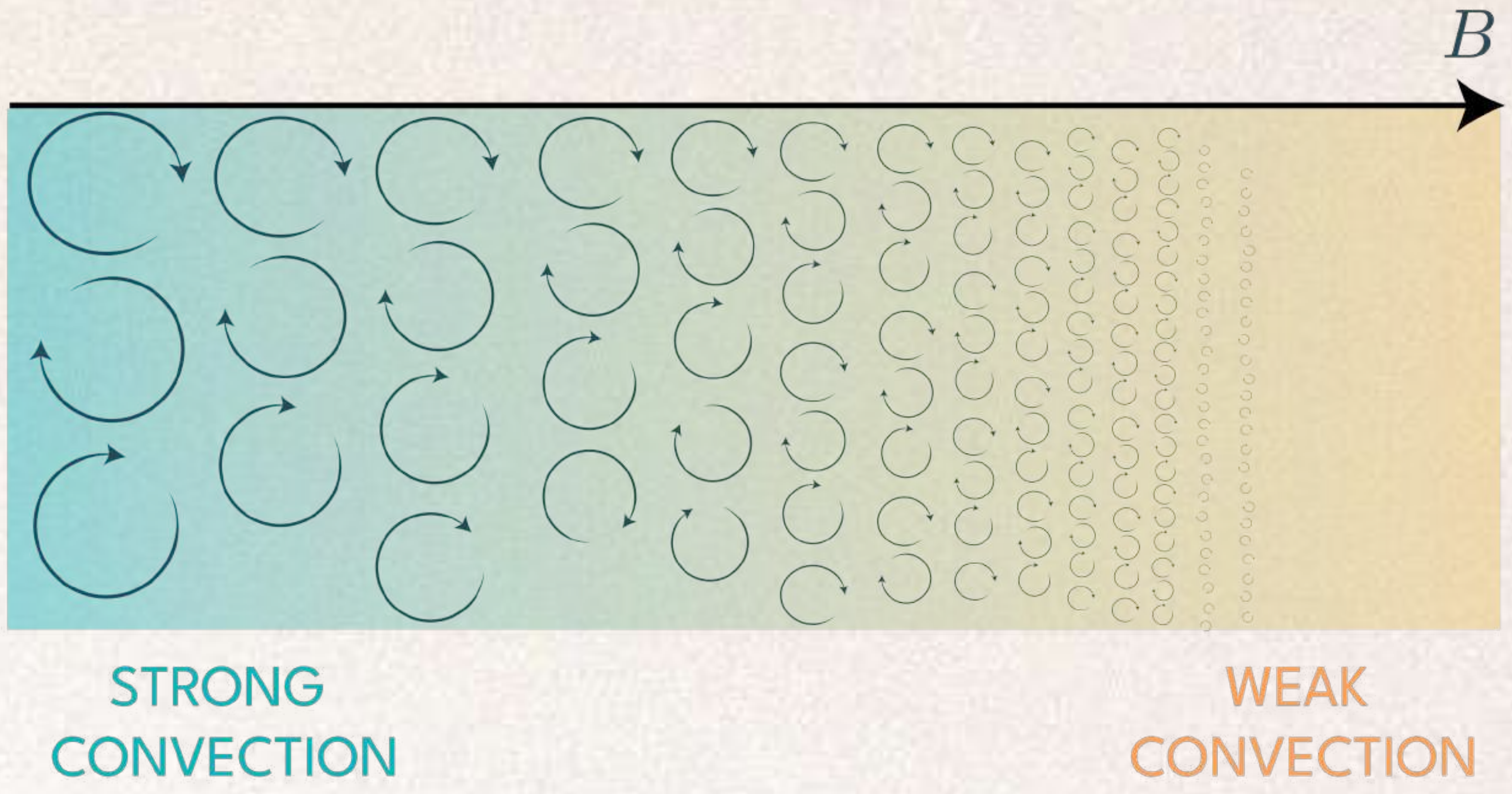
$$B_{\rm crit} = \sqrt{4\pi\rho c_s^2(\nabla_{\rm rad} - \nabla_{\rm ad})}$$

Above, convection is stable

MAGNETIC MIXING LENGTH THEORY

A PROGRESSIVE WEAKENING OF CONVECTION

B Stevenson 1979



DO BOTH APPROACHES AGREE ?

Critical magnetic field [Gough & Tayler 1966]

➠ 97% diminution of the convective velocity

[Stevenson 1979] [Bessila & Mathis 2024]

MAGNETISED CONVECTION

HEAT-FLUX MAXIMIZATION

$$\frac{\partial \boldsymbol{v}}{\partial t} + 2\Omega \times \boldsymbol{v} = \frac{-1}{\rho}\nabla p - \boldsymbol{g}\alpha\theta + \frac{(\nabla \times \boldsymbol{b}) \times \boldsymbol{B}}{\mu_0 \rho}$$

$$\nabla \cdot \boldsymbol{v} = 0$$

$$\frac{\partial \theta}{\partial t} = -\boldsymbol{\beta} \cdot \boldsymbol{v}$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \lambda \nabla^2 \boldsymbol{b} + \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$$

$$\nabla \cdot \boldsymbol{b} = 0$$

Boussinesq equations
With a **vertical magnetic** field

$$\mathcal{F}_B = \frac{\rho c_p}{\alpha g} \frac{\hat{s}^2}{k^2} \left(s + \frac{\omega_B^2}{\hat{s}} \right)$$

