

I'm human



Ladder operators in quantum mechanics pdf

Operators in quantum mechanics. Ladder operators in quantum mechanics. Number operator quantum mechanics. L operator quantum mechanics.

Let's explore the behavior of ladder operators on basis states for spin-1 systems. The operators J+ and J- act on the basis states |1, -1>, |1, 0>, and |1, 1> in a specific way, resulting in certain outcomes. In our discussion of angular momentum, let's shift focus to spin, which is an intrinsic property of particles that doesn't rely on spatial motion. Unlike orbital angular momentum, where the quantum number l can only take integer values, spin exhibits different properties. Recall that the eigenvalue equations for orbital angular momentum define states |l, m> with specific properties. The dimension of the Hilbert space of these states is determined by the orbital quantum number l, which is related to the magnetic quantum number m by the condition |m| ≤ l. This means that fixing l determines the dimension of the state subspace. Interestingly, while integer values of l make sense for classical mechanics and spherical harmonics, the algebra of angular momentum operators also allows for half-integer eigenvalues. The smallest possibility is 1/2, which corresponds to a 2-component object that transforms non-trivially under rotation - an idea that's unfamiliar classically. This leads us to appreciate the concept of spin-1/2 systems, which we've encountered in Stern-Gerlach experiments. It's remarkable how spin exhibits properties that defy classical intuition. This is a good opportunity to formally introduce the idea of complete sets of commuting observables (CSCOs), which are crucial for understanding quantum mechanics. Measuring every observable in our complete set of commuting observables (CSCO) means the system will collapse to a unique state which is simultaneously an eigenvector of all the observables. This CSCO represents the most things we can know about our system at once. Ideally, the Hamiltonian should be part of the CSCO for simple time evolution. However, choosing a CSCO isn't always unique. For example, using different angular momentum operators like Sx or Sz can label eigenstates equally well in certain systems. In such cases, we might include other operators to construct a complete set, even if they don't add new information. In quantum mechanics, combining two independent systems results in a new system with properties that are direct products of their constituent parts. The direct product is denoted by ⊗ and combines kets from different Hilbert spaces, such as position and spin states for an electron moving through space. This operation creates a new ket representing the combined state of the overall system. For instance, the state of an electron can be described as |ket(vec{x}), ψm> = |ket(vec{x})> ⊗ |ket{ψm}>, where vec{x} represents position and |ket{ψm}> denotes spin states. The direct product Hilbert space is spanned by all possible combinations of basis states from each individual system, resulting in a new state that reflects the combined properties. The dimensionality of this new Hilbert space is the product of the dimensions of its constituent spaces. This concept is illustrated with an example involving two spin-1/2 particles, where their combined state results in four basis states. In more complex scenarios, like preparing a system in a combination of energy eigenstates and an unknown spin wavefunction, the direct product is used to express the overall state. The probability of observing the ground state can be calculated by considering the overlap with the energy eigenstate, implicitly summing over the spin outcomes. The question of whether we have found a Complete Set of Commuting Observables (CSCO) is ultimately an experimental one. Performing different experiments and measuring various observables can reveal if there's missing information in a seemingly completely determined eigenstate. A good example is the hydrogen atom, which initially appears as a CSCO but experimentally shows the emission spectrum doesn't perfectly align with expectations due to hyperfine transitions. When atoms are placed in a weak magnetic field, the spectral lines corresponding to different quantum numbers begin to split into multiple energy levels. This suggests that our initial set of quantum numbers was incomplete and we need to consider the spin of orbital electrons as an observable. To explore this further, let's examine how rotations affect a system with two-dimensional angular momentum operators. In particular, if we fix our attention on the z-axis, we can describe the rotation operator using the expression Σ |n> |n>|ψ = exp(-iϕ/ħ) ψ. To verify that this is indeed a rotation operator, we examine its effect on the expectation value of the x-component of spin (Sx). We need to calculate the commutator of (Sx) with the rotation operator Σ |n> |n>|n. By expanding (Sx) as an outer product in the z-eigenbasis, we