CONVECTIVE HEAT-FLUX
to maximise

$$\hat{s}^4 + \hat{s}^2 \left(2\mathcal{P}^2 - \frac{(z^3 - 1)}{z^3} \right) + \mathcal{P}^4 - \mathcal{P}^2 \frac{(z^3 - 1)}{z^3} = 0$$

DISPERSION RELATION

MAGNETIC PARAMETER

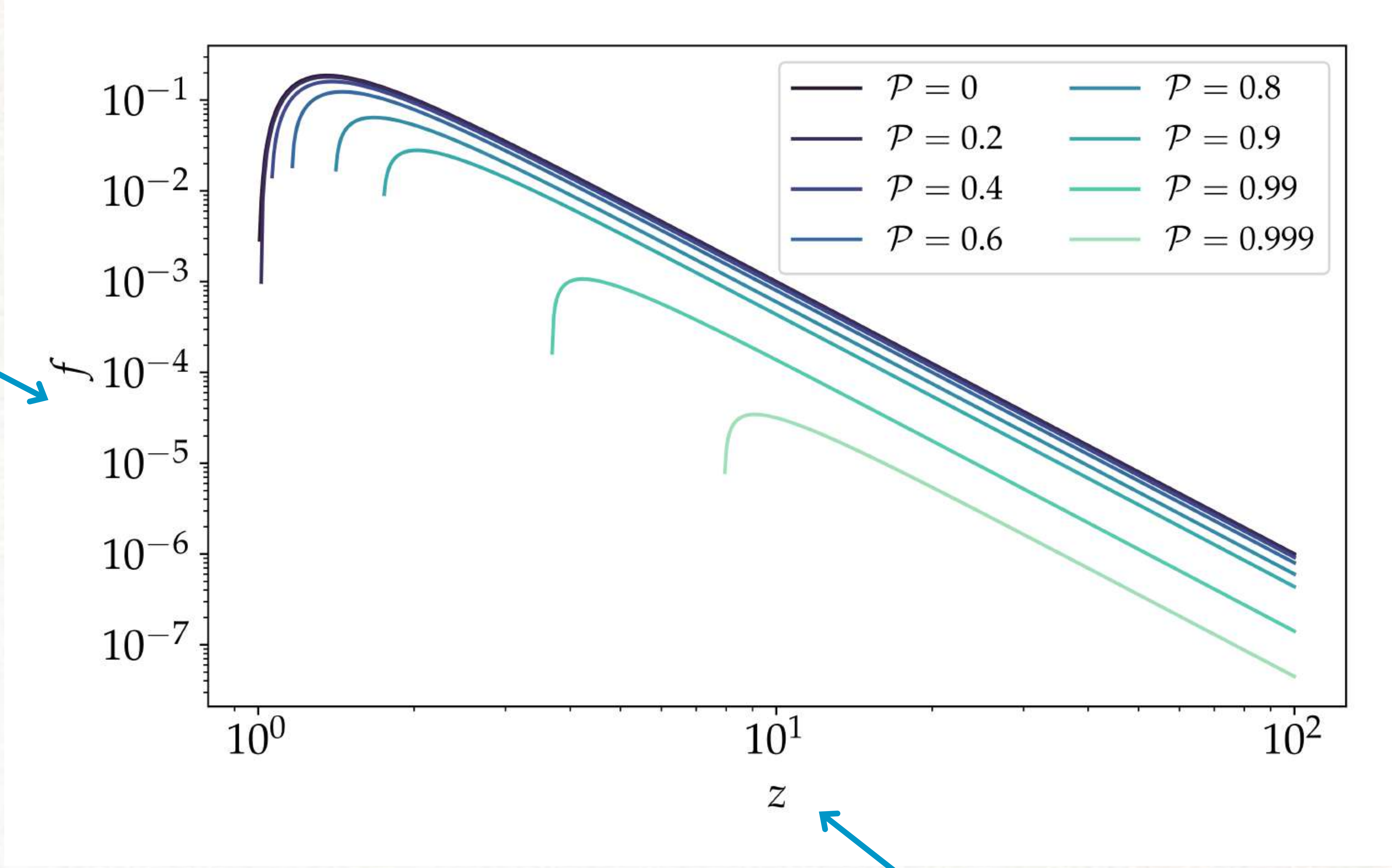
$$\mathcal{P}^2 = \frac{\omega_B^2}{N_*^2}$$

$$\omega_B^2 \equiv \frac{(\mathbf{k} \cdot \mathbf{B})^2}{\mu_0 \rho}$$

MAGNETISED CONVECTION

HEAT-FLUX MAXIMIZATION

Convective heat-flux



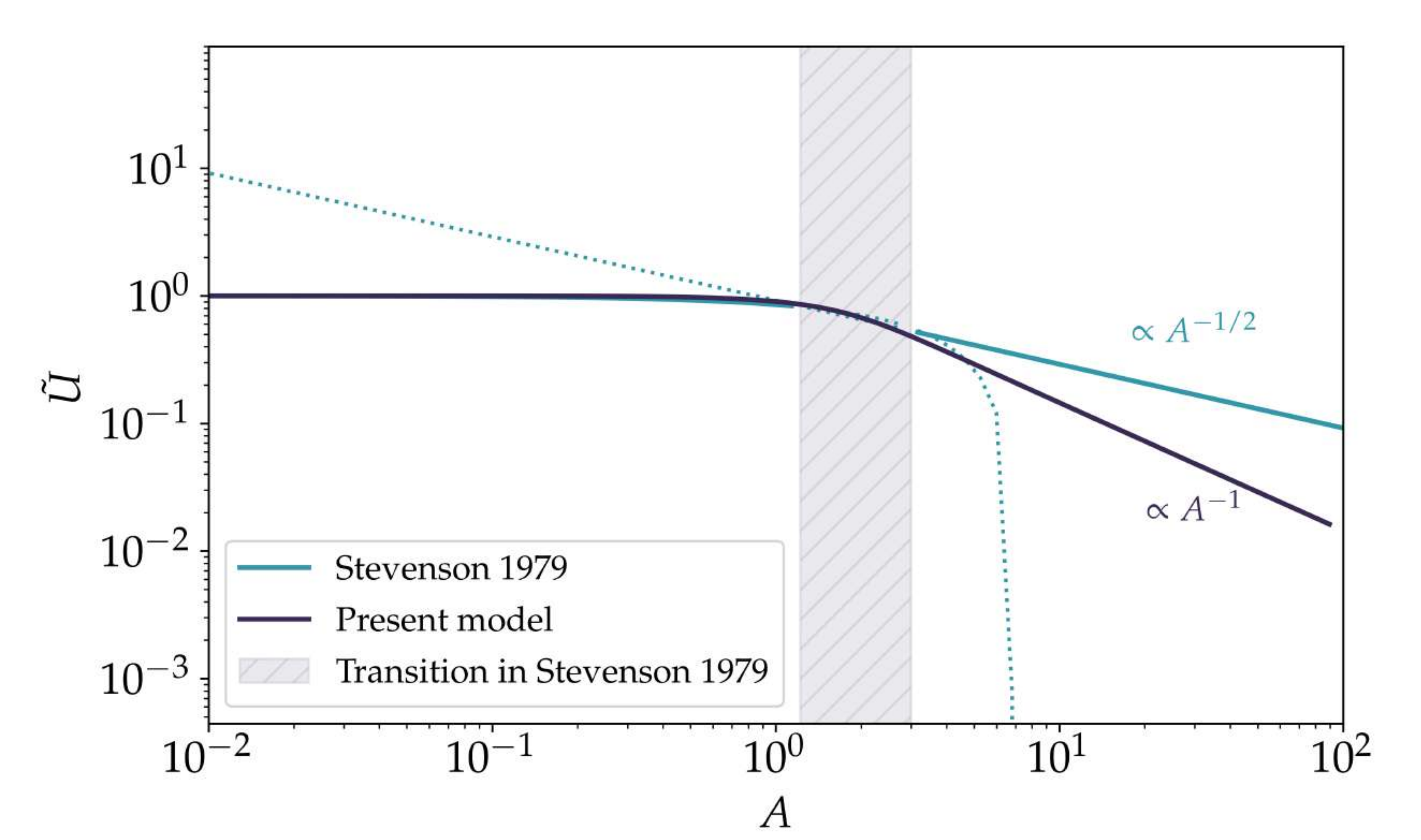
Magnetic field increases

Reduced wavenumber

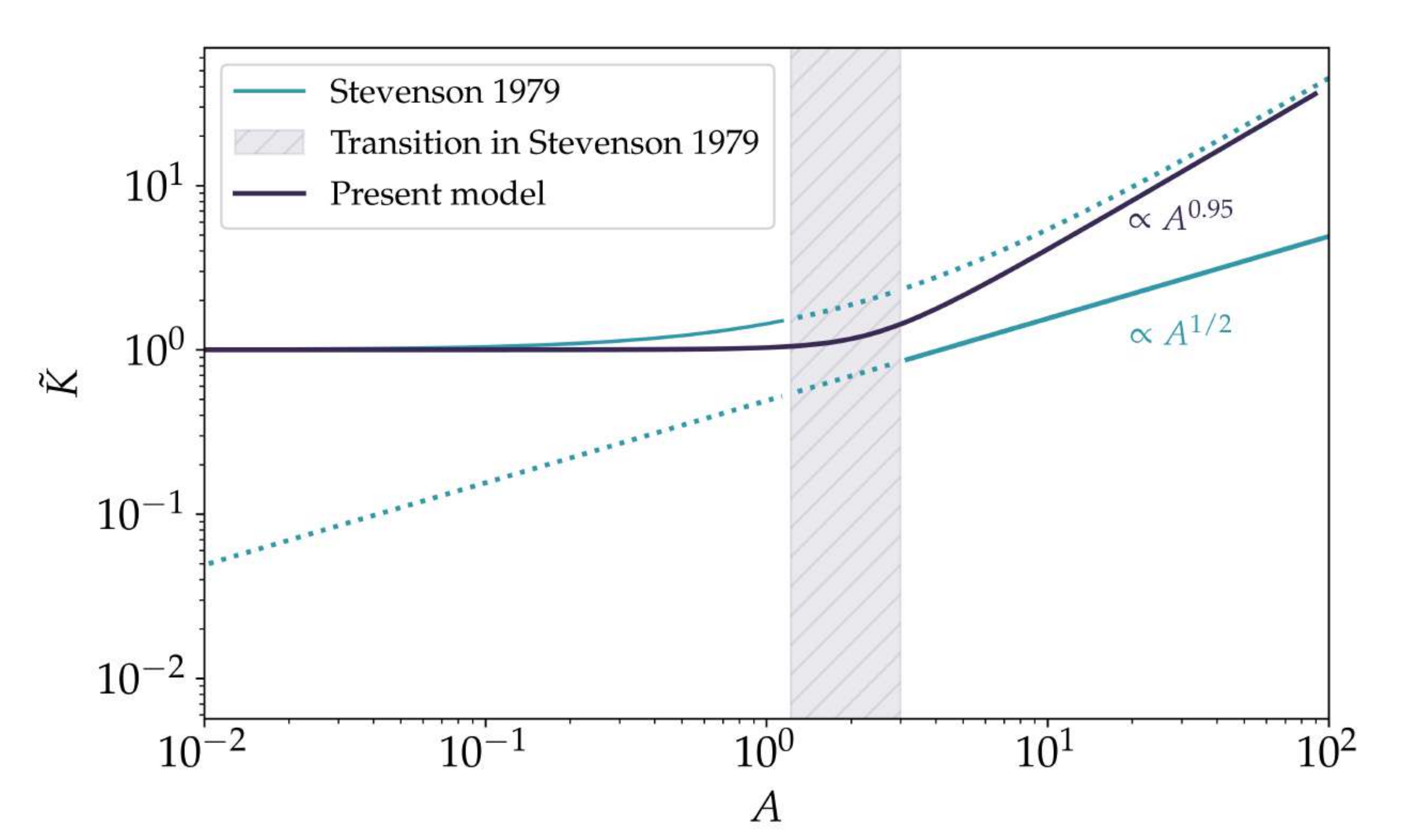
MAGNETISED MLT

WITH A VERTICAL MAGNETIC FIELD

Velocity modulation $\frac{v_B}{v_0}$



Wavenumber modulation $\frac{k_B}{k_0}$



INVERSE ALFVÉN NUMBER

$$A = \frac{B}{v_0 \rho \mu_0}$$

Alfvén velocity / Convective velocity

The **higher the magnetic field**, the **lower the convective velocity** and the **higher the convective wavenumber**