find that Σ |n> |n>|n(SxΣ |n>|n) = (Sx) cosϕ - (Sy) sinϕ. This matches the expected result for a rotation about the z-axis. This derivation relies on our prior knowledge of the two-state system, but we can generalize this result to any N using commutation relations and the Baker-Hausdorff formula. Similar results hold for other expectation values, including (Sk) rotating like an ordinary classical vector (Jk). However, there's a twist: what does the rotation operator do to an arbitrary ket? In a two-state system, we can expand it in eigenstates of (Sz). By doing so, we find that Σ |n> |n>|ψ = e^{-iϕ/2} |+>|ψ + e^{iϕ/2} |->|ψ. This result highlights the importance of considering arbitrary kets and their eigenstates when working with rotation operators in quantum mechanics. The fact that a rotation by 2π in our system changes the sign of the state could be problematic. We can see that a rotation by 4π has no effect, but this is due to the sign appearing twice. The question remains whether this sign change has any observable consequences. As we've seen before, if every ket in our Hilbert space picks up the same phase, then physical results remain unchanged. Therefore, to observe an effect, we need to compare a rotated state to an unrotated one. Spin and representations of rotation In previous notes on rotations, we discussed that the rotation group SO(3) only has odd-dimensional representations. However, we're now working in a two-dimensional Hilbert space, which is different from what we've seen before. The rotation matrices with half-integer angular momentum are not actually representations of SO(3)! To preserve the structure of SO(3), 2π Left-handed rotation as |left>|angle |hat{S} x |right>|angle |rightarrow |left>|angle |hat{S} x |right>|angle cos ϖ omega t - |left>|angle |hat{S} y |right>|angle sin ϖ omega t + |left>|angle |hat{S} y |right>|angle |rightarrow |left>|angle |hat{S} y |right>|angle cos ϖ omega t + |left>|angle |hat{S} x |right>|angle sin ϖ omega t + |left>|angle |hat{S} z |right>|angle |rightarrow |left>|angle |hat{S} z |right>|angle, where ϖ omega = ϖrac{|e|B}{m e c}. So application of a uniform magnetic field gives us a way to rotate the (average) electron spin orientation, as a function of time. This can be thought of, exactly as we derived above, a rotation of the state vector through angle ϖ phi = ϖ omega t. In fact, the state vector itself as a function of time is |ket{ψ(t)}> = e^{-i ϖ omega t/2} |ket{+}> |left>|angle + |ψ> |right>|angle + e^{i ϖ omega t/2} |ket{-}> |left>|angle - |ψ> |right>|angle. Once again, the rotation has to go twice as far to return the original state vector, τau S = 4π/ϖ omega, compared to the period of the observable spin precession τau P = 2π/ϖ omega. We can now construct an experiment to see the effects of rotation, and as usual, we'll turn to a simple two-slit interference setup. Sketch of a double-slit experiment with applied magnetic field. Nominally we use neutrons for this experiment, instead of electrons; neutrons also have spin-1/2 and a non-zero magnetic moment, but they don't have any charge we don't have to worry about the magnetic field significantly bending their trajectory. Denoting the amplitudes for traveling from A to B on the top and bottom paths as A, 1 and A, 2 respectively, we see that the effect of the magnetic field on the lower path will be A, 2 = A, 2(B=0) e^{i ϖ mp i ϖ omega T/2}, where T is the time spent in the magnetic field region, and the sign ambiguity is due to the fact that the neutrons have indeterminate spin. The resulting interference term at the screen is |extrm{Re}> (A, 1 A, 2) |sin ϖ cos |left(ϖrac{ϖ mp ϖ omega T}{2}> + ϖ delta |right), where ϖ delta is the phase shift when B=0. If we keep the time-of-flight in the magnetic field region fixed and vary ϖ omega, then we can change the interference term and thus the observed beam intensity; and the required change in order to give the same intensity back is ϖ Delta ϖ omega = ϖrac{4π}{T}. This is exactly the 4π we were looking for, and it has been seen conclusively in several experiments, another strong confirmation of quantum mechanics. Scattering experiments provide a more direct way to look at the angular properties of a particle, and to see that a spin-1/2 particle is something entirely new. We consider a double-scattering experiment, involving a series of spherically symmetric targets. Given text's complex theoretical concepts in higher dimensions are simplified in this experiment, where the scattered beam's intensity depends only on the angle between incident and deflected beams. However, introducing additional targets and a second scattering plane reveals polarization effects that depend on the relative orientation of the planes. X-rays exhibit a distinct polarization effect proportional to cos^2 ϖ phi, while electron beams show an intermediate result with intensity dependent on cos ϖ phi, indicating a spinor wavefunction rather than a vector or scalar one.