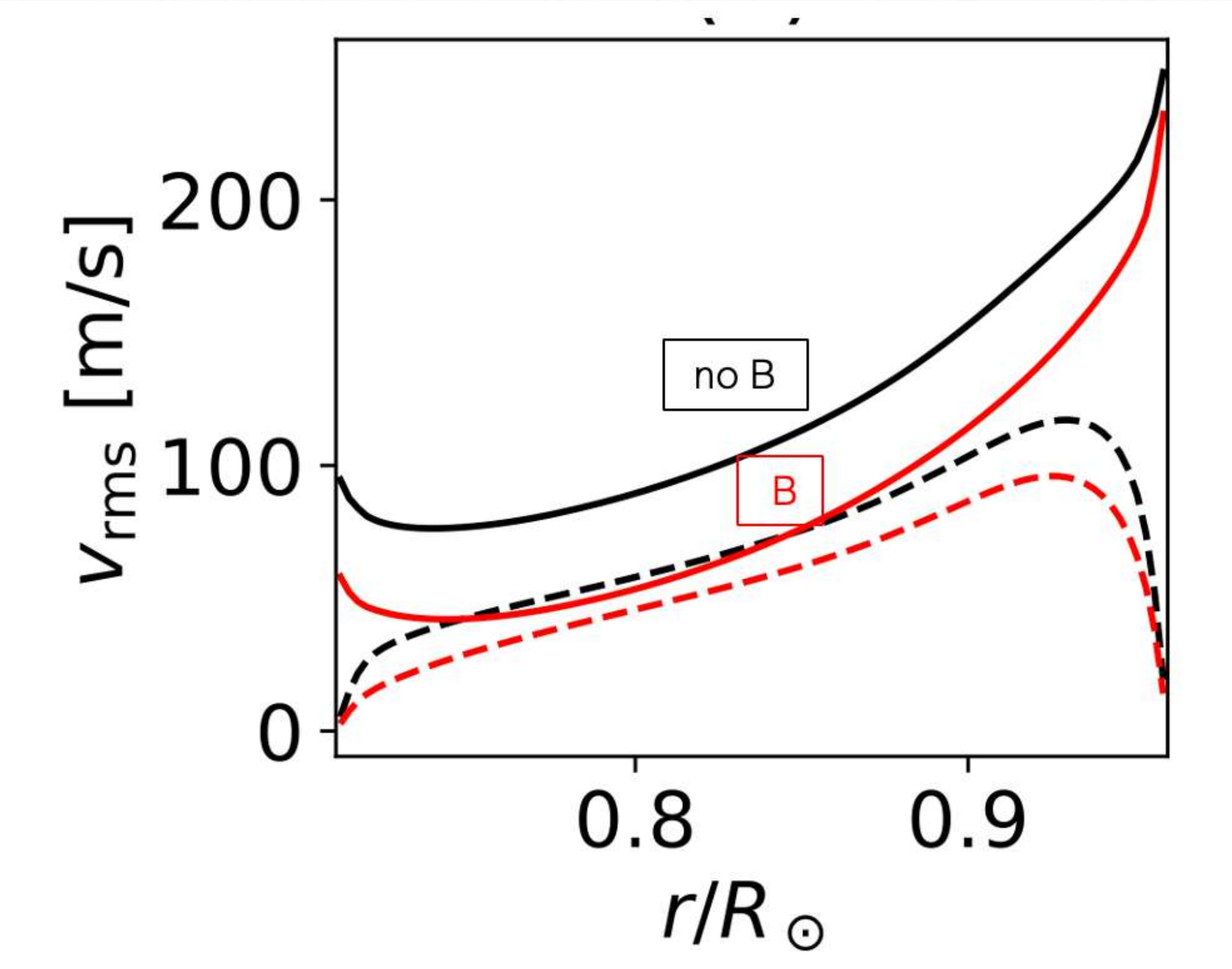
MAGNETISED CONVECTION IN NUMERICAL SIMULATIONS

QUALITATIVE DIMINUTION

Of the r.m.s. velocity in magnetohydrodynamic simulations

NO DIRECT COMPARISON

Between Magnetised MLT from Stevenson 1979 and numerical simulations



[Hotta et al. 2018]

MAGNETISED AND ROTATING CONVECTION

HEAT-FLUX MAXIMISATION

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$$\nabla \cdot \boldsymbol{v} = 0$$

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$$\mathcal{F}_B = \frac{\rho c_p}{\alpha g} \frac{\hat{s}^2}{k^2} \left(s + \frac{\omega_B^2}{\hat{s}} \right)$$

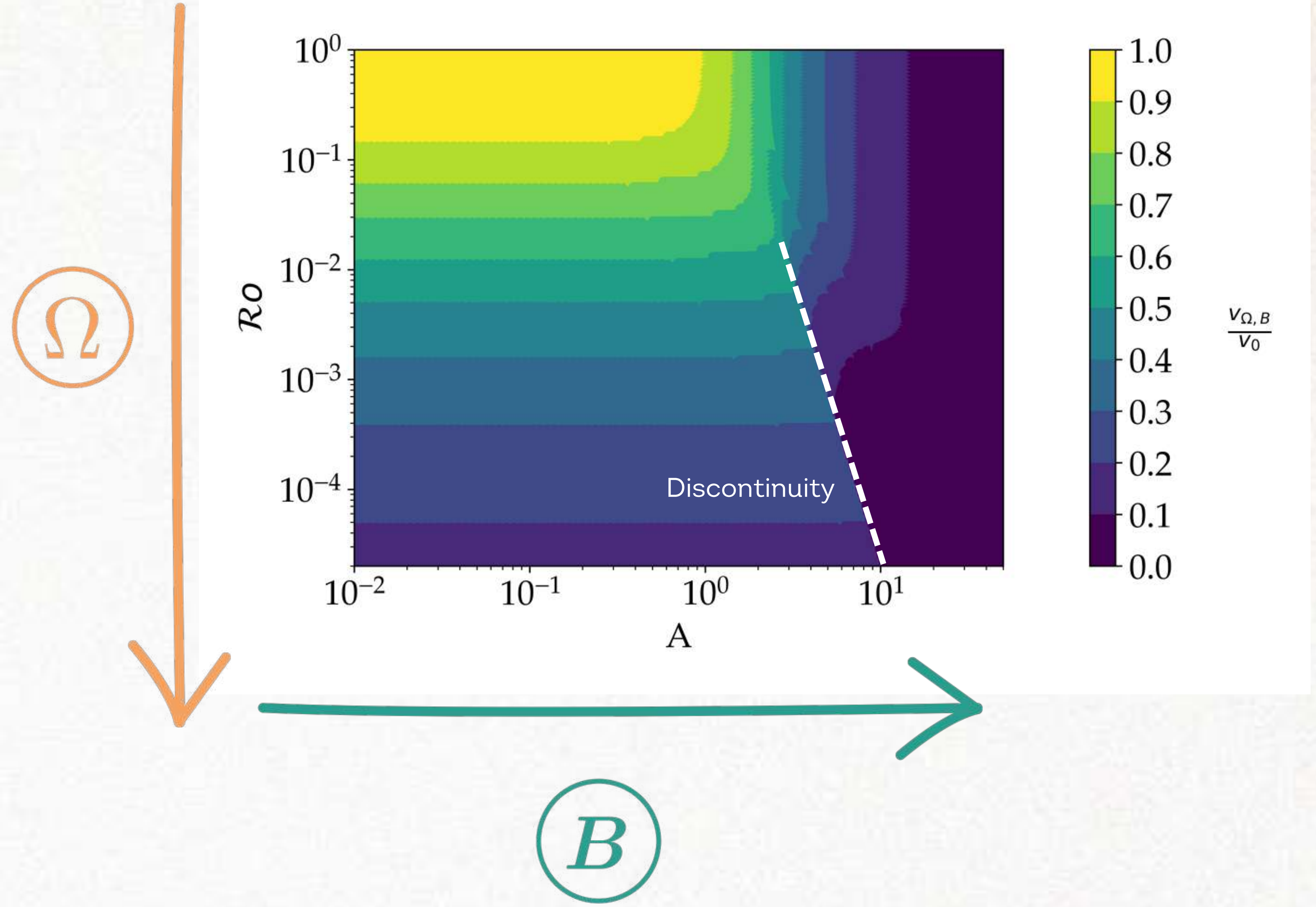
CONVECTIVE HEAT-FLUX
to maximise

$$\hat{s}^4 + \hat{s}^2 \left(2\mathcal{P}^2 + \mathcal{O}^2 - \frac{(z^3 - 1)}{z^3} \right) + \mathcal{P}^4 - \mathcal{P}^2 \frac{(z^3 - 1)}{z^3} = 0$$

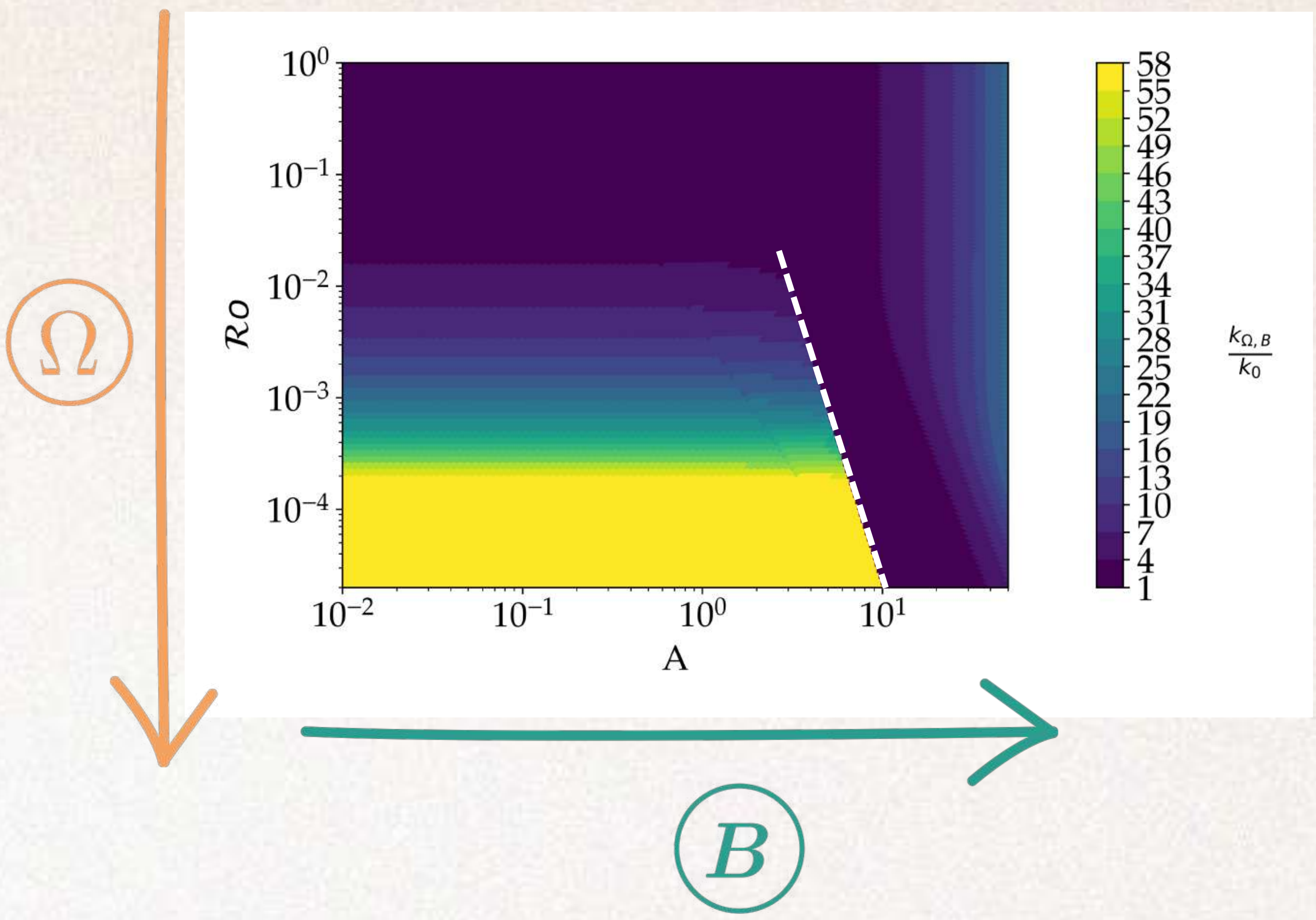
DISPERSION RELATION

MAGNETIC FIELD AND ROTATION

Velocity modulation $\frac{v_B}{v_0}$



Wavenumber modulation $\frac{k_B}{k_0}$

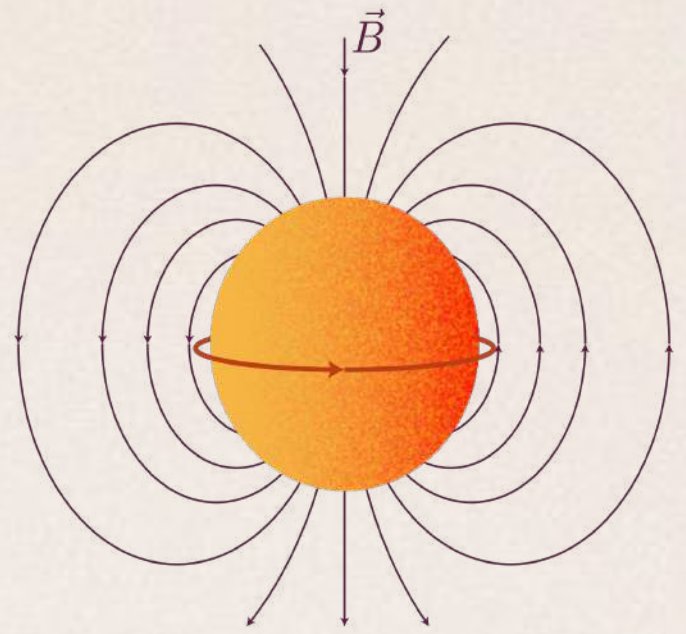


CONCLUSION

MIXING-LENGTH THEORY

Reduce the turbulent cascade to only **one size** and **velocity**

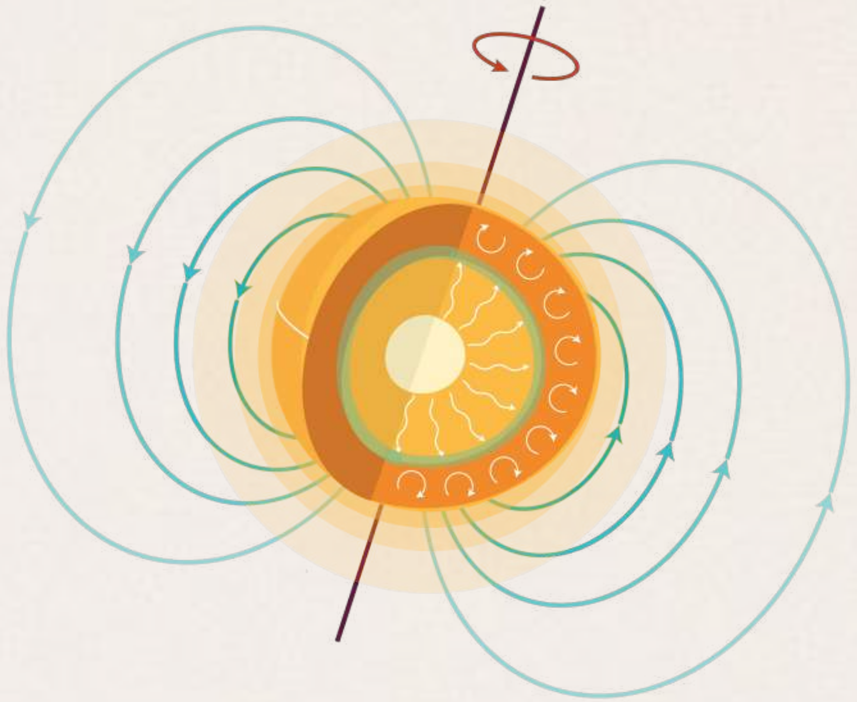
HOW IS IT MODIFIED BY ROTATION/MAGNETISM ?



Both the **convective velocity** and the **characteristic eddy size** are diminished

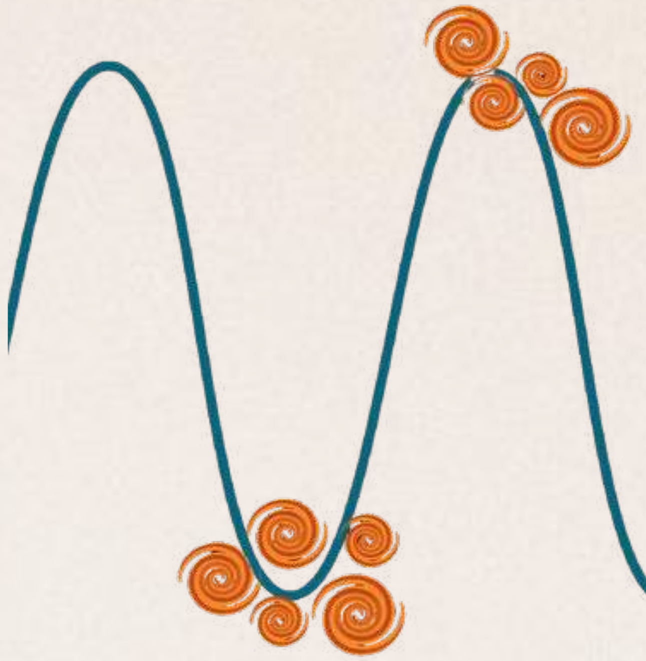
NEXT STEPS

STELLAR EVOLUTION AND STRUCTURE



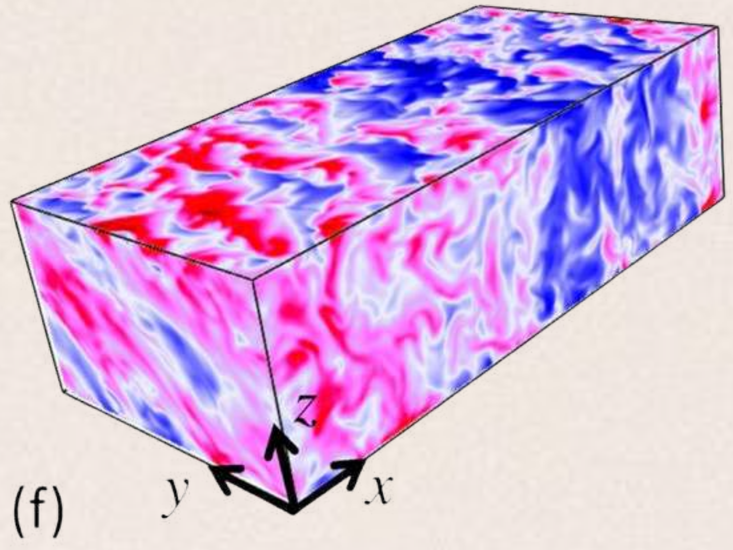
To be implemented in numerical evolution codes

WAVE EXCITATION BY CONVECTION



Acoustic waves, gravity waves, gravito-inertial waves...

IMPROVE AND TEST THE MODEL

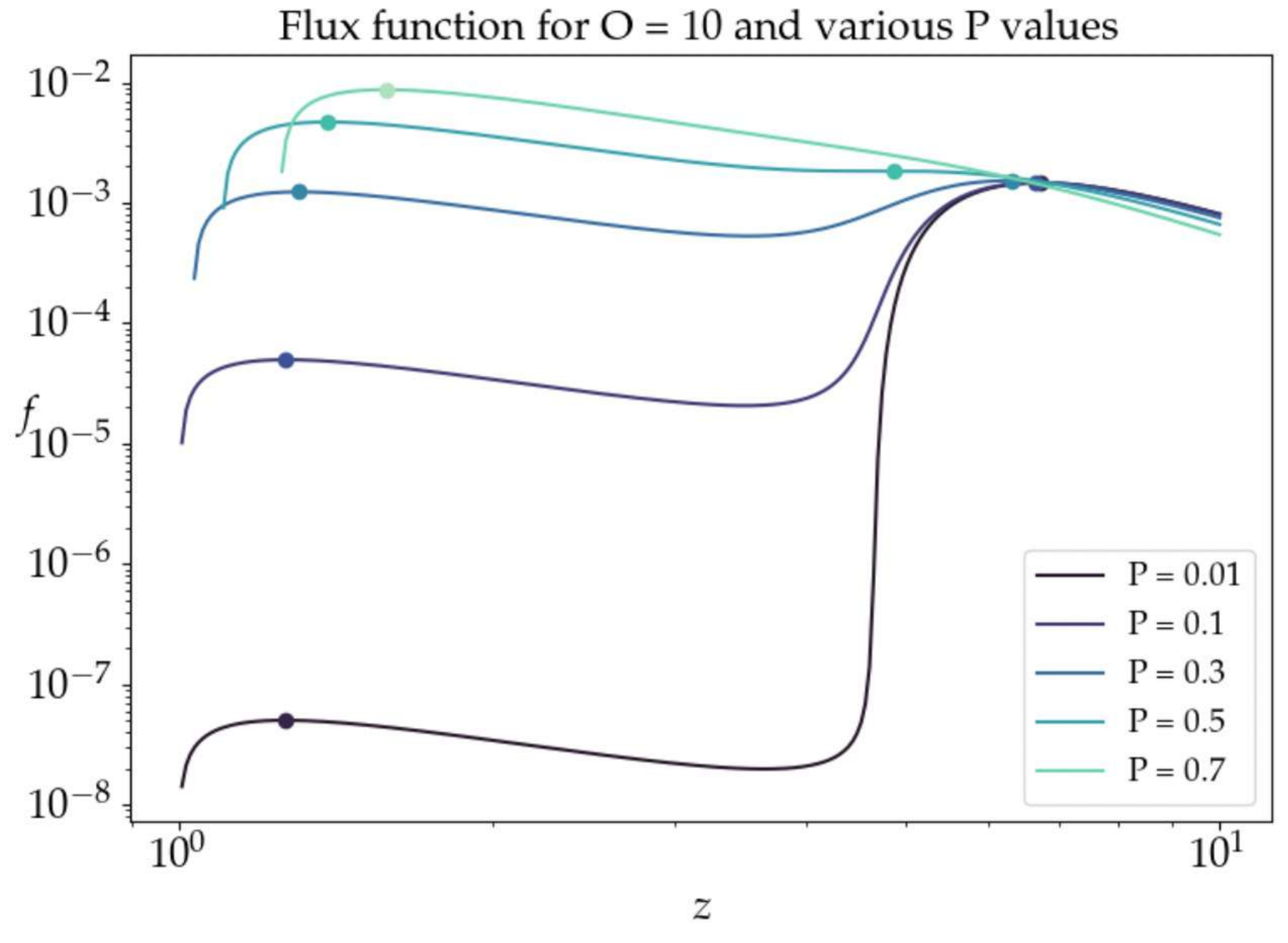


1. Include **diffusive processes**
2. More **complex geometry**
3. Test with **numerical simulations**

THANK YOU !

ROTATING AND MAGNETISED CONVECTION

THE DISCONTINUITY IS DUE TO A CHANGE IN THE MAXIMUM



Dots : local maxima